A SURVEY OF THEORETICAL ECONOMIC MODELS OF CONFLICT

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ABSTRACT

Among economists there is a growing interest on the multi-shaped aspect of conflict activities. Conflict, as a rational activity, plays a role in economic interactions as well as exchange. The acknowledged reference among economists is the pioneering work by Jack Hirshleifer. This paper is intended to survey the main contributions of a recent growing literature. It is organised as follows: a first section is devoted to analysis in depth the formal pillar of Hirshleifer (1988)'s seminal contribution. In the second section different contributions will be surveyed. In a third section attention will be paid on the impact of conflict on traditional exchange activity. A final section gives a summary and conclusions.

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I. HIRSHLEIFER'S BASIC FRAMEWORK TO MODEL CONFLICT

Among economists there is a growing interest on the multi-shaped aspect of conflict activities. Conflict, as a rational activity, plays a role as well as exchange in economic interactions. The pioneering work on conflict in recent economic literature is by Jack Hirshleifer whose foundations are in Hirshleifer (1987, 1988, 1989). In this view conflict is assumed to be a generalization of the famous concept of rent-seeking activities. Differently from rent-seeking models Hirshleifer proposes general equilibrium analysis. In particular he introduces the study of conflict by using four equations. (a) a *Resources Partition Equation*; (b) an *Aggregate Production Equation* that is characterized by constant returns to scale and constant elasticity of substitution (CES); (c) the *Contest Success Function* (hereafter CSF for brevity) determining the outcome of conflict; (d) an *income distribution equation*[i].

Within this basic framework two risk-neutral parties indexed by $i = 1,2$, make simultaneous once-and-for-all choices about their own allocation of resources between 'butter' and 'guns'. Each one is endowed with an initial positive endowment of resources, $n_i \in (0,\infty), i = 1,2$, which can be converted into 'guns', or 'butter' according a Resources Partition Equation defined by:

$$n_i = x_i + z_i, \forall i \tag{1}$$

where $x_i \in [0,\infty), z_i \in (0,\infty)$ respectively denote 'butter' and 'guns'. The CES function denoting the contestable output, that is the aggregate production function, denoted by Y, becomes a simple linear additive function:

$$Y = Y(x_1, x_2) = x_1 + x_2 \tag{2}$$

Then, the resources allocated to productive activities determine a total contestable output, say the 'pie', that is to be distributed according the resources allocated to 'guns'. The outcome of the conflict is determined through a CSF. It summarizes the relevant aspects of what Hirshleifer defines the *technology of* conflict. In particular, even if the CSF can take different forms, I apply the *ratio* form of the CSF[ii].

$$p_i = \frac{z_i}{z_i + z_j} \qquad \text{for } i = 1,2 \text{ and } j \neq i \qquad (3)$$

where, under the assumption of risk-neutrality p_i denotes the proportion of appropriation going to agent i for $i = 1,2$ and follows the conditions below:

$$\begin{cases} p_1 + p_2 = 1 & p_1 = 1 - p_2 \\ p(.,.) \text{ is twice differentiable} & \\ \partial p_i / \partial z_i > 0 & \partial p_i / \partial p_j < 0 \end{cases} \qquad (3.1)$$

Eventually, each agent's payoff function is given by:

$$U_i^{cc} = p_i Y = \frac{z_i}{z_i + z_j}(x_1 + x_2) \quad \text{for } i = 1,2 \qquad (4)$$

where the superscripts '*cc*' denote 'continuing conflict'. Hence, assuming a nash-cournot behaviour each opponent will maximize its own payoff expecting that the opponent is choosing the similar maximization strategy. The interior Cournot solution in the Hirshleifer basic model of 'continuing conflict' is then given by:

$$z_*^{cc} = z_1^{cc} = z_2^{cc} = (n_1 + n_2)/4 \qquad (5)$$

where the stars subscripted denote the equilibrium level. Indeed, whatever the amount of resources available for each player, the contenders allocate the same absolute value to fighting efforts. Therefore the income generated through the aggregate production function is equally divided between the two contenders. Formally we have:

$$U_*^{cc} = U_1^{cc} = U_2^{cc} = \frac{1}{4}(n_1 + n_2)\,; \tag{6}$$

However, in relative terms, the wealthier party devote fewer resources than poorer side into the contest. This is what Hirshleifer (1991) defined *The Paradox of Power* (POP). The initially disadvantaged group has an incentive to fight harder. Moreover, the less endowed side improves its position compared with the better-endowed rival. Hirshleifer, in particular, distinguishes between a strong form and a weak form of the POP. The strong form applies when contenders have an initial endowment of resources moderately unequal. In such a case, equilibrium can be found in the interior range. As the resources disparity increases, the strong form of the paradox no longer holds. When the resource asymmetry becomes sufficiently large, the opponents enter a corner-solution range where only a weak form of the paradox of power applies. It is defined 'weak' because, although no longer equalized, attained incomes could be less unequal than the initial resource endowment.

II. HIRSHLEIFER-STYLE MODELS: THE TECHNOLOGY OF CONFLICT

A large part of recent literature on conflict it can be considered as an offshoot of Hirshleifer seminal work. The main recent contributors are Michelle Garfinkel, Herschel Grossman, Stergios Skaperdas, Constantinos Syropoulos and Hugh M. Neary. The common base of their papers surveyed here is the 'focus' on the optimal choice of guns, feasible equilibria, and the different parameters which can induce new results, as compared to the basic model.

Skaperdas (1992) applies the common trade-off between productive and coercive activities in absence of property rights in order to examine the possible emergence of cooperation between two parties. In particular, the author states that conflict is not the necessary outcome of one-time interaction, given a room for cooperation depending upon the dominance of an actor over the other. A total available output is determined by agents' choice of productive activities. The investment in arms (which does proxy the efforts devoted to coercive activities) determines each agent's probability of winning a war. The winner receives the total product as prize or could divide the total product when a war does not occur in proportion to their respective probability of winning that represents a '*clear index of the agent's power*'[iii].

Each agent is endowed with one unit of inalienable resource that can be transformed into two inputs. Normalizing the resources constraint makes easier to concentrate on symmetric results of the model. The production technology is assumed to exhibit constant returns of scale. The assumptions on the conflict technology are sufficient for a pure strategy Nash equilibrium other than the worst case scenario when both parties devote all the resource endowment to 'guns'. The power of a player is increasing in its strategy and decreasing in the opponent's strategy. Moreover the power is assumed to be concave in a player's strategy when its strategy is greater than opponent's, and convex otherwise. Under these assumptions the author shows that three different Nash equilibria are feasible:

1) the *full cooperation* equilibrium, when there is no investment in arms on both sides. Note that this equilibrium is feasible only if a logit form of CSF is applied. In fact, in this case when $z_i = 0$, player 1 still retains a share of success. Otherwise no *full cooperation* (namely complete peaceful) equilibrium is feasible. In particular, a *full cooperation* equilibrium is more likely the more close to unity is the ratio of marginal products evaluated at the point of full cooperation, and the less 'effective' is the conflict technology. That is, given for example a large differential in armaments, only a small increase in winning probability emerges. Moreover it is necessary that the opportunity costs of investing in arms of both agents must be not so dissimilar.

2) a second feasible Nash equilibrium is the *partially cooperative equilibrium*. In such a case only one player devotes resources to the unproductive activity of conflict. That agent also receives her or his best payoff. To reach this, it is necessary that (i) the conflict technology is sufficiently ineffective even is not as much in full cooperation equilibrium; (ii) the marginal products are sufficiently diverse, with the more powerful having a lower marginal product.

3) a third feasible Nash equilibrium is the *conflict equilibrium.* It occurs when the effectiveness of conflict technology is sufficiently high. That is, when investment in arms easily increases power of the agents. Moreover, it is also necessary that the ratio of marginal products is sufficiently close to unity as well as in full cooperation equilibrium.

To summarise, the results show that the prevailing equilibrium depends upon the relationship between the effectiveness of the conflict technology and the ratio of marginal products. In particular the author points out that, considering imbalances of power *"cooperation itself can be consistent with domination of one party over another"*[iv]. In fact, since the more powerful agent invests more in arms, it must have a lower opportunity cost for that investment. Therefore, the marginal cost of arms is inversely related to each agent contribution to the total output.

Grossman (1991) develops a positive theory of insurrections and its deterrence or suppression. As economic activities, they compete with production. The story features a ruler and a large number of peasants' families. The ruler enforces the collection of land rents and or taxes on productive labour. In addition, the ruler is an employer of soldiers in order to reduce the probability of a successful insurrection. The ruler is also assumed to maximize his own income and that of his own clientele. Peasant families allocate labour time alternatively to production, soldiering or partaking in an insurrection. A successful insurrection is supposed to be a winner-take-all contest, namely insurgents gain all of the revenue of the ruler and his clientele. However, time and resources devoted to insurrection are considered socially wasteful.

Grossman claims his contrast with Hirshleifer (1988) and Garfinkel (1990) because the later works analyse the effect of technology on the allocation of resources between productive and appropriative activities, without analysing the behaviour of subjects. That is, in Hirshleifer and Garfinkel models competing parties are as rulers and no mention of subject is provided. The model is built on three fundamentals functions: (i) the income of ruler's clientele; (ii) the income of peasants' families; (iii) a technology of insurrection and its suppression. The latter, in particular, is a modified form of CSF:

$$\beta = \frac{I^{1-\theta}}{S^{\sigma} + I^{1-\theta}} \qquad \text{(gr.1)}$$

where I and S denote the fraction of peasant time allocated to insurrection and the fraction of time allocated to soldiering respectively. Of course, β is increasing in *I* and decreasing in *S*, and is bounded between zero and unity. The parameters σ and θ capture the technology of insurrection are both assumed to be bounded between zero and unity. The resulting equilibrium allocation of resources and probabilistic distribution of income depend on these parameters.

The results of the model show that a high tax rate imposed by the ruler to the peasant families can have two negative effects. First, it depresses the tax base; secondly it increases the probability of a successful insurrection. In fact, at a higher tax rate the fraction of time that peasants devote to insurrections become higher and higher. Even if peasants do not devote any time to insurrection, with a positive technology of insurrection, the threat of insurrection induces the ruler to demand for soldiering and to set a low enough positive tax rate in order to deter insurrection completely. Then, the technology of insurrection is the element driving the optimal choice both for ruler and peasants.

Although Grossman intended to contrast Hirshleifer, the model ends up with a very similar outcome. In fact, Hirshleifer (1988) stressed the importance of technology of conflict through the mass effect parameter, renamed 'decisiveness' parameter. In a similar fashion Grossman argues that technology of insurrection is the most important force driving towards an equilibrium of allocation between labour, soldiering and insurrections. However, through the CSF (gr.1) he distinguishes between the technology applied to defensive and appropriative efforts.

Garfinkel (1994) is intended to explore the interactions between domestic politics and international conflict. The main finding seems to recall the ideal of Kantian peace, also showing that electoral uncertainty in democratic states can reduce the severity of international conflict. In particular, it induces less arming.

The internal policy outcome is modelled by using a two-period protocol, which takes into account the optimization problem faced by voters (or consumers) and the political parties of one nation. In a first section, foreign nations' military expenditures are assumed to be given. In a second part, the model is extended to illustrate the linkage between international conflict and political competition. A limiting assumption is that nations are considered perfectly identical. Each nation chooses its military policy in an effort to secure a proportion of a given world resource. The defence technology is specified through a classical CSF. Comparing dictatorships and democracies it is shown that political competition reduces the severity of conflict between nations. This easing is supposed to be the lowering of the amount of resources allocated to military spending in both nations.

This result is driven through the fact that the party in office in the first period can choose the allocation of resources secured by current military spending among peaceful investment activities (goods and services provided by the government to the voters on a non-discriminatory basis) in the next period only if re-elected. The probability of not being reappointed produces a negative bias in military spending, while under autocracies the probability to stay in office exactly equals unity so as to rule out any uncertainty. In the latter case there is no negative bias on military spending.

However, the results of the model suggest that international cooperation, as an international disarmament, is more likely to be sustained in a cooperative equilibrium without threat and punishments[v]. Thus, Garfinkel model goes far beyond the implicit assumption of Hirshleifer considering the state as a unitary actor. In fact, in the basic model the trade-off between 'guns' and 'butter' is completely exhausted on the international scenario, since the agents only rationally simultaneously react to the opponent's rational optimal choice. This mechanism then is relaxed by Garfinkel even if with strict limiting assumptions (i.e. countries perfectly identical in resources, preferences etc...etc...). Note that in Garfinkel the 'pie' of conflict also changes. It is no longer the output produced through the joint production function, but a fraction of a contested resource which is to be secured in the next period.

This final is extended in Grossman and Kim (1995), which underlines the distinction between offensive weapons and instruments that can provide defence against predation. Many of the 'appropriative' activities are purely defensive and they are supposed to be a deterrent to predation. They explicitly consider the allocation of resources among productive and appropriative activities and the equilibrium security of claims of property. Moreover, the analysis does not address the possibility that a third a party, such a government, enforces claims of property. There are two risk-neutral agents (individuals, tribes or nation states) not integrated. They have initial resources endowments, which are claims to the property, subject to appropriation. Each agent chooses the allocation of resources following the sequence: 1) choice of defence, 2) allocation of resources between production and appropriation. Let h_i, z_i, x_i denote respectively the amount of defensive resources, the amount of offensive weapons and the share devoted to independent productive activities of consumables. Hence at the first step, each agent chooses h_i such that $0 \le h_i \le n_i$. At the second step each agent chooses x_i and z_i such that $x_i \ge 0, z_i \ge 0$. The resource partition equation is:

$$x_i + z_i = n_i - h_i \qquad \text{(gk.1.1)}$$

It is assumed for simplicity that the entire endowment of agent *i* is subject to appropriation by opponent, hereafter denoted by *j* ($j \ne i$), whereas the consumables are not. A particular formulation of CSF describes how the agent *i* retains a fraction p_i of its endowment.

$$p_i = \frac{1}{1+y_i}, \quad y_i = \theta \frac{z_j}{h_i} \qquad 0 \le p_i \le 1 \qquad \text{(gk.1.2)}$$

In this equation y_i captures the offensive strength of agent *j* relative to the defensive strength of agent *i*, and θ is a positive parameter that indicates the effectiveness of offensive weapons against defensive structure[vi]. It is affected by technical advancements, both positively and negatively. This formulation differs from CSF presented because y_i is assumed to be a homogenous function of degree zero in z_j and h_i and of degree one in the ratio z_j / h_i.

Predation is also assumed to be destructive. This implies that in any appropriative interaction the predator gains less than the prey loses. The net result of appropriative activities for agent *i* is a nonnegative wealth. The objective of each agent is to maximise the sum of its consumables (that are assumed not to be subject to appropriation) and the final wealth. Hence at the second stage agent *i* takes the level of defensive efforts as given and maximise its utility subject to the resource constraint and the non-negativity constraint for 'guns'.

To summarise, the findings show that the equilibrium security of the claims to property is independent of the size of endowments, but depends only upon the degree of effectiveness of offensive weapons and the destruction parameter. Under some circumstances the equilibrium is not aggressive and claims to property are considered fully secure. By contrast, there are equilibria less than fully secure where the claims to property are negatively related to the effectiveness of offensive weapons and also positively related to the destructiveness of struggle. In non aggressive equilibria the total cost of appropriative activities is lower than in any equilibrium with less than fully secure claims to property. For non-aggressive equilibria the welfare of agent *i* is negatively related to effectiveness of offensive weapons and positively related to destruction parameter, and negatively related to the total cost of appropriative activities. If agent *i* is relatively rich, its welfare is higher the smaller the effectiveness of offensive weapons and the larger the destruction parameter. (i.e. when the claims to property are more secure). If agent is relatively poor the welfare is higher the larger is effectiveness of offensive weapons and the smaller is the destruction parameter, i.e. if claims to property are less secure. This means that if offensive weapons are highly effective and predation is not too destructive, a relatively poor agent can be better off in an equilibrium with less secure claims to property.

Another prominent contribution is in Neary (1997) that, following Hirshleifer and Skaperdas, develops a model of a society where only armed self-enforcement of property rights is possible. The main parameters considered are the level of system total wealth and the initial asymmetrical distribution of this resource stock between the actors. The bilateral interaction is two-stage shaped. In the first stage each player has an initial stock of resource that is committed to an allocation between 'guns' and 'butter'. The allocation of resources to productive activities result in a consumable output, say the 'pie'.

In the second stage parties divide the 'pie' according to the investment in 'guns' chosen in the first stage. Then, given a positive initial total wealth, asymmetrically distributed according to a parameter $\sigma \in (1/2,1]$. The initial endowments are given by $n_1 = \sigma N$ for player 1 supposed to be the richer, and $n_2 = (1-\sigma)N$. In the first stage, the players must decide the level of 'guns'. In the second stage, the divide the 'pie' according to the outcome of a CSF in its logit form. The production function is assumed to be symmetric, concave and increasing in both arguments. Neary assume Nash behaviour by each player. Then, both parties maximize simultaneously their own payoffs subject to the constraint of productive resources.

There are four types of equilibrium: (i) *communal* equilibrium; (ii) *suzerainty* equilibrium; (iii) *hobbesian* equilibrium; (iv) *banditry* equilibria. These are depicted in figure 2.4

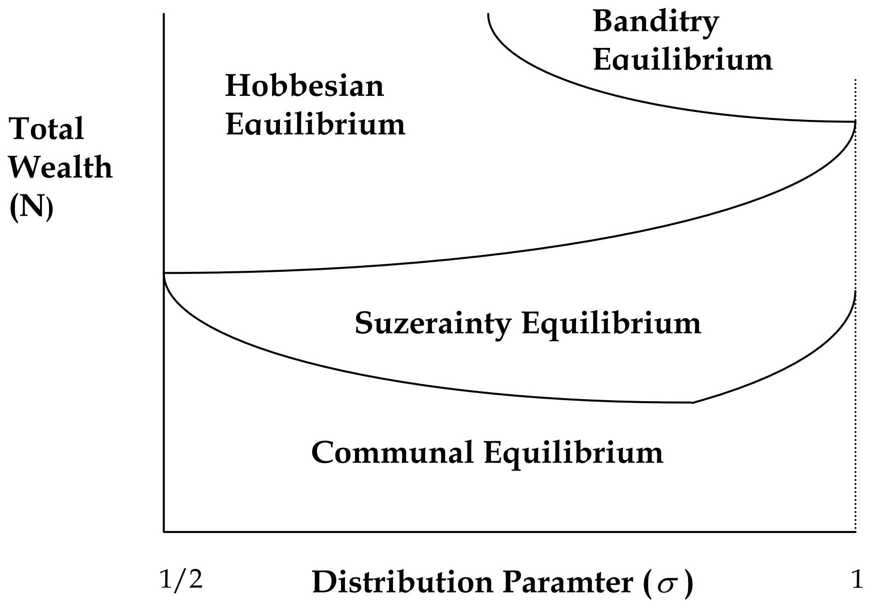

Figure 1
Equilibrium Regions in Parameter Space
(Neary, 1997)

1) The *communal equilibria.* In such a case parties do not devote any resource to 'guns', $z_1 = z_2 = 0$. Zero-guns allocations are chosen non-cooperatively simply because the marginal benefit of investment in butter outweighs its marginal cost. Thus, given by assumption that $p(0,0) = 1/2$ each party has an equal share of 'pie'. The communal are those defined 'full cooperation' by Skaperdas (1992).

2) The *suzerainty equilibria*. The richer party puts resources into both guns and butter. The poorer side only invest in 'butter'. In such a case a hierarchical relationship between parties exists. Neary cites as examples: the hierarchical relationship between an imperial centre and the periphery as tributary centres or colonies. This is also a reminiscent of authoritarian, despotic or feudal situations in which one group dominates another. As a special case when the production function is linear there are no Suzerainty equilibria.

3) *The Hobbesian equilibria*. In this case, both players devote an amount of resources to guns, according to $0 < z_i < n_i, i = 1,2$. The Hobbesian equilibria are interior, since the solution vectors lie in the interior strategy space. The European Great Powers are an example of Hobbesian equilibria, as well as Third Reich.

4) The *Banditry* equilibria is characterised by the poorer side devoting all resources to guns. This outcome reflects the fact that, as the less-endowed party it runs into a severe resource constraint as σ increases. This also involves a level of initial total wealth sufficiently high. Banditry equilibria do not occur at all. Neary shows that neither the Cobb-Douglass nor the CES forms of the production function allows banditry equilibria to exist. However, the banditry equilibrium concept is akin to that of Paradox of Power expounded by Hirshleifer. Neary also contradicts the intuition which suggests that increasing absolute scarcity increases the likelihood of conflict. In fact, as showed in *communal equilibria*, at very low levels of total wealth equilibria occur at zero-investment in arms. This outcome emerges simply because absolute poverty makes the return on arms expenditure unattractively low to both players; so neither player arms and each shares equally the output irrespective of the initial resource distribution.

Skaperdas and Syropoulos (1996) explore the possibility for cooperation in a long-term relationship. They show that in an inter-temporal model of conflict the possibility of opposite effect arises, i.e. the conflict intensifies. The basics of the model are: (1) a Cobb-Douglas specification of the functional form of production function exhibiting decreasing returns of scale. However, the authors suggest that the results of the model are expected to remain unaffected if other type of functions exhibiting constant returns to scale are adopted; 2) a logit form of the CSF; 3) an equal initial endowment of resources; 4) a common specification for the *income distribution equation.* Given the perfect equality of agents the equilibrium depends upon the mass effect parameter of CSF. This implies that with symmetric players a sufficiently low level of decisiveness of conflict lead to a *full cooperation* ($z_i = 0, i = 1,2$) Nash equilibrium.

When the conflict is decisive both parties invest a positive amount of resources to 'guns'.

To evaluate the impact that a long-term horizon can have on cooperation the authors focus on values of mass effect parameter and resource endowment which satisfies the condition for a *full cooperation* Nash equilibrium in the one-shot game. The second period game differs from the first only because of the resources endowment that is a function of past payoff weighted by a positive growth parameter. Each agent's total two-period payoff is the sum of utilities in the two periods. In particular, the second-period payoff is discounted through a discount parameter which lies in a half-open interval $(0,1]$. To solve the model, Skaperdas and Syropoulos employ the concept of the sub-game perfect equilibrium. First they solve for the symmetric equilibrium in the second period, and then they find the optimal first period investments in arms. There is still room for a full cooperation equilibrium in the two-period game. Both parties will refrain from investing in arms since the second period investment in arms is not influenced by changes in the first period arms investment. On the contrary, when the product of resource endowment, the growth and the mass effect parameter, there are three possible scenarios depending on other feasible combinations of discount factor and growth factor:

(i) parties undertake positive investment in the second period but not in the first. This is because future is not valued enough (the discount parameter lies between zero and a critical values which is a function of growth parameter);

(ii) if future payoffs are highly evaluated both parties undertake positive investments in arms only on the first period;

(iii) parties invest in arms in both periods. Taking into account any exogenous growth factor there exist a discount factor so that conflict activities occur in both periods.

The authors state that a longer shadow of the future may discourage cooperation and intensify conflict. However, the main result of the model is based upon the assumption a party's payoff today affects tomorrow's resources, and indirectly also tomorrow's payoff.

Grossman (1998) explores a series of general equilibrium models in which agents choose to be either producers or predators by allocating their own resources either to production or to guarding their production against predators. Agents' choices depend upon the level of consumption they would get. In the first model, given perfect equal productive opportunities among agents, it is showed that the technology of predation (depicted through a modified form of CSF), determines both the equilibrium ratio of predators to producers and the equilibrium amount of resources allocated to guarding against predators.

In the second model, an irreversible collective choice of the amount of resources invested in deterrence against a possible predation decreases the social cost of predation. In the third model, the basic analytical framework is extended. Agents are now well-endowed or poorly endowed. Since production is more productive, well-endowed agents have lower incentives to be involved in predation. Individual choice of guarding assumed and given only a small fraction of poorly-endowed agents within the system, it is expounded how all the poorly-endowed agents as well as some of the well-endowed would choose to be predators.

As the fraction of poorly endowed people increases, all of them still would choose to be predators whilst all the well-endowed agents would choose to be producers. The equilibrium ratio of predators to producers increase as well as the equilibrium amount of resources allocated to guarding against predators. In the last model the fraction of well-endowed people makes an irreversible collective choice of the amount of resources to allocate against predators. The amount of this allocation is enough to deter well-endowed people from becoming predators. Finally, the poorly-endowed people choose to be predators.

A related argument is then expounded by Grossman and Mendoza (2001). This model develops a general equilibrium approach in which rival rulers are engaged in economic competition for the allegiance of subjects and in military competition for the control of land. The rival rulers control a fraction of arable land. The production functions on the lands are assumed to be Cobb-Douglas with decreasing return of scale and land and labour force as inputs. The income for both rulers is simply determined as the difference between the total gain and the cost of labour force. If an amicable split of land is assumed, the rulers equally divide the land, and then number of farmers hired and the production. Indeed they equal their own income to the value of the marginal products of their lands, whereas the income of subject farmers equals their own marginal products.

The military competition is introduced through a CSF that does determine the amount of land which can be gained by both rulers. In the perfect symmetric case, however, the results do not change apart from the net income of rulers. The rulers simply divide equally the land, the production and the income of farmers are exactly the same, but because of the resources dissipated in military competition the incomes of rulers are lower than in the foregoing situation. Introducing the existence of soldiers as to proxy the military strength, the outcome of model changes. The soldiers have to be provided with a net income as well as the farmers. Given the symmetry of the model the authors show: a) the production on the lands of each ruler decrease since the people hired in the army cannot longer farm the land. Then, the subjects receive higher incomes than in economic competition only. In addition, the social cost of military competition is given by the foregone production of soldiers and the incomes of the rulers are decreased by more than this social cost. Moreover, allowing the military competition be destructive the results change and appear to be more realistic. As the destruction parameter increases, the value of the marginal product of farmers decreases as well as the incomes of the subjects of the rival rulers. As long as military competition is not too destructive the subjects of rival rulers have higher incomes with both military and economic competition than with economic competition alone.

Stauvermann (2002), intends to explore under what circumstances a total peace can occur. He notes that no model exists where a peaceful anarchy without arming constitutes an equilibrium. In fact, he argues that peaceful outcomes based upon deterrence are to be distinguished from peace without any investment in weapons. In other words, he recalls the 'full cooperation' scenario where no investment in any coercive unproductive is made. Differently form the most existing literature it is assumed that the agents act sequentially like in a Stackelberg game. Hence, one agent moves first as a Stackelberg leader choosing its optimal armament. The opponent observes the optimal choice of the leader and also chooses its optimal armament. The main result of the paper is that a war only occurs if both adversaries are sufficiently different concerning their production possibilities, effectiveness of their arms and their resources. Whenever rivals are relatively homogenous peace can take place.

Hirshleifer-style models are commonly static analyses. Reuveny and Maxwell (2001) produce a dynamic Hirshleifer-style model with two agents, dependent on a single renewable resource. Moreover, the authors underline that their work is also based on the ecological competition literature. In this latter body of literature, in fact, the recurring feature is represented by the competition between two interacting species that feed off the same renewable resource.

The dynamic models are commonly composed as a system of differential equations for the stocks of the species and a resources stock. A rise in the size of either species reduces the resource stock, whereas a rise in the size of one species reduces the size of the other. Then, the authors study the dynamic interaction between conflict, population, and resources in a lesser developed society. The model has five steady states. Four steady states exhibit no conflict because either one or both groups are extinct. A breakdown can depend upon parameters of the resource and population. Then, the authors focus on a fifth steady state that features conflict. The comparative static reveals that changes enhancing the resource stock or the population raise conflict. Whenever the conflict efficiency of one group relative to the other raises, this reduces its conflict effort. A rise in the model's decisiveness parameter generates an ambiguous effect on conflict, which also differs from Hirshleifer static model. The effect of raising harvesting efficiency on conflict is positive when the resource stock is high. Turning to the dynamics, the results fit with the stories of historical societies that exhibited a relatively brief flowering, followed by decay, all the while exhibiting conflict over the resource base.

III. EXCHANGE IN THE SHADOW OF CONFLICT

In the contributions presented in the first section, the focus was on the standard resource allocation problem: how many guns and how much butter? Then, only production and conflict were contemplated and the highest attention has been paid on technology of conflict. The works by Charles Anderton and his colleagues have introduced the emergence of exchange, '*in the shadow of conflict*'. They did this by enriching the Hirshleifer basic model, with an explicit consideration of prices and terms of trade, showing that terms of trade also depend upon the level of predation and conflict.

Anderton and Anderton (1997) present an integrated model of conflict, production and exchange and demonstrate how a shrinkage of the classical Edgeworth Box occurs. Two players labelled as 1 and 2, each with an endowment of player-specific resources use these resource endowment to produce two goods, say 'butter' and 'clothes'. Consider the Edgeworth box in figure 2. Assume for simplicity perfect specialization, namely player 1 produces only butter and player 2 produces only clothes. They are both better off if exchange at point E on the contract curve.

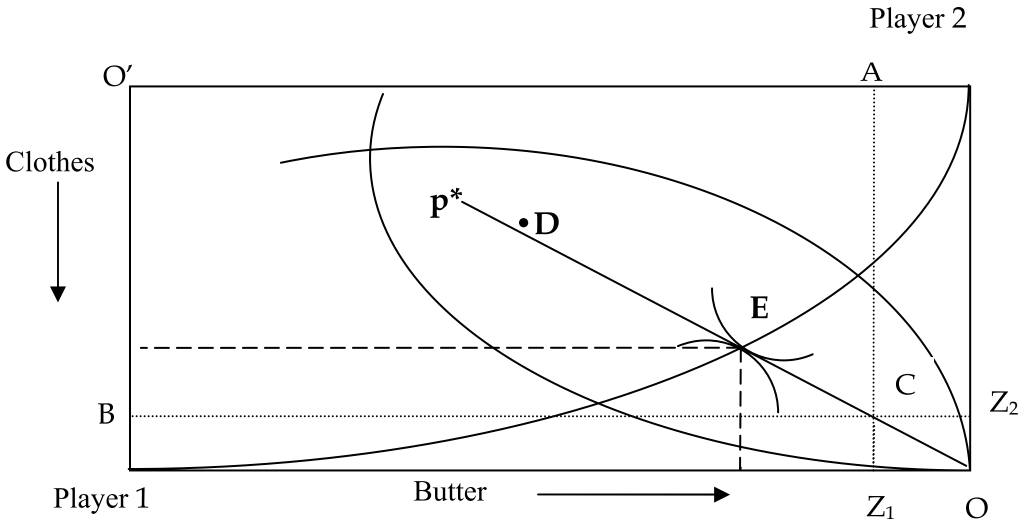

FIGURE 2
THE EDGEWORTH-BOX WITH CONFLICT
ANDERTON AND ANDERTON (1997)

Assume that both players can transform some or all of their resource into 'guns' and fight over the good that are produced in the system. First suppose that player 1 arms and player 2 does not. In such a case, the player 1's production of butter falls by **OZ**$_1$, but it can appropriate the clothes produced by player 2 and end up at a better point **A.** If player 2 arms and player 1 does not, in a similar way player 2 can end up at better point **B**. If player 1 and player 2 both arm, then the coordinate points (**Z**$_1$, **Z**$_2$) can be interpreted as the distribution of the total output according to the relative appropriative strengths. Whenever, players are equally strong, the players will split the common output, (see point **D**).

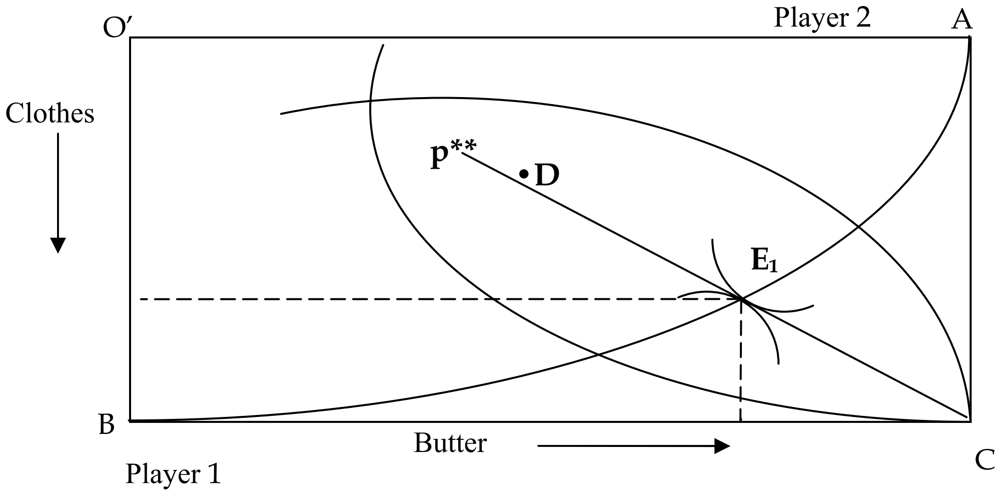

FIGURE 3
EXCHANGE OR CONFLICT IN THE
SMALLER EDGEWORTH-BOX
ANDERTON AND ANDERTON (1997)

In figure 3, the box now is smaller then that in figure 2 (note that the point C becomes the origin). If exchange is considered, then parties prefer to exchange at point $\mathbf{E_1}$. The exchange which occurs at price **p****, which depends upon the resources, the abilities of players as well as upon their appropriation/protection abilities and the technology of conflict. The end result is a shrinkage of Edgeworth Box.

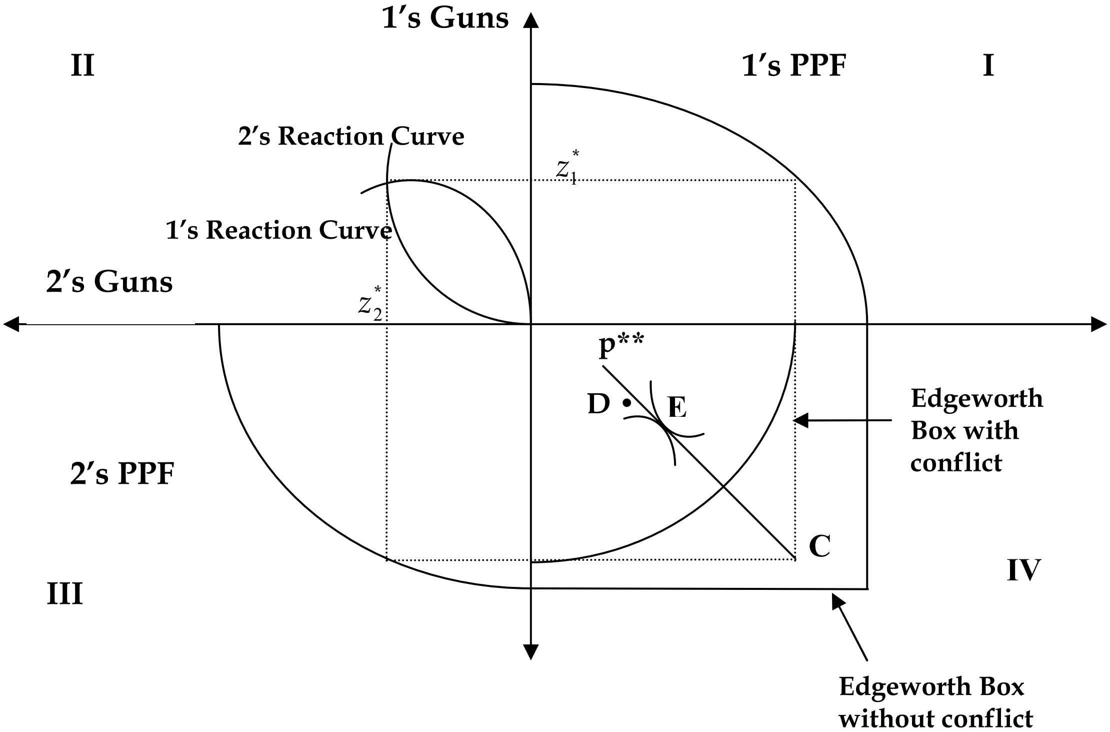

FIGURE 4
CONFLICT, PRODUCTION AND EXCHANGE
ANDERTON AND ANDERTON (1997)

In figure 4 the interdependence of conflict, production, and exchange. Quadrants I and III depict the production possibility frontiers (PPF) for both players. The limiting assumption of perfect specialization between players allows fro a bi-dimensional graph. In quadrant II the reaction functions for both players are drawn. The equilibrium level of 'guns' and the technology on conflict determine the distribution of consumer goods *in the shadow of conflict*, which is point **D**, in the quadrant IV. Thus, the level of production serves as the 'pie' to distributed under conflict, but also as to define the size of Edgeworth Box. In such a situation, Anderton and Anderton note that neither the first nor the second theorem of welfare hold. Whenever the production of 'guns' falls to zero, Pareto-optimal equilibria are still feasible. Otherwise they are not. Clearly, all the points in the smaller Edgeworth Box can be defined as sub-pareto-optimal.

Anderson and Marcouiller (1997) consider a general equilibrium trade model with endogenous predation. Such predation impedes specialization and gains from trade. The consider an anarchic world with agents that are rational utility maximizers allocate labour between predation e the production of two goods. Long-run static equilibria between two countries are analysed under the limiting assumption that each country is composed of agents with identical ricardian technology. Three types of equilibria may emerge: (i) autarky, with no predation and no defence; (ii) insecure exchange equilibria, with predation and defence; (iii) secure exchange equilibria (defence completely deters predation). The interaction of technologies of predation and defence is captured through a CSF akin to that presented in Grossman and Kim (1995). The trading equilibria, however, are supported only in a narrow range of security parameters. The interaction of predation and production and the general equilibrium interaction between two trading economies determine the terms of trade and gains form trade. In fact, since changes in technologies of defence and predation have terms of trade effects, some producers are hurt by enhanced security. This is idea of 'immiserizing security'. Since some larger and poorer countries can lose from improvements in the security of international exchange, immiserizing security suggests a potential for international conflict due to the opposing interests with respect to security.

Anderton et al. (1999) present integrated models of production, exchange, and appropriation. In particular, the common interpretation of the gains of trade is modified in order to include the detrimental effect of predatory activities. Since exchange and appropriation are intertwined, pure conflict and pure trade constitute only special cases of these new integrated models. The model shows how conflict can be overcome by gains form trade, but at a resource cost that modifies exchange itself. They model a one-time interaction between two agents as a sequential predator/prey game with the potential for Ricardian trade. The authors recall the story, famous among economists, of Robinson Crusoe and Friday.

Think of Crusoe as a potential prey and Friday as a potential predator. Crusoe moves first by allocating some resources to defensive fortifications to protect his remaining resources. Friday moves second and has two options: either allocates some resources to weapon in order to seize resources from Crusoe, or produce no weapons and move on the path of specialised production and trade.

The economic interactions depend upon the parameters: relative resources endowments, effectiveness of offensive weapons against defensive fortifications, and weapons and consumer goods productivities. Due to the complexity of the model, no general analytical solutions are derived. They present the results of simulations testing the comparative statics of the model. In the pure predator/prey scenario the equilibrium level of predation and final distribution depend upon the relative resource endowments and the effectiveness of attack relative to defence. They distinguish three equilibrium areas: pure predation, partial predation, and no predation. Whenever the relative effectiveness of attacker towards the defender lies in the range [0,1] and the relative resources endowment is not too high there is no predation. As the effectiveness increases, agents enter an area of partial predation up to a critical level of resources ratio. Integrating exchange into the model it is shown that it forestalls appropriation over a wide range of relative resource endowments and relative attack effectiveness. When the ratio of resource endowments and relative attack effectiveness are sufficiently high, pure predation forestalls exchange.

Anderton (2000), extends the foregoing model in order to take into account the different impact of different conflict function (i.e. ratio and logistic) and parameters. I report the functional forms adopted by the author: the ratio from is:

$$p_i = \frac{(\alpha_i z_i)^m}{(\alpha_i z_i)^m + (\alpha_i z_j)^m} \qquad \text{(A.1)}$$

and the logistic form is:

$$p_i = \frac{1}{1 + e^{m(\alpha_j z_j - \alpha_i z_i)}} \qquad \text{(A.2)}$$

The two functional forms of the CSF lead to very different results. This is the most important outcome of this study. The model is developed through simulations in which conflict only emerged under ratio conflict technology but it is much more costly to induce trade under logistic conflict technology. Moreover, it is found that a greater access to arms has no impact under ratio conflict technology, but a detrimental economic impact under logistic technology. When the relative resource endowment is highly unequal in favour of the defender trade gave away to conflict under the ratio technology but not under the logistic technology. Similar results are in Anderton (2003) showing how mutually beneficial exchange can subdue hawk playing. One of the results of the model is a nullifying productivity growth. That is, increases in the productivity of each party cause a fall in production and welfare per capita under certain conditions. After a transition cusp, however, such increases in productivity 'vaults' economy in a dramatically improved state. More precisely, increases in productivities of agents are expected to increase production and welfare, but the greater is the presence of hawks the higher the damage inflicted to economy due of dissipation of resources.

CONCLUSIONS

The recent theoretical literature on conflict, whose acknowledged forefather is Jack Hirshleifer, sheds light on a particular aspect of social interactions: the existence of force and violence that leads to different results of economic activity. This is particularly important when international economic relations are taken into account. Trade agreements and trade policies between countries constitute a noteworthy example. At the same time, theoretical analysis that consider conflict from the beginning can also pave the way for studies regarding the emergence of norms and institutions between actors in anarchic societies. Then, this kind of approach appears to be a future fruitful direction in economic modelling.

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NOTES

[i] The Contest Success Function is a mathematical relation that links the outcome of a contest and the efforts of the players. It is actually a founding pillar of many models. Selective seminal contributions are by Tullock (1980), O'Keeffe et al. (1984) and Rosen (1986). Dixit (1987) develops a general framework for contests using the general properties of logit functions. Hirshleifer (1989) focuses on a different form for the CSF: the ratio form and logit form. See also Skaperdas (1996) and Clark and Riis (1998) for a basic axiomatization.

[ii] Hirshleifer (1989) analyses the different impact of two different function form for CSF: the *ratio form* and the *logistic form*. In the first case, the contest outcome depends upon the ratio of the efforts applied, whilst in the second case it depends upon the difference between the resources committed.

[iii] Skaperdas (1992), p. 721.

[iv] Skaperdas (1992), p.733.

[v] In Garfinkel (1990) the author develops a positive economic theory of military spending. Then the analysis shows that if there are repeated interaction between nations, a game of threat and punishments usually does not support a disarmament outcome and that fluctuations in military spending can be an endogenous result of fluctuations in aggregate economic activity. Moreover military spending depends upon whether governments are acting cooperatively or opportunistically.

[vi] Dixit (2004) develops a simple model applying the basic framework of Hirshleifer (1995) enriched by the effectiveness parameter of offensive weapons proposed by Grossman and Kim (1995). In the resulting symmetric equilibrium, the resources spent both on defence and offence by both agents equal. As the technology favours defence over offence the productive use of resources increase.