Space, Growth and Technology:

an Integrated Dynamic Approach

Orlando Gomes*

Escola Superior de Comunicação Social [Instituto Politécnico de Lisboa] and Unidade de Investigação em Desenvolvimento Empresarial [UNIDE/ISCTE].

Abstract

Economic phenomena are interrelated. From a growth perspective, time analysis concerning the choices of present and future consumption and the choices between the allocation of scientific resources should be combined with a space analysis regarding the dissemination of economic activity through geographical locations. This paper intends to present such an integrated approach under a simple endogenous growth model. The determinants of growth are, on one hand, the decisions about how to allocate technological resources and, on the other hand, the strength with which productive activities can agglomerate in order to generate increasing returns to scale. We find that the long run steady state does not have to be a state of unchangeable geography – consumption and production conditions and technological progress not only determine long term growth but also the long term tendency for the economy to geographically concentrate or disperse.

Keywords: Optimal control, Economic growth, Technology, Agglomeration economies, Increasing returns.

JEL classification: O33, O41, R11, R12

* Orlando Gomes; address: Escola Superior de Comunicação Social, Campus de Benfica do IPL, 1549-014 Lisboa. Phone number: + 351 93 342 09 15; fax: + 351 217 162 540. E-mail: ogomes@escs.ipl.pt.

1. Introduction

Technological and location issues play a critical role on the growth process. The model presented in this text develops an endogenous growth setup where the determinants of long run growth are technology and the rate at which production activities concentrate or disperse over geographical space.

Technology has always represented, in growth explanations, a fundamental source for economic prosperity. The pioneer work by Solow (1956) and Swan (1956) highlighted the importance of technology – although exogenous, technology appeared as the rescuing growth factor in the presence of decreasing capital returns. Later, endogenous growth models have attributed to technology a crucial role, with authors like Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992, 1998) and Jones (1995, 2003), among many others, pointing out endogenous mechanisms through which technology generation is possible and promotes economic growth. A point in common of these studies concerns the inevitability of considering innovation an economic activity, where scarce resources are employed but where a special kind of output is produced.

The way in which we understand the mechanics of technology generation is of primary importance for the understanding of the growth phenomenon. Jaffe and Trajtenberg (2002) state that *"At the heart of this phenomenon lies a complex, multifaceted process of continuous, widespread and far-reaching innovation and technical change. Yet 'knowledge', 'innovation', and 'technical change' are elusive notions, difficult to conceptualize and even harder to measure in a consistent, systematic way. Thus, while economists from Adam Smith on have amply recognized their crucial role in shaping the process of economic growth, our ability to study these phenomena has been rather limited."* (page 1).

Our interpretation of endogenous technology intends to be simple yet original. Relying on a brief analysis of growth determinants by Nelson and Phelps (1966), we develop a dynamic model where basic science and applied technology conflict; to get more knowledge immediately available to the production process, research resources must be reallocated from basic research activities. Putting it simple, in an economy with limited scientific resources, the choice between research, on one hand, and development, on the other, is not unconstrained: some representative agent has to choose between faster and more efficient application of available knowledge to generate physical goods and allocating resources to amplify the innovation possibilities frontier. The intertemporal problem faced by the economic system concerning the allocation of technological resources culminates in a steady state where an optimal positive, constant and finite technology growth rate is compatible with an optimal constant percentage of theoretical knowledge being oriented to applied uses. The technology framework and some of the presented results relating it, were in a first phase derived in Gomes (2004).

The technology analysis serves the purpose of explaining the evolution of an efficiency index that reflects physical inputs productivity for a given aggregate production function. In our growth analysis, the main concern is with long term results, and thus aggregate consumption and capital variables are assumed for a competitive environment.

This paper also combines space and time analysis. While the growth analysis must be essentially an intertemporal one, location concerns are easily introduced in such a framework. In this way, one can evaluate how the economics of agglomeration influence the economics of growth. As with the technology analysis, the introduction of geographical concerns is simple and straightforward but relatively different from other studies of the kind. The simplicity is related to the assumption that location of economic activities has to do only with the conflict between increasing returns and transaction costs. The first are clearly a centripetal or agglomeration force; the second will be a centrifugal force under the assumption that will be imposed that consumers are dispersed across geographical space.

The new economic geography literature, which is highly indebted to the work of Krugman (1991), Krugman and Venables (1995), Venables (1996, 2001), Fujita, Krugman and Venables (1999) and Quah (2002), among many others, insists on the basic but essential tension between agglomeration forces and inertial factors that contradict such forces. Thus, we will generically call ‘increasing returns’ to all possible forces that generate the spatial agglomeration of production activities, and designate by “transaction costs” all the forces yielding the dispersion of production activities like the existence of immobile inputs, the pecuniary costs of mobility or the congestion economies that reverse the tendency for agglomeration.

The relation between technology and economic geography has been highlighted in the past few years through the notion of ‘death of distance’. Authors like Cairncross (2001) argue that technological progress may imply that the distance is no longer important for economic activity, what allows for the conclusion that there is no need for activities to agglomerate; the evidence does not support this kind of reasoning – the new economy, strongly linked to technological developments, is an economy of clusters, where the most geographically concentrated tasks are the ones relating R&D.

Fujita and Thisse (2002) address some of these main worries; they claim that *“Intuitively, it should be clear that the spatial configuration of economic activities is the outcome of a process involving two opposing types of forces, that is, agglomeration (or centripetal) forces and dispersion (or centrifugal) forces. The observed spatial configuration of economic activities is then the result of a complicated balance of forces that push and pull consumers and firms.”* (page 5).

These authors also address the new technological era, *"The increasing availability of high-speed transportation infrastructure and the fast-growing development of new informational technologies might suggest that our economies are entering an age that will culminate in the 'death of distance'. If so, locational difference would gradually fade because agglomeration forces would be vanishing."* (page 4), but they are clearly aware that *"technological progress brings about new types of innovative activities that benefit most from being agglomerated and, therefore, tend to arise in developed areas. Consequently, the wealth or poverty of nations seems to be more and more related to the development of prosperous and competitive clusters of specific industries."* (page 4), and thus space does matter to economic activity and more precisely to the growth process.

The way in which we have chosen to integrate increasing returns to scale in our production possibilities is different from conventional analysis. Rather than assuming a monopolistic competition (Dixit-Stiglitz) framework, as in the standard core-periphery models by Baldwin (1999), Ottaviano (2001), Forslid and Ottaviano (2003) and Pfluger (2004), increasing returns appear as an index (like the technology index) on an aggregate production function – the more firms are concentrated in space, the higher is the value of this scale index, what implies a higher income generation value for a same combination of inputs. This way of introducing increasing returns allows for the consideration of an economic space that has virtually infinite locations and infinite agglomeration possibilities and degrees. Furthermore, the long run steady state does not have to be one in which space stops to change – utility maximization may imply an everlasting rate of agglomeration or an everlasting rate of dispersion, at least as long as preferences or the shape of the production conditions do not change.

Theoretical studies in the location-growth field, as Martin and Ottaviano (1999), Baldwin and Forslid (2000) and Baldwin, Martin and Ottaviano (2001), tend to make use of factor mobility, and in particular of skilled labor related to the R&D sector mobility to establish a link between the two issues. The goal is to understand how mobility in space can change growth patterns. Our analysis is different – choosing to work with an aggregate setup we are interested in perceiving how an exogenously given effect of geographical location over increasing returns implies the choice of location dynamics and growth dynamics that best combine in contributing to an optimal intertemporal utility result.

Technology and geography appear as well in the literature linked together as fundamental elements for explaining economic phenomena besides growth. An explanation of international trade, under a Ricardian model of comparative advantages, where technological enhancements and agglomeration forces shape the pattern of trade, is offered in Eaton and Kortum (2002).

The paper is organized as follows. Sections 2 and 3 present our intertemporal optimization framework; section 2 is concerned with location and capital accumulation and section 3 characterizes the technological innovation framework. Section 4 solves the technology problem and section 5 integrates the technology dynamics in the space-growth setup; a steady-state analysis is then made relating the evaluation of growth rate dynamics. Section 6 concludes.

2. Geography, Capital Accumulation and Utility

The description of our model begins with the presentation of the production possibilities. An aggregate production function is considered,

$$Y(t) = b(z).f\left[A(t), K(t), L(t)\right] \tag{1}$$

In equation (1), $Y(t)$ represents output in time moment t, f is a production function with three inputs [technology, $A(t)$, physical capital, $K(t)$, and labor, $L(t)$], and $b(z)$ is a function that reflects the intensity of returns to scale. Function f is a usual neo-classical production function, that exhibits constant returns to scale (the function is homogeneous of degree one) and where the marginal returns of each input are positive but diminishing; we assume also that the production function is a labor-augmenting production function where technical progress is Harrod-neutral. Defining $k(t)\equiv K(t)/L(t)$ as the stock of physical capital per unit of labor (or per capita), we may present function (1) in intensive form:

$$y(t) = b(z).f\left[A(t), k(t)\right] \tag{2}$$

with $y(t)$ defining output per capita.

As stated, $b(z)$ translates the eventual existence of increasing returns to scale and, thus, it will be connected with the geographical distribution of economic activity through the physical space. Variable z is a measure of production concentration in space; we assume z=0 if there is no concentration, that is if economic activity distributes the most evenly possible through geographical space and $z\to +\infty$ means that economic activity tends to be fully concentrated in only one location of the multiple location sites that exist in the economy. With this definition of z, $b(z)$ will be an increasing concave function that obeys to $b(0)$=1 and $\lim_{z\to+\infty} b(z) = B > 1$. Explaining these limiting values is easy: when there is no concentration of economic activity, z=0, no increasing returns to scale can be enjoyed and thus production is a mere result of the combination of production inputs, while the higher the value of the concentration variable the higher will be the increasing returns effect, which allows to produce more with the same amounts of capital and technology available in the economy. The imposition of a finite limit for $b(z)$ simply means that even when production is fully concentrated in one location, there will be a finite value of production, that nevertheless is higher than any other output outcome generated by any other formula of distribution of the production process through space.

Hence, our first important assumption is that increasing returns contribute to a higher efficiency of the production inputs, and increasing returns are the direct result of concentrating economic activity.

Thus far, concentration has obvious advantages; it allows to capture scale economies. Although we have in $b(z)$ a centripetal force, centrifugal forces that push in the opposite direction must also be considered. This second set of forces is associated with transaction costs. Assuming that consumers are uniformly distributed in geographical space, the distribution of the productive activity in space eliminates transaction costs, that is, there is no need for regional trade to occur and thus setting z=0 benefits the consumers. If there is some concentration of economic activities, there is the need for trade between locations. The trade erodes the value of the goods as in the iceberg effect of Samuelson (1971). This erosion in the value of the goods implies a reduction of the utility of consumption dictated by the higher costs in the acquisition of the good. Transaction costs act in this way as a centrifugal force that collides with the centripetal force of scale economies.

We define the following utility function,

$$U(t) = g(z).u\left[c(t)\right] \tag{3}$$

In expression (3), $c(t)$ corresponds to per capita consumption and $g(z)$ is a location function such that $g'<0$, $g''>0$, $g(0)=1$ and $\lim_{z\to+\infty} g(z)=G<1$.[1] Thus, the thicker the concentration of economic activity, the lower will be the overall utility withdrawn from consumption. The utility function $u(.)$ has the usual properties of continuity and smoothness, being strictly increasing and concave.

Agglomeration and dispersion forces are simultaneously present in our model. Functions $b(z)$ and $g(z)$ imply an important trade-off in the economic system, and this is the first of three trade-offs that this paper deals with.

Trade-off number 1: increasing returns to scale and transaction costs conflict – if production concentrates geographically in order to gain with increasing returns, consumers will be penalized through larger transportation costs; if, on the other hand, production is dispersed in order to attenuate transaction costs, the consumer will loose in terms of generated output available to consume.

Our second trade-off is the trivial relation in growth models between present consumption and future consumption, that is, the trade-off between producing consumption goods and producing capital goods. This relation can be presented through a conventional capital accumulation dynamic equation, that we write in per capita values,

$$\dot{k}(t)=b(z).f\left[A(t),k(t)\right]-c(t)-(n+\delta).k(t),\ k(0)=k_0 \text{ given.} \quad (4)$$

[1] Note that we have to guarantee also that $G>0$ to ensure that the utility is always positive. If the concentration of activities is such that transaction costs lead to negative utility, trade will not take place.

with $\dot{k}(t) \equiv dK(t)/dt$, n a positive and constant growth rate for labor / population, and δ>0 a capital depreciation rate. Our second trade-off is the one implicit in (4),

Trade-off number 2: there is a conflict between consumption and capital accumulation. In deciding what to produce in each time moment, the economy has to ask how much it is ready to sacrifice in terms of present consumption in order to guarantee the fulfilment of future consumption needs.

The economic structure that was set forth allows to state our first optimization problem: this is an intertemporal optimal control problem, where a representative household maximizes consumption utility given the resource constraint (4). This is a growth problem, where location concerns are present. Capital is a state variable, while $c(t)$ and $z(t)$ are control variables – the representative agent chooses not only the time trajectory of consumption in order to maximize intertemporal utility, but also the degree of concentration of economic activities in order to attain the same goal.

The maximization problem is $\underset{c(t),z(t)}{Max}\int_0^{+\infty} g(z).u[c(t)].e^{-(\rho-n).t}.dt$ subject to (4).[2]

3. Technology Choices

To model technology choices we consider a central planner who has to decide how to allocate resources to innovative activities.[3] Following Nelson and Phelps (1966), we distinguish between two types of technology: a theoretical level or a technology possibilities frontier, which we designate by $T(t)$, and a level of ready-to-use

[2] We consider an infinite horizon problem, where future per capita utility is discounted at a rate ρ-n>0.

[3] Of course, not all research is public research. Nevertheless, the government has an important role in the economy concerning how technological resources are allocated. It is by having this role in mind that we develop the model in this section.

technology, that corresponds to the technology index present in production functions (1) and (2), $A(t)$. Given the basic science and the applied science indexes, the representative agent goal is the following: to achieve the highest possible values for variables $\tau(t) \equiv \dot{T}(t)/T(t) - a(.)$ and $\phi(t) \equiv A(t)/T(t)$. The first is the controllable part of the rate of technological progress and the second a measure of the gap between applied and basic technology.

We assume that the science frontier is, in part, controllable, that is, the representative agent can choose an allocation of resources that influences such frontier; the factors that cannot be controlled in the way technological progress happens are included in exogenous variable $a(.)$.

Given the representative agent goal, we define an objective function $v[\phi(t), \tau(t)]$ with the following properties: v is continuous, concave and smooth (infinitely many times continuously differentiable), the function is also homogeneous of degree one and the intertemporal elasticities of substitution are $v_\phi . \frac{\phi(t)}{v} = \theta$ and $v_\tau . \frac{\tau(t)}{v} = \mu$, with $\theta, \mu \in (0,1)$. Objective function (5) obeys the stated properties.

$$v[\phi(t), \tau(t)] = \phi(t)^\theta . \tau(t)^\mu \quad (5)$$

Next, we have to find which are the constraints to which the optimization of the objective function is subject to. A first condition relates to the time evolution of the technology frontier; this comes just from rewriting the previous definition of the controllable rate of growth,

$$\dot{T}(t) = [a(.) + \tau(t)]T(t), T(0) = T_0 \text{ given} \quad (6)$$

A second constraint establishes a link between basic and applied science, and takes the form,

$$\dot{A}(t) = h(.).[T(t) - A(t)],\ h(.) > 0,\ A(0) = A_0 \text{ given} \tag{7}$$

Equation (7) translates the notion of a convergence process: the lower the level of technology ready to use relatively to the benchmark level, the faster will grow the first. The time evolution of $A(t)$ depends on the proposed gap and on a series of other variables which are included in the exogenous variable $h(.)$.

Recovering the definition of $\phi(t)$ as the technology ratio, dynamic rules (6) and (7) give place to our technology model state constraint,

$$\dot{\phi}(t) = h(.).[1 - \phi(t)] - [a(.) + \tau(t)].\phi(t),\ \ \phi(0) = A_0 / T_0 \tag{8}$$

The proposed technology setup has an implicit third trade-off,

Trade-off number 3: Since technological resources are limited, the promotion of a higher technological frontier is an objective that conflicts with the incentive for a higher rate of application of existing technology to productive uses. The society must choose: to apply more knowledge directly to produce final goods, loosing in this way in terms of technology growth rate or, on the other hand, stimulate the expansion of the technology frontier neglecting in this way the adaptation of existent knowledge to productive tasks.

The technology choices debate leads us to the second intertemporal maximization problem, $\underset{\tau(t)}{Max}\int_0^{+\infty} v[\phi(t),\tau(t)].e^{-\rho(T).t}.dt$ subject to (8).

Note that, in this second problem, we continue to consider an infinite horizon and a discount rate $\rho(T)>0$, that does not have to be equal to the one in the problem concerning consumption utility. The present problem has only two endogenous variables, the control variable $\tau(t)$ and the state variable $\phi(t)$. These two variables synthesize the economic concerns about technology: to accelerate the pace of theoretical innovation and to reduce the gap between what is possible and what is effectively available.

Sections 2 and 3 have developed two independent optimal control problems. The first adds location concerns to a standard growth model and the second equates technology options. The link between the two is the technology variable $A(t)$, which is an input in the production of physical goods. In the next two sections, we study the models' dynamics. We begin by solving the technology problem, finding steady state values and discussing some features about transitional dynamics. The results of the technology model will allow then to find steady state and dynamics properties of the consumption-capital-location model; in particular, we search for results about the agglomeration level of economic activity that is optimal from the point of view of utility maximization, given the optimal rate of technological progress.

4. Technology Dynamics

In this and in the next section, we present a set of propositions relating to the technology model and to the location-growth problem. These propositions concern steady state results and transitional dynamics relations.

<u>Proposition 1.</u> In the long run, the state constraint $\tau(t) = h(.).\frac{1-\phi(t)}{\phi(t)} - a(.)$ intersects the indifference curve $\tau(t) = \frac{\mu}{\theta}.\left(\frac{\theta-\mu}{\theta}\right)^{\frac{\theta-\mu}{\mu}}.\frac{\rho(T)+h(.)+a(.)}{\left[\frac{\mu}{\theta}.\rho(T)+h(.)+a(.)\right]^{\frac{\theta}{\mu}}}.\left[\frac{h(.)}{\phi(t)}\right]^{\frac{\theta}{\mu}}$ in the unique steady state point $\{\bar{\phi},\bar{\tau}\} = \left\{\frac{(\theta-\mu).h(.)}{\mu.\rho(T)+\theta.[h(.)+a(.)]}, \frac{\mu}{\theta-\mu}.[\rho(T)+h(.)+a(.)]\right\}$.

Proof: The proposition reveals the existence of a unique steady state point concerning the optimal technology problem. The steady state corresponds to the point where the time derivative of the share variable equals zero: $\dot{\phi}(t) = 0$. In this point, the state constraint is the one in the proposition and, therefore, in the steady state, the optimal control problem reduces to a static optimization problem, where the maximization of *v* is subject to the referred state constraint.

Solving the static long run maximization problem, we get a second steady state relation between $\tau(t)$ and $\phi(t)$: $\tau(t) = \frac{\rho(T)+h(.)+a(.)}{\rho(T)+(\theta/\mu).[h(.)+a(.)]}.\frac{h(.)}{\phi(t)}$. The two relations between the endogenous variables of the technology model form a system from which we withdraw the two steady state values in the proposition.

Replacing the variables in objective function (5) by the computed steady state values, one obtains the following expression for *v*: $v^* = \bar{\phi}^{\phi}.\bar{\tau}^{\mu} = \frac{(\theta-\mu)^{\theta-\mu}.\mu^{\mu}.h(.)^{\theta}.[\rho(T)+h(.)+a(.)]^{\mu}}{[\mu.\rho(T)+\theta.(h(.)+a(.))]^{\theta}}$. The indifference curve that includes the steady state point is then given by $\phi(t)^{\theta}.\tau(t)^{\mu} = v^*$. Rearranging, we get the indifference curve in the form given in the proposition □

Relatively to the steady state point note that it has to obey to the condition $\bar{\phi} \in (0,1)$. For such, we just have to impose hereafter the inequality $\theta > \mu$. Note also that, given the state constraint, variables $A(t)$ and $T(t)$ have to grow in the long run solution at exactly the same rate, which is

$$a(.) + \bar{\tau} = \frac{\mu}{\theta - \mu}\left[\rho(T) + h(.)\right] + \frac{\theta}{\theta - \mu}.a(.) \tag{9}$$

We proceed to the characterization of transitional dynamics through the presentation of proposition 2.

<u>Proposition 2.</u> Given the condition $0 < \mu < \theta < 1$, the technology choice model exhibits saddle-path stability and the stable trajectory is negatively sloped, $\tau(t) - \bar{\tau} = -\zeta.(\phi(t) - \bar{\phi})$, with $\zeta = \dfrac{\frac{1-\theta}{1-\mu}.\frac{h(.)}{\bar{\phi}^2}.\bar{\tau}}{\frac{\theta}{\mu}.\bar{\tau} - \lambda_1}$ and where $\lambda_1<0$ is the negative eigenvalue of the Jacobean matrix that is derived from the linearization of the Hamiltonian system associated with the technology dynamic problem.

Proof: Using the tools of optimal control analysis, we present a Hamiltonian function, where $p(t)$ is a co-state variable associated to $\phi(t)$,

$$\aleph\left[\phi(t), \tau(t), p(t)\right] = v\left[\phi(t), \tau(t)\right] + p(t).\left\{h(.).\left[1 - \phi(t)\right] - \left[a(.) + \tau(t)\right].\phi(t)\right\} \tag{10}$$

The first order optimality conditions are

$$\aleph_{\tau} = 0 \Rightarrow v_{\tau} = p(t).\phi(t) \tag{11}$$

and

$$\aleph_{\phi} = \rho(T).p(t) - \dot{p}(t) \Rightarrow \dot{p}(t) = [\rho(T) + h(.) + a(.) + \tau(t)].p(t) - v_{\phi} \tag{12}$$

The transversality condition $\lim_{t \to +\infty} p(t).e^{-\rho(T).t}.\phi(t) = 0$ also applies. Rearranging (12), given (11), one obtains the growth rate of the co-state variable,

$$\dot{p}(t)/p(t) = \rho(T) + h(.) + a(.) - \frac{\theta - \mu}{\mu}.\tau(t) \tag{13}$$

From the previous conditions we derive the equation of motion of the controllable innovation rate:

$$\dot{\tau}(t) = \left\{ \frac{\theta}{\mu}.\tau(t) - \frac{1-\theta}{1-\mu}.\frac{h(.)}{\phi(t)} - \frac{\theta}{1-\mu}.[h(.) + a(.)] - \frac{\rho(T)}{1-\mu} \right\}.\tau(t) \tag{14}$$

Solving the system $[\dot{\phi}(t)\ \dot{\tau}(t)]' = 0$, we arrive to the same steady state result as in proposition 1. The motion properties of the system in the steady state vicinity are analyzed through the linearization of the model in the steady state neighbourhood. The linearized system is: $\begin{bmatrix} \dot{\phi}(t) \\ \dot{\tau}(t) \end{bmatrix} = J.\begin{bmatrix} \phi(t) - \bar{\phi} \\ \tau(t) - \bar{\tau} \end{bmatrix}$ with $J = \begin{bmatrix} \frac{\partial \dot{\phi}(t)}{\partial \phi(t)} & \frac{\partial \dot{\phi}(t)}{\partial \tau(t)} \\ \frac{\partial \dot{\tau}(t)}{\partial \phi(t)} & \frac{\partial \dot{\tau}(t)}{\partial \tau(t)} \end{bmatrix}_{|(\bar{\phi},\bar{\tau})} = \begin{bmatrix} -\frac{h(.)}{\bar{\phi}} & -\bar{\phi} \\ \frac{1-\theta}{1-\mu}.\frac{h(.)}{\bar{\phi}^2} & \frac{\theta}{\mu}.\bar{\tau} \end{bmatrix}$. Matrix J has two eigenvalues, which are

$\{\lambda_1, \lambda_2\} = \left\{ \frac{\rho(T)}{2} - \sqrt{\left(\frac{\rho(T)}{2}\right)^2 - |J|}, \frac{\rho(T)}{2} + \sqrt{\left(\frac{\rho(T)}{2}\right)^2 - |J|} \right\}$, with

$|J| = -\frac{\theta - \mu}{\mu.(1-\mu)}.\frac{h(.)}{\bar{\phi}}.\bar{\tau} < 0$ the determinant of the Jacobean matrix. It is straightforward to verify that $\lambda_1 < 0$ and $\lambda_2 > 0$ both real. Then, we confirm the saddle-path stability result.

With a saddle-path stable equilibrium, the stable arm underlying the system dynamics is found through the computation of the eigenvector associated to the negative eigenvalue of *J*. This eigenvector is *P*=[1 -*ζ*]'. Given that the second line of matrix *J* respects to the control variable the same is true for vector *P*. Hence, we can identify the second element of vector *P* as being the slope of the stable path □

Note the economic interpretation of the accomplished result: saddle path stability implies that, once on the negatively sloped stable trajectory, the variables evolve to the steady state following opposite directions - an increasing growth rate for technology implies a slower technology gap straightening and vice-versa, as the system adjusts to the long run locus.

With propositions 1 and 2 we have clarified how technology choices can be equated in a scenario where resources may be used to promote the expansion of the technology frontier or, alternatively, to stimulate the productive use of already available knowledge. We have found that optimal long run values for the technology gap and for the technology growth rate can be quantified; we have also verified that both technology aggregates grow, in the steady state, at a same rate, what is a simple corollary of a constant long run technology gap; and finally, a stable trajectory characterizes a convergence process to the steady state where an increasing level of applied technology (relatively to basic knowledge) can only be attained if simultaneously one observes a decreasing rate of growth of technology generation.

5. Location and Growth Dynamics

The technology problem is linked to the space-growth setup of section 2 through the applied technology variable, $A(t)$, which is an argument of the aggregate production function. To derive steady state results and dynamic relations from the space-growth maximization problem, we define for functions $f[A(t), k(t)]$, $u[c(t)]$, $b(z)$ and $g(z)$ explicit functional forms that obey the general properties in section 2. We write the following functions,

$$f\left[A(t),k(t)\right] = A(t)^{1-\alpha}.k(t)^{\alpha}, \quad 0<\alpha<1 \tag{15}$$

$$u\left[c(t)\right] = \frac{\sigma}{1-\sigma}.\left[1-\left(\frac{1}{c(t)}\right)^{(1-\sigma)/\sigma}\right], \quad 0<\sigma<1 \tag{16}$$

$$b(z) = \frac{1+b.B.z(t)}{1+b.z(t)}, \quad b>0 \tag{17}$$

$$g(z) = \frac{1+g.G.z(t)}{1+g.z(t)}, \quad g>0 \tag{18}$$

In equation (15), α represents the output-capital elasticity. In (16), parameter σ corresponds to the utility function elasticity of intertemporal substitution. In (17) and (18), B and G were defined as the values for $b(z)$ and $g(z)$ translating a maximum agglomeration degree of economic activities [note that what distinguishes (18) from (17) is that $B>1$ and $G<1$, although they are both positive values]. The other two parameters, b and g, reveal the intensity of the impact of the activity agglomeration level over increasing returns to scale and over transaction costs, respectively. For a given finite $z(t)>0$, the higher the value of b, the larger are scale economies, and the higher the value of g, the larger will be the transaction costs [the lower will be the value of $g(z)$].

With the assumed functions, proposition 3 can be stated.

Proposition 3. In a location-growth model where technology choices assume the form characterized in section 3 and where the assumption that output and physical capital grow at a same steady state rate holds, the steady state implies the following growth rate relation, $\gamma_k = \gamma_c = \frac{1}{1-\alpha}.\gamma_{b(z)} + a(.) + \overline{\tau}$, where variables γ_k, γ_c and $\gamma_{b(z)}$ are the steady state growth rates of capital per unit of labor, per capita consumption, and increasing returns, respectively.

Proof: In proposition 3 we make use of a common steady state property, which is that capital grows at a same rate as output.[4] Under this assumption, one is able to find a capital / consumption long run growth rate that corresponds to the technology growth rate plus a term that indicates that the higher the rate at which activities agglomerate and consequently the scale economies become stronger, the faster capital and consumption will grow. To prove this result, we have to consider production function (2) under functional form (15). Differentiating this function in order to time, one obtains the steady state growth rate relation in (19).

$$\gamma_y = \gamma_{b(z)} + (1-\alpha).\gamma_A + \alpha.\gamma_k \tag{19}$$

[4] See, for instance, the Caballé and Santos (1993, page 1047) definition of steady state for a growth model.

with γ_y and γ_A respectively the growth rates of per capita output and technology in the steady state. The second of these growth rates is known from the technology choices problem, while the first is said in the proposition to coincide with γ_k. Having these points in consideration, a simple rearrangement of (19) leads to the long run result concerning capital in the proposition.

To understand that the obtained rate is also the growth rate of per capita consumption, one has to differentiate in order to time equation (4). Since γ_k is a constant value, the differentiation of the right hand side of the equation must equal zero; computing the requested expression,

$$\left[\gamma_{b(z)} + (1-\alpha).(\gamma_A - \gamma_k)\right] b(\bar{z}).(\bar{A}/\bar{k})^{1-\alpha} - (\gamma_c - \gamma_k).(\bar{c}/\bar{k}) = 0 \qquad (20)$$

Note, in (20), that the first term is zero, given the derived capital steady state growth rate, and thus the only way for expression (20) to hold is to conclude that $\gamma_c = \gamma_k$, such that $\bar{c}/\bar{k}$ is a constant value□

Our analytical framework points to the general result in growth models (and particularly in endogenous growth models) that output, capital and consumption grow at a same positive and constant rate in the long run. This long run growth rate has two components: technology growth, which was endogenously determined, and the growth of spatial agglomeration. Note that if the economy is under a process of spatial agglomeration, such that increasing returns in production grow positively ($\gamma_{b(z)}>0$), then consumption and capital per capita will grow faster than the technology level; if spatial dissemination prevails, the economy looses in terms of increasing returns stimulus ($\gamma_{b(z)}<0$) what implies a per capita output / capital / consumption growth rate lower than technology growth.

More long run steady state results are achievable. Proposition 4 presents the long run growth rate as depending only upon technology growth.

<u>Proposition 4.</u> Under the same assumptions as in proposition 3, the steady state growth rate of the increasing returns function is $\gamma_{b(z)} = \left(\frac{2.\sigma - 1}{1 - \sigma.(1+\alpha)}\right).\left[a(.) + \bar{\tau}\right]$, and the per capita growth rate of the main economic variables can be written as $\gamma_y = \gamma_k = \gamma_c = \frac{\sigma.(1-\alpha)}{1-\sigma.(1+\alpha)}.\left[a(.) + \bar{\tau}\right]$.

Proof: This proposition states that the long term increasing returns growth rate is a function of technology growth, and therefore output, capital and consumption, all in per capita values, grow in the steady state at a rate that depends solely on technological progress and on two elasticity parameters.

To determine the above growth rates, one has to solve the optimal control problem. Defining *q*(*t*) as a co-state variable relating to state variable *k*(*t*), the following are optimality conditions,

$$g(z).c(t)^{-1/\sigma} = q(t) \tag{21}$$

$$1 - \left[\frac{1}{c(t)}\right]^{(1-\sigma)/\sigma} = \frac{1-\sigma}{\sigma}.\frac{b.(B-1)}{g.(1-G)}.A(t)^{1-\alpha}.k(t)^{\alpha} \tag{22}$$

$$\dot{q}(t) = \left[\rho + \delta - \alpha.b(z).(A(t)/k(t))^{1-\alpha}\right]q(t) \tag{23}$$

$$\lim_{t \to +\infty} q(t).e^{-(\rho-n).t}.k(t) = 0 \tag{24}$$

Differentiating condition (22) in order to time and evaluating the result in the steady state,

$$\overline{A}^{1-\alpha}.\overline{k}^{\alpha}.\overline{c}^{-(1-\sigma)/\sigma}=\frac{g.(1-G)}{b.(B-1)}.\frac{\gamma_k}{(1-\alpha).\gamma_A+\alpha.\gamma_k} \tag{25}$$

We know that the right hand side of (25) is constant, thus the left hand side has to be also constant, what implies the veracity of the condition $(1-\alpha).\gamma_A+\alpha.\gamma_k-\frac{1-\sigma}{\sigma}.\gamma_c=0$. With this condition and knowing that $\gamma_c=\gamma_k$, one arrives to the second equilibrium growth rate in the proposition. Replacing this equilibrium growth rate in the growth rate expression of proposition 3, it is straightforward to obtain the first growth expression in proposition 4 □

Steady state growth rates deserve some comments. Per capita output, per capita capital and per capita consumption grow at a positive rate if the condition $\sigma.(1+\alpha)<1$ is satisfied. For high values of elasticity parameters σ and α it is possible that the growth rate of the increasing returns function becomes negative, offsetting the effect of technological progress. We also regard that agglomeration forces that allow for a positive growth rate $\gamma_{b(z)}$ are present only for $½<\sigma<1/(1+\alpha)$. Otherwise, centrifugal forces prevail (z declines), lowering in this way the effect of scale economies. Note that if $\sigma=½$, we have a special case where $\gamma_{b(z)}=0$ and $\gamma_y=\gamma_k=\gamma_c=\gamma_A$. In this case, steady state is characterized by a constant z parameter. Such a situation allows for the computation of explicit equilibrium results for the ratios consumption-capital and technology-capital.

<u>Proposition 5.</u> Assume a steady state in which the agglomeration variable is a constant value (in this steady state there is no tendency for economic activities to concentrate or to disperse, relatively to an optimal agglomeration value). Under this assumption, the following ratios are attained,

$$\frac{\bar{c}}{\bar{k}} = \frac{1}{\alpha}.\left[2.a(.) + 2.\bar{\tau} + \rho\right] - n + \frac{1-\alpha}{\alpha}.\delta\,;$$

$$\frac{\bar{A}}{\bar{k}} = \left\{\frac{g.(1-G)}{b.(B-1)}.\left[\frac{1}{\alpha}.\left[2.a(.) + 2.\bar{\tau} + \rho\right] - n + \frac{1-\alpha}{\alpha}.\delta\right]\right\}^{1/(1-\alpha)}$$

Proof: The assumption that $\bar{z}$ is constant implies that σ=½ and that $\gamma_{g(z)}=\gamma_{b(z)}=0$. Therefore, the differentiation of (21) yields, in the equilibrium, $-2.\gamma_c=\gamma_q$; given that the growth rate of per capita consumption equals the growth rate of technology, in the steady state, and that the growth rate of the co-state variable is observable from (23), we get the long run relation

$$b(\bar{z}).\left(\frac{\bar{A}}{\bar{k}}\right)^{1-\alpha} = \frac{1}{\alpha}.\left[2.a(.) + 2.\bar{\tau} + (\rho + \delta)\right] \tag{26}$$

From (4) it is true that

$$b(\bar{z}).\left(\frac{\bar{A}}{\bar{k}}\right)^{1-\alpha} = \frac{\bar{c}}{\bar{k}} + n + \delta \tag{27}$$

Combining (26) with (27) one obtains the result $\bar{c}/\bar{k}$ in the proposition.

Note now that, for σ=½, expression (25) reduces to

$$\left(\frac{\overline{A}}{\overline{k}}\right)^{1-\alpha} \cdot \frac{\overline{k}}{\overline{c}} = \frac{g.(1-G)}{b.(B-1)} \qquad (28)$$

Replacing the result for $\overline{c}/\overline{k}$ in (28), the second steady state ratio in the proposition is attained □

Some important features characterize the derived ratios,

i) Technology growth, impatience to consume, low population growth rate and high capital depreciation rate are all factors that contribute to a high long term consumption level relatively to the accumulated capital level. These results are common to a great variety of growth models;[5]

ii) The technology-capital ratio depends on the same factors as the consumption-capital ratio plus the agglomeration parameters. The stronger the effect of agglomeration over transaction costs (given by g), the higher is the ratio value; the opposite for the effect of agglomeration over scale economies (given by b).

To end our analysis, we make a remark about transitional dynamics.

Proposition 6. Consider an unchangeable economic space (z constant). Under this assumption, the technology-growth model is a four dimensional system, where saddle-path stability holds. The stable trajectory is a dimension two space.

Proof: With a constant z, the model possesses two control variables, $c(t)$ and $\tau(t)$, and two state variables, $k(t)$ and $\phi(t)$. Variables $c(t)$ and $k(t)$ are not constant in the steady state, although they grow in the long run at a same rate. To study stability, we

---

[5] See, for instance, Barro and Sala-i-Martin (1995).

need variables that assume constant equilibrium values; thus, we define $\varphi(t)\equiv k(t)/T(t)$ and $\psi(t)\equiv c(t)/k(t)$. To these variables, the following pair of equations of motion apply,

$$\dot{\varphi}(t)=\left\{b(z).\left[\frac{\phi(t)}{\varphi(t)}\right]^{1-\alpha}-\psi(t)-(n+\delta)-a(.)-\tau(t)\right\}.\varphi(t) \tag{29}$$

$$\dot{\psi}(t)=\left\{\psi(t)-(1-\alpha.\sigma).b(z).\left[\frac{\phi(t)}{\varphi(t)}\right]^{1-\alpha}-\sigma.\rho+n+(1-\sigma).\delta\right\}.\psi(t) \tag{30}$$

Note that equation (30) is the result of the differentiation of (21), having in mind (23).

In possession of equations (8), (14), (29) and (30), the steady state vicinity linearized four dimensional system takes the form,

$$\begin{bmatrix}\dot{\phi}(t)\\ \dot{\tau}(t)\\ \dot{\varphi}(t)\\ \dot{\psi}(t)\end{bmatrix}=\Xi.\begin{bmatrix}\phi(t)-\overline{\phi}\\ \tau(t)-\overline{\tau}\\ \varphi(t)-\overline{\varphi}\\ \psi(t)-\overline{\psi}\end{bmatrix},\ \text{with }\Xi=\left[\begin{array}{cc|cc} \multicolumn{2}{c|}{J} & \multicolumn{2}{c}{\overline{0}} \\ \hline \varpi & -\overline{\varphi} & -\varpi.\dfrac{\overline{\phi}}{\overline{\varphi}} & -\overline{\varphi} \\ -\xi & 0 & \xi.\dfrac{\overline{\phi}}{\overline{\varphi}} & \overline{\psi}\end{array}\right]$$

$$\text{and } \varpi=(1-\alpha).b(z).\left(\frac{\overline{\varphi}}{\overline{\phi}}\right)^{\alpha},\ \xi=(1-\alpha).(1-\alpha.\sigma).b(z).\left(\frac{\overline{\phi}}{\overline{\varphi}}\right)^{1-\alpha}.\frac{\overline{\psi}}{\overline{\phi}}$$

In matrix Ξ, J corresponds to the Jacobean matrix of the technology problem and $\overline{0}$ is (2×2) null matrix. Two of the eigenvalues of Ξ are the $\lambda_1<0$ and $\lambda_2>0$ eigenvalues found for J. The signs of the other two eigenvalues can be computed through the evaluation of the (2×2) matrix in the right-low corner of Ξ. Denote this matrix by Ξ_{22}. Then, $\text{Det}(\Xi_{22})=-\alpha.\sigma.\varpi.\dfrac{\overline{\phi}}{\overline{\varphi}}.\overline{\psi}<0$. Thus, being λ_3 and λ_4 the two other eigenvalues of Ξ, we guarantee that $\lambda_3.\lambda_4<0$, and in this way they have to have different signs. Therefore, from matrix Ξ, four eigenvalues can be computed, being two positive values and the other two negative. As the proposition states, the stable trajectory has dimension two (the number of negative eigenvalues)□

6. Discussion

The proposed framework allows for an integrated approach about the relation between location, growth and technology. Our model may be understood as an endogenous growth model where long run positive growth is determined by endogenous decisions about technology generation and by an agglomeration / dispersion rate of economic activities that is set optimally in order to maximize consumption utility.

In a first moment, one has observed that the conflict in the allocation of scientific resources allows to determine a steady state optimal result where a particular share of applied technology is consistent with a constant growth rate for the technology variables. This rate is dependent on the rate at which future technological outcomes are discounted to the present, on the relative weight put on basic and applied science, and upon other factors that stimulate technology growth but that are left exogenous in our analysis, like human capital or the existence of a sound institutional environment.

The relevant result in the technology framework is precisely the possibility to encounter a constant positive long run rate for the technology index that is included in the definition of the aggregate production function of the economy. It is this technology index that constitutes the bridge between the technology framework and the growth setup. For a given set of explicit functional forms for production, utility, transaction costs and increasing returns, several equilibrium relations are found. As it is usual in growth models, the long run steady state is characterized by a same growth rate for output, capital accumulation and consumption. The rate of per capita growth is the rate of technological progress (determined endogenously) plus the rate at which scale economies grow (or decline). Increasing or decreasing scale economies are, in our framework, simply the result of the dominance of centripetal forces or, rather, the prevalence of centrifugal forces. These forces, in turn, are conditioned by the value of elasticity parameters regarding utility and production conditions.

Because the model points to a long term economic agglomeration that is directly dependent on technology growth, in the end the economy's per capita growth rate depends solely on technological progress – this apparently is nothing new relatively to conventional growth models, but there is an important difference: the economy's per capita growth rate can be higher or lower than the technology growth rate, depending on the way firms choose to locate in space. An agglomeration process stimulates growth above technological progress; otherwise, growth will be lower than the rate of technological progress. One might rush to the conclusion that since agglomeration benefits the creation of wealth, economic activity should concentrate the most in order to take full advantage of increasing returns; but remind the other side of the coin: although concentration allows for a higher output level, concentration implies also transaction costs that produce lower utility. This is the reason why the optimal long run rate of economic agglomeration is not necessarily a positive value. It can be indeed positive, but it may be also negative or zero, depending on parameter values.

In the specific case of absence of a tendency for the agglomeration degree to change, the present setup points to a long run result where per capita economic growth corresponds to the technology growth rate. Furthermore, in such a case it is easy to identify a saddle-path equilibrium that reflects the technology trade-off and the consumption-capital accumulation trade-off.

References

Aghion, Phillippe and Peter Howitt (1992). “A Model of Growth through Creative Destruction.” *Econometrica*, vol. 60, 323-351.

Aghion, Phillippe and Peter Howitt (1998). *Endogenous Growth Theory*. Cambridge, Mass.: MIT Press.

Baldwin, Richard E. (1999). “Agglomeration and Endogenous Capital.” *European Economic Review*, 43, 253-280.

Baldwin, Richard E. and Rikard Forslid (2000). “The Core-Periphery Model and Endogenous Growth: Stabilising and Destabilising Integration.” *Economica*, 67, 307-324.

Baldwin, Richard E.; Phillippe Martin and Gianmarco Ottaviano (2001). “Global Income Divergence, Trade and Industrialization: the Geography of Growth Take-offs.” *Journal of Economic Growth*, 6, 5-37.

Barro, Robert J. and Xavier Sala-i-Martin (1995). *Economic Growth*. New York: McGraw-Hill.

Caballé, Jordi and Manuel S. Santos (1993). "On Endogenous Growth with Physical and Human Capital." *Journal of Political Economy*, 101, 1042-1067.

Cairncross, Frances (2001). *The Death of Distance 2.0; How the Communications Revolution will Change our Lives*. Cambridge, Mass.: Harvard Business School Press.

Eaton, Jonathan and Samuel Kortum (2002). “Technology, Geography and Trade.” *Econometrica*, 70, 1741-1779.

Forslid, Rikard and Gianmarco Ottaviano (2003). “An Analytically Solvable Core-Periphery Model.” *Journal of Economic Geography*, 3, 229-240.

Fujita, Masahisa; Paul R. Krugman and Anthony J. Venables (1999). *The Spatial Economy. Cities, Regions and International Trade*. Cambridge, Mass.: MIT Press.

Fujita, Masahisa and Jacques-François Thisse (2002). *Economics of Agglomeration: Cities, Industrial Location and Regional Growth*. Cambridge, Cambridge University Press.

Gomes, Orlando (2004). "The Optimal Control of Technology Choices." in M. A. Ferreira, R. Menezes and F. Catanas (eds.), *Themes in Quantitative Methods 4*. Lisbon: Edições Sílabo. 255-273.

Grossman, Gene M. and Elhanan Helpman (1991). *Innovation and Growth in the Global Economy*. Cambridge, Mass.: MIT Press.

Jaffe, Adam and Manuel Trajtenberg (2002). *Patents, Citations and Innovations – a Window on the Knowledge Economy*. Cambridge, Mass.: MIT Press.

Jones, Charles I. (1995). "R&D-Based Models of Economic Growth." *Journal of Political Economy*, 103, 759-784.

Jones, Charles I. (2003). "Population and Ideas: A Theory of Endogenous Growth." in Aghion, Philippe; Roman Frydman; Joseph Stiglitz and Michael Woodford (eds.) *Knowledge, Information and Expectations in Modern Macroeconomics* (in honor of Edmund S. Phelps). Princeton, New Jersey: Princeton University Press. 498-521.

Krugman, Paul R. (1991). "Increasing Returns and Economic Geography." *Journal of Political Economy*, 99, 483-499.

Krugman, Paul R. and Anthony J. Venables (1995). "Globalization and the inequality of Nations." *Quarterly Journal of Economics*, 60, 857-880.

Martin, Phillippe and Gianmarco Ottaviano (1999). "Growing Locations: Industry Location in a Model of Endogenous Growth." *European Economic Review*, 43, 281-302.

Nelson, Richard R. and Edmund S. Phelps (1966). "Investment in Humans, Technological Diffusion and Economic Growth." *American Economic Review*, 56, 69-75.

Ottaviano, Gianmarco (2001). “Monopolistic Competition, Trade, and Endogenous Spatial Fluctuations.” *Regional Science and Urban Economics*, 31, 51-77.

Pfluger, Michael (2004). “A Simple, Analytically Solvable, Chamberlian Agglomeration Model.” *Regional Science and Urban Economics*, 34, 565-573.

Quah, Danny (2002). “Spatial Agglomeration Dynamics.” *American Economic Review*, 92, 247-256.

Romer, Paul M. (1990). "Endogenous Technological Change." *Journal of Political Economy*, 98, part II, S71-S102.

Samuelson, Paul A. (1971). “On the Trail of Conventional Beliefs about the Transfer Problem”, in J. Bhagwati et al. (eds.), *Trade, Balance of Payments and Growth*, Amsterdam, North-Holland.

Solow, Robert M. (1956). "A Contribution to the Theory of Economic Growth." *Quarterly Journal of Economics*, 70, 65-94.

Swan, Trevor W. (1956). “Economic Growth and Capital Accumulation.” *Economic Record*, 32, 334-361.

Venables, Anthony J. (1996). “Equilibrium Location of Vertically Linked Industries.” *International Economic Review*, 37, 341-359.

Venables, Anthony J. (2001). “Geography and International Inequalities: the Impact of New Technologies.” *Journal of Industry, Competition and Trade*, 1, 135-160.