Specification Searches in Spatial Econometrics: The Relevance of Hendry's Methodology*

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Abstract

This paper brings together a number of new specification search strategies in spatial econometric modeling. In the literature, experimental results for several forward stepwise strategies aimed at remedying spatial dependence, have been reported. Essentially, these strategies boil down to the expansion of a spatial linear regression model with spatially lagged variables, conditional upon the results of misspecification tests. We investigate a Hendry-like specification strategy, starting from the spatial common factor model and subsequently reducing the number of spatially lagged variables on the basis of significance tests. The experimental simulations pertain to various small to large sample sizes, with spatial processes modeled on regular lattice surfaces. Our main conclusion is that the classical forward stepwise approach outperforms the Hendry strategy in terms of finding the true data generating process as well as in the observed accuracy of the estimators for spatial and non-spatial parameters. It also dominates concurrent forward stepwise strategies recently suggested in the literature.

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1 Introduction

Over the last few decades an important part of the spatial econometric literature has been concerned with the development of tests for spatial (auto)correlation or dependence in linear regression models as well as the development of efficient and consistent estimators for these types of models. The oldest test statistics for spatial correlation are the Moran coefficient and the related Geary, and Cliff and Ord coefficients (Cliff and Ord, 1973, 1981). More recently, spatial variants of the Wald, Likelihood Ratio and Lagrange Multiplier tests have been developed within the framework of maximum likelihood theory (Anselin, 1988; Anselin and Bera, 1998).[1] Among the estimators used for spatial regression models, such as the spatial AR(1) model and the model with a spatially lagged dependent variable, maximum likelihood (ML) estimators have been predominant, as the spatial counterpart of easy-to-use estimated generalized least squares (EGLS) procedures do not possess the favorable properties encountered in a time series framework. However, even for relatively straightforward specifications, such as spatial MA and ARMA models, estimators are still not available or are not easy to implement (see e.g., Anselin, 2000).

In the spatial econometric literature the classical specification search approach has been predominant, while so far hardly any attention has been paid to the so-called Hendry approach. The former can be characterized as an approach based on "excessive pre-simplification with inadequate diagnostic testing" (Maddala, 1992, p. 494). In a spatial setting the classical approach is a sequential procedure, that typically includes the following steps:

- estimate the initial model (i.e., a model without spatially lagged variables that has a well-behaved disturbance term);
- test for a spatial autoregressive process; and
- if the null hypothesis of no spatial correlation is rejected, apply a remedial procedure.

Three types of remedial procedures can be distinguished (see Florax and Folmer, 1992 and the references therein): remedial action that consists of

[1] See Cliff and Ord (1981), Getis(1991), Anselin and Florax (1995), and Anselin et al. (1996) for an overview of various statistics. The latter two sources also provide information on the small sample distributions of various tests. For other, sometimes newly developed, variants see also Kelejian and Robinson (1992), Brett and Pinske (1997), and de Graaff et al. (2001).

some kind of transformation of the sample observations, leading to EGLS estimators or to variables with the autoregressive components filtered out; the use of ML estimators for various forms of spatial dependence, either as a substantive or a nuisance process (Anselin and Rey, 1991; Anselin and Florax, 1995a); and adjustments in the context of model specification, either through spatial expansion or spatial adaptive filtering.

Following amongst others Hendry (1979) and Mizon (1977) the classical approach is a 'specific to general' or 'bottom-up' approach. It has the following three main drawbacks:

- The significance levels of the (unstructured) sequence of tests actually conducted is unknown. For instance, if a model is estimated by ordinary least squares (OLS) and spatial autocorrelation among the residuals is detected, and subsequently the model is re-estimated on data adjusted for spatial autocorrelation, the significance levels of the estimated coefficients of the transformed model are unknown and frequently not analytically or numerically tractable.

- Every test is conditional on arbitrary assumptions, which may be tested at a later stage. If these assumptions are subsequently rejected, the inferences drawn earlier are invalidated.

- The classical approach does not always lead to the 'best' model, as one may get sidetracked by using inadequate diagnostic tests (see Maddala, 1992, p. 495, for an example).

The Hendry approach starts with a very general model that is overparameterized in the sense that spatial correlation among various variables is assumed *a priori*. The model is progressively simplified with a sequence of specification tests. The Hendry strategy is therefore essentially a backward stepwise regression approach. It starts from the spatial common factor model, which is subsequently reduced on the basis of significance tests. The Hendry approach can be characterized as "intended over-parameterization with data-based simplification" (Maddala, 1992, p. 494), and constitutes a 'top-down' or 'general to specific' approach.

In applied spatial econometric modeling the classical approach has been used almost exclusively. We are not aware of any applications of the Hendry approach. This may partly be due to lacking easy-to-use software. Contrary to the situation for most standard software packages, which include automated forward as well as backward regression specification strategies, the well-known spatial software package SpaceStat (Anselin, 1995) does not yet contain automated specification strategies geared to the specific spatial modeling context. Earlier work on alternative specification strategies, such as by Blommestein (1983), Bivand (1984), and Florax and Folmer (1992), has not been widely applied, except for the classical strategy. Following this earlier work on various classical forward stepwise regression approaches, and in view of the favorable theoretical properties of the Hendry approach, this paper sets out to compare the Hendry strategy to the classical approach in an experimental setting.

The organization of the remainder of this paper is as follows. Section 2 describes the differences between the different specification search strategies and spells out their specific implementation in the spatial context. Moreover, an alternative to the classical approach on the basis of tests robust to local misspecification (Anselin et al., 1996) is unfolded. In section 3 the experimental design is explained, and in section 4 simulation results are presented. Section 5 winds up the paper, and presents the general conclusions.

2 Spatial Specification Searches

The classical approach to econometric modeling in the context of spatial process models is explicitly outlined in Anselin and Rey (1991) and its performance is experimentally investigated in Florax and Folmer (1992). It can briefly be described as follows. Assume the following linear model is considered to adequately represent the data generating process:

$$y = X\beta + \epsilon \tag{1}$$

where y is a $(N \times 1)$ stochastic variate, X is a $(N \times k)$ matrix of non-stochastic variates, and ϵ is a $(N \times 1)$ error vector that is $NID(0, \sigma^2)$. Subsequently, it is investigated whether substantive spatial dependence (i.e., an autoregressive residual pattern due to the omission of a spatial lag) or a nuisance type of spatial dependence (i.e., an autoregressive error structure) occurs. These types of misspecification are likely to show up in misspecification testing if the true data generating processes are either the spatial lag model, given by:

$$y = \rho W y + X\beta + \mu \tag{2}$$

or the spatial AR error model, which is given by:

$$y = X\beta + (I - \lambda W)^{-1}\,\epsilon \tag{3}$$

where W is a row-standardized $(N \times N)$ matrix of exogenously determined elements representing the spatial morphology, and ρ and λ are scalar autoregressive parameters.

The diagnostic tools used to identify a spatially autoregressive error term or an erroneously omitted spatial lag are the well-known Lagrange Multiplier (LM) tests (see e.g., Anselin, 1988, Burridge, 1980):

$$LM_\lambda = \frac{\left(\hat{\epsilon}' W \hat{\epsilon} / \hat{\sigma}^2\right)^2}{T} \tag{4}$$

$$LM_\rho = \frac{\left(\hat{\epsilon}' W y / \hat{\sigma}^2\right)^2}{NJ} \tag{5}$$

with

$$J = \frac{1}{N\hat{\sigma}^2}\left[\left(WX\hat{\beta}\right)' M \left(WX\hat{\beta}\right) + T\hat{\sigma}^2\right] \tag{6}$$

where LM_λ has the spatial AR error model as the alternative hypothesis and LM_ρ a spatial AR lag model, $M = I - X(X'X)^{-1}X'$, T is the trace of the matrix $((W' + W)\,W)$, $\hat{\epsilon} = My$ are the OLS residuals, $\hat{\sigma}^2 = \hat{\epsilon}'\hat{\epsilon}/N$, and all other symbols are as previously defined.

It should be noted that three different estimators are appropriate depending on the model specification: OLS for the specification with an IID error term and without a spatially lagged dependent variable, and maximum likelihood estimators for the spatially autoregressive error model (MLERR) and for the model including a spatially lagged dependent variable (MLLAG); see Anselin (1988) and Anselin and Hudak (1992) for details on the estimation routines.

The classical approach towards specification can be summarized as follows:

1. Estimate the initial model $y = X\beta + \epsilon$ by means of OLS

2. Test the hypothesis of no spatial dependence due to an omitted spatial lag or due to spatially autoregressive errors, using LM_ρ and LM_λ, respectively.

3. If both tests are not significant, the initial estimates from step 1 are used as the final specification. Otherwise proceed to step 4.

4. If both tests are significant, estimate the specification pointed to by the more significant of the two tests. For example if $LM_\rho > LM_\lambda$ then estimate (2) using MLLAG. If $LM_\rho < LM_\lambda$ then estimate (3) using MLERROR. Otherwise, proceed to step 5.

5. If LM_ρ is significant but LM_λ is not, estimate (2) using MLLAG. Otherwise proceed to step 6.

6. Estimate (3) using MLERROR.

Following the general principles of specification testing with locally misspecified alternatives presented in Bera and Yoon (1993), experimental simulation results in Anselin and Florax (1995b) and Anselin et al. (1996) have shown that robust LM tests may actually have more power in pointing out the correct alternative than the tests given above. The robust tests are designed to work well under local misspecification, and there are therefore two variants. One is a test for spatial error autocorrelation in the presence of a spatially lagged dependent variable, and the other a test for endogenous spatial lag dependence in the presence of spatial error autocorrelation. Consequently, we can formulate an alternative strategy along the lines in the above scheme, although with the robust tests instead of the traditional tests. The robust tests are obviously similar to the ones given in equations (4) and (5), extended with a correction factor to account for the local misspecification. The test for the presence of a spatial AR error process, when the specification contains a spatially lagged dependent variable, denoted LM_λ^*, reads as:

$$LM_\lambda^* = \frac{\left(\hat{\epsilon}' W \hat{\epsilon}/\hat{\sigma}^2 - T(NJ)^{-1}\hat{\epsilon}' W y/\hat{\sigma}^2\right)^2}{T[1 - T(NJ)]^{-1}} \tag{7}$$

Alternatively, the test for a spatially lagged dependent variable in the presence of a spatial AR error process, denoted as LM_ρ^*, is given by:

$$LM_\rho^* = \frac{\left(\hat{\epsilon}' W y - \hat{\epsilon}' W \hat{\epsilon}/\hat{\sigma}^2\right)^2}{NJ - T} \tag{8}$$

The robust approach towards specification proceeds exactly along the same six steps as the classical approach, with the exception of the robust LM tests taking the place of their classical counterparts.

In addition to the robust approach, a hybrid specification strategy can be based on the combined use of the classical and robust tests as follows:

1. Estimate the initial model $y = X\beta + \epsilon$ by means of OLS.

2. Test the hypothesis of no spatial dependence due to an omitted spatial lag or due to spatially autoregressive errors, using LM_ρ and LM_λ, respectively.

3. If both tests are not significant, the initial estimates from step 1 are used as the final specification. Otherwise proceed to step 4.

4. If both tests are significant, estimate the specification pointed to by the more significant of the two *robust* tests. For example if $LM_\rho^* > LM_\lambda^*$ then estimate (2) using MLLAG. If $LM_\rho^* < LM_\lambda^*$ then estimate (3) using MLERROR. Otherwise, proceed to step 5.

5. If LM_ρ is significant but LM_λ is not, estimate (2) using MLLAG. Otherwise proceed to step 6.

6. Estimate (3) using MLERROR.

The Hendry approach to model selection can be characterized by means of six criteria (Gilbert, 1986, p. 289-295). Satisfactory models should meet the following criteria:

1. data admissibility: it should be logically possible that the data have been generated from the model under consideration;

2. consistency with theory: the hypothesized model should be acceptable from a theoretical point of view;

3. the regressors should be (at least) weakly exogenous (eventually in the reduced form), otherwise a simultaneous system should be modeled;

4. parameters should be constant over space and time;

5. data coherency: the difference between the observed and the expected value of the dependent variable should be random;

6. encompassment of a wide range of rival models: the hypothesized model should not only explain the data at hand but also encompass rival theories that are estimated with the same data.

It is typically criteria 5 and 6 that make up the difference with the classical approach outlined above.

The Hendry approach to specification in the context of spatial dependence models begins with a test of the so-called common factor hypothesis. This hypothesis can be obtained from the following expression of the spatial error model (3):

$$y = \lambda Wy + X\beta - WX\lambda\beta + \epsilon \tag{9}$$

The common factor hypothesis has as its null hypothesis:

$$\lambda \times \beta = -\lambda\beta \tag{10}$$

or that the product of the coefficient on the spatial lag of y times the coefficients on the original X variables is equal to the negative of the (composite) coefficient on the spatial lag of the X variables (omitting the constant). The common factor hypothesis has been suggested as a way to discriminate between the spatial error and spatial lag model (Burridge, 1981; Blommestein, 1983; Bivand, 1984). More specifically, rejection of the common factor hypothesis is taken as evidence against the spatial error specification, and in favor of the lag.

In the context of the models examined in this paper, however, a failure to reject the common factor hypothesis could also be consistent with the spatial independence model (1) since $\lambda = 0$. Consequently, we suggest the following modification of the Hendry approach for specification searches in spatial dependence models:

1. Estimate (9) using both restricted and unrestricted maximum likelihood estimators (see the below definitions);

2. Test the common factor restriction;

3. If the common factor restriction is not rejected go to step 5, otherwise go to step 4;

4. Estimate the spatial lag model using MLLAG and test the coefficient ρ. If the latter is significant, the final specification is taken to be the lag model, otherwise the final specification is the spatial independence model;

5. Estimate the spatial error model using MLERROR and test the coefficient λ. If the latter is significant, the final specification is taken to be the error model, otherwise the final model is the spatial independence model.

In step 1 of the Hendry strategy, the restricted version of (9) is obtained via estimation of the spatial error model (3). The unrestricted estimate is based on the maximum likelihood lag estimator applied to (9). The common factor hypothesis is then tested using a likelihood ratio statistic:

$$\chi^2_{(1)} = -2(L_r - L_{ur}) \tag{11}$$

where L_r (L_{ur}) is the value of the log likelihood function for the restricted (unrestricted) estimator.

The classical, robust and hybrid strategies have the attractive property of being based on the LM tests, which in turn require only the residuals from OLS estimation of (1). The Hendry strategy, by contrast, requires two different maximum likelihood estimations to form the first stage test of the specification strategy, substantially increasing the computational cost. In fairness, it should be noted that if the spatial independence model is rejected in the LM based strategies (classic, robust or hybrid) then a maximum likelihood estimation would be required to end the specification search, so the savings in computational cost due the LM tests is partly offset in the estimation phase of the strategy.[2] Nonetheless, the Hendry strategy also requires an additional ML estimator in subsequent stages, so that the overall computational cost of the Hendry strategy will exceed that of the LM based approaches.

3 Experimental Design

We apply the four specification search strategies outlined above (i.e., the classical, the robust, the hybrid, and the Hendry approach) to a variety of data generating processes. The basic data generating process, under the null as well as the alternative reads as:

$$y = \rho W y + X\beta + (I - \lambda W)^{-1} \mu \tag{12}$$

where y is an $(N \times 1)$ dependent variate vector, X an $(N \times k)$ design matrix containing a unit vector and two exogenous variables drawn from a uniform $(0, 10)$ distribution, and μ an $(N \times 1)$ error vector following a NID distribution with mean zero and variance 2.0. The value for the error variance ensures that the estimated R^2 is on average approximately 0.55 for the model under the null hypothesis of no spatial correlation. This is a realistic value frequently encountered in applied spatial research. The true values contained in the vector β are all set to 1.0, and the elements of X are held fixed between the experiments.

There are three different true models or data-generating processes (DGP) which can be specified as submodels of (12) by putting the following constraints on the parameters:

[2] Alternative estimators such as instrumental variables or generalized methods of moments could be used in place of ML estimators in the LM based strategies in the second stage. We use ML estimators in this study to provide a more direct comparison with the Hendry strategy which requires ML estimators.

- the AR *endogenous lag* model, when $\lambda = 0$:

$$y = \rho W y + X\beta + \epsilon \tag{13}$$

- the AR *error* model, when $\rho = 0$:

$$y = X\beta + (I - \lambda W)^{-1}\mu \tag{14}$$

- the *spatial independence model*, when $\lambda = \rho = 0$:

$$y = X\beta + \mu \tag{15}$$

The experiment consists of a comparison of the three variants of the LM based strategy (classic, robust, and hybrid) and the Hendry approach in terms of the probability of finding the true model, and the mean squared error (MSE) of the estimators of the coefficients. It should be noted that the estimators of the parameters and their variances are pre-test estimators. Consequently, the MSEs are averages with the probabilities of finding a specific model as the weight, for the vector β given in equation (12). Averaging over more parameters, in particular ρ and λ, is not in order due to scale differences. This also implies that the properties of the estimators as reflected by the MSE may markedly differ from the asymptotic properties of the standard estimators described in the literature, because they have been derived under the assumption of no pre-testing.

In order to be able to compare the results of the different approaches, the levels of the tests under the alternative strategies have to be fixed in such a way that the overall significance levels of the approaches are of comparable size. A fundamental problem with specification searches in practice is that the number of tests required is not known beforehand. However, in the context of this experiment and the strategies outlined above, the maximum number of tests carried out in any strategy is two in each case. The nominal significance level of all tests is therefore fixed at 0.025, leading to an approximate overall significance level of 0.05, if it is assumed that the tests are independent.[3] As the tests are all computed from the same realization, the statistics will be positively correlated, so that the overall significance levels are likely to be lower than indicated.

The number of replications is set to 1000 and the main dimensions over which the variation in power of the approaches and the distributions of the

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[3] The significance level for the jth test is $1-\prod_{i=1}^{I}(1-\alpha_i)$, where α_i denotes the significance level of the ith test in the sequence of tests.

pre-test estimators are investigated are the values of the spatial parameters (ρ and λ) and the sample size. For the spatial parameters we examine the parameter space from [0.0, 0.9] using increments of 0.1. Five different sample sizes are obtained from a regular lattice structure starting with a 5×5 structure (N=25). The dimensions of the lattice are increased to 7, 10, 14, and 20, roughly doubling the sample size in each case (i.e., N=49, 100, 196, 400). The associated spatial weight matrices are defined on these lattices using a first-order rook definition of contiguity. All matrices are then row-standardized.

4 Experimental Simulation Results

We focus first on the performance of the four testing strategies with regard to the selection of the true DGP. Subsequently, we examine the mean squared error of the pre-test estimators obtained from the different strategies.

4.1 Model Selection Results

Table 1 reports the probability of detecting the underlying DGP for the testing strategies for all sample sizes and values of λ. There are several striking results to be noticed in the table. Under the null hypothesis of no spatial dependence, one would expect a probability of finding the true DGP to be approximately one minus the overall α level. While this is generally the case for the LM based approaches, the Type-I error rates are much lower than expected for the Hendry strategy.

The classical and the hybrid strategy perform exactly the same with regard to both size and power. Another important feature is that the classical strategy has highest power in the majority of sample sizes and values of λ. The exceptions pertain to low values of the spatial parameter and small sample sizes (25 and 49) where the robust strategy displays somewhat higher power.

In sum, the main result is that all the LM based tests dominate the Hendry strategy with regard to selecting the DGP when spatial error autocorrelation is present, and the Hendry strategy dominates when no autocorrelation is present.

[Table 1 about here.]

Table 2 reports the probability of detecting the underlying DGP for the different testing strategies, for all sample sizes, and for different values of ρ. As was the case for the results in Table 1 the Hendry strategy has substantially lower than expected Type-I error rates, while those of the LM based strategies are in general agreement with prior expectations. For positive but small values of ρ the LM based strategies all dominate the Hendry strategy with regard to selecting the proper DGP. For large values of ρ the probability of finding the true DGP is equal to one for all strategies. With increasing sample size this tends to hold for lower values of the spatial autoregressive parameter.

The classical strategy dominates for most non-zero values of ρ and sample sizes. As was the case for the error model, the exceptions involve the smallest sample size and values of $\rho < 0.5$, where the robust strategy has the highest detection probabilities. The results for the classical and the hybrid approach are identical again.

For all specification strategies, the probability of detecting lag dependence is substantially higher than the probability of detecting spatial error dependence for the same sample size and level of dependence. For the LM based strategies, this can be attributed to the higher power of the tests for lag dependence over those for error dependence reported in previous Monte Carlo studies (Anselin et al., 1996; Anselin and Florax, 1995b; Anselin and Rey, 1991). For the Hendry strategy the same pattern is displayed, however this has not previously been reported in the literature.

[Table 2 about here.]

4.2 Mean Squared Error Results

Table 3 reports the Mean Squared Error (MSE) for the estimators of λ from the different specification search strategies. For $\lambda = 0$ and $\lambda = 0.1$ the Hendry approach outperforms the LM based strategies, except for $N = 400$. The same holds for $\lambda = 0.2$ and $\lambda = 0.3$, and for sample sizes $N = 25$ and $N = 49$. For values of $\lambda \geq 0.3$ the Hendry approach is dominated by the LM based strategies. As expected, the MLE generally declines with increasing sample size.

Under the null hypothesis of no spatial dependence the robust approach has lower MSE for λ than the other LM based strategies. Generally, for low values of λ the robust strategy has lower MSE than the classical pre-test estimator, although the advantage becomes less as the sample size increases. On balance the classical approach displays lower MSE than the robust approach in a wider number of cases. The results for the hybrid and classical approaches are identical, as expected.

[Table 3 about here.]

Table 4 reports the MSE for the estimators of ρ from the different specification search strategies. We observe that the MSE for the classical and robust strategies is virtually identical for values of $\rho > 0.6$ for all sample sizes, and for $\rho > 0.2$ for the largest two sample sizes. Except for $N = 25$, the classical approach dominates the robust approach, although there are some minor exceptions. As expected, the results for the hybrid approach and for the classical approach are again identical.

Under the null hypothesis of spatial independence the Hendry approach dominates the classical approach for all sample sizes. The same holds for $\rho = 0.1$ for the smallest sample size. For $\rho \geq 0.7$ the results for the Hendry and the classical approach are identical, for all sample sizes. This equivalence is obtained for smaller values of ρ with increasing sample size. For $N = 400$ the different (pre-test) estimators are already identical for values of $\rho \geq 0.3$. In all other cases the classical approach outperforms the Hendry approach.

The difference in performance between the Hendry and LM based approaches is less pronounced for the case of the error model than for the lag model. The MSEs for ρ are generally lower than the MSEs for λ for all estimators and sample sizes. This reflects the higher probability of detecting the true model when the underlying DGP is the autoregressive endogenous lag model rather than the autoregressive error model.

[Table 4 about here.]

Table 5 reports the MSE for the estimators of β from the different specification search strategies, under the alternative of the spatial autoregressive error model. There is a high degree of similarity in the performance of the estimators across sample sizes and levels of dependence. For values of $\lambda \geq 0.4$ the LM based approaches are slightly better than the Hendry approach. We obtain similar results for the classical and robust approaches. Moreover, for $N = 100$ the classical and the robust approach produce identical results for $\lambda \geq 0.7$. For larger sample sizes the same occurs, although at increasingly lower levels of spatial dependence. The hybrid and the classical approach coincide fully again.

[Table 5 about here.]

Table 6 reports the MSE for the estimators of β from the different specification search strategies, under the alternative of the spatial lag model. There is a higher degree of similarity in the performance of the estimators across sample sizes and levels of dependence than in the previous case of the spatial autoregressive error model. For the small sample sizes the four strategies produce identical results for $\rho \geq 0.6$. For larger sample sizes they produce identical results, almost regardless of the level of spatial dependence present (except under the null hypothesis). For small sample sizes the Hendry approach performs slightly better than the LM based approaches for $\rho \leq 0.1$. The reverse holds in the all other cases.

In comparing the MSE results between the autoregressive lag model and the autoregressive error specification no consistent pattern with regard to the mean squared error for the non-spatial coefficients can be detected.

[Table 6 about here.]

Figures 1-3 show the experimental simulation results in a nutshell for the intermediate sample size $N = 100$, except for the situation of no spatial dependence. Under the null hypothesis of no spatial dependence the Hendry strategy clearly dominates all other strategies. Figure 1 shows that in a comparison of the classical (or alternatively, the hybrid strategy that produces identical results), the robust and the Hendry strategy regarding the probability of finding the true DGP, the LM based strategies dominate. The results for the autoregressive error model (top) and the endogenous lag model (bottom) are similar, although the probability of detecting the latter is substantially higher due to the power of the tests. For the MSE of the estimated λ in Figure 2, we find a parabolic shape with relatively higher MSEs for intermediate values of the spatial parameter. The LM based tests clearly show lower MSE values compared to the Hendry approach. The accuracy, in terms of bias and variance, of the pre-test estimator for the endogenous lag model is substantially higher for the endogenous lag model than for the autoregressive error model. For the latter the MSEs are approximately six times as high. Finally, Figure 3 shows that in terms of MSE for the non-spatial parameters β the MSEs are much lower than for the spatial parameters, they are similar for the endogenous lag model and the autoregressive error model, and they are relatively constant for the entire range of the spatial parameter value, except for higher values of λ, where the Hendry strategy shows slightly higher MSEs.

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

5 Conclusion

This paper brings together four specification search strategies in spatial econometric modeling, and ties on to earlier work in the 1980s and 90s on various alternative forward stepwise specification strategies. We use the classical (forward stepwise) strategy as the benchmark strategy. It consists of the estimation of a standard model without without spatially lagged variables and a well-behaved error term as a first step. Subsequently, the models' residuals are checked for spatial autocorrelation. Finally, when the null hypothesis of no spatial dependence is rejected, a remedial procedure accounting for the spatial correlation is applied. The tests usually applied in the classical framework, except for the general Moran's I test that does not have an informative alternative hypothesis, are the LM tests for spatial error autocorrelation and the LM test for spatial lag dependence.

The second strategy we labeled the robust strategy differs from the classical approach in the sense that the LM tests for error autocorrelation in the presence of a spatially lagged dependent variable, and the spatial LM test for spatial lag dependence in the presence of spatial error autocorrelation, are applied rather than the abovementioned standard tests.

The third, or hybrid strategy, is based on the combined use of the traditional LM tests and their robust counterparts. The robust tests are used in the case where the standard LM tests show up to be significant.

The final strategy is the Hendry strategy, that starts off with the estimation of the spatial error and the spatial lag model. Next, the common factor restriction is tested. If this restriction is not rejected, the spatial autoregressive error model is estimated, and in the case of a significant spatial autoregressive parameter the error model is taken to be an adequate representation of the underlying DGP. Alternatively, the default case of the spatial independence model applies. If the common factor restriction is rejected, the spatial endogenous lag model is estimated. If the coefficient of the spatially lagged dependent variable is significantly different from zero, the final specification is taken to be the spatial endogenous lag model. Alternatively, the default of spatial independence applies again.

The LM based approaches (classical, robust and hybrid) dominate with respect to detecting spatial dependence, and they display lower mean squared errors for the spatial parameters. The corresponding differences in the mean squared errors of the non-spatial parameters are relatively constant over the entire range of the spatial parameters, and in general rather small and similar across strategies.

As a word of caution we should obviously observe that the results may not generalize under alternative experiments, such as alternative choices of the exogenous variables, and in particular deviations from our assumption that the coefficient of determination is approximately 0.55.

The performance of the hybrid strategy is identical to the performance of the classical strategy, which may partly be due to the way it has been designed. As an alternative the robust LM tests can be applied somewhat more extensively. In particular, the robust tests can also be used to indicate whether the standard independence model or the error or lag model applies in the case where only one of the traditional LM tests is significant. We expect this particular hybrid strategy to produce slightly different results. It is, however, clear from the experimental simulations presented in this paper, that this alternative hybrid strategy is very likely to show a performance that is again somewhere in-between the results for the classical and the Hendry strategy.

Our key findings can be summarized as follows:

- the LM based approaches dominate with respect to detecting spatial dependence;
- the LM based approaches generally display lower mean squared errors for the spatial parameters;
- the difference in mean squared error for non-spatial parameters across the different pre-test estimators is very small;
- the classical approach dominates the robust approach, in terms of power and accuracy;
- in terms of size the Hendry strategy clearly dominates the various LM based strategies; and
- All specification strategies are more sensitive to spatial lag dependence relative to spatial error dependence.

The reverse conclusions regarding the size and the power of the different strategies (Hendry outperforming the LM based strategies under the null hypothesis of no spatial dependence, and the reverse performance under the alternative hypothesis) can nevertheless be reconciled in a sound advice to the practitioner using spatial econometric techniques. Given the computational burden associated with the Hendry strategy which is not offset by its performance, and given the fact that dependence is the rule rather than the exception in spatial datasets, it is advisable to use the classical specification strategy in spatial modeling. It generally results in the highest accuracy for the spatial and non-spatial parameters of the chosen model. In those cases where the classical approach ultimately leads to the conclusion that the standard spatial independence model is the correct alternative, a cautious approach would be to check this result using the Hendry approach to spatial econometric modeling.

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List of Figures

Figure 1: Probability of Finding the True Model

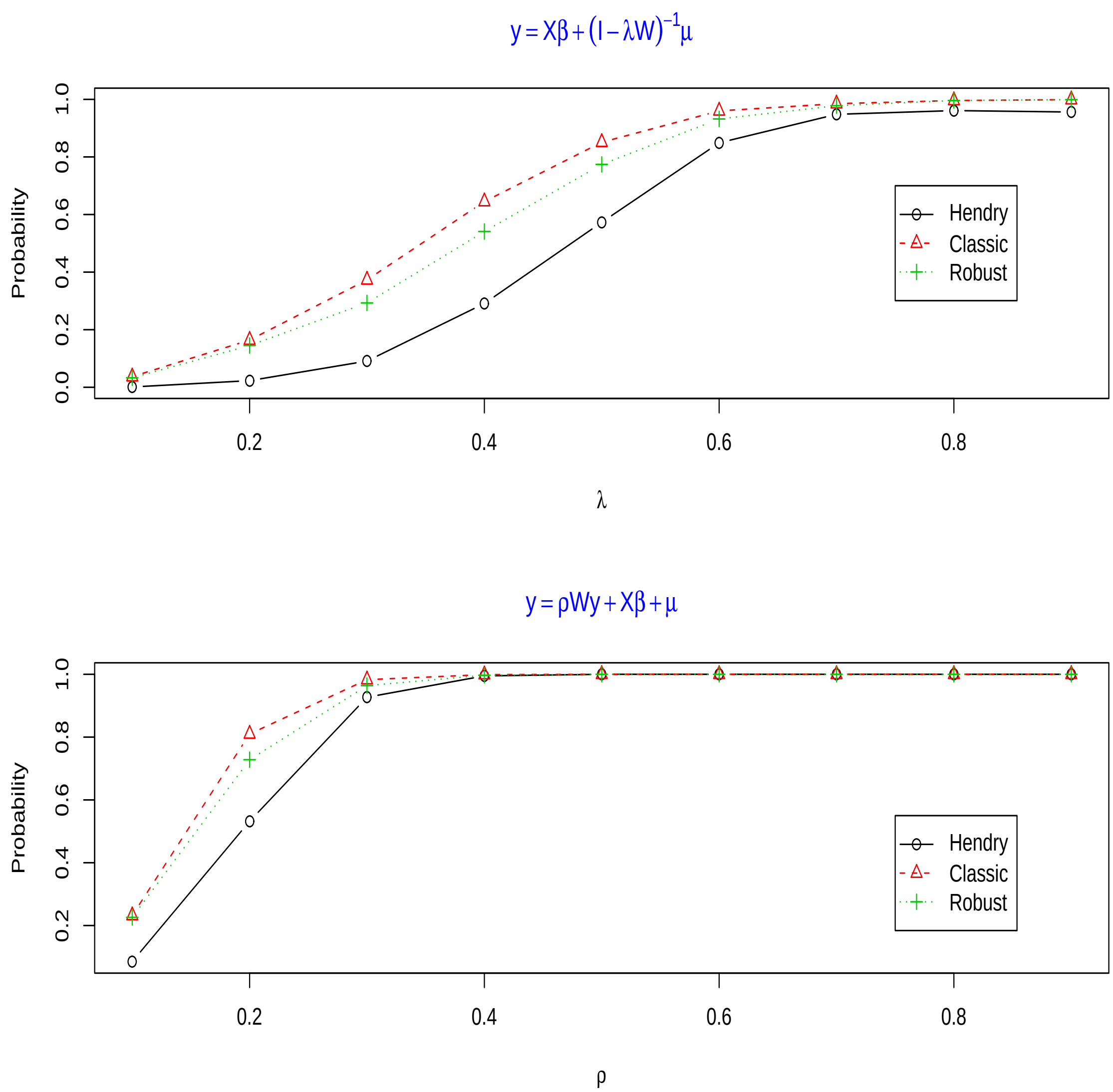


Figure 2: Mean Squared Error: Spatial Parameter Estimates

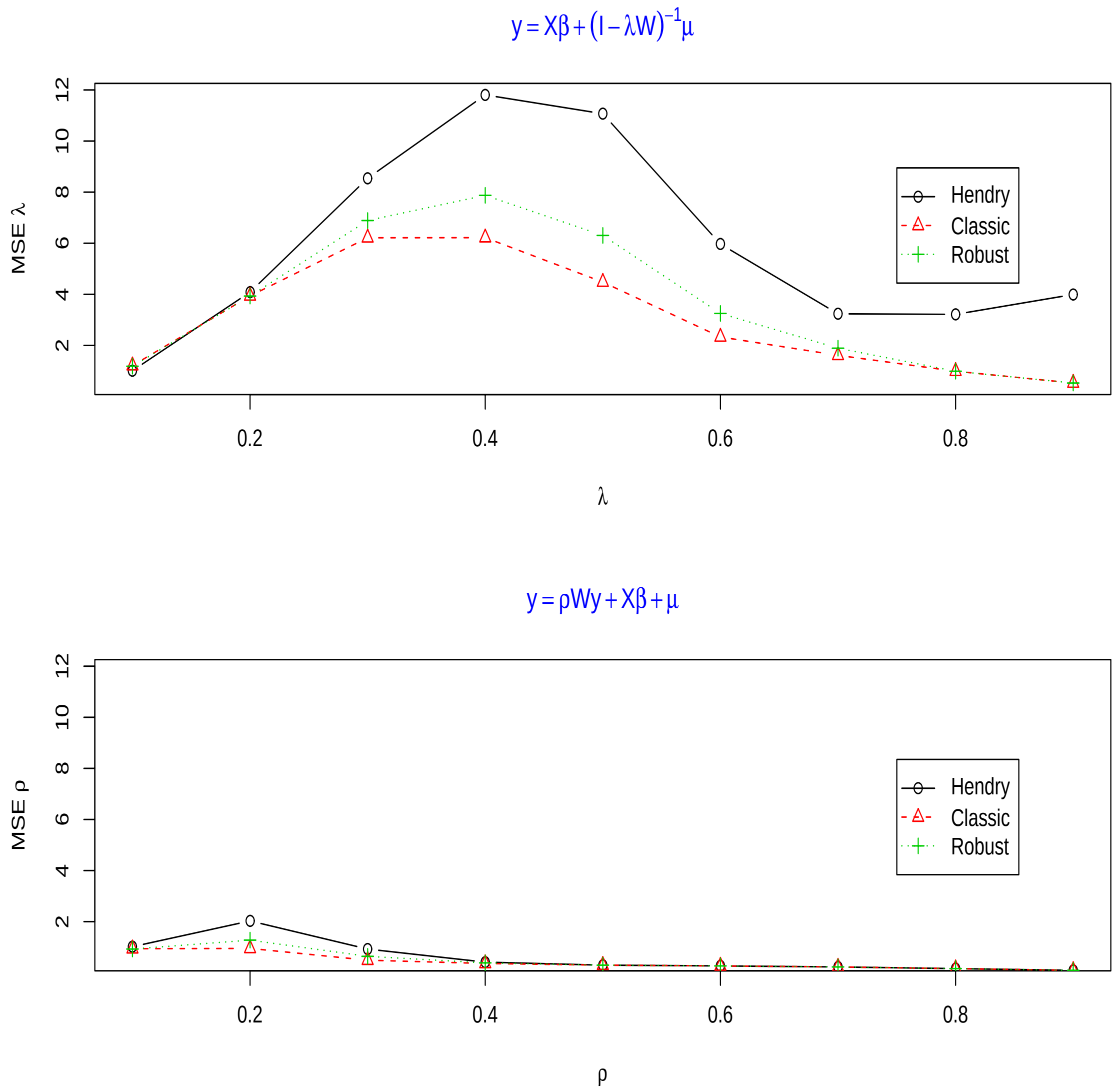
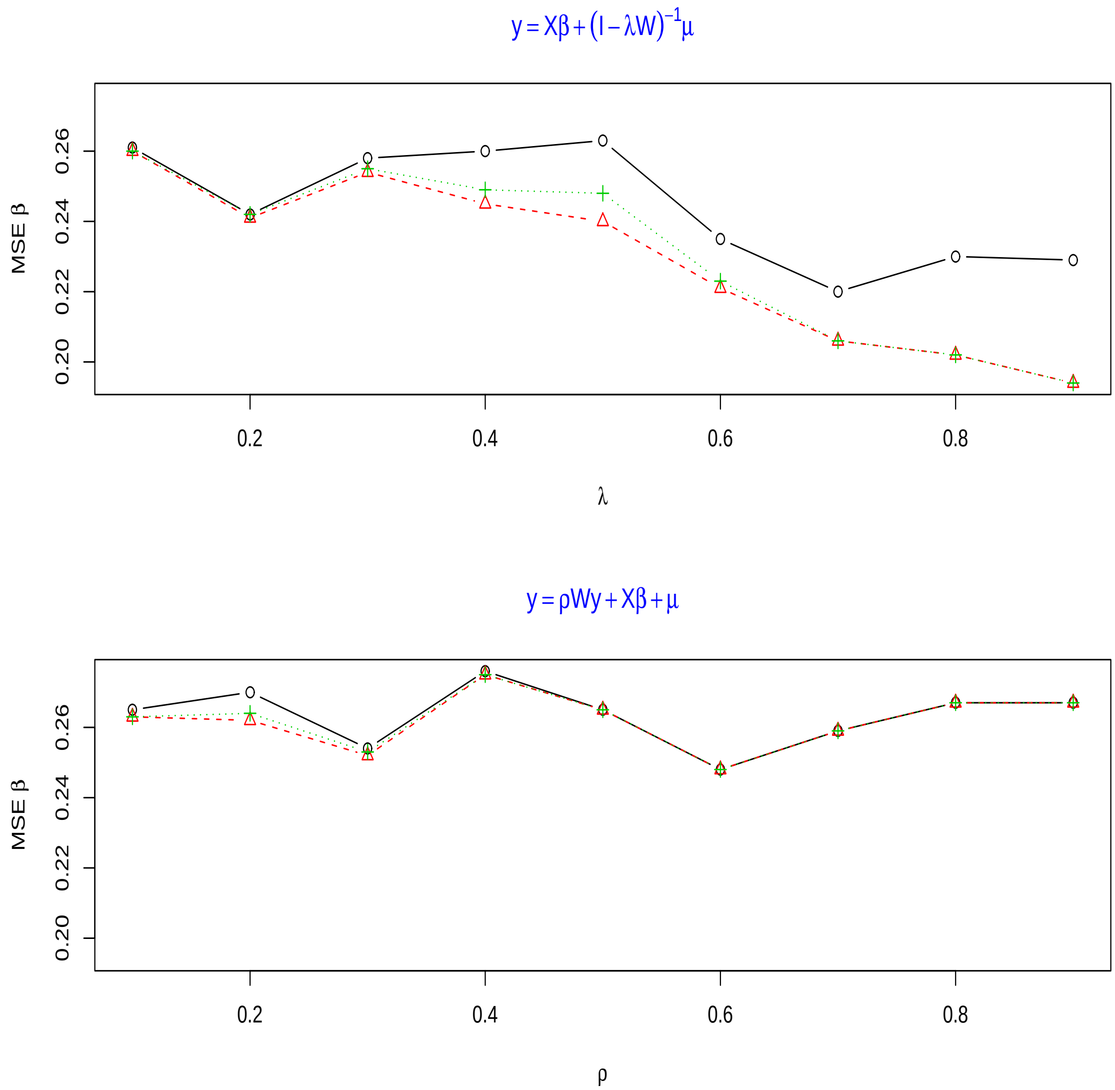


Figure 3: Mean Squared Error: β Estimates



List of Tables

Table 1: Probability of finding the true DGP for the different specification strategies, with five different sample sizes and ten different values for the spatial error autocorrelation parameter.

N	λ	Hendry	Classical	Robust	Hybrid
25	0.000	0.996	0.949	0.956	0.949
	0.100	0.000	0.007	0.016	0.007
	0.200	0.003	0.022	0.019	0.022
	0.300	0.014	0.033	0.036	0.033
	0.400	0.021	0.055	0.055	0.055
	0.500	0.066	0.142	0.104	0.142
	0.600	0.112	0.196	0.158	0.196
	0.700	0.231	0.343	0.223	0.343
	0.800	0.328	0.433	0.295	0.433
	0.900	0.526	0.584	0.384	0.584
49	0.000	0.995	0.957	0.950	0.957
	0.100	0.004	0.022	0.030	0.022
	0.200	0.009	0.051	0.056	0.051
	0.300	0.029	0.134	0.117	0.134
	0.400	0.088	0.265	0.215	0.265
	0.500	0.228	0.471	0.360	0.471
	0.600	0.416	0.640	0.554	0.640
	0.700	0.631	0.832	0.744	0.832
	0.800	0.819	0.918	0.872	0.918
	0.900	0.915	0.963	0.940	0.963
100	0.000	0.997	0.963	0.959	0.963
	0.100	0.001	0.037	0.033	0.037
	0.200	0.023	0.164	0.145	0.164
	0.300	0.091	0.373	0.293	0.373
	0.400	0.291	0.645	0.541	0.645
	0.500	0.573	0.851	0.774	0.851
	0.600	0.849	0.960	0.932	0.960
	0.700	0.948	0.985	0.978	0.985
	0.800	0.961	0.996	0.996	0.996
	0.900	0.956	0.999	0.999	0.999
196	0.000	0.996	0.954	0.952	0.954
	0.100	0.004	0.081	0.076	0.081
	0.200	0.057	0.347	0.293	0.347
	0.300	0.290	0.711	0.658	0.711
	0.400	0.667	0.927	0.888	0.927
	0.500	0.924	0.990	0.979	0.990
	0.600	0.970	0.995	0.995	0.995
	0.700	0.968	0.999	0.999	0.999
	0.800	0.975	1.000	1.000	1.000
	0.900	0.967	1.000	1.000	1.000
400	0.000	0.999	0.954	0.953	0.954
	0.100	0.015	0.162	0.137	0.162
	0.200	0.229	0.702	0.628	0.702
	0.300	0.746	0.971	0.952	0.971
	0.400	0.961	1.000	0.999	1.000
	0.500	0.973	1.000	1.000	1.000
	0.600	0.972	1.000	1.000	1.000
	0.700	0.971	1.000	1.000	1.000
	0.800	0.974	1.000	1.000	1.000
	0.900	0.972	1.000	1.000	1.000

Table 2: Probability of finding the true DGP for the different specification strategies, with five different sample sizes and ten different values for the spatial lag parameter.

N	ρ	Hendry	Classical	Robust	Hybrid
25	0.000	0.993	0.954	0.957	0.954
	0.100	0.064	0.088	0.102	0.088
	0.200	0.289	0.359	0.411	0.359
	0.300	0.687	0.757	0.792	0.757
	0.400	0.934	0.960	0.966	0.960
	0.500	0.992	0.996	0.995	0.996
	0.600	0.998	0.999	1.000	0.999
	0.700	1.000	1.000	1.000	1.000
	0.800	1.000	1.000	1.000	1.000
	0.900	1.000	1.000	1.000	1.000
49	0.000	0.994	0.957	0.962	0.957
	0.100	0.032	0.095	0.098	0.095
	0.200	0.209	0.387	0.393	0.387
	0.300	0.607	0.811	0.795	0.811
	0.400	0.915	0.972	0.959	0.972
	0.500	0.991	0.999	1.000	0.999
	0.600	1.000	1.000	1.000	1.000
	0.700	1.000	1.000	1.000	1.000
	0.800	1.000	1.000	1.000	1.000
	0.900	1.000	1.000	1.000	1.000
100	0.000	0.999	0.948	0.962	0.948
	0.100	0.085	0.232	0.226	0.232
	0.200	0.532	0.810	0.728	0.810
	0.300	0.927	0.983	0.964	0.983
	0.400	0.995	0.999	0.997	0.999
	0.500	1.000	1.000	1.000	1.000
	0.600	1.000	1.000	1.000	1.000
	0.700	1.000	1.000	1.000	1.000
	0.800	1.000	1.000	1.000	1.000
	0.900	1.000	1.000	1.000	1.000
196	0.000	0.998	0.948	0.938	0.948
	0.100	0.233	0.493	0.432	0.493
	0.200	0.944	0.994	0.982	0.994
	0.300	1.000	1.000	1.000	1.000
	0.400	1.000	1.000	1.000	1.000
	0.500	1.000	1.000	1.000	1.000
	0.600	1.000	1.000	1.000	1.000
	0.700	1.000	1.000	1.000	1.000
	0.800	1.000	1.000	1.000	1.000
	0.900	1.000	1.000	1.000	1.000
400	0.000	0.998	0.948	0.945	0.948
	0.100	0.467	0.777	0.694	0.777
	0.200	0.995	0.999	0.998	0.999
	0.300	1.000	1.000	1.000	1.000
	0.400	1.000	1.000	1.000	1.000
	0.500	1.000	1.000	1.000	1.000
	0.600	1.000	1.000	1.000	1.000
	0.700	1.000	1.000	1.000	1.000
	0.800	1.000	1.000	1.000	1.000
	0.900	1.000	1.000	1.000	1.000

Table 3: Mean Squared Error (×100) of estimators for λ for the different specification strategies, with five different sample sizes and ten different values for the spatial error autocorrelation parameter.

N	λ	Hendry	Classical	Robust	Hybrid
25	0.000	0.050	1.140	0.950	1.140
	0.100	1.000	1.320	1.540	1.320
	0.200	4.056	4.612	4.534	4.612
	0.300	9.151	9.199	9.286	9.199
	0.400	15.934	15.664	15.641	15.664
	0.500	23.742	22.010	22.734	22.010
	0.600	32.245	29.245	30.563	29.245
	0.700	37.908	32.466	38.306	32.466
	0.800	43.162	36.669	45.449	36.669
	0.900	38.879	34.590	50.612	34.590
49	0.000	0.000	0.641	0.590	0.641
	0.100	1.104	1.429	1.544	1.429
	0.200	4.116	4.313	4.280	4.313
	0.300	9.079	8.580	8.651	8.580
	0.400	15.067	12.459	13.178	12.459
	0.500	19.894	13.873	16.607	13.873
	0.600	21.409	13.567	16.641	13.567
	0.700	18.513	9.208	13.344	9.208
	0.800	12.324	6.578	9.319	6.578
	0.900	7.844	4.236	5.932	4.236
100	0.000	0.000	0.249	0.270	0.249
	0.100	1.011	1.209	1.185	1.209
	0.200	4.087	3.937	3.932	3.937
	0.300	8.543	6.213	6.889	6.213
	0.400	11.806	6.218	7.875	6.218
	0.500	11.070	4.479	6.305	4.479
	0.600	5.968	2.331	3.252	2.331
	0.700	3.237	1.612	1.891	1.612
	0.800	3.215	0.989	0.989	0.989
	0.900	3.993	0.527	0.527	0.527
196	0.000	0.000	0.168	0.155	0.168
	0.100	1.028	1.170	1.133	1.170
	0.200	3.940	2.957	3.153	2.957
	0.300	6.677	2.954	3.438	2.954
	0.400	5.630	1.772	2.345	1.772
	0.500	2.403	0.923	1.148	0.923
	0.600	1.615	0.780	0.780	0.780
	0.700	2.015	0.508	0.508	0.508
	0.800	1.949	0.354	0.354	0.354
	0.900	2.825	0.159	0.159	0.159
400	0.000	0.007	0.088	0.070	0.088
	0.100	1.027	1.009	1.012	1.009
	0.200	3.268	1.411	1.709	1.411
	0.300	2.484	0.604	0.758	0.604
	0.400	0.946	0.361	0.374	0.361
	0.500	1.018	0.360	0.360	0.360
	0.600	1.257	0.253	0.253	0.253
	0.700	1.627	0.210	0.210	0.210
	0.800	1.809	0.150	0.150	0.150
	0.900	2.337	0.070	0.070	0.070

Table 4: Mean Squared Error (×100) of estimators for ρ for the different specification strategies, with five different sample sizes and ten different values for the spatial lag parameter.

N	ρ	Hendry	Classical	Robust	Hybrid
25	0.000	0.047	0.112	0.114	0.112
	0.100	1.094	1.092	1.104	1.092
	0.200	3.122	2.859	2.657	2.859
	0.300	3.182	2.585	2.291	2.585
	0.400	1.624	1.241	1.163	1.241
	0.500	0.852	0.767	0.790	0.767
	0.600	0.684	0.648	0.623	0.648
	0.700	0.462	0.462	0.462	0.462
	0.800	0.373	0.373	0.373	0.373
	0.900	0.238	0.238	0.238	0.238
49	0.000	0.028	0.116	0.102	0.116
	0.100	1.081	1.132	1.109	1.132
	0.200	3.423	2.755	2.721	2.755
	0.300	3.838	2.081	2.248	2.081
	0.400	1.863	1.039	1.232	1.039
	0.500	0.799	0.621	0.605	0.621
	0.600	0.470	0.470	0.470	0.470
	0.700	0.397	0.397	0.397	0.397
	0.800	0.281	0.281	0.281	0.281
	0.900	0.203	0.203	0.203	0.203
100	0.000	0.000	0.078	0.049	0.078
	0.100	1.020	0.940	0.928	0.940
	0.200	2.029	0.952	1.277	0.952
	0.300	0.927	0.491	0.642	0.491
	0.400	0.415	0.363	0.386	0.363
	0.500	0.297	0.297	0.297	0.297
	0.600	0.267	0.267	0.267	0.267
	0.700	0.227	0.227	0.227	0.227
	0.800	0.162	0.162	0.162	0.162
	0.900	0.095	0.095	0.095	0.095
196	0.000	0.003	0.039	0.035	0.039
	0.100	0.837	0.584	0.644	0.584
	0.200	0.355	0.184	0.224	0.184
	0.300	0.167	0.167	0.167	0.167
	0.400	0.138	0.138	0.138	0.138
	0.500	0.137	0.137	0.137	0.137
	0.600	0.094	0.094	0.094	0.094
	0.700	0.074	0.074	0.074	0.074
	0.800	0.050	0.050	0.050	0.050
	0.900	0.025	0.025	0.025	0.025
400	0.000	0.002	0.021	0.022	0.021
	0.100	0.586	0.282	0.366	0.282
	0.200	0.124	0.111	0.114	0.111
	0.300	0.103	0.103	0.103	0.103
	0.400	0.087	0.087	0.087	0.087
	0.500	0.075	0.075	0.075	0.075
	0.600	0.060	0.060	0.060	0.060
	0.700	0.048	0.048	0.048	0.048
	0.800	0.028	0.028	0.028	0.028
	0.900	0.013	0.013	0.013	0.013

Table 5: Average Mean Squared Error (×100) of estimators for β (slopes only) for the different specification strategies, with five different sample sizes and ten different values for the spatial error parameter.

N	λ	Hendry	Classical	Robust	Hybrid
25	0.000	1.296	1.323	1.307	1.323
	0.100	1.315	1.324	1.326	1.324
	0.200	1.323	1.324	1.329	1.324
	0.300	1.398	1.404	1.386	1.404
	0.400	1.597	1.583	1.577	1.583
	0.500	1.593	1.592	1.537	1.592
	0.600	1.767	1.681	1.714	1.681
	0.700	1.739	1.675	1.757	1.675
	0.800	2.007	1.866	1.986	1.866
	0.900	1.884	1.727	2.047	1.727
49	0.000	0.487	0.493	0.493	0.493
	0.100	0.490	0.490	0.492	0.490
	0.200	0.497	0.497	0.496	0.497
	0.300	0.557	0.554	0.564	0.554
	0.400	0.555	0.545	0.545	0.545
	0.500	0.584	0.558	0.576	0.558
	0.600	0.609	0.576	0.611	0.576
	0.700	0.612	0.539	0.579	0.539
	0.800	0.561	0.498	0.529	0.498
	0.900	0.592	0.473	0.499	0.473
100	0.000	0.255	0.256	0.255	0.256
	0.100	0.261	0.260	0.260	0.260
	0.200	0.242	0.241	0.242	0.241
	0.300	0.258	0.254	0.255	0.254
	0.400	0.260	0.245	0.249	0.245
	0.500	0.263	0.240	0.248	0.240
	0.600	0.235	0.221	0.223	0.221
	0.700	0.220	0.206	0.206	0.206
	0.800	0.230	0.202	0.202	0.202
	0.900	0.229	0.194	0.194	0.194
196	0.000	0.123	0.124	0.124	0.124
	0.100	0.117	0.117	0.117	0.117
	0.200	0.120	0.121	0.120	0.121
	0.300	0.129	0.125	0.125	0.125
	0.400	0.125	0.118	0.119	0.118
	0.500	0.127	0.122	0.123	0.122
	0.600	0.124	0.119	0.119	0.119
	0.700	0.127	0.118	0.118	0.118
	0.800	0.130	0.114	0.114	0.114
	0.900	0.125	0.105	0.105	0.105
400	0.000	0.062	0.062	0.061	0.062
	0.100	0.059	0.059	0.059	0.059
	0.200	0.062	0.061	0.061	0.061
	0.300	0.062	0.060	0.060	0.060
	0.400	0.067	0.066	0.066	0.066
	0.500	0.058	0.057	0.057	0.057
	0.600	0.060	0.057	0.057	0.057
	0.700	0.058	0.054	0.054	0.054
	0.800	0.056	0.051	0.051	0.051
	0.900	0.060	0.051	0.051	0.051

Table 6: Average Mean Squared Error (×100) of estimators for β (slopes only) for the different specification strategies, with five different sample sizes and ten different values for the spatial lag parameter.

N	ρ	Hendry	Classical	Robust	Hybrid
25	0.000	1.018	1.024	1.021	1.024
	0.100	1.050	1.057	1.074	1.057
	0.200	1.195	1.170	1.186	1.170
	0.300	1.151	1.111	1.096	1.111
	0.400	1.153	1.142	1.136	1.142
	0.500	1.088	1.090	1.089	1.090
	0.600	1.136	1.136	1.135	1.136
	0.700	1.123	1.123	1.123	1.123
	0.800	1.099	1.099	1.099	1.099
	0.900	1.131	1.131	1.131	1.131
49	0.000	0.625	0.628	0.631	0.628
	0.100	0.661	0.665	0.668	0.665
	0.200	0.834	0.777	0.780	0.777
	0.300	0.782	0.688	0.695	0.688
	0.400	0.721	0.691	0.701	0.691
	0.500	0.672	0.666	0.665	0.666
	0.600	0.710	0.710	0.710	0.710
	0.700	0.710	0.710	0.710	0.710
	0.800	0.737	0.737	0.737	0.737
	0.900	0.730	0.730	0.730	0.730
100	0.000	0.248	0.250	0.250	0.250
	0.100	0.265	0.263	0.263	0.263
	0.200	0.270	0.262	0.264	0.262
	0.300	0.254	0.252	0.253	0.252
	0.400	0.276	0.275	0.275	0.275
	0.500	0.265	0.265	0.265	0.265
	0.600	0.248	0.248	0.248	0.248
	0.700	0.259	0.259	0.259	0.259
	0.800	0.267	0.267	0.267	0.267
	0.900	0.267	0.267	0.267	0.267
196	0.000	0.127	0.128	0.128	0.128
	0.100	0.129	0.126	0.126	0.126
	0.200	0.120	0.120	0.120	0.120
	0.300	0.118	0.118	0.118	0.118
	0.400	0.114	0.114	0.114	0.114
	0.500	0.126	0.126	0.126	0.126
	0.600	0.124	0.124	0.124	0.124
	0.700	0.121	0.121	0.121	0.121
	0.800	0.124	0.124	0.124	0.124
	0.900	0.125	0.125	0.125	0.125
400	0.000	0.060	0.060	0.061	0.060
	0.100	0.062	0.063	0.063	0.063
	0.200	0.063	0.063	0.063	0.063
	0.300	0.061	0.061	0.061	0.061
	0.400	0.061	0.061	0.061	0.061
	0.500	0.065	0.065	0.065	0.065
	0.600	0.063	0.063	0.063	0.063
	0.700	0.059	0.059	0.059	0.059
	0.800	0.062	0.062	0.062	0.062
	0.900	0.064	0.064	0.064	0.064