Location Dynamics and Knowledge Agglomeration

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Abstract

A simple economic activity location rule is considered. Under this rule, one regards that location decisions depend on the presence or the absence of agglomeration economies. Considering a three-location economy, the system that is built leads, under certain conditions, to a saddle-path equilibrium, relatively to which we verify that the most interesting dynamics are associated not with the eventual convergence to the steady state (the saddle-path), that occurs only under exceptional circumstances, but with the divergence process away from the steady state. To explain the dynamics of the agglomeration economies, a knowledge variable is assumed. Returning to a two location economy one is able to assess in graphical terms the relation between distribution of knowledge and location of economic activities.

Keywords: location decisions, dynamic systems, knowledge, technology, agglomeration economies.

JEL classification: O33, R11, R12

I. Introduction

An important part of economic decisions is related with location decisions. Although undeniable the growth of the *weightless economy*, it is crucial to understand the relevance of the geography of economic activities, that is, to understand the factors underlying economic location. This is true for all kinds of activities, not only the traditional but also the new intangible ones. As Quah (2000) refers, for the intangible sectors of the new economy location matters as well. The location of financial services, entertainment industries or pharmaceutical laboratories obeys to the same logic as the location of any other activity – the goal is always to maximize their economic performance given the centripetal and the centrifugal forces that push into or pull away, respectively, the economic activity from some given place. Perhaps the main feature regarding the *weightless economy* is that this does not have any more transportation costs as a form of centripetal force, nevertheless many other items play a role in the concentration, diffusion or clustering of economic activities through space.

The revival of geography economics is linked with the a-spatial characteristics of the new economy but also with the necessity to develop a new framework to relate trade (in particular, international trade) and economic growth. A static analysis of comparative advantages constitutes no longer a convincing explanation for a great part of the reality of international trade. There is a significant number of economic variables that must be considered simultaneously to understand the nature of economic relations. Venables (2001*a*) cites some, relating location, endowments, markets structure and size, on one hand, with flows of goods, foreign direct investment and workers migrations, on the other, to assess the main subject of sustained GDP growth, under an increasing trade relations scenario.

Putting it in simple terms, the globalization concept that seems to embrace so many issues of the contemporary economic system,[1] is nothing more than a set of economic issues that must be jointly analyzed. In concrete, the idea of increasing global relations can only be understood under a framework involving the following set: {economic growth, trade relations, location decisions, knowledge dissemination, new technologies}.

Here, one looks at location dynamics on an integrated way, regarding the previous set. This is the way these items are being approached in recent literature. For instance, Quah (2001) surveys economic growth under the new economy concept giving special attention to the spread of technological knowledge, and Venables (2001*b*) discusses international inequalities through space regarding the new information and communication technologies. On the other hand some of the most eminent economists begin to look with particular attention to the structure of economic spaces and respective determinants.[2] Following the same line of thought, Fujita and Thisse (2002) undertake a detailed analysis of how agglomeration economies relate to the organization of market structures and industries and to the growth of different regions.

A central piece of our analysis deals with the eventual existence of agglomeration economies.[3] Agglomeration economies are first treated under a *black-box* approach. That is, resorting to a simple rule of spatial activities allocation [a rule used in Fujita, Krugman and Venables (1999)], we analyze the way in which activities spread through space without giving details about the factors that determine the agglomeration / disagglomeration of activities. This first approach to location decisions considers a three

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[1] See, about the need to understand the true meaning of the globalization concept, e.g., Murteira (2002), Crafts and Venables (2001) and Baldwin and Martin (1999).

[2] See, in this respect, Lucas (2001) and Lucas and Rossi-Hansberg (2002).

[3] For a detailed analysis of spatial agglomeration dynamics, take a look at Krugman and Venables (1997) and Quah (2002).

point economy, relatively to which the underlying dynamics imply under some circumstances the existence of a saddle-path equilibrium. The most interesting point is that under the saddle-path dynamics we will not be concerned with the steady state result, that is accomplished only under some restrictive conditions but with the divergence process away from the cited point, a process that culminates in a kind of concentrated economic activity result, pointing eventually to a clustering of activities that is compatible with the analysis in Quah (2000).

In the second part of the paper, the black box is opened revealing a possible explanation for agglomeration dynamics. We consider the hypothesis that the state of knowledge determines the prevalence of centrifugal or centripetal forces. We assume that in a first stage knowledge activities need to be concentrated, but beyond a given state of accumulated knowledge, the agglomeration economies no longer exist – for high levels of technology, i.e., for the information and communication society that we live in today on the developed world, the resulting centrifugal forces tend to overcome pro-agglomeration factors. The considered assumption is useful to establish a phase diagram relation between activities location and the accumulation of knowledge.

The remainder of this document is organized as follows. Section II presents a three location economy under which the prevalence of agglomeration / disagglomeration economies determines the geographical distribution of some economic activity. Section III relies on the same location rule as in the previous section, nevertheless we now restrict the analysis to two points in space; a knowledge accumulation dynamic equation is added to the framework in order to relate location dynamics, agglomeration economies and the accumulation and diffusion of technical capabilities. Section IV develops some dynamic results of the knowledge-geography model. The last section concludes.

II. Location Dynamics on a Three Point Economy

A simple rule to characterize the distribution of any economic activity in space is given in Fujita, Krugman and Venables (1999). Let $z(t)$ be the time dependent share of economic activity (for example, in the form of accumulated capital or accumulated knowledge available to production) in location 1 of two possible locations, and a an agglomeration economies variable that for now we assume as a constant value. If a takes a positive value agglomeration economies exist; if a is a negative value then the forces pushing the economic activity away from the location are stronger. The referred rule is

$$\dot{z}(t) = a.[z(t) - 1/2],\ z(0)=z_0 \text{ given.} \tag{1}$$

Equation (1) gives a simple relation between agglomeration economies and the location of economic activities. There is a steady state point $\bar{z} = 1/2$; this is a stable point, to which the system converges, if a<0, meaning that under the disagglomeration economies scenario location 1 will retain, in the long run, one half of the economic activity. The other half will represent the other location long run concentration of activity ($1 - \bar{z}$). If a>0, the steady state point is not accomplished for $z_0 \neq \bar{z}$ and thus the economic activity will fully concentrate in one of the locations depending on z_0 being to the left or to the right of the steady state value.

This simple idea may be extended to more than two locations. For an interesting graphical analysis we assume a three location economy with $z_i(t)$ the share of economic activity allocated to geographical point i, i=1,2,3. Let a_i be the centripetal / centrifugal forces towards economic activity in place i. Once again the signs of the three a_i parameters will determine the kind of dynamics the model exhibits. Nevertheless, nothing is known about the economic forces promoting or not the concentration of activities.

The following assumption is that each geographical point share of economic activity is increased by its own centripetal forces and reduced with agglomeration economies on other economic spaces. Thus, for any i space:

$$\dot{z}_i(t) = a_i.[z_i(t) - 1/3] - \sum_{\substack{j=1 \\ j\neq i}}^{3} a_j.[z_j(t) - 1/3],\ z_i(0)=z_{i0} \text{ given.} \tag{2}$$

Because $z_3(t)$=1-$z_1(t)$- $z_2(t)$, the dynamic system that will be the target of our analysis is:

$$\begin{cases} \dot{z}_1(t) = (a_1 + a_3).z_1(t) + (a_3 - a_2).z_2(t) - \frac{1}{3}.(a_1 - a_2) - \frac{2}{3}.a_3 \\ \dot{z}_2(t) = (a_2 + a_3).z_2(t) + (a_3 - a_1).z_1(t) - \frac{1}{3}.(a_2 - a_1) - \frac{2}{3}.a_3 \end{cases} \tag{3}$$

System (3) is a linear system. Under matrix form:

$$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = J.\begin{bmatrix} z_1(t)-1/3 \\ z_2(t)-1/3 \end{bmatrix} \tag{4}$$

with $J = \begin{bmatrix} a_1+a_3 & a_3-a_2 \\ a_3-a_1 & a_2+a_3 \end{bmatrix}$. Note that the steady state corresponds to $\bar{z}_i = 1/3$, i=1,2,3; that is, the steady state describes the total absence of activity concentration. We know that this is a case attainable only when centrifugal forces dominate. Now, one may study rigorously the circumstances of perfect activity distribution and the circumstances that lead to activity concentration, through the examination of the contents of matrix J. The eigenvalues of J are $\lambda_1=a_1+a_2$ and $\lambda_2=2.a_3$. The system is globally stable if $\lambda_1<0$ and $\lambda_2<0$. Thus, in the case where $a_1<-a_2$ and $a_3<0$ the system converges to the absence of concentration point, independently of the initial values $\{z_1(0), z_2(0), z_3(0)\}$. The important point is that stability does not require all three agglomeration parameters to be negative: a_1 or a_2 may be positive values if properly compensated by the other parameter's value. Not to strong centripetal forces on the direction of one of the locations do not necessarily change the scenario of perfect dissemination of activities. The opposite case, global instability, has the symmetric consequences: not all the agglomeration parameters have to be positive but predominantly this must be the case.

The most interesting results are related to saddle-path equilibrium, i.e., to the case where one of the eigenvalues is positive and the other a negative value. In such case, there is a line through which convergence to the perfect dispersion of activities steady state is accomplished but this is a particular case that implies an initial point $\{z_1(0), z_2(0), z_3(0)\}$ that is already located on the stable arm. For any other case, the result is a divergence process away from the steady state and in the direction of activities concentration in one or two of the three locations. Thus, saddle-path stability implies that when centrifugal and centripetal forces coexist in a way that our two eigenvalues have opposite signs, a perfect distribution of activities in space may occur only in exceptional conditions. The rule is a divergence process that leads to concentration of activities, being the nature of this concentration of activities dependent on the initial locus regarding location.

A particular case and a numerical example allow to clarify several issues. Let $a_2<a_1<0<a_3$, $a_1+a_3>0$ and $a_2+a_3>0$. In this case, the system displays saddle-path stability, according to the eigenvalues signs. Under a saddle-path equilibrium stable and unstable trajectories correspond to lines in a two-dimensional referential. These are computed through the calculation of eigenvectors associated with the negative eigenvalue (stable arm) and with the positive eigenvalue (unstable arm). The two are, respectively:

$$S\text{: } z_2(t) = 2/3 - z_1(t) \quad (5)$$

$$U\text{: } z_2(t) = \frac{1}{3}.\frac{a_1 - a_2}{a_1 - a_3} + \frac{a_2 - a_3}{a_1 - a_3}.z_1(t) \quad (6)$$

Under our example, line S is negatively sloped and line U is positively sloped. Figure 1 displays the kind of dynamics that our assumptions over a_i parameters impose.

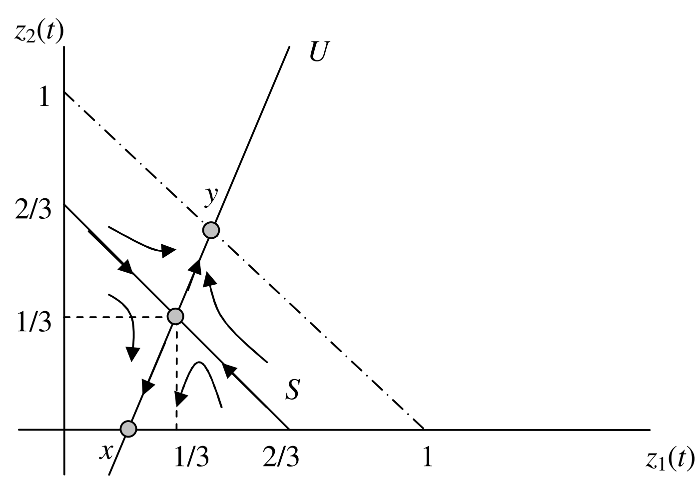

Figure 1 – Stable and unstable trajectories in the three-point location model

According to figure 1, if the distribution of activities is somewhere over S, it converges to the perfect distribution point, where to all locations will correspond 1/3 of the economic activity. If the initial point is any other, then the system tends to one of the two points, x and y, depending on initial conditions. Points x and y are activity concentration points where one of the locations is excluded from the economic activity. Point x concentrates the activity in locations 1 and 3 and point y in locations 1 and 2.[4]

The two specified points are:

$$x=(z_1, z_2, z_3)=\left(\frac{1}{3}.\frac{a_1-a_2}{a_3-a_2}, 0, \frac{3.a_3-a_2-a_1}{3.(a_3-a_2)}\right) \text{ and}$$

$$y=(z_1, z_2, z_3)=\left(\frac{3.a_3-2.a_1-a_2}{3.(2.a_3-a_1-a_2)}, \frac{3.a_3-a_1-2.a_2}{3.(2.a_3-a_1-a_2)}, 0\right).$$

We can concretize our specific case in a numerical example, taking for instance, a_1=-1, a_2=-3 and a_3=6. In this case, x=(2/27, 0, 25/27) and y=(23/48; 25/48; 0). Thus, under the particular case considered we conclude that:

- when there are agglomeration economies promoting economic activity to concentrate in location 3 and disagglomeration economies elsewhere, the steady state z_i=1/3, i=1,2,3, is unstable. Thus, two long run results are possible following the unstable path:

  - large economic activity concentration in location 3; residual economic activity in the other locations (point x);
  - zero economic activity in location 3; balanced distribution of activity in the other locations (point y).

The location decisions framework developed along this section has the importance of allowing to understand how the magnitude of forces pulling and pushing activities to / from a given location determines the correspondent spatial distribution. Nevertheless, it does not allow a perception of the driving forces behind what leads activities to concentrate or cluster. The next section intends to introduce an important factor at this level: technological capabilities.

III. Knowledge and Geography

Variable $z_i(t)$ as considered in the previous section is associated with economic activity in a vague sense. Now, we define a variable $z(t)$, recovering equation (1), that is

[4] Regard that activities may also concentrate in locations 2 and 3, what requires a less sloped U schedule. In some specific cases the concentration in only one location is also possible.

the share of technological knowledge allocated to production in location 1. With $A(t)$ the global level of technology, the level of technology available in location 1 corresponds to $A_z(t)=z(t).A(t)$. The second location will have the following availability of knowledge: $A_{1-z}(t)=[1-z(t)].A(t)$.[5]

Technology is produced. For both locations we assume that there are spillovers in knowledge accumulation: the same elasticity parameter describes the extent of marginal returns of a technology variable in both locations. For a $\delta>0$ technology obsolescence rate, the two technology production functions will come

$$\dot{A}_z(t) = g.A_z(t)^{\phi}.A_{1-z}(t)^{\eta} - \delta.A_z(t) \tag{7}$$

$$\dot{A}_{1-z}(t) = h.A_{1-z}(t)^{\eta}.A_z(t)^{\phi} - \delta.A_{1-z}(t) \tag{8}$$

with g, $h>0$ and ϕ, $\eta\in(0,1)$, i.e., equations (7) and (8) display diminishing returns in the accumulation of knowledge.

Having defined a relation between knowledge generation and the location of knowledge potential, it is possible to look to the issues of economic agglomeration. One assumes that a relation between technology levels and the agglomeration parameter a can be established. Our argument is that in both locations technology accumulation is positively related with the concentration of knowledge until a certain point where the accumulated knowledge represents a high degree of communication capabilities that make unnecessary the concentration of activities and thus centripetal forces no longer prevail.

For χ, $\omega>0$ arbitrary points that represent the turn over from the prevalence of the centripetal forces to the prevalence of centrifugal forces, the agglomeration parameter a is now written as a function of technology amounts:

$$a\left[A_z(t), A_{1-z}(t)\right] = A_z(t).\left[\chi - A_z(t)\right] + A_{1-z}(t).\left[\omega - A_{1-z}(t)\right] \tag{9}$$

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[5] This analysis implicitly considers that technology is not completely non rival (the same technology is not available in every location at any moment).

Equation (9) represents the economies of agglomeration as depending on technology levels. For low A_z, A_{1-z} (lower than χ and ω parameter values) one has $a>0$; for high A_z, A_{1-z}, then $a<0$ and the dispersion forces dominate.

Given equations (1), (7), (8) and (9) and the definitions of local technology availability we may reduce our problem to a two differential equations system, which is,

$$\begin{cases} \dot{z}(t) = \left\{\omega.A(t) + (\chi - \omega).z(t).A(t) - \left[z(t)^2 + (1 - z(t))^2\right].A(t)^2\right\}.[z(t) - 1/2] \\ \dot{A}(t) = (g + h).z(t)^{\phi}.[1 - z(t)]^{\eta}.A(t)^{\phi+\eta} - \delta.A(t) \end{cases} \tag{10}$$

The pair of equations in (10) relates the global level of technology and the location share variable. The dynamics of the model will allow to understand how the way in which the technology is allocated across the two geographical points is associated with the overall knowledge capabilities.

The steady state continues to be defined as the point where there is a perfect distribution of economic activity. In this case, $\bar{z} = 1/2$ implies that any of the two locations will have access to the same amount of knowledge in the long term. The steady state level of technology expression is a relation between the various parameters in (7) and (8): $\bar{A} = \left[\frac{g+h}{2^{\phi+\eta}.\delta}\right]^{1/(1-\phi-\eta)}$.

IV. DYNAMICS

The study of the dynamics of the (z,A) relation implies the linearization of system (10) around the steady state point. System (10) is equivalent to the following, in the steady state vicinity:

$$\begin{bmatrix} \dot{z}(t) \\ \dot{A}(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}.\bar{A}.(\chi + \omega - \bar{A}) & 0 \\ 2.\delta.\bar{A}.(\phi - \eta) & -[1 - (\phi + \eta)].\delta \end{bmatrix} . \begin{bmatrix} z(t) - 1/2 \\ A(t) - \bar{A} \end{bmatrix} \tag{11}$$

As in section II, the dynamic behaviour of the variables is related to the signs of the eigenvalues. For (11), these are $\lambda_1 = \frac{1}{2}.\bar{A}.(\chi + \omega - \bar{A})$ and $\lambda_2 = -[1 - (\phi + \eta)].\delta$.

Neither λ_1 or λ_2 have unambiguous signs under the imposed constraints for the values of parameters. Four cases are possible:

	$\overline{A} > \chi + \omega$	$\overline{A} < \chi + \omega$
$\phi + \eta < 1$	I	II
$\phi + \eta > 1$	III	IV

Case I is one of global stability (λ_1<0, λ_2<0). For any initial [$z(0)$, $A(0)$] the steady state point is always accomplished. Therefore, when the level of technology is high enough to overcome the necessity of knowledge concentration to generate more knowledge and when the returns on knowledge accumulation are relatively low then a perfect distribution of knowledge capabilities scenario is always found in the long run. The opposite case, IV, means instability independently from the initial point (λ_1>0, λ_2>0) and, thus, the concentration of the activity in one of the two locations. This case suggests low technological levels implying the predominance of agglomeration forces and high returns on technology accumulation.

The two other cases, characterized by eigenvalues with opposite signs, reflect saddle-path stability. In these cases the perfect dispersion of knowledge power across locations is possible but it is an exceptional situation. For a [$z(0)$, $A(0)$] point outside the stable arm the rule is the divergence process that leads the technology to concentrate in only one of the locations. Consider, in particular, case III, that may better characterize today's information society. In this case, the level of knowledge is high enough to allow to think that knowledge accumulation is well explained by a world where agglomeration economies do not need to prevail and the marginal returns on knowledge generation are relatively high. For this case we present the correspondent phase diagram.

The matrix in expression (11) suggests a $\dot{z}(t) = 0$ schedule that is vertical, in the relation between z and A, while the slope of $\dot{A}(t) = 0$ is conditioned by the relation between elasticities ϕ and η. Supposing $\phi > \eta$, the referred line is positively sloped. From the signs of the elements in the first column of the matrix in (11) it is possible to draw arrows pointing the dynamic behaviour of the considered endogenous variables. The result is a positively sloped unstable arm (U) which indicates that the system tends to one of two points: a (z=0, $A < \overline{A}$) point or a (z=1, $A > \overline{A}$) point.[6]

If the first of these points is reached, it implies that all the accumulated knowledge will concentrate on location 2 and the accumulated knowledge will be a lower quantity relatively to the amount of knowledge that would correspond to a perfect distribution among locations (steady state). The second point, that represents a concentration of knowledge in location 1 is characterized by the condition $A_z > \overline{A}$. The fact that it is preferable for economic activity to concentrate in location 1 from a global result in terms of knowledge accumulation point of view follows directly from the condition $\phi > \eta$. If the relation between elasticities were the opposite, then U would be negatively sloped and the result would be symmetric to the one exhibited in figure 2.

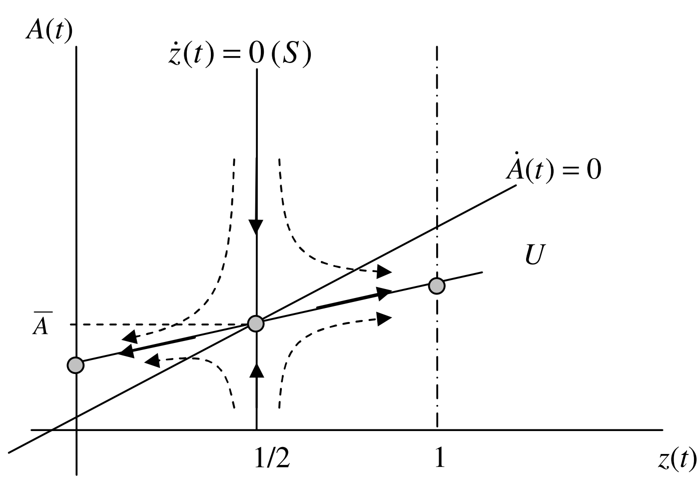

Figure 2 – Phase diagram for the location-technology relation

Figure 2 reflects an important puzzle of modern societies: in a world where high returns on technology development exist and where the technological capabilities allow for important centrifugal forces to gain weight relatively to conventional centripetal arguments (as transportation costs), still we observe that economic activity tends to cluster because of initial conditions. An economy that has initially a perfect dispersion of activities tends to maintain such dispersion, but if for historical reasons there is an

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6 The stable arm (S), corresponds to the line $\dot{z}(t) = 0$, as it is clear from the picture in figure 2.

unbalance in the distribution of knowledge across regions this tends to be perpetuated and accentuated.

The previous seems indeed an argument in favour of inertia as an important explanation for technological huge differences in our world. If we were able to set an even distribution of technology potential this would have a tendency to persist over time; nevertheless, differences tend to be reinforced over time under the assumptions that gave place to the graphic in figure 2.

V. CONCLUSIONS

This paper emphasizes the need to conjugate space and time analysis. Location decisions are dynamic and the concentration / dispersion of economic activity must be understood in such an evolving scenario. This perspective was highlighted in the first part of the document where a three location economy was presented. Depending on the intensity of agglomeration / disagglomeration factors, multiple results are possible. The steady state defines the situation of perfect dispersion of activities that may occur when centrifugal forces dominate. Nevertheless, the rule seems to be concentration because even in saddle-path conditions the divergence process pulling away from the steady state implies points where at least one of the locations will not benefit from the presence of the economic activity.

The driving forces of agglomeration economies are certainly related with the state of technology / knowledge. To illustrate this relation, it is assumed that the two locations share the existent technology. This is not a static distribution, but a distribution that evolves in time since rules about technology accumulation are given and because we make the parameter relating agglomeration economies to depend on technology levels. Two ideas become crucial under our arguments: although technology is rival in its use in each location, there are knowledge spillovers meaning that the accumulation of knowledge in each place is dependent on the other place level of knowledge; second, high levels of knowledge tend to promote the weightless economy in the sense that location becomes unessential or, in other words, the construction of a knowledge economy implies the triumph of centrifugal forces. Therefore, we build a strong biunivocal relation between location and knowledge accumulation. The most interesting result is that a world economy with high technology levels and strong marginal returns in the accumulation of knowledge is not necessarily an economy where economic activity tends to spread across space. A dispersion of activities is possible, but initial conditions may imply increasing asymmetries.

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