Considerations on Gravitational Wave in Economics

Ovidiu Racorean*

Abstract

In Regional Analysis and Projection Methods that occurs in Macroeconomics at late 19th century, an important section is occupied by Gravitational Type Models. In these models, the interactions between economic regions of an economy have an analogue relation with the relation of interaction between masses in Newtonian gravitational theory; this analogy gives the name of this models.
A proposal for a dynamical potential of population displacements (named gravitational potential) between economic regions will be given. For a particular ideal chosen case – an important event taken place in an region of reference attracting great amounts of population from all other regions-the gravitational potential is acting as a wave; so an equation of the wave form will be given for gravitational potential – **gravitational wave in economics**.
It remains for the practical experiments to confirm or to invalidate this proposal for dynamical systems.

1.Introduction

a. Concepts in Gravitational Type Models

The demonstration of gravitational types models begin considering the economic regions of an economy as being masses structured according certain methods. Under this condition, the interactions between regions behave similarly to the masses interactions in Newtonian universal attraction theory:

$$F = G\frac{m_1 m_2}{d^2}.$$

The determination of an analogue relation in macroeconomics needs the following assumptions:

-the economy has a population P that is structured in regions:

$$P = (P_1 P_2 P_3 ... P_n);$$

-the total number of population displacements from one region to another is known and is noted I;

* e-mail : decontatorul@hotmail.com

-the average number of one person displacements from one region to another is k_p :

$$k_p = \frac{I}{P} \ ;$$

-differences regarding the population structure evidenced by sex, age, profession and incomes are not especially considered; the population is homogenous.

For two referential regions i and j from the economy in consideration, the population displacement from region i to j is direct proportional with the population ponder in total population, and is called displacement probability-Dp.

$$Dp = \frac{P_j}{P} \ .$$

In these conditions the total displacements number of one person from the economical region i to j is:

$$t_{ij} = \frac{k_p P_j}{P} \ .$$

However, there are P_i persons in region i, and the total displacements number of population from the economic region i to j is:

$$I_{ij} = k_p \frac{P_i P_j}{P} .$$

The economic region j is placed at certain distance d_{ij} from region i and after some calculations and adjustments with statistical data collected in the field, the interaction between the regions i and j is calculated as follows:

$$I_{ij} = k_p C \frac{P_i P_j}{P d_{ij}{}^b} \ .$$

With k_p, C, P, constants, and G defined as:

$$G = k_p \frac{C}{P} \ ,$$

it will be:

$$I_{ij} = G \frac{P_i P_j}{d_{ij}{}^b} .$$

Meaning that interactions between two economic regions i and j are directly proportional with the population in this regions and inverse proportional with the distance between the studied regions. This relation is similar with Newton equation for gravity.

The total number of population in region i displaced in all other regions is:

$$I_{i1} + I_{i2} + ... + I_{in} = G\frac{P_i P_1}{d_{i1}{}^b} + G\frac{P_i P_2}{d_{i2}{}^b} + ... + G\frac{P_i P_n}{d_{in}{}^b} ,$$

or

$$\sum I_{ij} = GP_i \sum \frac{P_j}{d_{ij}{}^b} .$$

Taking ${}_iV = \frac{\sum I_{ij}}{P_i}$, the relation become:

$${}_iV = G\frac{\sum P_j}{d_{ij}{}^b} , \tag{1.a.1}$$

which represents the displacement potential of one person from region i to all other economic regions or **Gravitational Potential of region i**.

b. Concepts of Oscillations and Waves in Theoretical Physics

A harmonicas oscillatory move is described in theoretical physics by a function of the form:

$$y = A\sin(\omega t) \text{ or } y = \cos(\omega t) , \tag{1.b.1}$$

where y is the elongation and represent the position of the oscillator at a time t , A is the amplitude (the maximum of elongation) , t represent time and ω is pulsation which can be expressed in term of period T as:

$$\omega = \frac{2\pi}{T} . \tag{1.b.2}$$

Functions (1.b.1) can be written:

$$y = A\sin(\frac{2\pi}{T}t) \text{ and } y = A\cos(\frac{2\pi}{T}t) , \tag{1.b.3}$$

and can be represented graphically like :

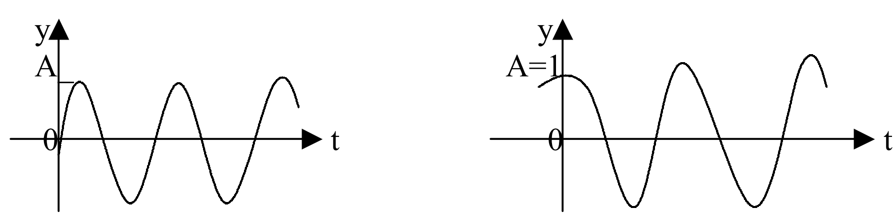

Fig.1.Sine and cosine functions.

These two functions are interchangeable following the relation:

$$\cos(\frac{2\pi}{T}t) = \sin(\frac{\pi}{2} + \frac{2\pi}{T}t) \ . \tag{1.b.4}$$

In a material medium, all its parts are in interactions. If one point oscillates than this oscillatory move is propagated in medium to all other points. After a time the last point will oscillate too. This propagation of an oscillatory move in the medium is called wave and is physically described by a relation of the form:

$$y = A\sin[\frac{2\pi}{T}(t-\tau)] \ , \tag{1.b.5}$$

where τ is the delay or the time needs for oscillation to propagate from one point to another ,in the medium. If the speed of wave is v and x is the distance that the wave is move in time τ than:

$$\tau = \frac{x}{v} \ . \tag{1.b.6}$$

Now, the wave relation can be written as:

$$y = A\sin[\frac{2\pi}{T}(t-\frac{x}{v})], \tag{1.b.7}$$

or, in other words, elongation in a point, say y_2 in space have the same value at time t as it has in a point y_1 at a time (t-τ).

2. Dynamical Systems

Introductory section has defined the gravitational potential of an economic region i as:

$$_iV = G\frac{\sum P_j}{d_{ij}{}^b} \ .$$

$_i$V is dependent of the population of all other economic regions and the distances between region i and all other regions. As long as no factor of time occurs in this relation, $_i$V is statically defined. Nevertheless, in time, the population of an economic region is modifying in time.

The following sections treat the variation of $_i$V in time and space, as result of variation of the population number in economic regions.

Consider, in the economy frame, that in region i of reference an important event take place periodically and periodically attract a great number of populations from all other economic regions.

This case represents a particular economic model of a dynamical system, which permits to study the evolution of gravitational potential of an economic region in other regions from economy, in time.

Some idealizations will simplify the rationalities and will make the analysis of dynamical systems more accurate:

1.only the time of population displacement between economic regions is of interest. The time that population is not moving can be omitted as being irrelevant for the rationalities that follow;

2.population is moving with the same constant speed, noted v;

3.each economic region has a large number of persons, which are structured as a whole. Individual behavior will be neglected;

4.the whole population from economic regions is moving, implying that there are moments in which no population exists in a region. Population not interested in the event, is neglected;

5.from reasons of psychological nature, personal order, or because of various factors the population is displacing continuously, from the first person to the last one. Population did not move between regions all in the same time.

In practical experience, these idealizations can be found only with a large degree of approximation.

a. The Oscillation of Gravitational Potential in an Economic Region i

First task is to determine the variation in time of region i gravitational potential.

Considering, at first, only one economic region, say region k, the gravitational potential of region i can be written as:

$$ {}_iV = G\frac{P_k}{{d_{ik}}^b}, \tag{2.a.1}$$

and depend, as it was showed, on population P_k from region k and on distance that separate, in space, these two regions. In time, only the population P_k varies and this aspect will be of interest. In which way the values of P_k evolutes it will be shown here.

Before the beginning of the event, according with idealization (5), region k population is displacing to region i continuous from the first person to the last one.

The time needs for a person to travel from region k to region i is:

$$ {}_kt = \frac{d_{ik}}{v} \quad . $$

Just to reach the region i, in time for the event, the last person that leaves, should do it, with a time $_k t$ before the starts of the event. After this moment, whole population is displaced, remaining none in the region k-see idealizations.

When the event ended the population P_k returns to the origin region k, populating the region k with its maximum population P_{kmax}. The displacement is also continuous.

The periodicity of the event imposes the cyclical displacements of the region k population.

A graphical representation of the population variation in region k, in respect with time is:

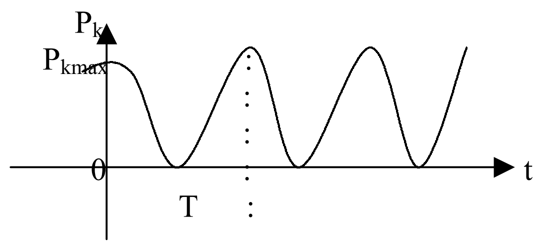

Fig.(2) Population Displacement Model P_k

The representation, as is shown in fig.2, is a cosine function, with the mention that it does not have negative values. In this case the cosine function suffer a modification, in respect with classical cosine, of the form:

$$y = A[1 + \cos(\frac{2\pi}{T} t)]. \tag{2.a.2}$$

In these conditions, the evolution in time of the population in economic region k, according to the relations mentioned in section concerning theoretical physics and relation (2.a.2), can be written taking P_k as elongation and P_{kmax} /2 as amplitude:

$$P_k = \frac{P_{k\max}}{2}[1 + \cos(\frac{2\pi}{T} t)], \tag{2.a.3}$$

where **T** is the **event period** that measures the time between two successive events happening in region i. The relation describes the oscillation of population from an economic region, in time.

Replacing (2.a.2) in (2.a.1), the gravitational potential of region i is calculated as follows:

$$_i V = G \frac{P_{k\max}}{2d_{ik}^{\ b}}[1 + \cos(\frac{2\pi}{T} t)]. \tag{2.a.4}$$

The meaning of this relation is that for this particular chosen case, the displacement potential of a person from region i in region j is oscillating. This relation describes the oscillatory move of the gravitational potential.

Relation (2.a.3) can be written in respect with sine function, according to equation (1.b.4) as:

$$ {}_iV = G\frac{P_{k\max}}{2d_{ik}{}^{b}}[1+\sin(\frac{\pi}{2}+\frac{2\pi}{T}t)]. \qquad (2.a.5) $$

The same oscillation law is applicable for any other economic regions in the economy.

The population number is different from one economic region to another imposing different amplitudes of gravitational potential oscillation.

b. The Gravitational Wave in Economy

In last section, the relation describing the gravitational potential of region i takes into account the influence of only one region, k. In this part, the relation will be extended considering the influence of all economic regions in the economy. The distances that separate economic regions will be taken into account.

The Oscillation Period

The oscillation period is defined as the time between two consecutive events, or the interval of time measured between the leave of first person and return of last person in an economic region.

One region population leaves and returns time depends on the distance up to the economical region where the event is produced.

In case the region from where the population leave is placed at a small distance from region i, the population travels on a small distance and the time between their leave and return is short.
The oscillation period of the population of the analyzed region is small and so is the oscillation period of the gravitational potential.

In case the economic region is further, the population travels on a large distance and the time between their leave and return is longer.
The oscillation period of the population of the analyzed region is bigger.

The oscillation period of the population of the analyzed regions is directly proportional with the distance.

$$T_k \sim d_{ik}$$

Population Wave Equation in Economy

The population from economy is moving to and from economic regions with a constant velocity v. Persons coming in region i, from further placed regions, begin to displace earlier.

Population that leaves first is coming from the furthest region-noted k.

To reach region i in time for the event, the last person from the region k should leaves, as it was showed in previous sections, with a time ${}_k t = \frac{d_{ik}}{v}$ before the beginning of the event.

Consider another region j, placed at a smaller distance from region i.

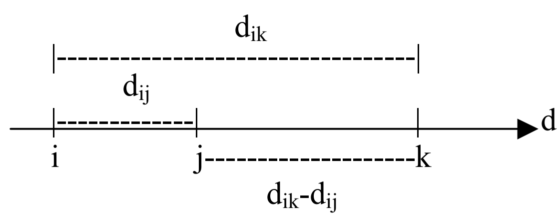

Fig.3. Distances between economic regions.

Population coming from region j begins to displace later because the distance is smaller. The last person from region j leaves region j with a delay τ in respect to the last person from region k that leaves region k.

However, in order to reach region i in time for the event, in the moment that last person from region j leaves, the last person from region k must be situated in region j either. The delay τ can be written:

$$\tau = \frac{d_{ik} - d_{ij}}{v}, \qquad (2.a.6)$$

and is the time interval needed by population to travel from region k to region j.

As the population belonging to economic region j, begins to move with a delay τ and applying equation (2.a.2) the relation for oscillation became in region j:

$$P_j = \frac{P_{j\max}}{2}[1 + \cos\frac{2\pi}{T_j}(t - \tau)]. \qquad (2.a.7)$$

These relation shows that at a time t the population belonging to economic region j has the same amplitude that the population belonging to region k has at time t-**τ.**

The economic regions are situated at different distances from region i and so the delay time τ is characteristic for each region.

Equation (2.a.5) becomes:

$$P_n = \frac{P_{n\max}}{2}[1+\cos\frac{2\pi}{T_n}(t-\tau_n)], \qquad (2.a.8)$$

which represents the ***Population Wave Equation in Economy***.
With the help of relation (2.a.4), equation (2.a.6) can be written:

$$P_n = \frac{P_{n\max}}{2}[1+\cos\frac{2\pi}{T_n}(t-\frac{d_{ik}-d_{ij}}{v})], \qquad (2.a.9)$$

which is the population wave equation function of distances.

The wave front came from the further regions to the region i where the event takes place. After the end of the event the wave front move is backward-from region i to the further regions.

The Equation of Gravitational Wave in Economy

As already was shown, the gravitational potential of region i is oscillating function of the population belonging to region n, as follows:

$${}_iV = G\frac{P_{n\max}}{2d_{in}{}^b}[1+\cos\frac{2\pi}{T}t].$$

Replacing in this relation the population belonging to region n, according with equation (2.a.7), the results is the mathematical form of the ***Gravitational Wave in Economy:***

$${}_iV = G\frac{P_{n\max}}{2d_{in}{}^b}[1+\cos\frac{2\pi}{T_n}(t-\tau_n)]. \qquad (2.a.10)$$

This last relation shows that at the moment t the gravitational potential of region i in region n has the same amplitude as it already have had at a former moment t-τ in region k.

Using the result (2.a.4) in (2.a.8), gravitational wave in economy is expressed function of distances:

$${}_iV = G\frac{P_{n\max}}{2d_{in}{}^b}[1+\cos\frac{2\pi}{T_n}(t-\frac{d_{ik}-d_{in}}{v})]\ . \qquad (2.a.11)$$

Expressing in term of sine function, the gravitational wave can be written:

$$_iV = G\frac{P_{n\max}}{2d_{in}{}^b}\{1+\sin[\frac{\pi}{2}+\frac{2\pi}{T_n}(t-\frac{d_{ik}-d_{in}}{v})]\}. \qquad (2.a.12)$$

The wave front came from the further regions to the region i where the event takes place. After the end of the event the wave front move is backward-from region i to the further regions.

This behavior is analogue with that of population wave.

3.The Gravitational Potential of Region i in all Other Economical Regions

The displacement potential of a person from region i in all other regions from economy, at any time moment, is found summing:

$$_iV = G\frac{P_{1\max}}{2d_{i1}{}^b}[1+\cos\frac{2\pi}{T_1}t]$$

$$_iV = G\frac{P_{i\max}}{2d_{i2}{}^b}[1+\cos\frac{2\pi}{T_2}t] \qquad :$$

$$\vdots$$

$$_iV = G\frac{P_{n\max}}{2d_{in}{}^b}[1+\cos\frac{2\pi}{T_n}t]$$

and is:

$$_iV = G\sum_{n=1}^{m}\frac{P_{m\max}}{2d_{im}{}^b}[1+\cos(\frac{2\pi}{T_m}t)]. \qquad (3.1)$$

This relation gives the gravitational potential of region i in all other regions, in this particular chosen case of a dynamical system.

4. Conclusion

This paper described an economic model with elements and concepts of theoretical physics demonstrating that the gravitational potential evolutes in time and space as a wave. The wave front is propagates from the furthest economical region k to the reference region i and back.

The validity of this model can be proved by experiments in a real economy with a sufficient large population, so that in each region the population displacement becomes a mathematically approximation modeled by a cosine function. Population displacement can also be expressed in term of sine function.

As in classical models of interactions of gravitational type, the masses, which are population here, can also be monetary movements, stock goods movements, or other economic categories.

In practice the distance exponent seams to have a value belonging to an interval:

$$1 \le b \le 3$$

As it has been demonstrated in the classical gravitational models, each category of economic phenomena has its own exponent for distance.

The detection of gravitational waves in economy can represent an important experimental project.

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