Rent stabilization and the long-run supply of housing[1]

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ABSTRACT

This study contributes to the understanding of how the construction and replacement of urban housing may be affected by rent stabilization. One of the primary insights is that neither the timing nor the density of development will be affected by rent stabilization because allowing perfectly flexible base rents permits landlords to capture all of the advantages of a rent growth control. However, redevelopment is hastened because rent stabilization complemented by vacancy decontrol increases the difference between rents before and after redevelopment, increasing the opportunity costs of postponing redevelopment. Extensions include an analysis of other common rent regulations and the impact of rent stabilization on the urban rent gradient.

I. Introduction

The notion that a rigid price ceiling discourages investment in rental housing has led to the introduction of more flexible rent controls over the past fifty years. One distinguishing characteristic of this "second generation" of rent regulations is that the upward adjustment of rents to follow market pressures is allowed. In the more strict ordinances, landlords may increase rents only if the cost of providing housing increases. A second characteristic that is common among these dynamically oriented rent controls is that landlords are allowed to charge the

[1] The author has benefited from the comments of Dennis Capozza, Philippe Thalmann, especially two anonymous referees, and participants of the Deuxième rencontres de recherches en développement régional (La Neuveville,

prevailing market rent on newly built or vacated apartments. The initial contract rent is not subject to regulation. Only the growth of rents is limited.

There are many countries in which rent regulations of this nature can be found. For example, Austria, Brazil, Belgium, Finland, France, India, the Netherlands, Portugal, Sweden[2], Switzerland, and the United States are all reported to have rent controls such that the base rent on private rental housing is unrestricted but increases are governed by statute (EEC 1991: 31-32 and Malpezzi 1993). Yet, the static rent ceiling is still used as a benchmark case for rent regulation in much of economic theory. Although sophisticated models of rent ceilings have been established (see Frankena 1975), models of flexible rent controls, probably more relevant, are rare. It is the purpose of this paper to contribute to the understanding of how the construction and replacement of urban housing may be affected by rent stabilization.

One of the difficulties of modeling flexible rent ceilings is that a dynamic approach is required. And adding the dimension of time will necessitate simplifications that may elide aspects of the housing market that are great importance. For example, I model the supply decisions of landowners independent of consumers and do not consider non-market means of allocating housing such as side payments and queuing. I believe that such a simplification is useful in order to isolate the incentives facing landlords but am conscious that it can also be hazardous. Nonetheless, this work is a step forward given the relative lack of material on the subject.

November 13-15, 1997), the ENHR workshop on Housing Economics (South Bank University, June 5-6 1998), and the ERES-AREUEA conference (Maastricht, June 11-13, 1998). I am responsible for any errors.

[2] In Sweden, the rent on private housing follows the rent on public housing which is negotiated by tenants' associations and municipal housing companies (Turner, 1988).

Other work on the subject is by Basu and Emerson (2000), Kutty (1996), and Murray et al. (1991). Basu and Emerson (2000) study the adverse selection that arises when asymmetric information is present. When a rent growth control with vacancy decontrol is in effect, landlords prefer tenants who plan to stay for a short time. However, if they do not know whether a potential tenant is planning on a short or long tenure, they are not able to discriminate. Thus, the longer staying tenants benefit and the shorter staying ones are harmed by vacancy decontrol.

Murray et al. (1991) simulate the effects of rent stabilization ordinances of the city of Los Angeles, which cap the growth of rents of occupied apartments but allow landlords to raise rents when the apartment is vacated. They find that the average rent will decline as a result of the ordinance but that the proportion of the rent reduction that is due to under maintenance increases over time. Thus, tenants benefit most from rent stabilization early on, when the gains from a reduction of the price of housing services substantially outweigh the costs of deterioration.

In a model of a landlord's investment decisions, Kutty (1996) examines a wide variety of rent regulations and their effect on maintenance. A significant finding is that if re-investment is allowed a market return, then maintenance will occur as if there were no rent regulation. However, a policy that does not allow the market return, even if it allows some growth will have an effect on maintenance that is equivalent to a rent ceiling if the growth is not linked to housing quality.

The work in this paper is based primarily on the models of urban growth with durable housing by Amin and Capozza (1993) and Capozza and Li (1994). These models are distinguished by the "open city" assumption and exogenous growth of consumers' incomes. Models of land conversion have been used to examine the impact of policies, especially taxation. For studying rent controls these same models may seem inappropriate because they do not address the market disequilibrium that may occur as a result of rent control that is modeled as a price ceiling. The implication of the open city assumption is that if consumers do not earn a certain reservation utility that can be earned elsewhere, then they will emigrate. Given perfect mobility and zero search costs, the demand side of the model becomes superfluous. Heffley (1998, p. 757) observes that distributional issues would be obscured by an open city model. While this is a valid and important criticism, the alternative "closed city" assumption is just as extreme.

A second criticism could be that the focus of such models is the development of vacant land, which is not a concern of most rent control studies. However, it should not be ignored. Second generation rent control programs are designed as long-run policies, ones that are not supposed to deter new construction. Thus, their properties in this regard are worthy of investigation. A model of the effect of rent stabilization on investor behavior and some general conclusions are presented in Section II; the spatial implications of rent regulation are presented in Section III; a discussion of other rent regulations in sections IV and V; and a summary of the results in Section VI.

II. A model of investment in land in the presence of rent regulation

In this section, I present the basic model of Amin and Capozza (1993) that I use to analyze landowners development and redevelopment decisions throughout the rest of the paper. The following notation is used:

q_i	space (housing) per unit of land; i.e., the density of development after *i* conversions, *i* ranges from zero to infinity
k_i	capital-land ratio employed in the i^{th} conversion
T_i	time of the i^{th} conversion
r	real interest rate
c	cost of one unit of capital
$R(t)$	market rent for land at time t
$\overline{R}(t, T_i)$	regulated rent at time t on one unit of housing on land last converted at time T_i
$P(t)$	price of land at time t
$\overline{p}(t, T_i)$	present value, at time t, of regulated rents from land converted at T_i over an infinite horizon. The variable $p(t)$ represents the present value of unregulated rents.

z distance of the plot of land from the center of the city
τ cost of transport per unit of distance

All pecuniary variables are real. The following assumptions apply to the production of housing:

A1. Capital is durable and does not depreciate.
A2. The existing capital is costlessly demolished when redevelopment takes place.
A3. The quantity of housing produced is a function of land, *L*, and capital, *K*. Normalizing by *L* yields the housing per unit of land, *q*, which is a function of the capital-to-land ratio: $Q(K,L)/L = Q(K/L,1) \equiv q(k)$. The production function of density is such that $q'(k) > 0$ and $q''(k) < 0$. The production function for housing, $Q(K,L)$, is homogeneous of the first degree. terms of density, the housing-to-land ratio and capital-to-land ratio.
A4. The cost of development is linear in the quantity of capital employed; i.e., it costs *ck* dollars to construct *q(k)* units of space.

Assumption A1, that capital does not depreciate abstracts from maintenance, the focus of much of the rent control literature (see for example Olsen 1988 and Kutty 1996). An implication of the assumption is that rent on a unit of housing would be equivalent to the revenue from a unit of housing (see Frankena 1975 for a discussion of this distinction). According to assumption 2, the only type of reinvestment is redevelopment. Buildings are not abandoned. They are demolished and rebuilt. The following are assumptions concerning the market and consumers

A5. $R(t)$ is a deterministic and non-decreasing, unbounded function of time such that $R(t,0) \le \hat{R} + R(0,0)e^{gt}$ for constants $\hat{R}$ and g such that $\hat{R} \ge 0$ and $g<r$[3].
A6. The market for housing is perfectly competitive.
A7. The city is an open city.
A8. Tenants are perfectly mobile.
A9. Tenants know their rights and there are no transaction costs to contesting an excessive rent level or excessive rent increase.

The policy of rent stabilization is as follows: there is a ceiling on the growth of rents; there is no ceiling on the initial rent level, which is determined at the outset of the rental contract; and

[3] The assumption that the growth rate is less than the interest rate guarantees that the present value of future rents is finite, i.e., that $R(t)\cdot e^{-rt}$ approaches zero as *s* approaches infinity.

eviction of tenants is illegal as long as they pay the regulated rent. The rent paid by the tenant at any time is defined by

$$\overline{R}(t,T_i) = \overline{R}(T_i,T_i) + \int_{T_i}^{t} \theta(s)ds \qquad 1$$

where T_i is the time that a new rental contract takes force, t is the current time, $\overline{R}(T_i,T_i)$ is the base rent, and $\theta(s)$ is the maximum rent growth permitted at time s. The rent growth restriction is parameterized by the variable θ, which represents the fraction of market growth permitted. The regulated rent growth at time t is then $\theta(t) = \theta \cdot R'(t)$, $0 \le \theta \le 1$. Strict rent stabilization is represented by a value of zero. An unregulated rent is represented by a value of one. An implicit assumption is that the growth control is binding. Values of θ between zero and one, must be interpreted with some care. Literally, a fractional value represents a policy that allows rents to increase by a proportion of what would be the market increase in rents. When growth is geometric then θ denotes a ceiling on the rate of growth and when growth is linear, θ is a ceiling on the size of the increase.

A. The market rent

Development of land yields an annual urban rent, $R(t,z)$, that declines over distance from the center, z, and increases over time, t. As in Capozza and Helsley [1989], it is assumed that residents consume one unit of housing, have identical incomes and preferences, and that the city is open. Each period the consumer earns income, y, rents one unit of housing at the price R, buys x amount of the consumer good whose price is normalized to one, and pays a transport cost τ for every unit of distance traveled to the city's central business district. In most of this paper, I assume that transport costs are equal to zero. Given a utility function, $u(x)$; a budget constraint, $y(t) \ge R + x + \tau z$; and the open city assumption that utility is equal to a world reservation level of utility, $u(x)=u^{world}$, then the rent on one unit of housing is given by $R(t,z)=y(t)-\bar{x}(t)-\tau z$, where $y(t)$ is an exogenously growing income and $\bar{x}(t)$ is the level of consumption needed to attain the reservation level of utility. A formal derivation of this rent function can be achieved by substituting the budget constraint into the open city constraint and applying the implicit function theorem (see Capozza and Helsley, 1989).

B. The negotiated rent

If the starting rent of a rental contract were not mandated by statute but instead negotiated between landlord and prospective tenant, then renters would be asked to pay for the pecuniary advantages of a rent-controlled apartment. A landlord would rent to the tenant willing to bid up the initial rent until the surplus from the rent control is entirely capitalized into the initial rent. Thus, when the initial rent is competitively determined, the net present value of rental income over the leasing period would be the same with the rent control as without. To formalize this insight, suppose that the consumer desires a lease that maximizes the discounted value of utilities during the period of the lease, from T_i. to T_{i+1}:

$$U\left(T_i, T_{i+1}\right)=\int_{T_i}^{T_{i+1}} u(x(s))\cdot e^{-\rho(s-T_i)}ds ,$$

where ρ is the consumer's discount rate. Assuming that the consumer is not constrained in borrowing. The budget constraint over the period of the lease is:

$$\int_{T_i}^{T_{i+1}} \left(y(s)-\overline{R}(s,T_i)\right)\cdot e^{-r(s-T_i)}ds \geq \int_{T_i}^{T_{i+1}} x(s)\cdot e^{-r(s-T_i)}ds .$$

where r is the interest rate. Since the evolution of rents is governed by regulation, renters and landlords can choose only the base rent. Arbitrage is possible if there are unregulated sectors in the housing market or if the tenant can move to another city. Individuals will choose the contract that minimizes the net present value of rent paid to the landlord. Thus, the initial negotiated rent must satisfy the following condition that the present value of the stream of rents under rent regulation must not be larger than the present value of the future stream of market rents:

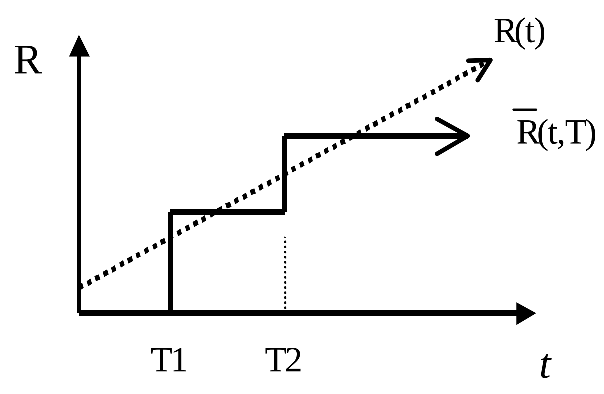
$$\int_{T_i}^{T_{i+1}} \overline{R}(s,T_i)\cdot e^{-r(s-T_i)}ds \le \int_{T_i}^{T_{i+1}} R(s)\cdot e^{-r(s-T_i)}ds .$$

Otherwise, tenants would be willing to pay market rents. The initial rent would be bid up to the point where consumers are indifferent between rent-controlled housing and its alternative. Integrating by parts and rearranging yields an expression for the initial rent:

$$\overline{R}(T_i,T_i) = R(T_i) + \frac{1}{1-e^{-r(T_{i+1}-T_i)}}\cdot \int_{T_i}^{T_{i+1}} \left(R'(s)-\theta(s)\right)\cdot \left(e^{-r(s-T_i)} - e^{-r(T_{i+1}-T_i)}\right)ds . \qquad 2$$

The negotiated initial rent under rent stabilization equals the market rent at the time the contract was made plus a premium reflecting the present value of the future rent growth prevented by the control. Initial rents under regulation overshoot the market rent. Strong evidence of the rent premium that tenants are willing to pay is presented in the empirical work of Nagy (1997), who assesses New York City's rent stabilization program. An illustration of the evolution of rents where no growth is allowed is depicted in Figure 1 below.

Figure 1 The evolution of rents when initial rents are negotiated and no growth is allowed



It is clear from equations 1 and 2 that the market rent will eventually overtake the regulated rent between T_i and T_{i+1} such that $R(T_{i+1}) > \overline{R}(T_{i+1}, T_i)$ as shown in Figure 1. The movement of initial contract rents is given by the following equation:

$$\frac{\partial \overline{R}(T_i, T_i)}{\partial T_i} = \theta(T_i) + \frac{r}{1-e^{-r(T_{i+1}-T_i)}} \cdot \int_{T_i}^{T_{i+1}} \left(R'(s) - \theta(s)\right) \cdot \left(e^{-r(s-T_i)} - e^{-r(T_{i+1}-T_i)}\right) ds . \qquad 3$$

The above expression is positive, which implies that the base rent increases over time. By waiting, the base rent will change for three reasons. First, there is an increase of the market rent by $R'(T_i)$. Second by waiting, the growth restriction premium falls by $R'(T_i) - \theta(T_i)$. Third, the second term in equation 3 reflects an increase in the discounted value of the rent control premium by developing later. The impact of developing later on rents after conversion is given by:

$$\overline{R}_{T_i}(t, T_i) = \frac{r}{1-e^{-r(T_{i+1}-T_i)}} \cdot \int_{T_i}^{T_{i+1}} \left(R'(s) - \theta(s)\right) \cdot \left(e^{-r(s-T_i)} - e^{-r(T_{i+1}-T_i)}\right) ds \text{, for } t > T_i \qquad 4$$

Rents after the conversion also increase with the time of the conversion, but the increase is not as great as for the base rent.

C. Conversion of land

The goal of the land developer is to maximize the value of a plot of unbuilt land, *P(t)*, by choosing the optimal development strategy. The maximization problem of the landlord is

$$P(t) = \max_{T_i, k_i, N} \sum_{i=1}^{N} \left\{ q(k_i) \int_{T_i}^{T_{i+1}} \overline{R}(s, T_i) \cdot e^{-r(s-T_i)} ds - ck_i \right\} \cdot e^{-r(T_i - t)} \qquad 5$$

The variables of choice are the timing, density and number of conversions (one development, and *N-1* redevelopments). The optimal timing, T_i, of the *i*th conversion is given by the following first-order condition:

$$P_{T_i} = q_{i-1} \cdot \overline{R}(T_i, T_{i-1}) - q_i \cdot \overline{R}(T_i, T_i) + q_i \cdot \int_{T_i}^{T_{i+1}} \overline{R}_{T_i}(s, T_i) e^{-r(s-T_i)} ds + rck_i = 0 \qquad 6$$

The benefit of developing sooner is an earlier receipt of revenue from the new project (the second term). The opportunity costs are a loss of the revenue from the existing structure (first term), the interest cost of the investment (fourth term), and a "withholding premium" (third term). This last opportunity cost stems from the rent increase in all periods from a later conversion (see equation 4) introduced by the rent regulation. The first-order condition with respect to capital intensity is:

$$P_{k_i} = q'(k_i) \cdot \int_{T_i}^{T_{i+1}} \overline{R}(s, T_i) \cdot e^{-r(s-T_i)} ds - c = 0 \qquad 7$$

The first-order condition implies that the value of the marginal product of capital must equal the marginal cost. Optimality requires that the second-order conditions are negative. For capital, this requires that the marginal product of capital is diminishing (A3). For timing, the second-order condition will hold if the benefits of developing earlier are growing faster than the costs. This will be the case as long as rents are growing as described in assumption A5. Now we can examine the impact of the regulation on the timing and density of conversion.

<u>Proposition 1</u> *The timing of the initial development is neutral with respect to rent growth restrictions when there is no statutory limit on the base rent and when other development decisions are exogenous. However, redevelopment is hastened by rent growth restrictions under the same conditions.*

<u>Proof</u>: The impact of a change in the rent growth limitation on both development and redevelopment is given by the implicit function theorem: $\partial T_i / \partial \theta = - P_{T_i\theta} / P_{T_iT_i}$. The denominator, which is the second-order condition is negative. The impact of the change in allowed rent growth depends upon $P_{T_i\theta}$, given below:

$$P_{T_i\theta} = q_{i-1} \cdot \frac{\partial \overline{R}(T_i, T_{i-1})}{\partial \theta} - q_i \frac{\partial \overline{R}(T_i, T_i)}{\partial \theta} + q_i \int_{T_i}^{T_{i+1}} \frac{\partial \overline{R}_{T_i}(s, T_i)}{\partial \theta} e^{-r(s-T_i)} ds$$

The impact of restricting the rent growth has two impacts on timing. First, it increases the difference between the current revenue of the future and present project. This acts to hasten development. Second, it increases the withholding premium, which leads to a postponement of development. Substituting the derivatives of rent, given in equations 2 and 3, with respect to the growth restriction into the above expression, the net effect is given by

$$P_{T_i\theta} = q_{i-1} \cdot \frac{1}{1 - e^{-r(T_{i+1}-T_i)}} \int_{T_i}^{T_{i+1}} R(s) \left(e^{-r(s-T_i)} - e^{-r(T_{i+1}-T_i)} \right) ds . \qquad 8$$

For initial development (i=1), q_{i-1}= 0 so that $P_{T_i\theta} = 0$. The rent regulation has no impact on the timing of development. However, for all redevelopments (i>1), $P_{T_i\theta} > 0$, which implies that tightening the rent growth restriction will hasten development.

The first result, that the timing of development is neutral with respect to the policy when bargaining is allowed, is intuitive. A flexible initial rent allows landlords to capture the lost rent from the rent growth control. Given that the present value of the project is the same, investment decisions are not affected. Skelley (1998) also finds that when tenants and landlords are free to make contracts with one another, the effect of a rent control can be neutralized. The second result, that the rent regulation hastens development, stems from the discontinuity in the evolution of regulated rent (see Figure 1). The greater gap between the regulated rent on the current project and the future project will act as incentive to develop sooner.

<u>Proposition 2</u> *The capital intensity of development is not affected by rent growth limits when there are no restrictions imposed upon the initial contract rent and when all other development decisions are exogenous. The same holds for the capital intensity of redevelopment.*

Proof: The impact of a change in rent growth limits on the intensity of conversion is given by the implicit function theorem: $\partial k_i / \partial\theta = -P_{k_i\theta} / P_{k_i k_i}$. The denominator, which is the second-order condition, is negative. The impact of the change in rent growth allowed depends upon $P_{k_i\theta}$:, which is given by

$$P_{k_i\theta} = q'(k_i)\cdot \int_{T_i}^{T_{i+1}} \frac{\partial \overline{R}(s,T_i)}{\partial\theta} \cdot e^{-r(s-T_i)} ds \, .$$

The derivative of $\overline{R}(s,T_i)$, defined by equations 1 and 2, with respect to θ yields the result that $P_{k_i\theta} = 0$. When the initial rent is flexible, the net present value of regulated rents from T_i to T_{i+1} is equal to that of market rents. Although rent stabilization will affect the level of rents at different points in time, it does not affect the present value of the sum of rents upon which investment depends. Thus, the density of development and re-development will not be affected by the rent growth control.

Proposition ??? *There is an infinite number of optimal conversions under rent stabilization with fully flexible base rents.*

Proof: This argument can be made by contradiction as in Amin and Capozza (1993). Suppose that there are only *N* developments, the density and timing of the final development being q_N and T_N respectively. To show that redevelopment will continue ad infinitum, it has to be shown that one more conversion must be profitable for any q_N . The (*N+1*)th conversion will occur if the increase in the value of the property from redevelopment is greater than the cost of redevelopment. This condition is expressed as

$$q_{N+1} \cdot \overline{p}(T_{N+1}, T_{N+1}) - ck_{N+1} > q_N \cdot \overline{p}(T_{N+1}, T_N) \, .$$

using the notation introduced on page 4. Integrating by parts, the price of space under regulated rents can be expressed as $\bar{p}(t,T) = (1-\theta)\cdot p(T) + \theta \cdot p(t)$, where $p(t)$ is the present value of market rents over an infinite horizon. Insert this into the above condition and re-arrange to get

$$(q_N - \theta q_N)\cdot\left(\frac{q_{N+1} - \theta q_N}{q_N - \theta q_N}\cdot p(T_{N+1}) - p(T_N)\right) > ck_{N+1}$$

As long as rents grow without bound (A5), then the present value of space will also increase without bound and redevelopment will eventually become profitable. Note, however, that the above condition is satisfied only if $q_{N+1} - \theta q_N$ is positive, which can be seen from the first-order condition with respect to timing after substituting the expressions for regulated rent. That redevelopment to a lower density may characterize an optimal redevelopment is due to the incentive effect of vacancy decontrol. As Olsen (p. 295, 1988) observed, "…rent control ordinances that increase the rent ceiling on an apartment generously when it is upgraded…will lead to greater landlord maintenance…" Note that as long as capital intensity is a decision variable, then redevelopment to a higher density will occur when rents and thus prices of developed land grow without bound.

D. Conversion of land and joint decisions

While we are able to make statements concerning the effect of rent regulation on the conversion of vacant land when investment decisions are independent of one another, the simultaneous choice of the density and timing for all conversions in addition to the number conversions is more difficult to analyze. Brueckner (2000) outlines two approaches to render the problem tractable. The first is to assume a stationary economy as in Brueckner (1981). Choosing the timing of redevelopment becomes equivalent to choosing the frequency of redevelopment. A second approach (Brueckner, 1980) is to assume that the developers are myopic: they assume that each conversion is the last one. Alternative approaches to simplifying the problem are to assume specific functional forms (Braid, 2001) and to treat certain decision variables as exogenous (Amin and Capozza, 1993). In this paper, two simplifications will be considered separately: myopia on the part of developers and an exogenous number of developments.

1. Myopic developers

Myopic developers do not foresee future redevelopment. This limits the decision variables to the timing and density of the next conversion[4]. The impact of a change on the rent growth restriction on timing, T_i, and capital density, k_i, is given by the following equations:

$$\frac{\partial T_i}{\partial \theta} = J^{-1} \cdot \left(P_{k_i T_i} P_{k_i \theta} - P_{T_i \theta} P_{k_i k_i}\right) \qquad 9$$

$$\frac{\partial k_i}{\partial \theta} = J^{-1} \cdot \left(P_{T_i \theta} P_{k_i T_i} - P_{T_i T_i} P_{k_i \theta}\right). \qquad 10$$

<u>Proposition 3</u> *Rent stabilization affects neither the timing of development nor the density, when the initial contract rent is fully flexible and developers are myopic.*

<u>*Proof*</u>: As already shown, $P_{T_1 \theta} = 0$ and $P_{k_1 \theta} = 0$ for *i*=1. Substituting these into equations 9 and 10, for *i*=1 one finds that $\partial T_1 / \partial \theta = 0$ and $\partial k_1 / \partial \theta = 0$. This is an intuitive result: since the effect of the rent regulation is neutral when the investment decisions are exogenous of one another, there will not be any interaction effects when density and timing are joint decisions.

<u>Proposition 4</u> *When developers are myopic, restricting rent growth will hasten redevelopment and reduce capital intensity when initial contract rents are unregulated.*

<u>*Proof*</u>: The expression, $P_{T_i \theta}$, is positive for *i*>1 (see equation 8), $P_{k_i \theta}$ is equal to zero for all *i* (see Proposition 2), the Jacobian matrix, $J = P_{T_i T_i} P_{k_i k_i} - P_{k_i T_i}{}^2$, which must be positive for optimality,

and the two second-order conditions are negative. Substituting these values into equation 9 confirms that the impact of permitting rent growth delays development. The explanation of this result to is equivalent to the one provided in Proposition 1 concerning redevelopment. To determine the effect on density, the derivative of P_{k_i} with respect to T_i is required:

$$P_{k_iT_i} = q'(k_i)\cdot\left(-R(T_i)\cdot e^{-r(T_{i+1}-T_i)} + \int_{T_i}^{T_{i+1}} R'(s)\cdot\left(e^{-r(s-T_i)} - e^{-r(T_{i+1}-T_i)}\right)ds\right). \qquad 11$$

The sign of this term is ambiguous and depends upon the time elapsed between T_i and T_{i+1}. If the difference is small, then the expression is negative. If it is large, then the expression is positive. In the case of the myopic developer, T_i is the final conversion so that T_{i+1} is equivalent to the end of the building's lifetime. Assuming an infinite time horizon, $\partial k_i/\partial\theta$ is positive. Restricting rent growth reduces density. By hastening redevelopment, the developer advances the conversion to a time when the present value of rents is lower and the high density is not warranted. If the lifetime were finite, then the effect of rent stabilization would be ambiguous.

2. Endogenous conversions

An alternative approach of determining the impact of the rent regulation is to assume that the number of conversions is fixed. I assume that there are two and examine two scenarios: first, when development and the following redevelopment are endogenous and second, when two successive redevelopments are determined jointly, but the initial development is exogenous.

<u>Proposition 5</u> *If development and redevelopment are endogenous, then limiting rent growth hastens development and redevelopment when the base rent is fully flexible. Restricting rent growth has no effect on the density of development, but will reduce the density of redevelopment.*

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[4] Note that there is a corner solution for the timing of a one-shot development if rent growth is geometric and the production technology is Cobb-Douglas.

Proof: The comparative statics of a four-equation system (see Silberburg, p.133) is governed by:

$$\begin{bmatrix} P_{T_iT_i} & P_{T_ik_i} & P_{T_iT_{i+1}} & P_{T_ik_{i+1}} \\ P_{k_iT_i} & P_{k_ik_i} & P_{k_iT_{i+1}} & P_{k_ik_{i+1}} \\ P_{T_{i+1}T_i} & P_{T_{i+1}k_i} & P_{T_{i+1}T_{i+1}} & P_{T_{i+1}k_{i+1}} \\ P_{k_{i+1}T_i} & P_{k_{i+1}k_i} & P_{k_{i+1}T_{i+1}} & P_{k_{i+1}k_{i+1}} \end{bmatrix} \begin{bmatrix} \partial T_i/\partial\theta \\ \partial k_i/\partial\theta \\ \partial T_{i+1}/\partial\theta \\ \partial k_{i+1}/\partial\theta \end{bmatrix} = \begin{bmatrix} -P_{T_i\theta} \\ -P_{k_i\theta} \\ -P_{T_{i+1}\theta} \\ -P_{k_{i+1}\theta} \end{bmatrix}$$

Substitute zero for those terms that are equal to zero: $P_{T_ik_{i+1}}$, $P_{k_ik_{i+1}}$, and symmetrically for $P_{k_{i+1}T_i}$ and $P_{k_{i+1}k_i}$, $P_{k_i\theta}$, and $P_{k_{i+1}\theta}$. The comparative statics are given by the following:

$$\partial T_i/\partial\theta = |J|^{-1}\cdot P_{k_ik_i}P_{k_{i+1}k_{i+1}}\cdot\left(P_{T_iT_{i+1}}P_{T_{i+1}\theta} - P_{T_i\theta}P_{T_{i+1}T_{i+1}}\right), \qquad 12$$

$$\partial k_i/\partial\theta = |J|^{-1}\cdot P_{T_i\theta}P_{k_iT_i}P_{T_{i+1}k_{i+1}}P_{k_{i+1}T_{i+1}}, \qquad 13$$

$$\partial T_{i+1}/\partial\theta = |J|^{-1}\cdot P_{k_ik_i}P_{k_{i+1}k_{i+1}}\cdot\left(P_{T_i\theta}P_{T_{i+1}T_1} - P_{T_iT_i}P_{T_{i+1}\theta}\right), \qquad 14$$

$$\partial k_{i+1}/\partial\theta = |J|^{-1}\cdot P_{k_iT_i}P_{k_{i+1}T_{i+1}}\cdot\left(P_{T_{i+1}\theta}P_{T_ik_i} - P_{T_{i+1}k_i}P_{T_i\theta}\right) \qquad 15$$

The effect on the timing of the first conversion is given in equation 12. When the first conversion is the initial development (*i*=1), then $P_{T_i\theta}$=0. The second-order conditions are negative for all decision variables, $P_{T_iT_{i+1}} = q(k_i)\cdot\overline{R}_{T_i}(T_{i+1},T_i)\cdot e^{-r(T_{i+1}-T_i)} > 0$, and $P_{T_{i+1}\theta} > 0$ as in equation 8. Thus, reducing the rent restriction hastens development. The effect on the density of the first conversion is given by equation 13, which is equal to zero when *i*=1. The impact of the rent growth ceiling on the timing of the second development is given by equation 14. By substituting $P_{T_{i+1}\theta} > 0$ from equation 8, relaxing the rent regulation is shown to delay redevelopment. The effect of the rent growth control on the capital intensity of the second conversion is given by equation 15: allowing rent growth diminishes the density of the first redevelopment when *i=1*.

The qualitative impact of rent stabilization on redevelopment when development and redevelopment are endogenous is the same as in the case of the myopic developer. However, now that development and redevelopment are jointly determined, an earlier redevelopment creates an incentive to develop earlier in order to lengthen the period for which the initial project collects rents. The only result that is not intuitive is the neutrality of the density of development. The implication of $P_{T_1\theta} = 0$ is that both T_1 and T_2 are distorted so as not to change the present value of rents from T_1 to T_2.

Proposition 6 *If initial development decisions are exogenous and the base rent is fully flexible, then a tightening of the rent growth restriction will hasten both the first and second redevelopments, but have an ambiguous effect on the capital intensity of both conversions.*

Proof: The proof follows from equations 12, 13, 14, and 15. The complication is that when *i>1*, $P_{T_1\theta} > 0$. For the effect on the timing of the first redevelopment, given by equation 12, the impact of easing the rent growth restriction is to delay both conversions. The effect on the density of the first redevelopment, equation 13, is ambiguous because the sign of $P_{k_iT_i}$ is ambiguous (see equation 11). The effect on the timing of the first redevelopment, equation 14, of easing the rent growth restriction is to delay development. To determine the effect of easing the rent restriction on the density of the second (and last) redevelopment, the following equation is needed:

$$P_{T_{i+1}k_i} = q'(k_i)\cdot \overline{R}(T_{i+1},T_i),$$

which is positive. Substituting this result into equation 15, it is evident that the effect of the rent growth restriction on the density of the final redevelopment is ambiguous. From the below results, it is not entirely clear what the effect of the rent regulation would be on housing stock. An increase is imaginable and such a case will be examined in the next section.

Table 1 The impact of tightening the rent growth restriction ($\theta\downarrow$) on development and redevelopment

	Development is first conversion (i=1)		Redevelopment is first conversion (i>1)	
	Myopic developers	***Joint decisions***	***Myopic developers***	***Joint decisions***
Timing of first conversion (T_i)	neutral	hastening	hastening	hastening
Density of first conversion (k_i)	neutral	neutral	reduction	ambiguous
Timing of second conversion (T_{i+1})	hastening	hastening	hastening	hastening
Density of second conversion (k_{i+1})	reduction	reduction	reduction	ambiguous

III. Extensions of the Model

A. The spatial structure of rents and asset prices

Introducing space into the model allows one to make statements concerning the impact of rent stabilization on the spatial structure of rents and asset prices, citywide averages, and housing supply. Suppose that the city is monocentric and that proximity to the central business district (CBD) is the only characteristic that distinguishes parcels of land. The marginal cost of transport is assumed to be constant, at τ dollars per unit of distance. The market rent can be expressed as $R(t,z) = R(t,0) - \tau \cdot z$ (see page 7 for a discussion of the derivation).

Given the structure of the model, the city will expand outwards from the CBD in a continuous manner. Land being developed currently is land that is located at the current urban-rural boundary, Z. Following Capozza and Helsley (1989), the rent at the time of development in terms of the urban boundary at time t is defined as:

$$R(T,z) = R(t,Z(t)). \qquad 16$$

Add $\tau(Z(t) - z)$ to both sides of the expression and re-arrange for a definition of market rents in terms of the urban-rural boundary: $R(t,z) = R(T(z),z) + \tau(Z(t) - z)$. As the city grows, so does rent on urban land at all locations. For the regulated rent, substitute the latter spatial definition of the unregulated rent into equation 1 for a spatial definition of the regulated rent:

$$\overline{R}(t,T,z) = \overline{R}(T,T(z),z) + \theta\tau(Z(t) - z). \qquad 17$$

If rent stabilization were strict (no rent growth allowed), then under linear rent growth, the rent gradient would be flat. As the restriction is loosened, the rent gradient will turn around a point, z^*, given by $Z(t) - \int_{T(z^*)}^{\infty} Z'(s) \cdot e^{-r(s-T(z^*))} ds$, and attains a steeper downwards slope.

Throughout this section, we assume the easy to analyze case of one-shot development of fixed density and fixed construction cost, *C*. The market rent at the time of development would be given by $R(T,z) = rC$ from the first-order condition. When the timing of the initial conversion is the only endogenous variable, the timing of development is neutral with respect to the rent growth control (see Proposition 1). Thus, the city boundary as defined in equation 16 will not be altered by rent stabilization. However, rents and asset prices, and their citywide averages, will be affected. Start with the spatial definition of regulated rents (equation 17).

$$\overline{R}(t,T(z),z) = R(T(z),z) + \int_{T(z)}^{\infty} \left(R'(s) - \theta(s)\right) e^{-r(s-T(z))} ds + \theta\tau\left(Z(t) - z\right) \qquad 18$$

The derivative of the above expression with respect to distance yields the rent gradient. Obtaining from equation 16 that $T'(z) = \tau / R_T(T,z)$, we find that

$$\frac{\partial \overline{R}(t,T(z),z)}{\partial z} = -\theta \cdot \tau + (1-\theta) \cdot \frac{\tau}{R_T(T,z)} \cdot \int_{T(z)}^{\infty} R''(s) e^{-r(s-T(z))} ds \,.$$

The rent regulation has two effects. First, the rent gradient flattens as landlords are denied the full growth of the value of proximity. Second, the initial rent paid by consumers increases as growth is restricted. The value of the regulation to consumers and thus on the initial regulated rent may change over time. Base rents rise over time when market rents accelerate. If the restriction were strict, $\theta = 0$, then the rent gradient would bend upwards from the center because land that is farther out and developed later will earn higher initial rents. As we ease the restriction, the rent gradient becomes U-shaped. With linear growth, only the first effect would hold as there is no change in the growth premium over time.

The impact of a flexible rent control on the price of land is less obvious. We have to consider both developed and undeveloped land. The price of land developed at time T (no redevelopments) at unit density is given by $P^d(t,T,z) = \bar{p}(t,T,z) \equiv \int_t^{\infty} \bar{R}(s,T,z)e^{-r(s-t)}ds$, which I re-express as

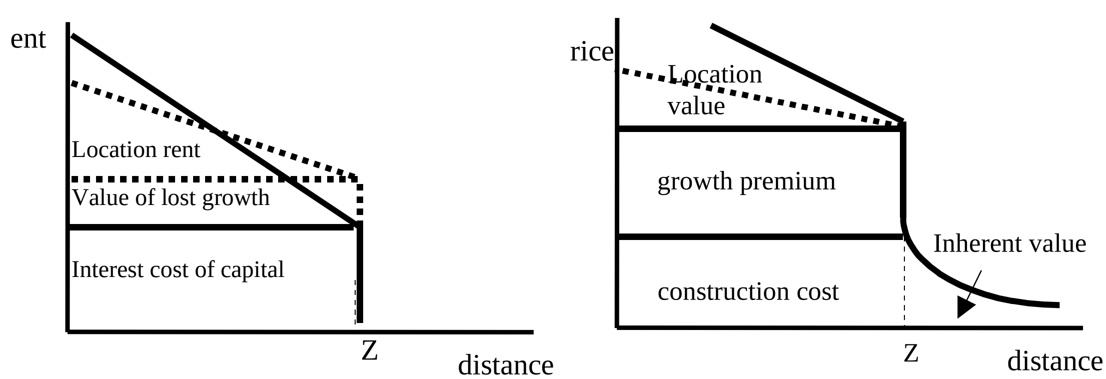
$$\bar{p}(t,T,z) = (1-\theta)\cdot p(T,z) + \theta \cdot p(t,z)$$

The impact of changing rent stabilization is then $\partial P^d(t,T,z)/\partial\theta = p(t,z) - p(T,z)$. At the time of development, there is no effect. Immediately afterwards, permitting rent growth limit raises land prices, an effect that increases over time. Expand the price equation:

$$P^d(t,z) = C + \theta(\tau(Z(t)-z)) + (1-\theta)\cdot\int_T^{\infty} R'(s)e^{-r(s-T)}ds + \theta\cdot\int_t^{\infty} R'(s)e^{-r(s-t)}ds.$$

The price of developed land consists of three elements: the cost of construction, the value of location, and the growth premium. The growth premium is the sum of the last two terms:

Figure 2 Effect of relaxing rent regulation on the structure of rents and asset pries when rent growth is linear (vertical axes not on the same scale)



the last is actual growth and the third term is the compensation paid by the tenant for lost growth. Note that if growth were linear, then the regulation would only affect location value (see Figure

2). To complete the picture, we need the price of vacant land, P^v. Given the current assumptions, the price of land (see equation 5) can be expressed as:

$$P^v(t,z) = \left(\bar{p}(T,T,z) - C\right) \cdot e^{-r(T(z)-t)} .$$

The inherent value of vacant land, the terms within the brackets, can be found by substituting in the first-order condition and the definition of regulated rents to get

$$P^v(t,z) = \left(\frac{1}{r} \cdot \int_{T(z)}^{\infty} R'(s) \cdot e^{-r(s-T(z))} ds \right) \cdot e^{-r(T(z)-t)}$$

The change in vacant land prices with respect to distance from the center is then $\partial P^v(t,z) / \partial z = -T'(z) \cdot R'(T(z)) \cdot e^{-r(T(z)-t)} = -\tau \cdot e^{-r(T(z)-t)}$, asymptotically approaching zero.

The price of vacant land is insensitive to the rent regulation as is the price of newly developed land. Additionally, there is no deadweight loss because, in this case, there are no distortions of developers' behavior. However, at any given location, *z*, within the urban boundary, the price of land is always lower under rent growth control. Owners of developed land are harmed because they are not able to raise rents to reflect growth in demand. Only the landowner at the time of the payment benefits from the compensation by the tenant.

Finding the impact of rent stabilization on the average urban rent, *AR*, involves integrating rent as defined in equation 18 over distance and dividing the total by the area of the city, $\pi Z(t)^2$:

$$A\bar{R}(t) = \frac{1}{\pi Z(t)^2} \cdot \int_0^{Z(t)} 2\pi z \cdot q(k(z)) \cdot \bar{R}(t, T(z), z) dz .$$

I consider the case of linear growth, $R'(t) = g$; fixed and unit density, $q=1$; and lump sum costs of conversion, *C*. The average rent in a regulated city is then

$$A\bar{R}(t) = rC + \left(1-\theta\right) \cdot \frac{g}{r} + \frac{\theta \tau Z(t)}{3} .$$

The effect of relaxing rent growth controls on average rents is given by

$$\frac{\partial A\overline{R}(t)}{\partial\theta} = -\frac{g}{r} + \frac{\tau Z(t)}{3} + \frac{\tau\theta}{3}\cdot\frac{\partial Z(t)}{\partial\theta}$$

The impact of the policy on the average rent is ambiguous and depends upon the size of the city. In a small (or young) city, the regulated city is likely to have higher average rents. As time passes and location rents surpass the withholding premium, average rents will be larger in the unregulated city. Even in a city where the size is restricted by regulatory or natural boundaries, this trend would continue as economic growth drives market rents.

B. Housing Supply

When redevelopment and development are independent of one another, we saw that the rent regulation would have no impact of on the supply of urbanized land. As we introduce more decision variable, ambiguity arises. To illustrate this, I examine the case of joint timing decisions: the second conversion is assumed to be the final redevelopment and denser than the previous conversion. The impact on the timings of conversion when density is exogenous are given by:

$$\partial T_1/\partial\theta = |J|^{-1}\cdot\left(P_{T_1T_2}P_{T_2\theta} - P_{T_1\theta}P_{T_2T_2}\right). \text{ and } \partial T_2/\partial\theta z = |J|^{-1}\cdot\left(P_{T_1\theta}P_{T_2T_1} - P_{T_1T_1}P_{T_2\theta}\right)$$

For development (*i*=1), $P_{T_1\theta}$=0 and for redevelopment (*i*=2), $P_{T_2\theta}$>0 (equation 8). The second-order conditions are negative for all conversions. Finally, $P_{T_1T_2} = q(k_1)\cdot\overline{R}_{T_1}(T_2,T_1)\cdot e^{-r(T_2-T_1)} > 0$, and $P_{T_2T_1} = q(k_1)\cdot\overline{R}_{T_1}(T_2,T_1) > 0$. Thus, the impact of reducing the rent restriction is to hasten development and redevelopment. As explained, the hastening of the redevelopment has an indirect effect on the timing of development.

To link the above result with location, consider the definition: $\overline{R}(T_i,T_i,z) = \overline{R}(t,t,Z_i(t))$, which is used to construct an implicit function. The impact of the rent regulation on the boundary is

$$\frac{\partial Z_i(t)}{\partial \theta} = \frac{\partial \overline{R}(T_i, T_i, z) / \partial T_i \cdot \partial T_i / \partial \theta}{\partial \overline{R}(t, t, Z_i(t)) / \partial Z_i(t)}.$$

Loosening the regulation will reduce the urban boundary and the quantity of redeveloped land given that $\partial \overline{R}(T_i, T_i, z)/\partial T_i > 0$ (see equation 3), that $\partial T_i / \partial \theta > 0$, and $\partial \overline{R}(t, t, Z_i(t))/\partial Z_i(t) < 0$ (see equation 17). Because permitting more growth delays development, the rents will be higher than otherwise. To maintain the above definition of the boundary, the boundary itself must be reduced. This nonintuitive result that limiting the growth of rents leads to an increase in the supply of housing should not be taken as a general one. If density were endogenous, then it is probable that a decrease in density would lead to a net decrease in the supply of housing.

C. Credit constraints

I have assumed that tenants and real estate investors face the same interest rate. However, it is possible that consumers face credit constraints such that they do not have the option to lend or to borrow. While extreme, this assumption helps make the point that incomplete capital markets would prevent tenants from bidding for rent-controlled apartments as they would not be able to borrow to pay the high initial rent.

To simplify the problem, we assume a money-metric utility function per period $u(t) \equiv y(t) - \overline{R}(t, T)$, and that consumers are credit constrained. Suppose that there is only one conversion and that density is exogenous. The utility for an infinite length of stay is:

$$U(t, \infty) \equiv \int_t^\infty u(s) \cdot e^{-\rho(s-t)} ds = \int_t^\infty \left(y(s) - \overline{R}(s, T) \right) \cdot e^{-\rho(s-t)} ds ,$$

where ρ is the consumer’s discount rate. Comparing the utility from living in rent-controlled housing with that of an unregulated unit, the consumer would be willing to pay a regulated rent of

$$\overline{R}(t,T) = R(T) + \int_T^\infty \left(R'(s) - \theta(s)\right) \cdot e^{-\rho(s-T)} ds + \int_T^t \theta R'(s) ds$$

before other alternatives. Note that we are required to assume that the net income (in an unregulated apartment) must be greater than the growth premium for the base rent to be within the means of the household. The timing of development is determined by the first-order condition, equation 6. Inserting the above definition of rents, we arrive at:

$$P_T = -q \cdot R(T) + q \cdot \left(\frac{\rho - r}{r}\right) \cdot \int_T^\infty (1-\theta) R'(s) \cdot e^{-\rho(s-T)} ds + rck = 0 .$$

The change is given by the implicit function theorem: $\partial T / \partial \theta = -P_{T\theta} / P_{TT}$. Since the second order condition P_{TT} is negative, the direction of the change depends upon the sign of

$$P_{T\theta} = q \cdot \left(\frac{r - \rho}{r}\right) \cdot \int_T^\infty R'(s) \cdot e^{-\rho(s-T)} ds$$

Since rent growth is positive, the direction of the effect on the timing of development is determined by the difference between r and ρ: $\partial T / \partial \theta < 0,$ if $\rho > r$; $\partial T / \partial \theta = 0,$ if $\rho = r$; and $\partial T / \partial \theta > 0,$ if $\rho < r$. Restricting rent growth (θ↓) delays development when consumers face higher discount rates than landlords do. As a result of the higher discount rate, the premium that consumers are willing to pay is less than the landlords' present value of market growth lost from the regulation. Development is delayed because the landlords’ loss is not substantially offset by a higher initial ren. If the rates are equal, then the neutrality result holds. If the consumers' discount rates are lower than the landlords', then the consumers' rent growth premium will be greater than the landowners'. In this case, restricting growth increases the value of the investment for the developer and hastens development.

D. When the initial rent is regulated

Whether the base rent is negotiable is crucial in determining the impact that rent stabilization will have on conversion. Instead, suppose that the initial rent level is determined by statute and not by bargaining between the landlord and the tenant. The regulated rent is as in equation 1 but the base rent would be exogenously given. The initial rent may be fixed, but is more likely to be a function of market conditions. Let the base rent $\overline{R}(T)$ be defined as $\beta \cdot R(T)$, where $\beta > 0$. The regulated rent is thus $\overline{R}(t,T) = (\beta - \theta)R(T) + \theta R(t)$. The former expression is positive as long as rent growth and β are positive. The maximum level that tenants would tolerate would be the negotiated base rent. The general properties of conversion activity under rent stabilization when the initial rent level is statutory are summarized by the following propositions.

<u>Proposition 7</u> *When timing is exogenous, the density of all conversions under rent stabilization when the initial rent is determined by statute is lower than when rents are unregulated.*

The proof follows from the first-order condition for density, equation 7. The value of the marginal product of capital is lesser under rent stabilization, which leads to investment at a lower intensity. To see this, inspect the present value of rents as given by

$$\overline{p}(T_i, T_{i+1}) = \frac{R(T_i)}{r} - \frac{\left((1-\theta)R(T_i) + \theta R(T_{i+1})\right)}{r} e^{-r(T_{i+1}-T_i)} + \frac{1}{r}\int_{T_i}^{T_{i+1}} \theta R'(s) e^{-r(s-T_i)} ds$$

The derivative of the present value of regulated rents with respect to θ is, after rearranging,

$$\frac{1}{r}\int_{T_i}^{T_{i+1}} R'(s) \cdot \left(e^{-r(s-T_i)} - e^{-r(T_{i+1}-T_i)}\right) ds$$

Since there is positive growth and $T_{i+1} \geq s$, increasing θ will raise the value of the above expression. For the first-order condition with respect to capital intensity at time T_i to hold, the capital intensity must rise (given decreasing marginal product).

<u>Proposition 8</u>: The first conversion occurs later under rent stabilization with no bargaining when density is exogenous.

Consider the first-order condition for the timing of conversion, equation 6. At the first conversion, *i*=1, q_{1-1} is 0. The implicit function theorem yields:

$$\frac{\partial T_1}{\partial \theta} = -\frac{\partial P_{T_1} / \partial \theta}{\partial P_{T_1} / \partial T_1} = \frac{q(k_1) R'(T_1)/r \cdot \left(1 - e^{-r(T_2 - T_1)}\right)}{P_{T_1 T_1}} < 0 .$$

Relaxing the regulation hastens the first conversion. The above derivative is less than zero because the numerator is positive from positive rent growth and the denominator, which is equivalent to the second-order condition with respect to timing, must be negative to ensure optimality. As the rent regulation is less strict, the opportunity cost of developing earlier is lower, which justifies earlier development. By choosing when to build and lease housing, a landlord is explicitly choosing the base from which the regulated rent will evolve. By waiting, the landlord can “lock in” to a higher rent. This creates a withholding premium reflecting the additional opportunity cost of developing sooner. When timing and intensity are joint decisions, the impacts of the rent regulation on timing and density are ambiguous, as can be seen from the comparative statics given in equations 9 and 10. First, raising the growth restriction creates the incentive to postpone. Depending upon the pattern of rent growth (accelerating, constant, or decelerating), the delay may development to a time when more intense development is warranted. At the same time, restricting rent growth will act as an incentive to reduce the intensity of development. These counteracting effects on density, and thus indirect effects on timing, prevents us from making an unconditional statement as to the impact of the rent growth control on timing or density of the initial development[5].

The effect of rent stabilization on the timing of redevelopment when initial rents are regulated is unclear. Inspecting equation 6, the first-order condition with respect to timing, two opposing effects are evident. First, the greater difference between the rent on the present and future project caused by restricting rent growth and allowing vacancy decontrol will hasten development. However, the withholding premium will delay development. Given that the net effect on timing is ambiguous, so may be the effect on the density of redevelopment. It may be the case for a rent ordinance that the base rent on the initial project is completely flexible but that there are restrictions on the amount that rents may increase immediately after redevelopment. If this restriction is binding, then the effect on the timing of redevelopment is ambiguous and not hastened as in section II.

IV. The Rent Ceiling

Unlike rent stabilization, a rent ceiling is a fixed superior limit on the rent *level*. A rent ceiling will eventually absorb a growing market rent but as long as the market rent is below the ceiling it can take on any value. The time, $\bar{t}$, when rents pass the rent ceiling is defined as $R(\bar{t}) = \overline{R}$ or alternatively $\bar{t}\left(\overline{R}\right) = R^{-1}\left(\overline{R}\right)$. Thus, the regulated rent is

$$\overline{R}(t) = \begin{cases} R(t), & \text{if } t < \bar{t} \\ \overline{R}, & \text{if } t \geq \bar{t} \end{cases}$$

Correspondingly, the price of space (present value of rents) is:

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[5] The case of linear rent growth is straightforward because the present value of linear rent growth does not change over time. Thus, the effect of restricting rent growth would be to delay the timing of development and not effect density.

$$\overline{p}(t,t') = \int_t^{t'} \overline{R}(s)e^{-r(s-t)}ds = \begin{cases} \left(R(t) - R(t') \cdot e^{-r(t'-t)} + G(t,t')\right)/r, & \text{if } t' \le \bar{t} \\ \left(R(t) - \overline{R} \cdot e^{-r(t'-t)} + G(t,\bar{t})\right)/r, & \text{if } t < \bar{t} < t' \\ \overline{R} \cdot \left(1 - e^{-r(t'-t)}\right)/r, & \text{if } t' \le t \end{cases} \qquad 19$$

where $G(t_1,t_2) = \int_{t_1}^{t_2} R'(s) \cdot e^{-r(s-t)}ds$. The first-order conditions with respect to time is

$$-\left(q_i - q_{i-1}\right) \cdot \overline{R}(T_i) - rck_i = 0\,. \qquad 20$$

as long as the ceiling is non-binding. If this is the case, then $\overline{R}(T_i) = R(T_i)$. Thus, the rent ceiling has no effect on timing as long as $T_i < \bar{t}$ and density is exogenous This agrees with the insight described in Arnott (1997) that a moderate rent ceiling can be less detrimental to investment than a strict rent growth control (see Proposition 8 concerning rent growth controls with non-negotiated rent).

If market rents have surpassed the rent ceiling, then optimal redevelopment is characterized by

$$\left(q_i - q_{i-1}\right) \cdot \overline{R} \ge rck_i \qquad 21$$

A local optimum of development timing is not determined when market rents are above the ceiling. Instead, there is a corner solution such that development occurs immediately.

The first-order condition with respect to capital is the same as in equation 7, except that the price of space is as defined in equation 19. The rent ceiling affects the capital intensity of conversion by imposing a limit on the price of space. If the time of the next redevelopment has not yet reached $\bar{t}$, then the intensity of the current conversion will not be affected by the rent control (equation 19). If, however, the next investment occurs after $\bar{t}$, then the density of development will be reduced. Thus, the final redevelopment is always less intense under a rent ceiling.

By examining the first-order condition for timing, we have seen that redevelopment will eventually cease because of a rent ceiling. To determine the long-run effects of the rent ceiling, we can ask when and at what density the final development occurs.

<u>Proposition 9</u>: *In the presence of a rent ceiling, the capital intensity of conversion will not exceed a maximum level implied by the rent regulation.*

What follows is the sketch of a proof from Amin and Capozza (1993). Substitute the Cobb-Douglas production function, $q(k) = k^{\gamma}$ $(0 < \gamma < 1)$, into the first-order condition given in equation 7. Rearrange for a solution to the capital intensity of the i^{th} conversion:

$$k_i = \left(\frac{\gamma \overline{p}(T_i, T_{i+1})}{c} \right)^{\frac{1}{1-\gamma}}$$

Since the highest value of the price of space is $\overline{p}(\bar{t}, \infty) = \overline{R}/r$, the maximum intensity of development is

$$k_i \le k_{max} = \left(\frac{\gamma \overline{R}}{rc} \right)^{\frac{1}{1-\gamma}}, \text{ QED.}$$

Given that there is a maximum density that is reached by development at or after $\bar{t}$, it would follow that the number of conversions is finite. The question remains as to when the last development occurs: before or after $\bar{t}$?

<u>Proposition 10</u>: *Conversion activity ceases when the market rent surpasses the rent ceiling.*

First, consider the second-order condition for timing

$$-\left(q_i - q_{i-1}\right) \cdot \overline{R}'(T_i) < 0 \qquad 22$$

Given that $\left(q_i - q_{i-1}\right) > 0$, then rents must be growing at the time of development for development to be optimal. Since the ceiling halts rent growth, then the development must occur before $\bar{t}$. However, this is not a complete proof because it leaves out corner solutions. Suppose that there is a finite number, *N*, of developments. To show that the above proposition holds for corner solutions, one must show that a final development after $\bar{t}$ is not profitable. The price of this land developed for a last time at T_N is

$$P(t) = q_{N-1} \cdot \int_t^{T_N} \overline{R}(s) e^{-r(s-t)} ds + q_N \cdot \int_{T_N}^{\infty} \overline{R}(s) e^{-r(s-t)} ds - ck_N e^{-r(T_N - t)}$$

Suppose that the last development occurs at or after $\bar{t}$ so that $t < \bar{t}$ and $T_N \geq \bar{t}$. The price of land can be rewritten as

$$P(t) = q_{N-1} \cdot \left(\frac{R(t) + G(t,\bar{t})}{r} \right) + \left((q_N - q_{N-1}) \cdot \frac{\overline{R}}{r} - ck_N \right) e^{-r(T_N - t)}$$

The time of the final development appears only once in the above equation and affects the land value negatively. Since the quantity in round brackets is positive (equation 21), hastening its reception will increase the present value. Thus, no final development will occur after $\bar{t}$. This result distinguishes the rent ceiling from rent stabilization. When rent growth is unbounded, then redevelopment will occur ad inifinitum (see Amin and Capozza, 1993). While rent stabilization may alter the timing of redevelopment, it allows growth and thus will not halt redevelopment. While the rent ceiling stifles redevelopment, rent stabilization allows the process to continue.

In the remainder of this section, I investigate the effects of the rent ceiling on the first development when timing and intensity of development are joint decisions. The first- and second-order conditions for timing are as in equations 20 and 22 except that $q_{i-1} = 0$. The first-order conditions with respect to capital are the same as in equation 7 except that $T_{i+1} = \infty$. Examine the case of the nonbinding rent ceiling. I have postulated that the rent control will have no effect on timing as long as development occurs when rents are below the ceiling. Consider the following:

<u>Proposition 11</u>: *Lowering the rent ceiling will hasten development and reduce its density.*

Proof: The change in timing from an increase of the rent ceiling as given by $\partial T / \partial \overline{R} = J^{-1}(P_{kT} P_{k\overline{R}} - P_{T\overline{R}} P_{kk})$. The additional expressions needed are:

$$P_{kT} = q'(k) \cdot G(T, \bar{t}) > 0 ,\ P_{T\overline{R}} = 0 , \text{ and } P_{k\overline{R}} = q'(k) \cdot \frac{\overline{p}(T)}{\partial \bar{t}} \cdot \frac{\partial \bar{t}}{\partial \overline{R}} ,$$

where $G(T, \bar{t})$ is the present value of rent growth. The latter equation can be simplified by substituting in

$\frac{\overline{p}(T)}{\partial \bar{t}} = \frac{R'(T) \cdot e^{-r(\bar{t} - T)}}{r}$ and $\frac{\partial \bar{t}}{\partial \overline{R}} = \frac{1}{R'(T)}$ to get $P_{k\overline{R}} = q'(k) \cdot \frac{e^{-r(\bar{t} - T)}}{r} > 0$.

Given that $J > 0$, $P_{kT} > 0$, $P_{k\overline{R}} > 0$, and $P_{T\overline{R}} = 0$, raising the rent ceiling will delay development.

Capital intensity is encouraged by growth, and higher construction costs, from higher density delay, development. Thus, the conversion of vacant land is postponed as a consequence of raising the rent ceiling and allowing the landowner more growth.

The effect on density is given by $\partial k / \partial \vec{R} = J^{-1}(P_{T\overline{R}} P_{kT} - P_{TT} P_{k\overline{R}})$. Substituting the appropriate equations yields $\partial k / \partial \vec{R} > 0$. With a higher rent ceiling, development occurs at a higher density. Allowing more rent growth increases the price of space and encourages capital-intensive land conversion. While a stricter rent ceiling always reduces capital intensity and hastens development, the effects of rent stabilization can be the opposite or neutral. This example should illustrate the hazards of approximating a dynamic rent control by a static one.

V. Ceiling on the rate of return

A standard practice associated with the regulation of the rental sector is a ceiling on the rate of return that an investor may receive. In the context of a model of one-shot development, the constraint imposed by the maximum rate of return regulation can be expressed as

$$\frac{R(t) - rck}{rck} \leq \overline{\rho} .$$

It follows from the above definition that the maximum rate of return, $\bar{\rho}$, is equivalent to having a rent ceiling equal to $(1+\bar{\rho})\cdot rck$. The idea of such a policy is to allow property owners freedom in managing their property but to deny them of "unfair" profits. Indeed, in a model where density is exogenous, the policy has no effect on the optimal timing decision: $R(T) = rck < (1+\bar{\rho})\cdot rck$. However, when capital intensity is variable, the policy becomes distortionary. The rate of return cap creates an additional benefit to capital intensity. By spending more on development, the developer raises the effective rent ceiling and thus the length of time that market rents can legally be received. The time at which the regulation becomes binding is given by $\bar{t} = R^{-1}((1+\bar{\rho})\cdot rck)$

The effect on timing is indirect via the impact on capital intensity. The comparative statics of a change in the fair rate of return are not as straightforward as for the previous regulations analyzed. Intuitively, raising the rate of return cap would be expected to increase the growth premium by raising the rent at which the regulation becomes binding and thus increase density and delay development. At the same time, raising the maximum rate of return lessens the incentive to build at a high capital intensity in order to raise the effective rent ceiling. This counters the first effect because spending less on construction hastens development. The implicit function theorem yields the effect of the rate cap on the lock-in time $\partial\bar{t}/\partial\bar{\rho} = rck/R'(\bar{t}) > 0$.

The net effect of raising the cap is ambiguous. However, one lesson is that when growth is high, development is more likely to be postponed and at a higher capital intensity. When rent growth is high, rents reach the effective ceiling faster. Thus, the benefits of capital intensity to extend the period during which the landlord can collect growing rents investing would be more substantive.

Consider the first-order condition with respect to capital:

$$P_k = q'(k)\cdot\left(\int_t^{\bar{t}(\bar{\rho},k)} R(s)e^{-r(s-T)}ds + \int_{\bar{t}(\bar{\rho},k)}^{\infty}(1+\bar{\rho})rcke^{-r(s-T)}ds\right) - c + q(k)\cdot\left(\int_{\bar{t}(\bar{\rho},k)}^{\infty}(1+\bar{\rho})rce^{-r(s-T)}ds\right).$$

The last term represents the extra benefit from density, which is the increase in the ceiling. The derivative of the first-order condition with respect to the fair rate of return is:

$$P_{k\bar{\rho}} = \left(1+\gamma(k) - \frac{r}{R'(\bar{t})/R(\bar{t})}\right)\cdot q(k)\cdot c\cdot e^{-r(\bar{t}(\bar{\rho},k)-T)}.$$

where $\gamma(k)$ is the elasticity of output with respect to capital. This is an ambiguous term because the growth of rents was assumed to be less than the real interest rate (A5). To summarize, there are two effects on capital: first the ceiling raises the value of rents. Second, this will delay the lock-in time and so the above will be worth less.

VI. Conclusion

We have explored the effects of rent stabilization on the conversion of urban land. One of the primary insights is that because allowing fully flexible base rents permits landlords to capture all of the advantages of a rent growth control, neither the timing nor the density of development will be affected by rent stabilization. However, redevelopment is hastened because rent stabilization complemented by vacancy decontrol increases the difference between rents before and after redevelopment, increasing the opportunity costs of postponing redevelopment. Advancing the timing of redevelopment may have the indirect effect of hastening development and thus the somewhat counterintuitive result of stimulating construction activity.

Although rent stabilization does not appear to have deleterious effects on investment activity, it fails to benefit the tenants that it is supposed to protect (see Nagy 1997 for a discussion of this point). An obvious solution is to regulate the base rent in addition to the growth of rents. We considered the case where the base rent is set at rent levels corresponding to the unregulated sector. This would confer some consumer surplus, but at the cost of altering development activity. The dynamic nature of the regulated base rent control creates an opportunity cost of development: by developing now landlords lock themselves into lower rents than if they had waited. However, because landowners will delay development and wait for higher rents to compensate them for the additional opportunity costs, the intensity of development will not necessarily be negatively affected by this more rigid form of rent stabilization. Finally, we considered the effects of a rigid rent ceiling on development and redevelopment activity. Unlike rent stabilization, a rent ceiling will eventually halt the redevelopment process. Second, while the effects of different types of rent stabilization on the intensity of development are often ambiguous, a rent ceiling will always hasten development and reduce its density.

This work can be extended in a number of directions. First, one may ask whether considering rent processes governed by different assumptions would change the theoretical results. For example, the incorporation of a stochastic process into the rent function would approximate the uncertainty that developers face (see Capozza and Li, 1994). This would not be a trivial exercise given the dynamic nature of the rent growth control. It is the author's guess that there would not be any interaction between the volatility of rents and the rent regulation. According to a "bad news principle" (Dixit and Pindyck 1994, p. 40), it is downward movements in prices that lead investors to postpone investment, and not upwards ones. Second, a number of testable hypotheses were put forward in this paper. Examination of construction data in cities where the majority of housing is regulated would establishing a framework for explaining development behavior under rent stabilization. Finally, there is the issue of a fuller treatment of the demand-side. While it would not be easy to include a more complete model of consumer behavior, further explorations of the consequences of doing so are worthwhile.

VII. Bibliography

Amin, Kaushik and Dennis Capozza, 1993, Sequential Development, *Journal of Urban Economics*, 34(2): 142-158.

Arnott, R.J., 1997, Rent control, mimeo, to appear in *The New Palgrave Dictionary of Law and Economics*.

Arnott, Richard, 1995, Time for revisionism in rent control?, *Journal of Economic Perspectives*, vol. 9: 99-120.

Basu, K. and P.M. Emerson, 2000, The economics of tenancy rent control, *Economic Journal*, 110, 939-962.

Braid, Ralph M., 2001, Spatial growth and redevelopment with perfect foresight and durable housing, *Journal of Urban Economics*, 49, 425-452.

Brueckner, Jan, 1980, A vintage model of urban growth, *Journal of Urban Economics* 8, 389-402.

Brueckner, Jan, 1981, A dynamic model of housing production, *Journal of Urban Economics* 10, 1-14.

Brueckner, Jan, 2000, Urban Growth Models with Durable Housing: An Overview, in *Economies of Cities: Theoretical Perspectives*, J. M. Huriot and J.-P. Thisse, eds., Cambridge University Press.

Capozza, Dennis R. and Robert W. Helsley, 1989, The Fundamentals of Land Prices and Urban Growth, *Journal of Urban Economics* 26: 295-306.

Capozza, Dennis and Yuming Li, 1994, The Timing and Intensity of Investment: The Case of Land, *American Economic Review*, 84: 889-904.

Dixit, A.K. and R.S. Pindyck, 1994, *Investment under uncertainty*, Princeton University Press, Princeton, USA..

European Economic Commission, 1991, *Politique des loyers dans les pays de la CEE*, United Nations, Geneva, Switzerland.

Frankena, Mark, 1975, Alternative Models of Rent Control, *Urban Studies*, 12, 303-308.

Heffley, Dennis, 1998, Landlords, tenants and the public sector in a spatial model of rent control. *Regional Science and Urban Economics*, 28, 745-772.

Kiefer, David, 1980, Housing deterioration, housing codes, and rent control, *Urban Studies*, 17: 53-62.

Kutty, Nandinee K., 1996, The Impact of Rent Control on Housing Maintenance: A Dynamic Analysis Incorporating European and North American Regulations, *Journal of Housing Studies*, 11: 69-88.

Malpezzi, Stephen, 1993, Can new York and Los Angeles Learn from Kumasi and Bangalore? Costs and Benefits of Rent Controls in Developing Countries, *Housing Policy Debate*, 4, 589-626.

Murray, Michael P., C. Peter Rydell, C. Lance Barnett, Carol E. Hillestad, Kevin Neels, Analyzing Rent Control: The Case of Los Angeles, *Economic Inquiry*, Vol. 29, October 1991, 601-625.

Nagy, John, 1997, Do Vacancy Decontrol Provisions Undo Rent Control?, *Journal of Urban Economics*, 42, 64-78.

Olsen, Edgar, 1988, What do economists know about the effect of rent control on housing maintenance?, *Journal of Real Estate Economics and Finance*, 1: 295-307.

Silberberg, Eugene, 1978, *The Structure of Economics: A Mathematical Analysis*, McGraw-Hill, Inc.

Skelley, Chris, 1998, Rent control and complete contract equilibria, *Regional Science and Urban Economics*, 28, 711-743.

Spiegel, Murray R., 1990, *Theory and Problems of Advanced Calculus*, *Schaum's Outline Series*, McGraw-Hill, Inc., United States of America.

Turner, B., 1988, The politics and economics of rent control, *The Journal of Real Estate Economics and Finance*, 1, 257-276.