Rent Control Evasion: Effects on Income and Deadweight Loss *

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Abstract

When ability to avoid rent control is randomly distributed over landlords, housing may rise above its free market level. Landlords evading part of rent-control are like suppliers selling to a perfectly price-discriminating monopsonist. A continuum of evasive abilities gives a range of low rent landlords, charging different prices and to whom tenants flock. High housing supply comes at the expense of deadweight loss due to adverse selection of landlords with high costs and good evasive abilities.Rent-control produces an equilibrium *range* of rents and raises what some tenants pay above that which they would pay in an unregulated market. *Keywords: Rent control, informal sector, deadweight loss J.E.L.* classification: D61, R52. Running head: Rent control evasion

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I. Introduction

The present paper suggests that non-compliance with rent control can change the equilibrium of housing markets in surprising ways: Rent-control in the presence of landlords who can evade a part of the control may reduce rental units by very little. In some special cases rent-control may *increase* rental units above the free market level. This increase comes at a cost of adverse selection of tenants. The tenants who get rent-controlled housing may not be those who value it most. Rent-control in the presence of evasion produces a range of equilibrium rents. After rent-control some tenants end up paying *more* than the free-market rent. Seeds of these results are scattered throughout the rent-control literature but none are planted in the field of non-compliance.

The results I present are based on the following reasoning: some landlords with low costs may have a very poor ability to evade rent controls. An excess supply of tenants flocks to these landlords. Tenants who do not manage to get housing with these prize landlords then flock to landlords of equal or slightly higher cost but who have a slightly better ability to charge above the rent ceiling. The remainder of tenants seeks out progressively higher cost landlords until a critical underground rent is reached at which the supply of housing units equals the demand. Tenants who are lucky enough to get low rent apartments mimic how a price discriminating monopsonist would treat a landlord. The mimicry is not perfect, because all tenants do not pay a different price. At some point tenants become scarce and landlords with strong evasive abilities find themselves offering the same underground rent for the remaining tenants. Price discrimination of this sort moderates the depressing effects that rent- controls can have on housing markets. The dispersion of underground rents emanating from price discrimination also forces some tenants who find housing to pay more than what they would pay in a free market, as some tenants with low reservation rents have crowded out some tenants with high rents from landlords offering low rents. This creates an artificial excess demand which pushes up the range of underground rents above the free market rent level. This phenomenon calls for a revision of how we look at the income redistribution effects of rent controls.

Moderate reductions in housing that may result from rent control come at the cost of adverse selection. Some landlords with high costs but with a strong ability to evade rent controls will displace landlords with a low costs but with poor evasive abilities. This deadweight loss due to adverse selection may be of the same order of magnitude as the traditional triangle loss from imposing maximum prices.

II. The Standard case

This section lays out a basic model of equilibrium in the housing market. The goal is to establish a benchmark free market rent, housing level, and deadweight loss against which results from later modifications to the model can be compared.

Consider an industry with a continuum of tenants, each renting and infinitessimal amount of housing dt. The size, or "measure," of the market T is the sum of all these rentals. If we were considering a world of a discrete number of tenants, T would be the potential number of renters in the market. The reservation rent of the tenant is r. Each tenants draws his or her particular reservation rent r from a uniform distribution over the range [0 , r_{max}]. Faced with a market rent of R for unit unit of housing (which means Rdt for dt units of housing) the proportion of tenants demanding a unit of housing is $Pr(r \geq R)$. With a market of potential size T the demand for rental housing $H_d(R)$ comes out to:

$$H_d(R) = T \times Pr(r \geq R) \tag{1}$$

$$= T\left(1 - \int_0^R \frac{1}{r_{max}} dr\right) \tag{2}$$

$$= T\left(1 - \frac{R}{r_{max}}\right) \tag{3}$$

There is a continuum of landlords each renting dt units of housing. The measure of pontetial landlords is L. All rental units have the same level of quality, which for convenience I set at one. A particular landlord's cost of supplying his unit of rental housing is c and is drawn from a uniform distribution on the interval [0 , c_{max}]. Since quality is one for all units, the cost of providing the rental unit is also the cost of providing a unit of quality. The proportion of house owners choosing to rent is $Pr(c \leq R)$. Since there are L potential landlords, aggregate housing supply $H_s(R)$ is

$$H_s(R) = L \times Pr(c \leq R) \tag{4}$$

$$= L\int_0^R \frac{1}{c_{max}}dc \tag{5}$$

$$= \frac{L}{c_{max}}R \tag{6}$$

The market rental price "sweeps out" a supply curve. As the price rises, some homeowners convert their dwellings to rental units. It is also possible that landlords already renting, build additions to their apartments. This setup does not allow for continuous increases or decreases in housing by any one landlord. A landlord must build in increments of one unit. This unit's costs must be above the going market price, otherwise the landlord would already have built the addition (this reasoning also preserves the smoothness of the supply function).

I have used the somewhat awkward terms "measure of landlords" and "measure of tenants" where it would have been more natural simply to speak of the number of landlords and tenants. I do this to point out that there is no randomness in the model I present and that no one in the market is big enough to influence prices. A continuum of landlords and tenants allows me to convert distributions of reservation prices and costs into deterministic levels of supply and demand.

The equilibrium free market rent R_{free} for one rental unit comes from setting housing demand and supply equal to each other. It is simple to show that in equilibrium this rent is

$$R_{free} = \frac{c_{max} \times r_{max}}{T \times c_{max} + L \times r_{max}} T \tag{7}$$

Figure 1 illustrates these demand and supply curves, with their slopes and intercepts labeled. Note that I speak of rent for one unit. The amount any given tenant pays is $R_{free} \times dt.$ From now on to ease notational burden I shall stop making this fine distinction.

When rent control is imposed in the housing market, and everyone complies, the number of units offered for rental falls. This fall produces two types of deadweight loss. The first and best known loss is the traditional triangle deadweight loss. By restricting housing, rent control drives a wedge between what the marginal tenant is willing to pay and what the marginal landlord is willing to be paid. The sum of such wedges between housing under rent control $H_s(R_{con})$ and housing in the free market $H_s(R_{free})$ is the triangle loss. We need not worry that owners of vacant lots will sell their houses and build units in uncontrolled markets because I have assumed that all markets are under rent control. The best these rental unit owners can do is enjoy the alternate uses of their housing, perhaps living in it themselves, or allowing friends or family to live there.

The triangle loss just described is calculated by first figuring the value renters would have attached to living in the rental units that have been pulled off the market due to rent control. The maximum price any particular tenant is willing to pay is his reservation rent r. From equation (1) it is easy to see that the reservation rent (the height of the demand curve) at some quantity of housing demanded H is $r = r_{max}(1 - H/T)$. So lost tenant welfare due to the rent control is the area under the demand curve between $H_s(R_{free})$ and $H_s(R_{con})$

$$lost \;\; benefit \quad = \quad \int_{H_s(R_{con})}^{H_s(R_{free})} r_{max} \left(1 - \frac{H}{T}\right) dH \tag{8}$$

$$= \; r_{max}\left[H_s(R_{free}) - H_s(R_{con}) - \frac{1}{2T}[H_s(R_{free})^2 - H_s(R_{con})^2]\right] \quad (9)$$

The saved cost to landlords having produced this reservation rent must be subtracted from the above lost benefit to calculate triangle loss. The saved cost is the area under the supply curve between the same limits of integration as above:

$$saved \quad cost \;=\; \int_{H_s(R_{con})}^{H_s(R_{free})} \frac{c_{max}}{L} H dH \quad (10)$$

$$= \frac{c_{max}}{2L}[H_s(R_{free})^2 - H_s(R_{con})^2] \quad (11)$$

where I get the height of the supply curve from isolating from the supply equation (6) the minimum rent a landlord would be willing to accept. Subtracting these saved costs from lost tenant benefits gives the triangle deadweight loss. Triangle loss is illustrated in Figure 1 as area A. This very general discussion of triangle loss is in the tradition of Olsen (1972) and keeps in the background the complex dynamics that that Arnott et al. (1983) among others have shown may underlie changes in supply.

There is another deadweight loss from rent control that everyone knows about but until recently no one has bothered to analyse. This is the loss that Arnott (1995), in a survey of the social costs of rent control, describes as the case of "the proverbial elderly widow living in the same large, rent-controlled apartment long after her family has left home, at the same time that there is an acute housing shortage." Recently, Glaeser and Luttmer (1997) have injected precision into the analysis of this, the loss from the adverse selection of tenants in the lottery for low rent apartments. The derivation I present below is my own but I claim no originality for the idea. It will instead serve as a benchmark for comparison with the deadweight losses from adverse selection due to rent-control evasion.

The basis of adverse selection losses due to rent control is that tenants lucky enough to find a dwelling are not necessarily those with the highest reservation rents, that is, reservation rents between $[r', r_{max}]$, where r' would be the reservation rent of the marginal tenant if only the highest reservation rent tenants obtained rent-controlled housing units (see Figure 1). Rather, the tenants who get units flock from the broader range of reservation rents $[R_{con}, r_{max}]$. This broadness is what produces the deadweight loss from adverse selection Each dwelling seeker has an equal chance of getting a rent-controlled place because none can outbid the other's rent. The average reservation rent $\overline{r}$ of any tenant seeking a rent-controlled dwelling is:

$$\begin{aligned} \overline{r} &= \int_{R_{con}}^{r_{max}} \frac{1}{r_{max} - R_{con}} r dr && (12) \\ &= \frac{r_{max}^2 - R_{con}^2}{2(r_{max} - R_{con})} && (13) \end{aligned}$$

The average is taken using the conditional probability of being one of the tenants who gets the apartment. The conditional probability $1/(r_{max} - R_{con})$ comes from my assumption of a uniform distribution of reservation rents on the interval $[0, r_{max}]$ combined with the fact that only tenants in the range of reservation rents $[R_{con}, r_{max}]$ will seek rent-controlled units. The benefit to this group is then $H_s(R_{con})\overline{r}$.

If there had been no lottery for the dwellings then the dwellings would have gone only to those who valued them most. In that case the consumer benefit would have been the area under the demand curve between $[0, H_s(R_{con})]$:

$$\int_0^{H_s(R_{con})} r_{max} \left(1 - \frac{H}{T}\right) dH = r_{max} H_s(R_{con}) \left[1 - \frac{1}{2T} H_s(R_{con})]\right] \qquad (14)$$

Subtracting from this benefit the reduced benefit that consumers get by being chosen in the lottery for rental dwellings gives the deadweight loss from adverse selection:

$$H_s(R_{con}) \left[r_{max} \left(1 - \frac{H_s(R_{con})}{2T}\right) - \overline{r}\right] \qquad (15)$$

This deadweight loss appears as the shaded area B in Figure 1.

Table 1, column (4) shows the adverse selection loss, simulated for decreasing levels of rent control. The parameters of demand and supply are $T = 1000$, $L = 1000$, $r_{max} = 20$, $c_{max} = 20$. Equilibrium rent in an unregulated market would be \$10 and the number of rentals would be 500. To make rents more realistic I could have assumed higher reservation prices and costs, but for the sake of minimizing on the number of zeros carried throughout the paper I have chosen these low figures. They are purely illustrative. When a rent control is imposed, the number of rentals can be read from the housing supply function (column 2). Triangle deadweight loss (column 5) rises with rent control, as we would expect from the well-known properties of this form of loss. The loss from adverse selection (column 4) at first rises, but then falls with the rent control. This is bound to happen. Adverse selection disappears when there are no rental units left. The incomes of tenants lucky enough to find rent controlled units rises by $H_s(R_{con}) \times (R_{free} - R_{con})$. Column (7) shows that the deadweight loss generated per dollar increase in incomes of tenants who found a rental unit, rises with rent control.

II. Evasion by Tenants

The deadweight loss from random adverse selection becomes more systematic when evasion comes into the picture. This is a feature not considered by Glaeser and Luttmer (1997) in their general equilibrium treatment of adverse selection loss. Evasion is a broad term that covers any behavior by tenants or landlords to get around the controlled rent. Landlords may evade by baldly raising their rents above the officially mandated ceiling, by charging supra-market rents on furnishings, and by accepting key money. Or they may neglect building codes and service standards. Tenants may evade by providing landlords with in-kind services such as properly maintaining their dwellings (see Olsen 1988), by or simply by being of a certain race or social standing which allows the landlord to indulge his taste for discrimination. It is unusual, and to my best knowledge, unheard of in the economic literature to treat tenants as the ones who evade the controlled rent. Both popular lore and academic research take it as a matter of fact that landlords are the ones who evade. The point does not seem controversial, and no effort has been made to see whether the identity of the evader matters. This indifference is perhaps due to the insight from public finance that the object of a tax has nothing to do with who ends up paying it. As this, and the following section will illustrate, no such irrelevance result holds for the case of rent controls.

Consider the case where the tenant is able to offer the landlord more than the controlled rent. As in the previous section a tenant can be thought of as drawing his or her reservation rent r from a range $[0, r_{max}]$. The new assumption is that the tenant can evade a part e of the controlled rent, so that the maximum price the tenant can bid for the rental unit is $R_{con} + e$. Evasive ability e is drawn from a uniform distribution over the range $[0\ , E]$. For example, a tenant may draw $r = 8$ and $e = 3$. If the controlled rent is \$4, then this tenant can bid up to \$7, which is above the controlled rent but less than his or her reservation rent. Evasive ability is beyond the tenant's control. As mentioned earlier, it may be that feature of the tenant which predisposes him to the landlord, or it may be his ability to provide the landlord with in-kind services. I assume that evasive ability and reservation rents are independently and uniformly distributed in order not to stack the results in favor of any particular outcome.

Supply and demand respond to an underground rent R_u. Tenants will use their evasive abilities to express their reservation rents. Equilibrium comes, as before, where supply equals demand and will produce an equilibrium underground rent of R_u^*. Supply is the same as in equation (6). Demand depends not only on whether a tenant's reservation rent exceeds the underground rent ($r \geq R_u$) but also on whether he or she has sufficient evasive talent to bid at least the underground rent ($R_{con} + e \geq R_u$). This means that demand is

$$
\begin{aligned}
H_d(R_u) &= T \times Pr(r \geq R_u) Pr(e \geq R_u - R_{con}) && (16)\\
&= T\left(1 - \int_o^{R_u} \frac{1}{r_{max}} dr\right)\left(1 - \int_0^{R_u - R_c} \frac{1}{E} de\right) && (17)\\
&= \frac{T}{E \times r_{max}}(r_{max} - R_u)(E + R_{con} - R_u) && (18)
\end{aligned}
$$

Equation (18) shows that the new demand curve is demand without evasion $T \times Pr(r \geq R_u)$ multiplied by a fraction less than one $Pr(e \geq R_u - R_{con})$ that accounts for evasive ability. This is not a demand curve as we usually understand it because the ordering of tenants from left to right on the horizontal axis is not based only on reservation rents, but on a combination of reservation rents and an ability to avoid rent control.

The reader may verify that when the underground rent is equal to the controlled rent, the above demand equation reduces to the demand equation in an unregulated market. Figure 2 shows the demand curve without rent control (the same demand equation as in Figure 1), and the demand curve for a controlled rent of \$5, and maximum evasive ability of E = \$10. The maximum rent that anyone will bid is the controlled rent plus the maximum evasive ability in the population of potential tenants E. This means the intercept of the controlled rent demand curve is R_{con} + E = \$15. The equilibrium underground rent R_u^* is above the controlled rent but below the free market rent. For lower levels of controlled rent the demand curve bows further inward and a lower underground rent establishes equilibrium. At underground rents equal to or lower than the controlled rent, evasion becomes irrelevant and both demand curves merge. The bowed demand curve reflects that when tenants can avoid part of the controlled rent, the controlled rent becomes "woven" into the demand curve, as one of its parameters. This is what leads to a deadweight loss from adverse selection.

As in section II, the deadweight loss from rent control has two components. The triangle loss described in the previous section, and the loss from the adverse selection of tenants. The adverse selection in this case though arises out of circumstance that differ from those of section II. Now, some tenants with high reservation rents will not be able to bid for a rental unit at the underground equilibrium rent R_u^* because they have very poor evasive skills. Such tenants will be pushed out of the running by tenants who derive less surplus from renting but are endowed with greater evasive abilities. The deadweight loss from adverse selection is the difference between what would have been the benefit to tenants of renting the equilibrium number of units H_u^* in Figure 2 had there been no rent control and the actual benefit to those who wind up getting the H_u^* units. Without rent control, the benefit to tenants would have been the area under the demand curve without evasion in the range [0 , H_u^*]. This term was derived in equation (14) of section II and takes the form:

$$\int_0^{H_u^*} r_{max}\left(1-\frac{H}{T}\right)dH = r_{max}H_u^*\left(-\frac{1}{2T}H_u^*\right) \tag{19}$$

The benefit tenants actually end up getting falls out of a lottery in which tenants in the range of reservation rents $[R_{con}, R_u]$ each have an equal chance of getting a rental unit. Call the average reservation rent to this group $\overline{r}$. The total benefit to this group is the amount of housing they consume times their average reservation rents $H_u^*\overline{r}$:

$$H_u^*\overline{r} = H_u^*\int_{R_{con}}^{R_u}\frac{1}{R_u - R_{con}}r\,dr \tag{20}$$

$$= \frac{H_u^*}{2}\frac{R_u^2 - R_c^2}{R_u - R_c} \tag{21}$$

The deadweight loss due to adverse selection is the difference between benefits without adverse selection and benefit with adverse selection, namely the difference between equations (19) and (20):

$$H_u^*\left[r_{max}\left(1-\frac{H_u^*}{2T}\right)-\overline{r}\right] \tag{22}$$

This deadweight loss has exactly the same functional form as adverse selection loss when there was no evasive ability for which to account (equation 15). The expressions differ in the ranges of reservation rents on which each is based. Without evasion, the lottery for apartments is open to a higher end of reservation wages than without evasion. The upper bound on evasive ability limits the potential benefits of the lottery by excluding the high reservation rent tenants.

Table 2 shows how deadweight losses and rentals change as the controlled rent falls. The parameters are the same as in the no evasion example with the addition of an upper bound on evasive skills of $E = 10$. The controlled rent can fall to as low as \$0 without closing down the market. As it falls, so does the underground rent as shown in column (2). The costs of redistribution are larger in this case than in the first section where evasion was not possible. When controlled rent is \$5, 407 units are rented and each dollar of income is redistributed at a deadweight loss of \$5.26. In the case without evasion, when 400 units were rented under a rent ceiling of \$8, the deadweight loss was only \$1.25. The reason for the higher deadweight loss is that under evasion, the lottery now depends not just on whether your reservation rent is above the controlled rent, but also on your evasive skills. Those with high reservation rents, but poor evasive skills are barred from this lottery.

IV. Evasion by Landlords

When evasive abilities are distributed among landlords, rent control affects housing supply and deadweight loss differently than when evasive abilities are distributed among tenants. The main result is that a range of rents will develop, some tenants will end up paying more than they did without rent control, and there will be an adverse selection of both tenants and landlords. It is even possible under certain distributions of evasive talents that the controlled rent *increases* the availability of rental units above the free market level.

To understand the intuition for the result that a controlled rent in general may reduce rental units by very little, or may even increase rental units in special circumstances, it first helps to take an extreme, and elementary example. Consider again Figure 1, which portrays ordinary supply and demand curves for rental units. The regular result is that equilibrium is the place where the two curves meet- --at a rental units level of 500 (recall that there are 1000 tenants and 1000 landlords). Reservation rent in such a market could be greater than this. Suppose the tenant with the highest reservation rent were matched to the landlord with the highest cost. Match the tenant with the next highest reservation rent to the landlord with the next highest cost. Keep doing this and what you get is that all 1000 landlords rent out and 1000 tenants rent. Rental units are many, but the surplus produced by this market is zero. Matching of this sort violates what markets are supposed to do, namely, to maximize economic surplus. Surplus is maximized by grouping people who value highly the good or service in question with people who can supply it efficiently. The matching scheme I have just outlined produces a lot of rentals but makes a mess of grouping people efficiently. This example is extreme because it rests on the assumption that tenants with the highest reservation rents get the highest cost rental units. In the less extreme example I develop through the rest of this paper, all tenants have an equal shot at the low cost rentals. This still generates some of the price discrimination described above, but mismatching does not go haywire to the point where all surplus is dissipated. The result is that under very general circumstances, the controlled rent may put much less of a dent in rental units than the standard analysis of rental units markets suggests.

It is important to note here that in contrast to previous efforts to model the evasion of price ceilings, the present model leads to a single underground market in which all transactions are carried out at a single price. There is no legal market in my model which anyone respects. Browning and Culbertson (1974) consider a market in which only firms are punished for evading, all firms have the same evasive abilities, and the same good sells at a higher price in the black market than in the legal market. Raymon (1983) considers what happens under a price ceiling when producers are able to chose the quality of their product and when there is no entry or exit to the market. As mentioned earlier, changing quality can be seen as one way around rent- control. My method of modeling differs in that I take as given by how much landlords can vary quality and I consider the consequences of allowing these evasive abilities to differ. Loikkanen (1985) models a landlord's decision to search for tenants of his pleasing. His analysis differs from mine in that he model's one individual's strategy. I model an entire market of such individuals. Note that landlord discrimination between tenants is really a form of evasion by tenants, because the market outcome depends on how their personal characteristics are distributed. The broader view I take of evasion encompasses the evasive abilities of landlords and tenants. As will be seen, who does the evading can lead to some striking contrasts in the types of equilibria generated.

1. The Punctured Demand Curve

How does equilibrium come about in the rental market with landlords who evade? We might guess that there will be one underground rent at which demand and supply of rental units are equal to each other. This guess would be wrong. Imagine gathering all tenants and landlords in the same room and telling them that only one underground rent will hold. An auctioneer then starts announcing rents and stops where demand is equal to supply. The problem with this auction is that after the hammer falls there will be landlords with costs below the underground rent whose apartments go unrented because they have poor evasive abilities. These frustrated landlords are bound to be very popular with tenants. Tenants will at first not even bother to participate in the auction. Instead $H_d(R_{con})$ of them will flock to landlords with the poorest evasive abilities and the lowest costs (e = 0 and $c \leq R_{con}$). The lowest rent being offered will be the controlled rent (at lower rents than this, evasive ability is irrelevant and landlords are able to bargain with tenants for higher rents). The proportion of landlords offering their services at R_{con} will be the proportion whose cost is lower than the controlled rent $Pr(c \leq R_{con})$ times the frequency $f(e) = 1/E$ of landlords having precisely no evasive abilities in a narrow band de. Call this proportion $\alpha(e)$. The total number of tenants to get units with low rental firms who have zero evasive abilities ($e = 0$) is $L\alpha(0)$:

$$L\alpha(0) = L \times Pr(c \leq R_{con}) f(e) de \tag{23}$$

$$= \frac{L}{E}\frac{R_{con}}{c_{max}} de \tag{24}$$

The remaining tenants flock to landlords with slightly better evasive abilities ϵ to pay an underground rent of $R_{con} + \epsilon$. To figure out how many such tenants make these demands we have to note that holes have been punched into the demand curve at even intervals between reservation rents of r_{max} to R_{con}. The weight of these holes in the demand curve is the fraction of tenants who found units with landlords charging R_{con}, namely $L\alpha(0)$, to the number of tenants who tried to get these units $H_d(R_{con})$. So rental units demand is now

$$\text{É}_1(R_{con} + \epsilon) = \left[1 - \frac{L\alpha(0)}{H_d(R_{con})}\right] H_d(R_{con} + \epsilon) \tag{25}$$

Note that the new "punctured" demand curve is the regular demand curve, multiplied by a factor less than one which accounts for the attrition of tenants along the length of reservation rents between $[R_{con}, r_{max}]$ who managed to get a rental unit with a low charging, zero evasive ability landlord. I call the new "punctured" demand curve after this first iteration É $_1$. The subscript is very important to understand. Further iterations raise the subscript and change the functional form of demand. This means that É $_1$ and É$_2$ are different. Their relation lies in the recursiveness that follows from the exercise of calculating demand at ever greater increments of ϵ until the epsilons sum to e. The final iteration gives rental unit demand at $R_{con} + e$.

Next, the remaining excess demand of tenants focuses on landlords offering 2 ϵ more than the controlled rent. How many tenants? Once again we have to note that some of the tenants who earlier flocked to landlords charging $R_{con} + \epsilon$ were successful at finding a rental unit. The fraction of such successes was $L\alpha(\epsilon)/$É$_1(R_{con} + \epsilon)$ of the demand curve É$_1(R_{con} + \epsilon)$. It is by this fraction that we must multiply what was left of the demand curve after tenants flocked to landlords offering R_{con}, to give us the proportion of the demand curve that has been punctured away by the time remaining tenants make their demands at $R_{con} + 2\epsilon$. This second iteration of the punctured demand curve has the following form:

$$\text{É}_2(R_{con} + 2\epsilon) = \left[1 - \frac{L\alpha(\epsilon)}{\text{É}_1(R_{con} + \epsilon)}\right] \text{É}_1(R_{con} + 2\epsilon) \tag{26}$$

After n iterations, punctured rental units demand is

$$\text{É}_n(R_{con} + n\epsilon) = \left[1 - \frac{L\alpha((n-1)\epsilon)}{\text{É}_{n-1}(R_{con} + (n-1)\epsilon)}\right] \text{É}_{n-1}(R_{con} + n\epsilon) \tag{27}$$

Demand É(R_u) at some underground rent $R_u = R_{con} + e$ is the limit of the above term as n tends to infinity and $n\epsilon$ tends to e. This produces a hybrid of what we would ordinarily consider the demand equation for rental units, with a factor that accounts for the attrition of tenants of different reservation rents being plucked randomly from the range of tenants bidding for rental units.

2. Equilibrium Range of Rents

Equilibrium comes at the critical underground rent R_u^* (or identically at the critical sum of epsilons added to the controlled rent) where there is no further excess demand. Calculating this equilibrium is a simple matter of equating the punctured demand curve É(R_u) expressed in equation (27) to the supply curve. The supply curve is made up of landlords whose costs are below the underground rent $c \leq R_u$ and whose evasive abilities are strong enough to let them charge the underground rent $R_{con} + e \geq R_u$:

$$\begin{aligned} H_s(R_u) &= L \times Pr(c \leq R_u) Pr(e \geq R_u - R_{con}) && (28) \\ &= L \times \int_0^{R_u} \frac{1}{c_{max}} dc \times \left(1 - \int_0^{R_u - R_{con}} \frac{1}{E} de\right) && (29) \\ &= \frac{L}{c_{max} E} R_u (E + R_{con} - R_u) && (30) \end{aligned}$$

The equilibrium will have the feature that there is a range of underground rents from R_{con} to R_u^* offered by different landlords, and then one rent R_u^* offered by the remainder of landlords. This remainder is the group who have among them the highest evasive talents and are left to bid down against one another, once the low cost, evasively inept landlords have all rented their units to the tenants who flocked to them in an excess of demand. Total units rented is calculated as $H_s(R_u^*)$, the units rented by landlords in the group who all face the same rent, plus Ë(R_u^*), the sum of rentals by landlords with low (hence the symbol Ë) evasive abilities who faced a range of rents and were responsible for puncturing the demand curve. These landlords operated in the range of rents [R_{con}, R_u^*]. Their number depends on the proportion who have costs below any particular rent in this range $c \leq R$ and the proportion who have evasive abilities such that R_u is the maximum they can charge $R_{con} + e = R_u$:

$$\begin{aligned} \text{Ë}(R_u) &= L \int_{R_{con}}^{R_u} Pr(c \leq R_u) f(R_u - R_c) dc && (31) \\ &= L \int_{R_{con}}^{R_u} \frac{1}{c_{max}} \frac{1}{E} R_u dR_u && (32) \end{aligned}$$

$$= \frac{L}{2c_{max}E}(R_u^2 - R_{con}^2) \tag{33}$$

The above can be recognized as the number of landlords L times the integral of α with the limits of integration recast into rents instead of evasive abilities. The result is identical in either case. Provided that demand and supply meet, total rental units at the equilibrium underground rent is $H_s(R_u^*) + \ddot{E}(R_u^*)$. If the controlled rent is very low relative to evasive abilities, there will be no equilibrium rent at which demand and supply cross, only an equilibrium upper bound $R_{con}+E$ to the above integral, at which all landlords who participate in the market will have been faced by an excess demand for rental units. In that case, rentals are simply $\ddot{E}(R_{con} + E)$. These points are illustrated in the examples that follow.

3. Two Examples of Equilibrium

There is no obvious solution to the recursion relation for punctured demand. Appendix I shows that a lower bound to punctured demand at some rent R_u is simply the ordinary "unpunctured" demand curve at that rent less the number of tenants who paid within the range $[R_{con}, R_u^*]$. I explain there why this lower bound produces underestimates of equilibrium rental units. It is these underestimates, detailed in Table 3, I use in the remaining discussion. The use of underestimates will overstate the degree to which rent-control reduces rents. As I discuss in the Appendix though, these understatements are of a second order of importance.

Figure 3 looks at the equilibria that fall out of controlled rents of \$9 and \$1 (the demand curve used is the underestimate of demand described in Appendix I). The supply curve in Figure 3 is taken from equation (30), with the same parameters used in the previous sections. Recall that without a controlled rent, market equilibrium would produce a rent of \$10 and rental units of 500. We can see that supply is backward bending. The upward sloping part of the curve at rents below R_{con} is the piece of the "ordinary" supply curve below the controlled rent, as given by equation (6). The downward sloping part of the supply curve comes from equation (30) and incorporates the fact that above the controlled rent some landlords can evade. When the controlled rent rises in this range, two contradictory forces bear down on supply. Some landlords withdraw their supply because the rent is higher than their evasive ability. Other landlords with high costs but good evasive abilities come onto the market. Eventually the first effect dominates and higher prices disqualify suppliers because they are not sufficiently able to avoid the controlled rent. This is when the supply curve begins to bend backward. We normally understand a supply equation to be an ordering of landlords according to their costs. The upper part of the supply equation in Figure 3 is a mix of cost and *ability to evade* the controlled rent. This is what gives the upper part of the supply curve its downward slope.

At a controlled rent of \$9 (recall that the free market rent is \$10), demand and supply meet and as Figure 3(a) shows, the upper bound of underground rents R_u^* is \$10.03. Equilibrium rental units in this case are found by plugging \$10.03 into the supply function for rental units, $H_s(10.03) = 49$, and adding to this the number of landlords Ë(10 .03) = 450 who were constrained to accept lower rents spanning from R_{con} to R_u^*. The result is a rental units level of 499. This total cannot be read off the diagram because, as explained, the crossing of supply and demand determine only the cutoff rent R_u^* and the resulting amount supplied (49) by landlords who do not face an excess demand from tenants. The total is tantalizingly close to the free market rental units level of 500. Referring to Table 1 we see that without evasion, only 450 units of housing would be supplied and that if tenants evaded, at controlled rent of \$9 only 483 units would have been supplied.

When R_{con} = 1 the shape of both supply and demand curves changes. The change reflects that the controlled rent is a parameter woven not just into the supply curve, but also into the punctured demand curve. In Figure 3(b), where $R_{con} = 1$, supply and demand no longer meet. The fall in the controlled rent has created a situation where no rise in the underground rent will reduce the excess demand for rental units over some positive level of supply. In this case rental units are just $\ddot{E}(R_{con} + E) = 300$, which cannot be read from the graph. What takes some getting used to is that there is an equilibrium *even though demand and supply do not cross*. What also takes getting used to is that there is an equilibrium *range* of rents. In columns (1) and (2) of Table 3 we see a sequence of controlled rents, and the upper bounds of rents these imply. The highest of these upper bounds is \$12.95 (see column 2) and occur when the minimum rent is \$12.50. For any controlled rent lower than this, demand and supply do not meet, which explains why at $R_{con} = 1$ the curves demand and supply curves part ways entirely.

4. Adverse Selection

The low drop in rental units comes at the cost of adverse selection. Think again of the example of forced matching at the beginning of this section. In such a case all tenants are housed, but no surplus is created. The case under consideration is similar. As in the previous section, there is adverse selection of tenants, but now there is also adverse selection of landlords. Let us start with landlords first. The trick as in the case of evasion by tenants is to calculate the actual cost incurred by landlords and compare it to what the cost would have been without adverse selection. The actual cost has two components. First there is the cost generated by landlords to whom tenants flock in excess demand. At each underground rent level R_u between $[R_{con}, R_u^*]$ there are landlords in the cost range $[0, R_u]$ who *may* produce, as highlighted in equation (31). The proportion of all such landlords whose evasive abilities is precisely R is $1/E$. The costs of these landlords are

$$L\int_0^{R_u}\frac{1}{c_{max}}\frac{1}{E}cdc=\frac{L}{2c_{max}E}R_u^2 \tag{34}$$

The above is a small part of the total number of landlords finding themselves beset by an excess of tenants. These tenants pay rents rising in the range [R_{con}, R_u^*], so that the total costs of landlords serving this excess demand are the integral of the equation (34) over the range $[R_{con}, R_u^*]$:

$$\int_{R_{con}}^{R_u^*}\frac{L}{2c_{max}E}R_u^2dR_u=\frac{L}{6c_{max}E}(R_u^{*3}-R_{con}^3) \tag{35}$$

The second component of costs comes from those landlords who face a dearth of tenants and all rent at the same cut-off underground rent R_u^*. All of these landlords have costs less than R_u^* and some range as high as low as 0. Their combined costs summed over this range and weighted by the proportion who are sufficiently able to evade the controlled rent $(R_{con}+e\geq R_u)$ comes to

$$L\int_0^{R_u^*}\frac{1}{c_{max}}Pr(R_{con}+e\geq R_u^*)cdc \tag{36}$$

$$=\int_0^{R_u^*}\frac{L(E+R_{con}-R_u^*)}{c_{max}E}cdc \tag{37}$$

$$=\frac{L}{2c_{max}E}(E+R_{con}-R_u^*)R_u^{*2} \tag{38}$$

Equations (35) and (38) are added to get actual cost. The cost that would have been incurred by the same number of landlords in an unregulated market is the integral of the height of the supply curve over the range of rental units [0 , $H_s(R_u^*)+\ddot{E}(R_u^*)$]. As shown earlier, the height of the supply curve is $H\times c_{max}/L$ so that the minimum cost way of producing $H_s+\ddot{E}$ units of housing is

$$\int_0^{H_s+\ddot{E}}\frac{c_{max}}{L}HdH=\frac{c_{max}}{2L}(H_s+\ddot{E})^2 \tag{39}$$

The difference between actual and minimum cost appears in column (6) of Table 3. This is the deadweight loss from the adverse selection of landlords.

The cost of adverse selection of tenants is calculated in a similar manner as above. First sum the reservation rents of tenants who obtained a unit to get their total benefit from renting. Then compare this to the sum of reservation rents for a similar level of rental units in an unregulated market---which should be larger than the sum of actual reservation rents. The sum of reservation rents has two components. Some of the tenants who flocked in excess to low charging landlords will have found rentals in the range of prices [R_{con}, R_u^*]. At each level of rent R_u in this range there are, as explained earlier in reference to equation (31), $L \times Pr(c \leq R) f(R_u - R_{con}) dR_u$ such landlords. The average reservation rent of tenants flocking to these landlords will be

$$\int_{R_u}^{r_{max}} \frac{1}{r_{max} - R_u} r dr = \frac{r_{max}^2 - R_u^2}{2(r_{max} - R_u)} \tag{40}$$

The sum of such reservation rents is then taken over all *landlords* who found themselves beset by an excess of demand by tenants. Recall that these tenants paid rents in the range $[R_{con}, R_u^*]$. The total reservation rents of this group are

$$\int_{R_{con}}^{R_u^*} L \times Pr(c \leq R_u) f(R_u - R_{con}) \frac{r_{max}^2 - R_u^2}{2(r_{max} - R_u)} dR_u \tag{41}$$

$$= \int_{R_{con}}^{R_u^*} L \frac{R_u}{c_{max}} \frac{1}{E} \frac{r_{max}^2 - R_u^2}{2(r_{max} - R_u)} dR_u \tag{42}$$

$$= \frac{L}{2c_{max}E} \int_{R_{con}}^{R_u^*} \left(\frac{r_{max}^2 R_u}{r_{max} - r} - \frac{R_u^3}{r_{max} - R_u} \right) dR_u \tag{43}$$

Equation (43) is easily solved by reference to a table of integrals, but the expression is lengthy and is not presented here. It is however spelled out, in the spreadsheet used for simulations in the present paper and available from the author on request.

The second component of reservation rents is that of tenants who together all bid R_u^* and together rent $H_s(R_u^*)$ units. Their reservation rents lie in the range [R_u^*, r_{max}] so that the total benefit that they derive from renting is:

$$H_s(R_u^*)\int_{R_u^*}^{r_{max}} \frac{1}{r_{max}-R_u^*}dR_u = H_s(R_u^*)\frac{r_{max}^2 - R_u^{*2}}{2(r_{max}-R_u^*)} \tag{44}$$

The two above components are summed to get total benefit to tenants. We then subtract this actual benefit from the benefit that would have been derived by tenants selected to live in the first H_s + Ë units of housing in a market without rent controls. This benefit was derived in an earlier section and is represented by equation (14). The resulting difference is social loss from the adverse selection of tenants and is represented in column (7) of Table 3 for varying levels of the controlled rent. The reader will note that the sum of adverse selection and triangle deadweight losses as given column (9) of Table 3 are similar to their counterparts in column (7) of Table 2, where tenants evaded. This is not a meaningful comparison. A better idea of the social cost of the controlled rent is had when we put it in relation to how much the incomes of tenants rise. This is the purpose of the following section.

5. Income Redistribution

One of the striking things about equilibrium, as Figure 4 illustrates schematically, is that it takes place over a range of rents. There is an equilibrium price dispersion that has nothing to do with search, or uncertainty about the parameters of demand and supply. Prices are dispersed because the differing evasive abilities of landlords interact with the controlled rent in such a way as to give tenants some ability to act as price discriminating monopsonists. An equilibrium of this sort has a surprising consequence for the incomes of tenants. As Table 3 showed, provided evasive talents are large enough (so that $R_{con}+E$ is greater than the free market rent), the equilibrium range of rents [R_{con}, R_u^*] rises above the free market rent. In other words, some tenants "lucky" enough to find a rental unit may discover they are paying rents above what they would be paying in a free market. Nothing like this happened when evasive talents were distributed among tenants. The intuition is that tenants lucky enough to get into low rent units are drawn from a broader range of reservation rents than would be the case without the controlled rent. Some tenants in low rent units will be from the low end of this broadened band of reservation rents. This crowds high some higher reservation rent tenants into the contest for higher cost rental units, where they cause an excess of demand. The excess pushes rents for the remaining tenants upwards, and above what rents would be in a free market.

Having tenants who pay more than what they would pay in a free market means that the deadweight losses generated by the controlled rent may be high relative to the amount of redistribution achieved. But what is the amount of redistribution achieved? The standard way of proceeding, as explained by Gyourko and Linneman (1989), is to estimate a hedonic rent function for the uncontrolled sector. This function relates the characteristics of rental units Z, and length of tenant occupancy T, to the final rental price R. In their notation, in the uncontrolled sector, for any given tenant

$$R = h(Z, T; B) + e \tag{45}$$

Where e is an error term. The trick is to estimate the regression parameter vector B for the uncontrolled sector and then plug in characteristics of the controlled sector to get at R : what rents would have been in the controlled sector without rent-control. The resultant rise in the income of tenants who manage to get rental units is ($R - R_{con}) \times H_s(R_{con})$.

The analysis of non-compliance by landlords suggest that the above approach has to be modified. Not all tenants who get apartments will experience a fall in their rental rates. As we saw in the previous section, the rents of some tenants *increase*. First though let us examine how the incomes change of those tenants lucky enough to find units renting for an underground rent less than the free market rent. These tenants rent units in the rent range

$[R_{con}, R_{free}]$ (where R_{free} is what the rent would be in a free market). The formula for calculating the rise in income for the group is

$$\underbrace{R_{free}\int_{R_{con}}^{R_{free}} L\alpha(R_u)dR_u}_{free \quad market \quad payment} - \underbrace{\int_{R_{con}}^{R_{free}} L\alpha(R_u)R_u dR_u}_{actual \quad payment} \tag{46}$$

$$= R_{free}\int_{R_{con}}^{R_{free}} L\frac{1}{c_{max}}\frac{1}{E}R_u dR_u - \int_{R_{con}}^{R_{free}} L\frac{1}{c_{max}}\frac{1}{E}R_u^2 dR_u \tag{47}$$

$$= \frac{L}{c_{max}E}\left[\frac{1}{2}(R_{free}^3 - R_{free}R_{con}^2) - \frac{1}{3}(R_{free}^3 - R_{con}^3)\right] \tag{48}$$

The first term of (46) is the amount of rent tenants would have paid in a free market. The number of tenants is calculated by noting that they lie in the reservation rent range $[R_{con}, R_{free}]$ and that this is a subset of the range $[R_{con}, R_u^*]$ in which an excess demand of tenants flocked to landlords. This is why the α term becomes pertinent. Recall that $L\alpha(R_u)$ is the number of landlords forced to accept exactly R_u, where $R_u \leq R_u^*$. We sum these landlords over the range $[R_{con}, R_{free}]$ to get the total number of tenants in this range (see column 2 of Table 4). Had they each been paying the free market rent, the first term then represents what their total payments would have been. The second term is what their payments actually are. Column (3) of Table 4 shows the rents of tenants averaged in the range $[R_{con}, R_{free}]$. Note that this average lies below the free market rent. The difference between the first and second terms is then the income gain to this group of lucky tenants.

Some tenants lose from rent control. These are the ones who end up paying more than the free market rent. Their rents lie in the range $[R_{free}, R_u^*]$ (we know that $R_u^* > R_{free}$ because the maximum evasive ability of landlords in the simulations presented in Tables 3 and 4 is $R_{con} + E$ = \$15 which exceeds the free market rent of \$10). Their loss is the difference between what they would have paid for their rental units under the free market rent (the first term of the equation below) and what they actually end up paying (the second term of the equation below):

$$\underbrace{\left[\int_{R_{free}}^{R_u^*} L\alpha(R_u)R_u dR_u + R_u^* H_s(R_u^*)\right]}_{actual \quad payment} - \underbrace{R_{free}\left[\int_{R_{free}}^{R_u^*} L\alpha(R_u)dR_u + H_s(R_u^*)\right]}_{free \quad market \quad payment} \tag{49}$$

$$= \quad \frac{L}{c_{max}E}[\frac{1}{3}(R_u^{*3} - R_{free}^3) - \frac{R_{free}}{2}(R_u^{*2} - R_{free}^2) + (R_u^{*2} - R_u^* R_{free})(E + R_{con} - R_u^*)]$$

The first term of actual payment is the total amount paid by tenants who rented from those landlords beset by an excess demand who rented in the range [R_{free}, R_u^*]. The second term of actual payment is what the remaining tenants paid to landlords facing an excess supply unless they all kept their rents at R_u^*. Column (5) of Table 4 shows the average rent paid by these tenants for different levels of the controlled rent. Note that this average payment is above the free market rent and that as the controlled rent rises, the average payment of these tenants rises. The reasons for this were discussed earlier. Free market payments are simply what the free market rent would have been, multiplied by the actual number of tenants (see column 4 of Table 4). Column (8) of Table 4 shows the deadweight loss per dollar of redistribution for different levels of the controlled rent. For comparable levels of output, this column shows lower costs of redistribution per dollar redistributed than when tenants evaded (Table 2). The intuition is that when producers evade, those with low costs but poor evasive abilities are not excluded from the market but instead are the object of many renters' efforts to rent. Some exclusion does take place, but that is for those landlords all renting at R_u^*. In the case where tenants evaded, those with high reservation rents but poor evasive abilities were excluded from the market. The deadweight losses are always higher when producers evade than in the case where no evasion was possible (Table 1). The reason is that some exclusion of low cost landlords does take place, whereas in the case of no evasion, all tenants with reservation rents above the controlled rent could present themselves for the rental lottery.

VI. Conclusion

This paper has presented a model of the rental units market in which either tenants or landlords evade a part or all of the controlled rent. Three results emerged.

- *Rental units effects:* Under the most general assumptions about evasive abilities, the controlled rent at levels close to the free market rent has minimal effect on rental units. Rental units are resistant to the controlled rent because in the presence of landlords with differing abilities to evade, the controlled rent allows tenants to act as if they were part of a price discriminating monopsonistic collective. This leads to an equilibrium dispersion of rents.

- *Income redistribution*: When landlords evade, market equilibrium is not represented by a single rent, but by a *range* of rents. Price dispersion has nothing to do with search costs or imperfect information about demand and supply parameters. Some rents in the range lie *above* what rents would be in a free market. Tenants who find rental units in the upper end of rents will find that their incomes have dropped from what they were before the arrival of the controlled rent.

- *Deadweight loss:* The controlled rent creates a deadweight loss from the adverse selection of landlords and tenants. Survival in the model presented here depends not just on having a low cost, or having high reservation rent. Survival depends also on how well one can avoid the controlled rent. This second criterion for market selection is the cause of adverse selection.

Wherever there is a deadweight loss, an institution lurks that tries to moderate the damage. Traditional triangle losses are inevitable in the struggle to redistribute money between interest groups. The only way to reduce these losses in the rental market is to loosen the controlled rent. The deadweight losses from adverse selection are not inevitable. These can be shrunk without changing the controlled rent. In this sense, adverse selection deadweight losses are "non-antagonistic." Future research should ask what housing market rules could lessen non-antagonistic losses. A law giving incumbent tenants priority to rental units might help. Tenants in the housing market before rent control were those with the highest reservation rents. Adverse selection is created in part by new tenants drawn to the market by the lottery for a low-rent unit.

APPENDIX

The purpose of this appendix is to derive a lower bound to the exact expression for rental units demand and to develop a technique for more closely approximating rental units demand than this lower bound allows. The need to work with such bounds comes out of equation (27) in the main body of the text:

$$\acute{E}_n(R_{con} + n\epsilon) = \left[1 - \frac{L\alpha((n-1)\epsilon)}{\acute{E}_{n-1}(R_{con} + (n-1)\epsilon)}\right] \acute{E}_{n-1}(R_{con} + n\epsilon) \tag{50}$$

Let us take the first four terms of this expression:

$$\acute{E}_0(R_{con}) = H_d(R_{con}) \tag{51}$$

$$\acute{E}_1(R_{con} + \epsilon) = \left[1 - \frac{\alpha(1)L}{\acute{E}_0(R_{con})}\right] \acute{E}_0(R_{con} + \epsilon) \tag{52}$$

$$= \acute{E}_0(R_{con} + \epsilon) - \alpha(1)L\frac{\acute{E}_0(R_{con} + \epsilon)}{\acute{E}_0(R_{con})} \tag{53}$$

$$\acute{E}_2(R_{con} + 2\epsilon) = \left[1 - \frac{\alpha(2)L}{\acute{E}_1(R_{con} + \epsilon)}\right] \acute{E}_1(R_{con} + 2\epsilon) \tag{54}$$

$$= \acute{E}_1(R_{con} + 2\epsilon) - \alpha(2)L\frac{\acute{E}_1(R_{con} + 2\epsilon)}{\acute{E}_1(R_{con} + \epsilon)} \tag{55}$$

$$\acute{E}_3(R_{con} + 3\epsilon) = \left[1 - \frac{\alpha(3)L}{\acute{E}_2(R_{con} + 2\epsilon)}\right] \acute{E}_2(R_{con} + 3\epsilon) \tag{56}$$

$$= \acute{E}_2(R_{con} + 3\epsilon) - \alpha(3)L\frac{\acute{E}_2(R_{con} + 3\epsilon)}{\acute{E}_2(R_{con} + 2\epsilon)} \tag{57}$$

The ratio of É′s on the right hand side of the above expressions is less than one. If we were to replace this ratio by one, then each of the above expressions would be an underestimate of supply. With this modification the above could be written out as

$$\text{É}_0(R_{con}) = H_d(R_{con}) \tag{58}$$

$$\text{É}_1(R_{con}+\epsilon) = \text{É}_0(R_{con}+\epsilon) - \alpha(0)L \tag{59}$$

$$\text{É}_2(R_{con}+2\epsilon) = \text{É}_1(R_{con}+2\epsilon) - \alpha(1)L \tag{60}$$

$$\text{É}_3(R_{con}+3\epsilon) = \text{É}_2(R_{con}+3\epsilon) - \alpha(2)L \tag{61}$$

Repeated substitutions show that

$$\text{É}_3(R_{con}+3\epsilon) = H_d(R_{con}+3\epsilon) - \alpha(0)L - \alpha(1)L - \alpha(2)L - \alpha(3)L \tag{62}$$

If we iterated this process infinitely until the epsilons summed to some non- infinitesimal number e', we would find the underestimate of the punctured supply curve to be

$$\text{É}(R_{con}+e') \geq H_d(R_{con}+e') - L\int_{R_{con}}^{R_{con}+e'} \alpha(e)de \tag{63}$$

The final term in the above expression is simply the amount of rental units employed by tenants who are the subjects of price-discrimination. In the text an operator $H(R_{con}+e')$ was derived for the amount of rental units hired by these tenants. What this says is that our underestimate of the punctured supply curve is the unpunctured supply curve, less the number of landlords who represent the sum of punctures at some underground rent $r = R_{con} + e$.

$$\text{É}(R_u^*) \geq H_d(R_u^*) - \int_{R_{con}}^{R_u^*} \alpha(R_u)dR_u \tag{64}$$

The critical R_u^* at which supply and demand cross is calculated by equating the above expression with supply as defined by equation (30) and solving for the rent which is the upper bound of rents offered to landlords to whom an excess of tenants flock. By using this underestimate of demand we can expect the above approximation of the demand curve to show the controlled rent having a more adverse effect on rental than it really does. In other words, the estimates in the paper of the negative effect of rent- control on rental supply are over-estimates.

Another way of estimating rental units demand is to treat the epsilon terms as discrete units. If I want to estimate rental units demand at $R_{con}+e$ I can divide e into 40 parts Ée and take 40 iterations of the equation (27). This is easily done with a spreadsheet. The results produced (not shown here but available, with spreadsheet, on request) by this technique produce an equilibrium rental units demand of two or three landlords above the lower bound estimate of rental units supply. For some controlled rents rental units are above the free market level. This result cannot be taken to mean that the model shows that controlled rents can raise rental units above the free market level. Raising the iterations reduces the equilibrium rental units demand, and it is possible that with an infinite number of iterations, equilibrium rental units demand would be less than the free market level.

REFERENCES

Arnott, Richard (1995). "Time for Revisionism on Rent Control?" *Journal of Economic Perspectives*, 9:99-120.

Arnott, Richard, Russel Davidson, and Davide Pines (1983). "Housing Quality, Maintenance, and Rehabilitation." *Review of Economic Studies.* 50:467-494.

Browning, Edgar K. and William P. Culbertson (1974). "A Theory of Black Markets Under Price Control: Competition and Monopoly." *Economic Inquiry*, 12:175- 187.

Glaeser, Edward L. and Erzo F. Luttmer (1997). "The Misallocation of Housing Under Rent-Control." NBER Working Paper #6220.

Gyourko, Joseph and Peter Linneman (1989). "Equity and Efficiency Aspects of Rent Control: An Empirical Study of New York City." *Journal of Urban Economics*, 26:54-74.

Loikkanen, Heikki A. (1985). "On Availability Discrimination under Rent Control." *Scandinavian Journal of Economics*, 87:500-520.

Olsen, Edgar O. (1972). "An Econometric Analysis of Rent Control." *Journal of Political Economy*, 80:1081-1100.

Olsen, Edgar O. (1988). "What do Economists Know about the Effect of Rent Control on Housing Maintenance?" *Journal of Real Estate Finance and Economics*, 1:295-307.

Raymon, Neil (1983). "Price Ceilings in Competitive Markets with Variable Quality." *Journal of Public Economics*, 22:257-264.

TABLE 1

Deadweight Loss and Rental Effects of Controlled Rent R_{con}
Without Evasion

(1) R_{con}	(2) $H_d(R_{con})$	(3) $H_s(R_{con})$	(4) Adverse selection loss	(5) Triangle loss	(6) Total deadweight loss (4)+(5)	(7) Deadweight loss per dollar of income increase (6) $\div(R_{free} - R_{con})H_s(R_{con})$
10	500	500	-	-	-	-
9	550	450	450	50	500	1.11
8	600	400	800	200	1,000	1.25
7	650	350	1,050	450	1,500	1.43
6	700	300	1,200	800	2,000	1.67
5	750	250	1,250	1,250	2,500	2.00
4	800	200	1,200	1,800	3,000	2.50
3	850	150	1,050	2,450	3,500	3.33
2	900	100	800	3,200	4,000	5.00
1	950	50	450	4,050	4,500	10.00
0	998	3	25	4,950	4,975	200

Note: The parameters of supply and demand used in the above simulations are L = 1000, T == 1000, r_{max} = 20, c_{max} = 20. The rent that would prevail in an unregulated market is \$10 and the number of units rented would be 500.

TABLE 2

Deadweight Loss and Rental Effects of Controlled Rent R_{con} When Tenants Evade

(1) R_{con}	(2) R_u^*	(3) H_u^*	(4) Housing supply with no evasion	(5) Adverse selection loss	(6) Triangle loss	(7) Total deadweight loss (5)+(6)	(8) Income increase $(R_{free} - R_{con}) \times H_u^*$	(9) Deadweight loss per dollar of income increase (7) $\div$ (8)
10.00	10.00	500	500	0	0	0	0	0.00
9.00	9.66	483	450	2,821	6	2,827	165	17.17
8.00	9.30	465	400	3,115	24	3,139	324	9.68
7.00	8.93	447	350	3,380	57	3,437	477	7.20
6.00	8.54	427	300	3,612	106	3,719	622	5.98
5.00	8.14	407	250	3,809	173	3,983	757	5.26
4.00	7.72	386	200	3,968	261	4,228	881	4.80
3.00	7.28	364	150	4,084	370	4,454	990	4.50
2.00	6.82	341	100	4,154	505	4,659	1,084	4.30
1.00	6.35	317	50	4,175	666	4,841	1,159	4.18

Note: The parameters of supply, demand, and evasion used in the above simulations are T = 1000, L = 1000, r_{max} = 20, c_{max} = 20, E = 10. The rent that would prevail in an unregulated market R_{free} is \$10 and the number of units rented would be 500.The equilibrium underground rent is R_u, H_u is the number of equilibrium underground rental units.

TABLE 3

Deadweight Loss and Rental Effects of Controlled Rent R_{con}

When Landlords Evade

(1) R_{con}	(2) R_u^* (see note)	(3) Units offered by low charging landlords ($\ddot{E}(R_u^*)$ in equation 33)	(4) Units offered by high charging landlords (see equation 30)	(5) Total rental units (3)+(4)	(6) Adverse landlord selection loss	(7) Adverse tenant selection loss	(8) Triangle deadweight loss	(9) Total social loss (6)+(7)+(8)
10	10.00	500	0	500	0.00	0.00	0.00	0.00
9.50	10.01	475	25	500	-0.10	3.10	0.00	3.00
9.00	10.03	450	49	499	-0.83	12.30	0.03	11.50
8.50	10.06	425	72	497	-2.80	27.47	0.19	24.86
8.00	10.11	399	96	494	-6.60	48.50	0.62	42.52
7.50	10.18	373	118	491	-12.81	75.33	1.61	64.13
7.00	10.27	346	141	487	-21.93	107.91	3.56	89.53
6.50	10.38	318	164	481	-34.41	146.29	7.05	118.93
6.00	10.51	289	186	475	-50.53	190.59	12.91	152.97
5.50	10.67	258	209	467	-70.43	241.09	22.28	192.95
5.00	10.86	225	232	457	-93.92	298.22	36.80	241.10
4.50	11.08	189	256	446	-120.38	362.71	58.71	301.04
4.00	11.35	150	282	432	-148.52	435.68	91.25	378.41
3.50	11.67	107	310	417	-175.92	518.92	139.08	482.08
3.00	12.05	57	340	398	-198.34	615.22	209.21	626.09
2.50	12.50	0	375	375	-208.33	729.17	312.50	833.33
2.00	12.00	0	350	350	-208.33	841.67	450.00	1,083.33
1.50	11.50	0	325	325	-208.33	929.17	612.50	1,333.33
1.00	11.00	0	300	300	-208.33	991.67	800.00	1,583.33
0.50	10.50	0	275	275	-208.33	1,029.17	1,012.50	1,833.33
0.00	10.00	0	250	250	-208.33	1,041.67	1,250.00	2,083.33

Note: The parameters of supply and demand used in the above simulations are L = 1000, T = 1000, r_{max} = 20, c_{max} = 20. Below a controlled rent of \$12.50 there is no longer an equilibrium upper rent bound R_u which tenants are *willing* to offer. For lower controlled rents than \$12.50 all landlords face an excess demand of tenants. What then becomes relevant in determining how many landlords will offer rental units is the upper bound of possible rents they are *able* to offer, namely $R_{con} + E$. This is what appears in column (2) of the above table for values of $R_u <$ 12.50.

TABLE 4

Redistributive Effects of Rent Control when Landlords Evade

(1) R_{con}	(2) Number of Low paying tenants (see equation 45)	(3) Per capita average rent of low rent tenants	(4) Number of High rent tenants (see equation 46)	(5) Per capita average rent of high rent tenants	(6) Total rise in income of of lucky tenants	(7) Total fall in income of unlucky tenants	(8) Deadweight loss per dollar increase in income of lucky tenants
10.00	0	na	500	10.00	0.00	0.00	0.00
9.50	24.70	9.63	475.30	10.01	6.04	3.05	0.50
9.00	48.82	9.25	451.18	10.03	23.33	11.86	0.49
8.50	72.43	8.88	427.58	10.06	50.63	25.95	0.49
8.00	95.60	8.51	404.43	10.11	86.67	44.76	0.49
7.50	118.43	8.14	381.65	10.18	130.21	67.69	0.49
7.00	141.02	7.76	359.16	10.26	180.00	94.03	0.50
6.50	163.50	7.39	336.85	10.36	234.79	122.91	0.51
6.00	186.05	7.02	314.60	10.49	293.33	153.27	0.52
5.50	208.87	6.65	292.24	10.63	354.38	183.71	0.54
5.00	232.23	6.28	269.61	10.79	416.67	212.36	0.58
4.50	256.49	5.91	246.45	10.96	478.96	236.63	0.63
4.00	282.11	5.53	222.46	11.14	540.00	252.84	0.70
3.50	309.72	5.15	197.23	11.30	598.54	255.54	0.81
3.00	340.24	4.77	170.22	11.39	653.33	236.46	0.96
2.50	375.00	4.38	140.63	11.30	703.13	182.29	1.19
2.00	350.00	4.72	110.00	11.03	746.67	113.33	1.45
1.50	325.00	5.11	80.63	10.77	782.71	61.88	1.70
1.00	300.00	5.55	52.50	10.51	810.00	26.67	1.95
0.50	275.00	6.06	25.63	10.25	827.29	6.46	2.22
0.00	250.00	6.67	0.00	na	833.33	0.00	2.50

Note: The parameters of supply and demand used in the above simulations are T = 1000, L = 1000, r_{max} = 20, c_{min} = 20. Lucky tenants are defined as those whose rents fall after the introduction of rent control. Unlucky tenants are those who can only find rentals at an underground rent greater than the free market rent once rent control is imposed.

Figure 1: Deadweight Loss and Rental Effects of Rent Control: No Evasion

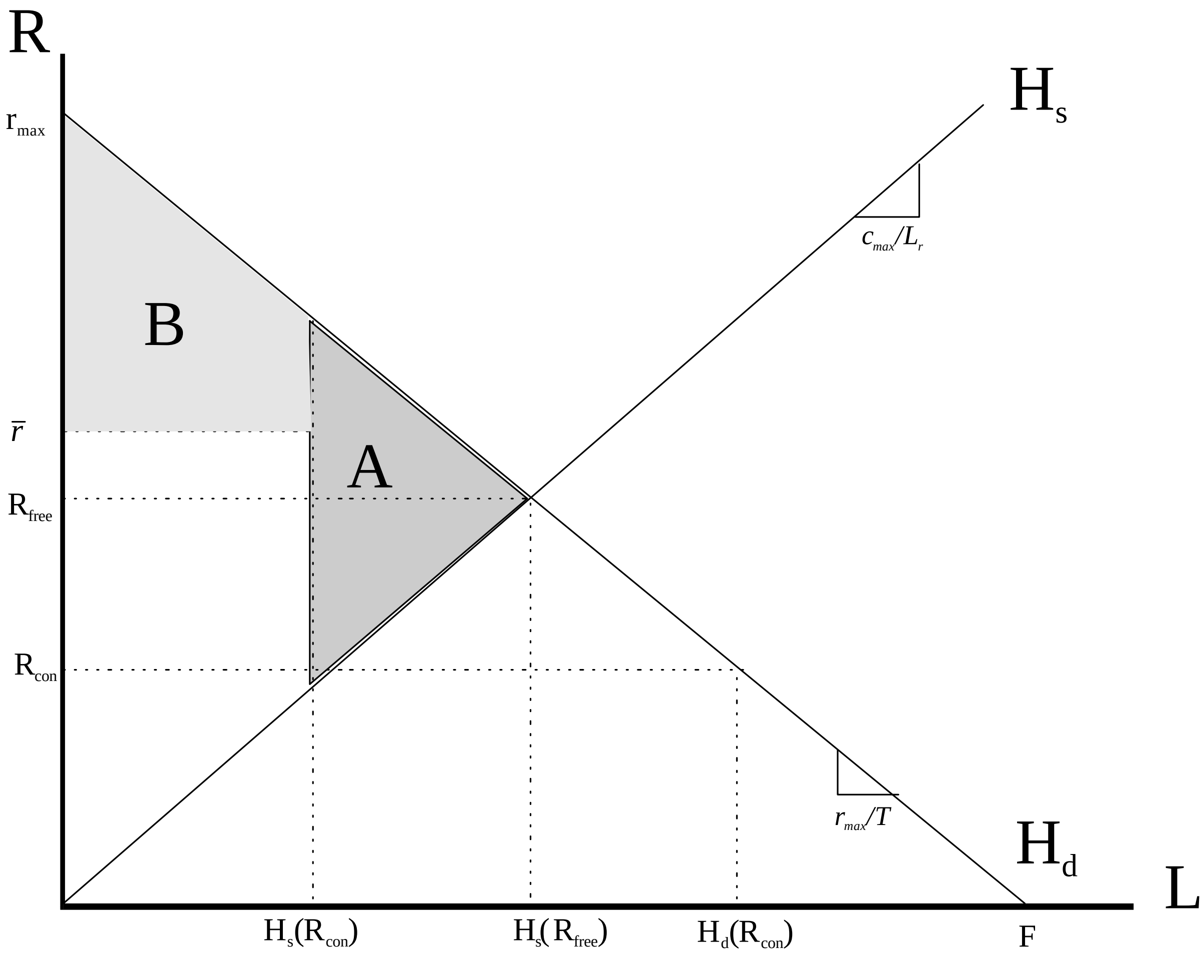
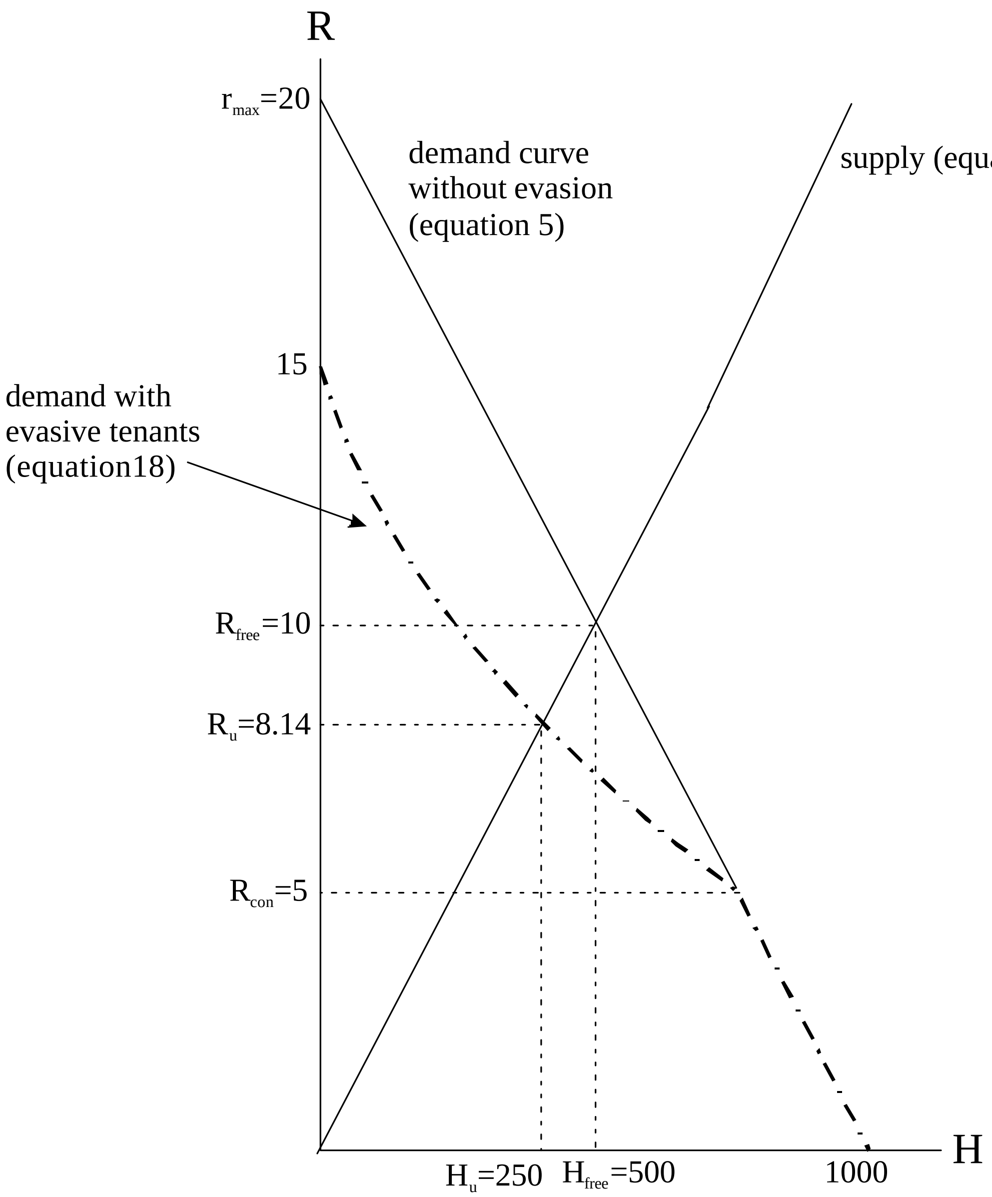


Figure 2: Underground Rent When Controlled Rent is R_{con}=5



Note: The above curves are not schematic drawings but precise graphings based on simulations.

Figure 3(a): Demand and supply when R_{con}=9

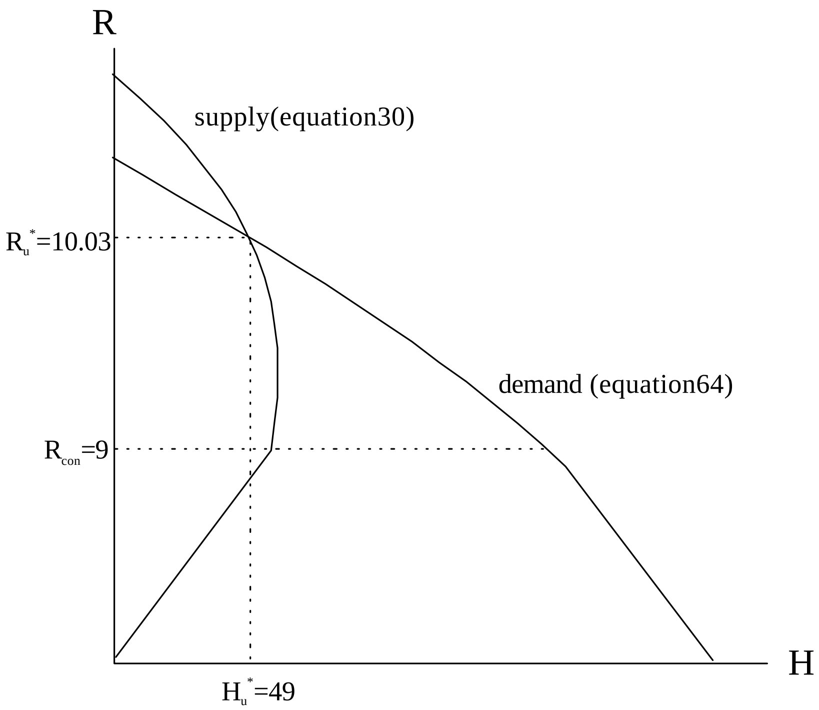
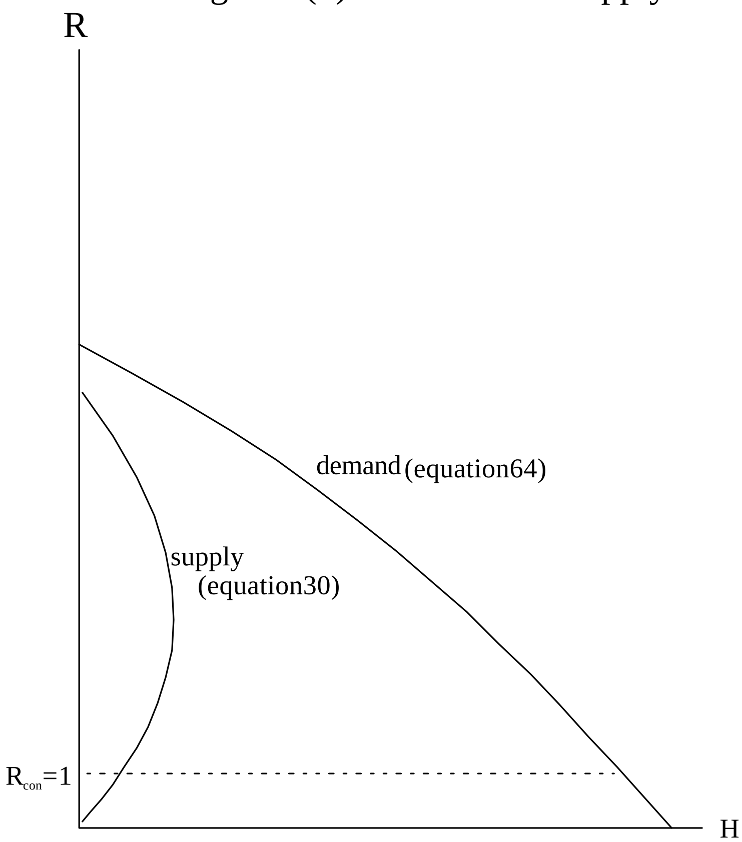


Figure 3(b): Demand and supply when R_{con}=1



Note:Thecurvesintheabovefiguresarenotschematicdrawingsbut preciserenderingsgenerated from closed form solutions derivedinthe mainbodyofthetext.

Figure 4: Range of rents