σ-Convergence in the Presence of Spatial Effects

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Abstract

This paper explores the implications that spatial effects can hold for the application of measures of σ-convergence. The bias of a common indicator of σ-convergence is examined for a family of spatial process models including: [a] spatial lag, [b] spatial error, and [c] spatial moving average. We show that the measure of σ-convergence is sensitive to a number of distinct influences including global dispersion, spatial dependence, and a variety of forms of spatial heterogeneity. We suggest a decomposition of the convergence indicator into two components: one reflecting global dispersion, and one reflecting the influence of spatial effects. We then illustrate this approach with a case study of the U.S. states over the 1929-2000 period.

Key Words: convergence, spatial dependence, spatial heterogeneity.

1 Introduction

Over the past 15 years the number of empirical studies of regional convergence has exploded.[1] Different notions of regional convergence abound in the literature (Quah, 1993; Sala-i-Martin, 1996) and much controversy exists surrounding the connections between alternative definitions of convergence and the empirical strategies used to examine regional growth patterns (Jones, 1997), as well as the evaluation of competing growth theories in light of existing evidence (Bernard and Durlauf, 1996; Galor, 1996; Esteban, 2000). Additionally, the empirical evidence is not robust to the underlying research design as the conclusions from cross-sectional studies (Barro and Sala-i-Martin, 1992; Sala-i-Martin, 1997; Tsionas, 2000) differ from those

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[1] Recent surveys of this literature can be found in De la Fuente (1997); Durlauf and Quah (1999); Temple (1999); Florax et al. (2002); Rey and Janikas (2003); Magrini (2004).

generated by time-series (Bernard and Jones, 1996a,b) and panel data approaches (Islam, 1995; Evans and Karras, 1996; Lee et al., 1998).

The empirical literature on growth can be split into two sets of studies. The first is confirmatory in orientation where formal growth theories are used to specify an econometric equation that is estimated using observations on a cross-section of national or regional economies. The second group of studies is more exploratory in nature and has generated a number of novel techniques for uncovering empirical patterns in the same growth data sets used in the confirmatory studies. In large part the findings of these two sets of studies tend to be conflicting (Bernard and Durlauf, 1996), although some have stressed their complementarities (Sala-i-Martin, 1996).

While the role of spatial effects in confirmatory analysis of the convergence question has begun to attract attention (Rey and Montouri, 1999; Fingleton, 2000), the implications of spatial dependence and heterogeneity for the new exploratory methods of convergence have been largely ignored.[2] An implicit assumption underlying most of this work is that each economy represents an independent observation providing unique information that can be used to estimate the speed of income or productivity convergence within a geographic system. In essence the cross-sectional observations at one point in time are viewed as a random sample from a univariate distribution. This stands in stark contrast to the empirical reality of strongly spatially autocorrelated income series (Rey and Montouri, 1999; Le Gallo and Etur, 2003).

In addition, exploratory analysis in the case of the U.S. (Rey, 2001) reveals that the level of spatial autocorrelation has varied substantially over time. The causes of the dynamic variation in spatial clustering remain unknown. Furthermore, regional dummy variables and other proxies for steady-state values (used in, for example Attfield et al. (2000); Barro and Sala-i Martin (1995)) may not sufficiently control for spatial effects that may be caused by substantive processes which are not temporally stationary. Consequently, the mixture of temporal and spatial characteristics of the data underlying convergence analysis introduces new complications that require attention.

This paper takes up the issue of the role of dynamic spatial effects in the exploratory analysis of regional income convergence. In section 2 we focus on the conventional measure of σ-convergence and highlight several problems with its use on geographically referenced data. Section 3 presents a family of spatial process models for such data, and in section 4 we explore the

[2] For a recent exception related to the research presented here see Gustafsson (2003).

behavior of the σ-convergence indicator across these alternative processes to examine the effects of spatial heterogeneity and dependence. In section 5 we suggest a decomposition of the sample variance in an attempt to distinguish between a-spatial dispersion and dispersion that is an outcome of spatial heterogeneity and/or dependence. We then illustrate this decomposition in a case study of US regional incomes over the 1929-2000 period. The paper closes with some summary comments and directions for future research.

2 σ-Convergence

Regional income distributions have been analyzed using a variety of methodologies. The earliest work examined the concept of σ-convergence in which the focus was on how the level of cross-sectional dispersion, measured using the sample variance, changed over time (Easterlin, 1960; Williamson, 1965). Downward trends in the indicator are viewed as evidence that a convergence process exists. This form of convergence has continued to attract much attention in recent years (Coughlin and Mandelbaum, 1988; Fan and Casetti, 1994; Carlino and Mills, 1996; Bernard and Jones, 1996c; Azzoni, 2001).

To fix ideas, we focus on the use of the sample variance as the convergence indicator for a set of n regional economies:

$$s_t^2 = 1/(n-1)\sum_{i=1}^{n}(y_{i,t} - \bar{y_t})^2 \tag{1}$$

where $y_{i,t}$ is per capita income of region i in time period t. σ-convergence is reflected in (1) declining over time, while increasing values indicate divergence in the cross-sectional distribution.[3]

Despite their popularity, measures of σ-convergence suffer from a number of limitations. From a geographical perspective their focus on dispersion will mask nontrivial geographical patterns that may themselves also be fluctuating over time (Rey and Montouri, 1999). Additionally, changes in this measure over time present an aggregate view of the transitional dynamics in the regional distribution. No information is provided on the movement of individual economies within the distribution. Finally, other characteristics of the distribution (skewness, modality etc.) are ignored.

Although employed as an exploratory measure, it is important to note that the interpretation of changes in the sample variance as an indicator of

[3] In what follows we omit the time period subscript as it is assumed the reference point is one time period, unless otherwise noted.

convergence rests on implicit assumptions about the underlying data generating process (DGP). More specifically, the observations on regional incomes are treated as if they represent a random sample, that is a collection of observations from independent and identically distributed (i.i.d.) random variables:

$$y_i \tilde{} N(\mu_i, \sigma_i^2) \tag{2}$$

with $\mu_i = \mu_j = \mu$, $\sigma_i^2 = \sigma_j^2 = \sigma^2$ and $\sigma_{i,j} = 0 \; \forall \; i, j$. If the underlying DGP is in fact the i.i.d. model, then the use of the sample variance as an estimator of the dispersion parameter (σ^2) of the DGP is appropriate. However, regional income data often display a high degree of spatial autocorrelation as well as various forms of spatial heterogeneity. The i.i.d. model fails to satisfy the "data admissibility" criteria (Florax et al., 2003) for such series since the spatial effects can not be formally represented in the DGP.

In these cases the direct linkage between the sample variance and the dispersion parameter of the DGP is broken. Consqeuently, a more general DGP needs to be adopted in order to account for the spatial characteristics. One such approach is a multivariate normal DGP:

$$y \tilde{} N(\mu, \Sigma) \tag{3}$$

where y is an $n \times 1$ vector of random variables with the expected value vector μ and a variance covariance matrix Σ. This allows for a richer set of spatially explicit models in regional convergence studies since both spatial dependence and spatial heterogeneity can be incorporated. With regard to spatial dependence, the strength of the spatial interactions can be reflected in some, or all, of the off-diagonal elements of Σ taking on non-zero values. Various forms of spatial heterogeneity can be represented as well. For example, mean heterogeneity is now possible:

$$\mu_1 \neq \mu_2 \neq \mu_3 \ldots \neq \mu_n. \tag{4}$$

Variance heterogeneity (heteroskedasticity) can also be reflected as:

$$\sigma_{1,1}^2 \neq \sigma_{2,2}^2 \neq \sigma_{3,3}^2 \ldots \neq \sigma_{n,n}^2. \tag{5}$$

As we explore more fully below, there are multiple specifications of each type of spatial effect (dependence or heterogeneity) that are possible. Moreover, a given data series might reflect both spatial heterogeneity and dependence. In any of these cases the use of the sample variance to provide insight on the dispersion property of the underlying DGP becomes problematic. This can be seen by decomposing the variance covariance matrix of the income series as:

$$\Sigma = \sigma^2 \Omega \tag{6}$$

where σ^2 is the scalar *global dispersion parameter* and Ω is a general $n \times n$ matrix. In the context of exploratory analysis of σ-convergence the global dispersion parameter is of key interest, and the implicit strategy adopted in most empirical studies is the use of the sample variance as an estimator for this parameter. This estimator has an expected value of:[4]

$$E\left[s^2\right] = \frac{1}{n-1}\left(\sum_{i=1}^{n}\left(\mu_i^2 + \sigma_{i,i}\right) - \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\mu_i\mu_j + \sigma_{i,j}\right)\right) \tag{7}$$

with $\sigma_{i,j} = \sigma^2\omega_{i,j}$. The sample variance will only be unbiased for the global dispersion parameter if both mean and variance homogeneity hold *and* all covariances are zero. When any of these conditions are violated the sample variance will continue to provide insights on the sample dispersion, however, it will provide only indirect, rather than direct, insights on the global dispersion parameter σ^2. This is because the sample variance will be affected by not only the global dispersion parameter, but also the level and form of any spatial heterogeneity and/or dependence.

As we mentioned above, there are multiple forms for each of these spatial effects, which if present, would violate the conditions for the sample variance to be unbiased for the global dispersion parameter. It is important to understand how each of these types of spatial effects influences the bias of this convergence indicator. This is for two reasons. First, the magnitude of the problem caused by spatial effects for analysis needs to be recognized so that the inferences regarding σ-convergence can be evaluated appropriately. The second, and perhaps more important, reason is that understanding the precise mechanisms by which spatial heterogeneity and spatial dependence, as well as global dispersion, influence sample variation may open up new avenues to a richer theoretical literature on regional growth empirics. We now turn to an investigation of the sampling behavior of the σ-convergence indicator for alternative spatial process models.

3 Spatially Dependent Processes

A number of specifications to represent spatial dependence have been suggested in the literature. Here we focus on the more commonly encountered

[4]For more details, see appendix.

forms.[5]

3.1 Spatial Lag Model

$$y = \rho W y + X\beta + \epsilon \tag{8}$$

where ρ is a scalar spatial autoregressive parameter, W is an $n \times n$ spatial weights matrix, X is a matrix of n observations on a set of k exogenous variables, with an associated $k \times 1$ parameter vector β and ϵ is a well behaved error term:

$$\epsilon\tilde{~}N(0, \sigma^2 I) \tag{9}$$

Under a spatial lag DGP, the dependent variable has the following multivariate normal distribution:

$$y\tilde{~}N((I - \rho W)^{-1} X\beta, \sigma^2 (I - \rho W)^{-1} (I - \rho W')^{-1}) \tag{10}$$

The form of spatial correlation is extensive under the LAG model as all pairs of locations exhibit non-zero correlation. This type of spatial covariance has been labeled as *global covariance* by Anselin (2003).

3.2 Spatial Error Model

A second specification is the so-called spatial error model:

$$y = X\beta + \nu, \nu = \rho W \nu + \epsilon \tag{11}$$

This specification results in the following joint distribution:

$$y\tilde{~}N(X\beta, \sigma^2 (I - \rho W)^{-1} (I - \rho W')^{-1}) \tag{12}$$

The covariance matrix for y is the same for the lag and error dependence models, however, the expected value term differs.

3.3 Spatial Moving Average

The third spatial DGP we examine is the moving average (MA) model:

$$y = X\beta + \rho W \epsilon + \epsilon \tag{13}$$

Under the MA specification the distribution for y is:

$$y\tilde{~}N(X\beta, \sigma^2 (I + \rho W) (I + \rho W)') \tag{14}$$

The SMA shares the same expected value as the ERROR specification, however it generates a much more localized form of spatial dependence than the ERROR (or LAG) specifications.

[5] For more detailed discussions of these spatial processes see Anselin (2003).

4 s^2 in the presence of spatial effects

In this section we explore the impact of various spatial effects on the bias of the sample variance as an estimator for the global dispersion parameter σ^2. This is accomplished by substituting the relevant moments of the DGP under consideration into (7). We examine four general cases:

1. Spatial Dependence
2. Variance Heterogeneity
3. Mean Heterogeneity
4. Spatial Dependence + Mean Heterogeneity

The results for each case are based on the following design: our spatial configuration is a simple regular lattice with 5 rows and columns for a sample size of $n = 25$. Our weight matrix is based on a simple rook contiguity criterion. The value of the global dispersion parameter (σ^2) is set equal to 0.12987 which corresponds to the variance of US state incomes when normalized to the mean for 1929.

4.1 Bias under spatial dependence

Case 1 allows for a comparison of the different forms of spatial dependence and their effects on the bias of s^2. Since mean homogeneity holds under this model, the spatial error and spatial lag specifications are identical, so only the lag and moving average models are compared. The level of dependence across each of these specifications is reflected in varying the value of ρ from -0.9 to 0.9 in increments of 0.1.

Figure 1 displays the results of this first set of comparisons. In general terms, the bias tends to be low for small absolute values of the spatial parameter for each of the two DGPs. However, as the absolute value of this parameter increases the patterns are more distinct between the DGPs. For the lag model, the bias of the sample variance is a smooth increasing function of $|\rho|$, although the bias is higher for negative values of the parameter. The pattern for the MA model suggests a much weaker bias effect due to spatial dependence. This is in line with expectations as the MA model generates a much more localized form of dependence than the LAG model. The asymmetry between the bias effect for negative versus positive values of ρ is also evident for the MA model although to a lesser extent than is the case for the LAG model.

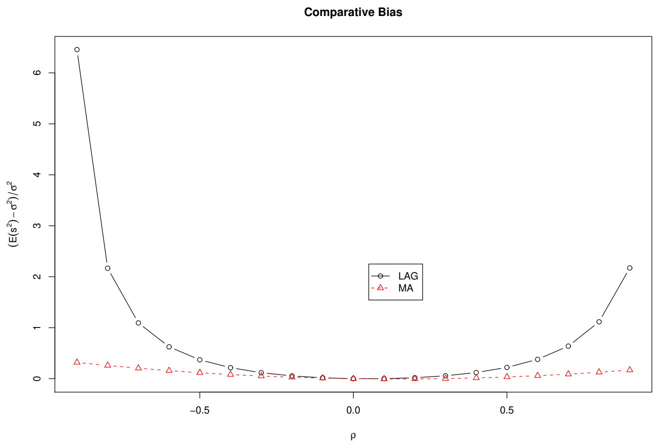

Figure 1: Bias of s^2 for alternative spatial process models.

To understand the asymmetric bias pattern consider the expected value of the sample variance under mean homogeneity:

$$E\left[s^2\right] = \frac{1}{n-1}\left(\sum_{i=1}^{n}\left(\sigma^2\omega_{i,i}\right) - \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\sigma^2\omega_{i,j}\right)\right) \tag{15}$$

Spatial dependence, of any form, will impact not just the right hand side of (15) through the covariances, but also induces a form of heteroskedasticity reflected in the diagonal elements no longer being constant: $\omega_{i,i} \neq \omega_{j,j}$. Rewriting (15) we have:

$$E[s^2] = \frac{\sigma^2}{n-1}\left(\sum_{i=1}^{n}\omega_{i,i} - \frac{1}{n}\sum_{i=1}^{n}\omega_{i,i} - \frac{2}{n}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\omega_{i,j}\right). \tag{16}$$

Each of the terms inside the parentheses will affect the bias in different ways. Focusing on the first two terms, and assuming non-zero covariances for the moment, the difference between these terms needs to equal $n-1$ for the sample variance to be unbiased. For this to occur, trace of Ω has to equal n. Under homoskedasticity this will be the case, however in most cases where heteroskedasticity is present it is unlikely that this condition will hold.[6]

[6]The exception would be a form of heteroskedasticity where the value of the global variance parameter was equal to the average of the individual regional variances.

Turning to the third term, non-zero covariances will cause a downward bias in the sample variance when the covariances are positive, but an upward bias when the covariances are negative. It should be noted, however, that when the spatial parameter is negative, some, but not all, of the covariances will be negative. When the spatial parameter is positive all the covariances will be positive.

4.2 Bias under variance heterogeneity

Unlike what holds in time series processes, where a model that has dependence but no heteroskedasticity is possible, for the spatial processes models we presented above, heteroskedasticity will always be present whenever there is spatial dependence. This is because the heterogeneity is induced by the dependence. It then becomes important to understand the bias effects due to true heteroskedasticity and how they may differ from those due to apparent heteroskedasticity induced by the spatial dependence.

Figure 2 displays the effect of heteroskedasticity alone on the bias of the sample variance. These values were obtained by setting the off-diagonal elements of the covariance matrices used in the previous cases to 0. A comparison of this figure with that for the bias due to dependence reveals a very similar pattern. Upon closer examination, some important differences can be discerned.

The first difference is that the bias pattern is much more symmetric for the two DGPs, in contrast to what is the case under spatial dependence. The second difference can be seen in Figure 3 which isolates the bias effects due to variance heterogeneity and dependence for the LAG model. For positive values of ρ the bias effect due to dependence is much lower than that due to heteroskedasticity alone. This indicates that the positive spatial covariances are working to pull the expected value of the sample variance downwards, as was suggested previously. In contrast, for negative values of ρ, the bias effect due to dependence is slightly larger than that due to heteroskedasticity alone. In this case the negative spatial covariances are augmenting the upward bias in the sample variance associated with the heteroskedasticity.

4.3 Bias under mean heterogeneity

We explore this in a fairly simplified manner in which our sample locations are partitioned into two regimes: regime A consisting of observations 1-12, and regime B composed of observations 13-25. We then vary the ratio of

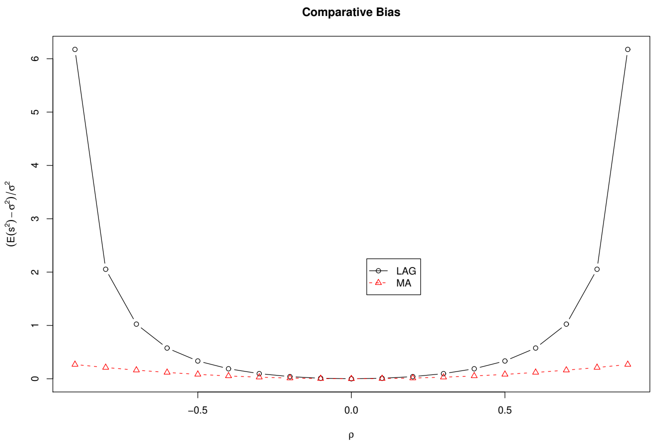

Figure 2: Bias of s^2 due to variance heterogeneity.

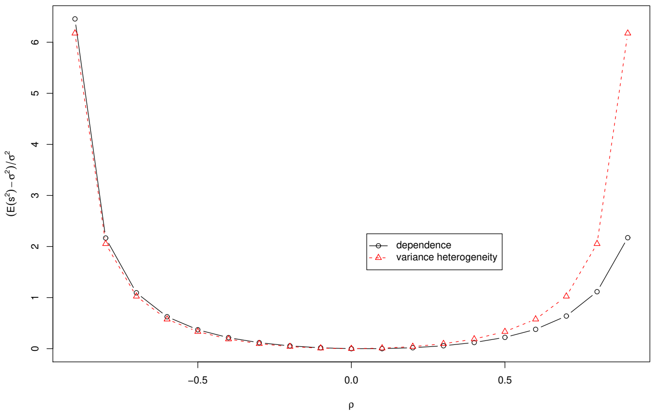

Figure 3: Bias of s^2 due to variance heterogeneity versus dependence (LAG model).

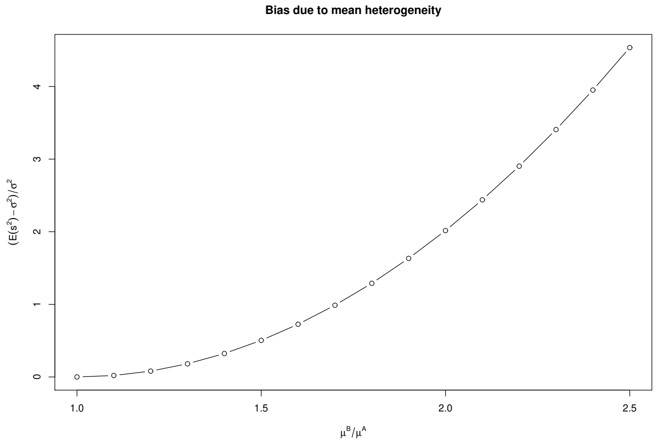

Figure 4: Bias of s^2 due to mean heterogeneity (LAG model).

their regime means: μ^B/μ^A, and explore the impacts on the bias of the sample variance.[7] To isolate the effects of mean heterogeneity we set the spatial dependence parameter to 0 for all DGPs. Under such a parameter setting, all the DGP will have the same expected value for the sample variance, so we limit our attention to the LAG model.

Figure 4 demonstrates the impacts that mean heterogeneity has on the bias of the sample variance. The bias is a smooth monotonic function of the level of mean heterogeneity. Mean heterogeneity impacts the bias through two channels. The first is similar to the way in which variance heterogeneity works, as can be seen in the first term of equation (7). A comparison of Figure 4 with Figure 2 suggests that while increases in the heterogeneity of either the means or variances increases the bias, the nature of the bias increase is qualitatively different between the two sources of heterogeneity. The difference is due to the cross-product term of the means appearing in the second term of (7).

4.4 Bias under dependence and mean heterogeneity

When mean heterogeneity and spatial dependence are both present the bias effects are quantitatively distinct across the three DGPs. This is displayed in Figure 5 for the same levels of mean heterogeneity from the previous section

[7]The upper bound for the ratio reflects the range in the differences between US state means based on BEA regionalization schemes over the 1930's.

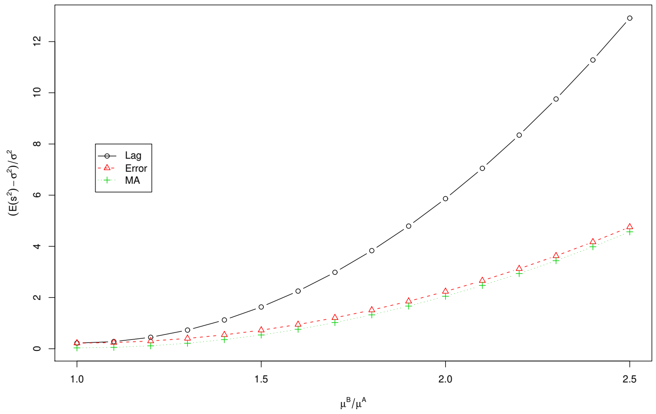

Figure 5: Bias of s^2 due to mean heterogeneity and dependence ($\rho = 0.5$).

and a moderate value of the spatial dependence parameter ($\rho = 0.5$). The pattern of increasing bias with larger mean heterogeneity is replicated in the presence of spatial dependence. However, there is a clear difference between the behavior of the sample variance across the ERROR and LAG models. This reflects the spatial inverse operator appearing in the expected value of the LAG model, but not for the ERROR model. This operator works to amplify the mean heterogeneity effect in the LAG specification.

The behavior of the sample variance for the MA and ERROR DGPs is very similar with the bias being higher under the latter specification. This small difference reflects the subtle influence of spatial dependence in the two DGPs, while the overall trend of the bias is clearly dominated by mean heterogeneity, which influences the bias in a similar fashion for both of these specifications.

5 Decomposition of σ-Convergence

The results of the preceding section demonstrate that spatial effects can lead to substantial bias in the sample variance as an indicator of global dispersion. Substantial can mean overestimation of σ^2 by a factor of twelve in the most extreme cases. The bias of s^2 in the presence of spatial effects would seem to suggest that its use in exploratory analysis of regional income convergence is limited. However, because s^2 will reflect the effects of changes in the global variance parameter σ^2 and the level (and type) of spatial dependence or spatial heterogeneity, it may be fruitful to explore whether s^2 can be decomposed to reflect the contributions of these different sources of changes in empirical dispersion of incomes.

Our point of departure is a general multiplicative decomposition of the sample variance of the regional income series as:

$$s^2 = \sigma^2\theta \tag{17}$$

with:

$$\theta = \frac{1}{n-1}\Bigg(\sum_{i=1}^{n}\left(\mu_i^2 + \omega_{i,i}\right) - \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\mu_i\mu_j + \omega_{i,j}\right)\Bigg). \tag{18}$$

The parameter θ reflects the combined effects of any spatial heterogeneity (mean or variance) as well as dependence, while the influence of a-spatial dispersion is captured in the global parameter σ^2.

There are a number of approaches that could be used to operationalize this decomposition. The first are the so-called spatial filtering schemes (Getis, 1995; Tiefelsdorf and Griffith, 2002) that attempt to separate a spatial series into trend, signal and noise components. The latter component should represent random white noise which could be used to estimate σ^2. With this estimate in hand, together with the sample variance, an estimate of θ could then be derived:

$$\hat{\theta} = \frac{s^2}{\hat{\sigma^2}} \tag{19}$$

An alternative approach to spatial filtering would be for a modeler to posit an underlying DGP so as to fully specify the structure of θ and then apply an appropriate econometric method to directly estimate all of the parameters in the DGP. Here there are a wealth of specifications that are possible, only a few of which we have outlined above. Additionally, the spatial econometrics literature offers a rich set of alternative estimation methods (Anselin et al., 2004).

The choice of which approach to base the empirical decomposition upon should be guided by a sound understanding of the sampling properties of each candidate under different DGPs. Unfortunately, a comprehensive analysis of these issues does not currently exist and is clearly beyond the scope of this paper. It should be noted, however, that either the filtering or direct representation approaches are likely to offer clear improvements over current practice in the σ-convergence literature where the implicit assumption underlying most empirical analyses is:

$$s^2 = \sigma^2 \tag{20}$$

which implies $\theta = 1$. In other words, spatial effects are ruled out.

Although a full evaluation of the merits of alternative approaches to decomposition awaits future research, here we illustrate a simple case study involving regional income data series from the lower 48 US states over the period 1929-2000. We compare three alternative approaches towards estimating the global dispersion parameter:

1. The conventional use of the sample variance,
2. Ordinary least squares estimation of the spatial lag model (8), and
3. Maximum likelihood estimation of the spatial lag model.

For the second two approaches the estimation of the global variance parameter is based on the estimate of the error variance parameter. For both specifications we use a simple row-standardized contiguity matrix to implement the decomposition. The income series are first mean normalized:

$$y_{i,t} = \frac{I_{i,t}}{1/n \sum_{i=1}^{n} I_{i,t}} \tag{21}$$

where $I_{i,t}$ is the original per capita income of state i in year t. This normalization rests on the assumption of mean homogeneity, so our analysis in turn focuses on the role of spatial dependence alone in the decomposition. Analysis of departures from the mean homogeneity assumption will be pursued in future research.

Figure 6 displays the estimates of the dispersion parameter for each year of the sample for the three approaches. Several patterns emerge. First, for each year the conventional estimate is above those of the other two approaches. The relative magnitude of this difference is not constant over the period, being largest for the first half of the sample and smaller in the second half.

The second finding is the estimates of the global variance parameter from OLS lag and ML lag are very similar in value, with the OLS estimate being slightly lower in most years. The similarity of the OLS and ML dispersion estimates is rather striking, given that the OLS is known to be inconsistent for the other parameters in the spatial lag model.[8]

[8] Examination of the estimates reveals that the inconsistency for β and ρ work in a counterbalancing manner. OLS generates higher estimates of spatial parameter than compared to ML lag. In some cases the OLS values are outside the feasible region. Larger values for the spatial parameter estimates are compensated for by lower values of the intercept estimate. These two points together explain why OLS and ML are so close for estimates of global variance parameter. A more extensive evaluation of these patterns is under investigation.

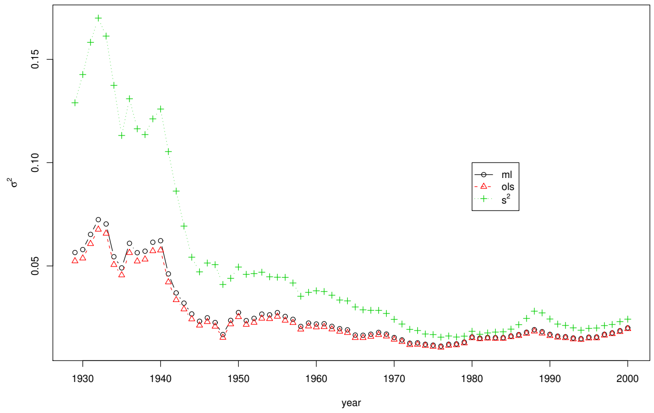

Figure 6: Estimates of U.S. σ-convergence

Although the estimates of the global dispersion parameter are similar, Figure 7 indicates that the estimates of the spatial component θ do differ across the OLS and ML approaches. The ML estimates are always lower than the OLS ones. The two approaches are in agreement in representing a general trend of an overall decline in the strength of the spatial component. At the same time, however, the spatial component is clearly the larger of the two multiplicative components for the sample variance over the entire period, suggesting that movements in the traditional measure of σ-convergence are masking important spatial dynamics that need to be better understood.

6 Conclusion

Cressie (1991, p. 1410) argues that purely spatial models should be used for data that have achieved temporal stationarity. Our results suggest that the level of spatial dependence varies rather substantially over time in the US space economy. This raises an interesting set of problems. On the one hand, the assumption of temporal stationarity clearly does not hold, bringing into question the utility of spatial econometrics for convergence analysis. On the other hand, there is a significant level of spatial dependence for each year of the sample - suggesting that taking a univariate approach to study income convergence risks ignoring important spatial covariance which appears to be at work in the data. We believe a fruitful avenue of research is adopting a

Figure 7: Estimates of the spatial component of σ-convergence.

perspective where the outputs from the spatial econometric analysis become inputs into a higher order study in which the dynamics of both the income distribution and the level of spatial clustering are treated jointly.

Observance of trends in the sample cross-sectional variance of regional per capita income distribution is an often used indicator of income convergence. This paper has shown that this σ-convergence indicator is sensitive to changes in spatial dependence and spatial heterogeneity, as well as a-spatial dispersion. Adopting a multivariate, rather than a univariate, perspective this paper provides a general theoretical framework for decomposing how spatial effects influence the σ-convergence indicator. Using this general framework, and assuming a DGP formulated as a spatial LAG model and a spatial MA model, we show with numerical results that bias levels change substantially as spatial effects change. Specifically, the sample variance as an estimate of global dispersion can be biased when any of the following three assumptions of the common, a-spatial, univariate approach fail to hold: 1) mean homogeneity, 2) variance homogeneity, and 3) zero covariance.

In empirical analysis, the univariate approach to study income convergence risks ignoring any spatial covariance or heterogeneity which may be present. Using alternative estimators for a spatial lag specification, we decompose the convergence indicator into two components: global dispersion and spatial effects. We illustrate this approach and the potential significance of spatial effects with a case study of the U.S. states over 1929-2000. The results reveal how traditional exploratory analysis of this data, without decomposing spatial effects, may miss how large portions of the observed trend towards (away from) σ-convergence may actually be attributed to changes beyond global dispersion. Specifically, variations in the cross-sectional sample variance of per capita personal income may be related to variations in spatial autocorrelation.

To conclude, it is important to understand how spatial effects influence the bias of the sample variance as a common indicator of σ-convergence. Our results demonstrate how evolving spatial effects can cloud the analysis of trends in the σ-convergence indicator. Care must be taken, however, not to translate these specific results based on US states into inferences about different growth theories. For instance, spatial dependence may itself be endogenous within the convergence process and thus might be incorporated into growth models. Therefore a second reason for understanding spatial effects is to encourage an explicit conceptualization of space within convergence measures which may lead to a richer theoretical literature on regional growth empirics.

Appendix

Let y_i, $i = 1, 2, ...n$ be a collection of random variables, with expected values $E[y_i] = \mu_i$, variances $V[y_i] = \sigma^2_{i,i}$ and covariances $C[y_i, y_j] = \sigma_{i,j}$. Define the sample variance as:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 \tag{22}$$

with $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$. The expected value of s^2 can be written as:

$$\begin{aligned}
E[s^2] &= E\left[\frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2\right)\right] && (23)\\
&= \frac{1}{n-1}E\left[\left(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2\right)\right] && (24)\\
&= \frac{1}{n-1}\left(E\left[\sum_{i=1}^{n} y_i^2\right] - E\left[n\bar{y}^2\right]\right) && (25)\\
&= \frac{1}{n-1}\left(\sum_{i=1}^{n} E\left[y_i^2\right] - nE\left[\bar{y}^2\right]\right) && (26)\\
&= \frac{1}{n-1}\left(\sum_{i=1}^{n}\left(\mu_i^2 + \sigma_{i,i}\right) - nE\left[\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} y_i y_j\right]\right) && (27)
\end{aligned}$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n}\left(\mu_i^2+\sigma_{i,i}\right)-\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}E\Big[y_i y_j\Big]\right) \quad (28)$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n}\left(\mu_i^2+\sigma_{i,i}\right)-\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\mu_i\mu_j+\sigma_{i,j}\right)\right) \quad (29)$$

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