Does a non-linear mean reverting process characterize real GDP movements? Results from non-linear unit root tests.

Dimitris K. Christopoulos
Department of Economic and Regional Development, Panteion University,
Athens, Greece.

Abstract

This paper uses non-linear unit root tests to investigate non-stationarity of real GDP per capita for seven OECD countries over the period 1900-2000. Non-linear unit root tests are more powerful than traditional ADF statistics in rejecting the null unit root hypothesis. Empirical results show that, contrary to what the linear ADF statistics suggest, stationarity characterizes five out of the seven countries. This finding stands at variance with other recent studies which conclude that movements in real GDP per capita can be characterized as a non-stationary process.

Keywords: Unit root tests, non-linear models, real GDP.

JEL Classification: C22, E1.

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Address for correspondence:
Dr. Dimitris K. Christopoulos,
Department of Economic and Regional Development,
Panteion University, Leof. Syngrou 136, 17671, Athens, Greece.
Tel. (++30) 210-9224948, Fax: (++30) 210-9229312,
e-mail address: *christod@panteion.gr*

1. Introduction

An important feature of business cycle theory is the trend stationarity of real output. This means that shocks have only a transitory impact on real output movements leading the economy towards an equilibrium value. However the stationarity of real output is an unlikely possibility. A substantial number of empirical studies give support to the contention that real output levels are non-stationary: see, for example, Nelson and Plosser (1982), Wasserfallen (1986), Cheung and Chinn (1996) and Rapach (2002). This finding has important implications for business cycle theory as it suggests that real factors such as technology shocks play an important role in economic fluctuations. Within this context, business cycle theory no longer implies stationary fluctuations around a deterministic trend. Finally, the presence of a unit root in output movements has consequences for the way we forecast economic activity and evaluate the role and importance of macroeconomic stabilization programs.

The empirical literature cited above reached the conclusion that real GDP levels are non-stationary by using either univariate unit root statistics (Cheung and Chinn, 1996) or panel unit root tests (Rapach, 2002) along the lines of the augmented Dickey-Fuller (ADF) statistic. The key feature of all these tests is that they work upon the hypothesis that a symmetric adjustment process exists. However, a very recent and expanding empirical literature allows for non-linear dynamics for unit root testing procedures: see for example Enders and Ludlow (2002), Caner and Hansen (2001), Shin and Lee (2001), He and Sandberg (2003) and Kapetanios *et al.* (2003). According to Enders and Granger (1998) all standard linear unit root tests have lower power in the presence of misspecified dynamics. They reviewed many important examples of asymmetric adjustment of economic variables.

The aim of the present paper is to find evidence for a non-linear mean reversion for real GDP using a battery of unit root methods with non-linear dynamics. These methods are more suitable for detecting non-stationarity in the levels of real GDP per capita.

2. The Caner and Hansen threshold autoregressive (TAR) model

Caner and Hansen (2001) consider a threshold autoregressive (TAR) model to provide statistical tests for the joint null hypothesis of linearity and stationarity. This model is given by:

$$\Delta y_t = \delta(L)\Delta y_t + \rho_1' x_{t-1} I_{\{Z_{t-1}<\lambda\}} + \rho_2' x_{t-1} I_{\{Z_{t-1}>\lambda\}} + e_t \qquad (1)$$

t=1,2,…..T

where $\delta(L)$ is a lag polynomial of order p, $x_{t-1} = (1, t, y_{t-1})'$, $Z_t \equiv y_t - y_{t-m}$ for some $m \geq 1$ (m is the delay parameter which in our application is set equal to one), $I(.)$ is the indicator function, λ is the threshold parameter and e_t is an i.i.d. error. We suppose that the threshold variable Z_{t-1} is predetermined and strictly stationary, while λ is unknown and takes values in the interval $\lambda \in \Lambda = [\lambda_1, \lambda_2]$ where λ_1 and λ_2 are chosen so that $\Pr ob(Z_t \leq \lambda_1) = \pi_1 > 0$ and $\Pr ob(Z_t \geq \lambda_2) = \pi_2 < 1$[1]. In so far as the description of the empirical methodology is concerned, it is convenient to consider the vectors $\rho_1 = (\alpha_1, \beta_1, \gamma_1)'$ and $\rho_2 = (\alpha_2, \beta_2, \gamma_2)'$ where α_1 and α_2 are the intercepts, β_1 and β_2 are the trend slopes and γ_1 and γ_2 are the slope coefficients on the lagged levels.

OLS can be used to estimate model (1) for a fixed $\lambda \in \Lambda$. Letting $\hat{\sigma}^2(\lambda) = T^{-1}\sum_{1}^{T} \hat{e}_t(\lambda)^2$ the computed residual variance, the LS point estimates of the threshold λ and the corresponding vectors ρ_1 and ρ_2 are found by minimizing $\hat{\sigma}^2(\lambda)$:

$$\hat{\lambda} = \arg\min_{\lambda \in \wedge} \hat{\sigma}^2(\lambda)$$

To test for linearity $(H_0 = \rho_1 = \rho_2)$ in a TAR model against the alternative of a threshold effect we can use a Wald statistic, $W_T(\lambda)$

$$W_T(\lambda) = T(\hat{\sigma}_0^2 / \hat{\sigma}^2(\lambda) - 1)$$

where $\hat{\sigma}^2(\lambda)$ is the generated residual variance from model (1) for a fixed threshold λ and $\hat{\sigma}_0^2$ is the residual variance from OLS estimation of the null linear model $(\rho_1 = \rho_2)$. Since $W_T(\lambda)$ is a decreasing function of $\hat{\sigma}^2(\lambda)$ the Wald statistic is

$$W_T = W_T(\hat{\lambda}) = \sup_{\lambda \in \Lambda} W_T(\lambda)$$

[1] Since only the magnitude of the change in the deviation matters and not the sign, we consider the absolute value of change as the switching variable. Therefore we treat π_1 and π_2 symmetrically so that $\pi_2 = 1 - \pi_1$ $\lambda_1 = -\lambda_2 = \lambda$. This makes our TAR a two-regime symmetric model.

However, since the distribution of the W_T is not identified under the null Caner and Hansen (2001) prove that although an asymptotic distribution can be generated they suggest the use of a model-based bootstrap approach.

In model (1) the parameters γ_1 and γ_2 control the stationarity of the process y_t. A leading case is when y_t is a unit root process. To test for unit roots, we use the one-sided formulation of Caner and Hansen (2001), namely $H_0 : \gamma_1 = \gamma_2 = 0$ versus the alternative $H_1 : \gamma_1 < 0$ or $\gamma_2 < 0$. The test statistic is a two sided Wald test of the form $R_T = t_1^2 + t_2^2$ where t_i signifies the t-ratio for $\hat{\gamma}_i$ from OLS regression in the TAR model. Exact probabilities values for this test can be computed using a bootstrap approach.

3. A general test of non-linear mean reversion

Enders and Ludlow (2002) and Ludlow and Enders (2000) follow a different strategy from Caner and Hansen (2001) to construct non-linear unit root tests. In particular, they adopt the following extension to the standard linear AR (1) model:

$$y_t = \alpha(t) y_{t-1} + e_t \qquad \text{t=1,2,…..T} \qquad (2)$$

where y_t is a time series variable, e_t is a white noise disturbance term and $\alpha(t)$ is a deterministic but unknown function of time. $\alpha(t)$ can also be a p-th order difference equation, a threshold function or a switching function. They showed that, although a sufficiently long Fourier series represents the function $\alpha(t)$ exactly, equivalent results can be obtained by considering only a single frequency so that:

$$\alpha(t) = \varphi_o + \varphi_1 \sin\frac{2\pi k}{T} \cdot t + \varphi_2 \cos\frac{2\pi k}{T} \cdot t \qquad (3)$$

where k is an integer in the interval 1 to T/2.

If $\varphi_1 = \varphi_2 = 0$ then a sufficient condition for stationarity (and in this case for linear mean reversion) is $|\varphi_o| < 1$. However, with $\varphi_1 \neq 0$ and $\varphi_2 \neq 0$, $|\varphi_o| < 1$ is neither a necessary nor sufficient condition for mean reversion of the y_t series.

The main advantage in using (3) to detect non-stationarity in comparison to Caner and Hansen's (2001) non-linear unit root test is that we do not need to specify the precise adjustment mechanism or the nature of the asymmetry. Thus, we are not forced *a priori* to impose a particular dynamic structure on the adjustment coefficient[2]. All we need to find are the most appropriate values of the coefficients φ_i (i =0,1,2) and k. In this context Enders and Ludlow (2002) proved that a necessary and sufficient condition for mean reversion is that $\varphi_o > 0$ and $\varphi_o < 1+\frac{r^2}{4}$ where $r < 2$, $r = \sqrt{\varphi_1^2 + \varphi_2^2}$. Finally, for various values of φ_o we can derive the adjustment paths of the y_t series.

In Figure 1, we present the various adjustment paths of the $\{y_t\}$ sequence for positive values of φ_0. The arc eb is constructed so that $\varphi_0 = 1+\frac{r^2}{4}$ shows the boundary line of decay. Any (r, φ_0) combination above this locus implies a divergent sequence. The restriction $r < 2$ limits the region of decay to the area $oebf$. On the other hand, a standard Dickey – Fuller test restricting the value of r equal to zero, considers only the line segment oe. Inside region $oebf$ we identify four separate areas of interests corresponding to four different types of decay.

- Direct Decay: Within region oec, $\varphi_0 > r$ and $\varphi_0 + r < 1$. The value of $a(t)$ is never negative and never greater than unity. Decay towards the attractor is direct although the speed of adjustment changes over time.
- Explosive Decay: Within region ebc, $\varphi_0 > r$ so that $a(t)$ is never negative. Since $\varphi_0 + r > 1$ and $\cos(2\pi kt)$ can equal unity, there will be values of t such that $a(t)$ is greater than unity. Although the $\{y_t\}$ sequence exhibits periods of exploding behavior, the overall process ultimately reverts towards the attractor.
- Non-explosive Decay with Oscillations: Inside region ocg, $\varphi_0 + r < 1$ so that there are no explosive periods. Given that $\cos(2\pi kt)$ takes the value –1, there will be periods such that $a(t)$ is less than zero. During these periods, $\{y_t\}$ will exhibit oscillating decay towards the attractor.
- Oscillations and Explosive Decay: Within region $bfgc$, $\varphi_0 < r$ and $\varphi_0 + r > 1$. There will be periods such that $a(t) < 0$ and others such that $a(t) > 0$. The $\{y_t\}$ sequence will exhibit periods of oscillating reversion in conjunction with periods of explosive behavior.

[2] In a recent paper Blake *et al.* (2003) consider a unit root test based on a neural network pure significance test, thus avoiding the specification of an alternative hypothesis. An anonymous referee has suggested this to me.

[Insert Figure 1]

Given that equation (3) is not known to the investigator we need to derive parameter estimates for φ_i, i=0,1,2 and k as well as their statistical levels of significance by performing the following steps.

To find the most appropriate value for the integer k we estimate the following regression in first differences using each integer value of k in the interval 1 to T/2.

$$\Delta y_t = \left[\mu + \varphi_1 \sin \frac{2\pi k}{T} \cdot t + \varphi_2 \cos \frac{2\pi k}{T} \cdot t \right] y_{t-1} + e_t \qquad (4)$$

Parameter μ in equation (4) corresponds to the value $(\varphi_o - 1)$ in equation (3). Reversion requires $|\mu| < \frac{r^2}{4}$ and $r < 2$.

The optimal level k^* is chosen so as to minimize the sum of squared errors. The coefficients that correspond to the optimal k^* are denoted by μ^*, φ_1^* and φ_2^*. Based on the estimated coefficients of (4) we can now test the following hypotheses:

- $\mu^* = 0$ (5)
- $\varphi_1^* = \varphi_2^* = 0$ (6)
- $\mu^* = \varphi_1^* = \varphi_2^* = 0$ (7)
- $\mu^* = \frac{(r^*)^2}{4}$ (8)

where $(r*)^2 = (\varphi_1^*)^2 + (\varphi_2^*)^2$

The null hypothesis (5) corresponds closely to the Dickey-Fuller unit root test. Within this context, rejection of the null hypothesis (5) in favor of the alternative $\mu^* < 0$ is a necessary condition for decay only when $\varphi_1 = \varphi_2 = 0$. However, rejecting the null hypothesis (5) in favor of the alternative $\mu^* < 0$ with $\varphi_1 \neq 0$ and $\varphi_2 \neq 0$, *is not a necessary* condition to guarantee decay, see Enders and Ludlow (1999, p. 11). Rejection of the null hypothesis (6) indicates that any adjustment towards an attractor is non linear. Within this context the implied value of $r^* = \sqrt{(\varphi_1^*)^2 + (\varphi_2^*)^2}$ yields the ordinate on line of in Figure 1; as such it can be considered as a measure of the degree of non-linearity in the data. Non-rejection of the null hypothesis (7) implies that the series y_t contains a unit root. Finally, rejection of the null hypothesis (8) guarantees decay. In other words rejecting the null of the $\mu^* = \frac{(r^*)^2}{4}$ in favor of the alternative $\mu^* < \frac{(r^*)^2}{4}$ is a necessary and sufficient condition for decay of the $\{y_t\}$ sequence. Further rejection of the null hypothesis (8) implies that the larger region $oebf$ of Figure 1 describes better the area of decay than the space oe considered in the standard linear Dickey –Fuller[3]. A t-statistic can be used to test the null hypothesis (5) while an F- statistic is necessary for the null hypotheses (6)-(8). Monte Carlo simulations approximate all these empirical distributions and are tabulated in Enders and Ludlow (2002).

4. Empirical results.

Table 1 reports single country ADF linear tests using annual data on real GDP per capita in logarithmic form for seven OECD countries, namely, Denmark, Finland, France, Italy, the Netherlands, the USA and the UK over the period 1900-2000. Data description and sources are given in Maddison (1995)[4]. The unit root null hypothesis is not rejected at conventional levels of significance for any country, except for the USA at the 5% level. This finding is consistent with the real GDP unit root literature.

[Insert Table 1]

We now apply the potentially more powerful non-linear unit root statistic of Caner and Hansen. Before applying this test we tested whether the data set is trended. We concluded in favor of the presence of a time trend in the fitted regression (1). In the following Table 2 we present the threshold test W_T (column 2). We see that the threshold statistic W_T rejects the null hypothesis of no threshold for only three countries out of seven, namely, Denmark, Finland and the Netherlands. For the remaining four countries, that is, France, Italy, the UK and the USA, a threshold effect cannot be established. This means that for these four countries we cannot reject the linear AR model in favor of the TAR model (1). Thus, for France, Italy, the UK and the USA a linear representation is sufficient to describe the adjustment of the real GDP towards the equilibrium level. Therefore, for these four countries we do not proceed with the estimation of the TAR model (1).

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[3] In fact the null hypothesis (8) tests whether μ^* and r^* lie in area $oebf$ of Figure 1.

[4] The data can be downloaded from the Maddison web site. http://www.eco.rug.nl/˜Maddison/

[Insert Table 2]

In Table 2 (column 3) we also display the unit root tests R_T for the three countries where the TAR model (1) remains a valid specification. Parameter estimates of this model are presented in Table 3.

[Insert Table 3]

The R_T test indicates that the null hypothesis of a unit root can be rejected for Denmark and the Netherlands, but not in Finland where a unit process seems to characterize real GDP movements. Therefore we can conclude that for Denmark and the Netherlands the R_T statistic gives evidence of a stationary process. However, in both countries the individual $t_i -$ratios ($i = 1,2$) identify the presence of a unit root only in one of the two regimes. Further, the OLS parameter estimates shown in Table 3 indicate that the the dynamics of the real GDP movements are different depending on whether the change in real GDP is above or below the estimated threshold value.

Next, we apply the Ludlow and Enders (1999) unit root test which does not place a priori restrictions on the exact form of the adjustment mechanism or the nature of the asymmetry. Given that our series y_t contains a trend we regress the y_t series on the trend attractor $z + z_1 t$ and save the residuals, thus generating a new variable which is de-meaned and de-trended. Next we replace y_t by the corresponding residual series and estimate equation (4) for each integer value of k in the interval 1 to (101/2). Having identified the optimal k^* we present in Table 4 the values of the test statistics for the hypotheses (5)-(8) while in Table 5 we report parameter estimates for equation (4) along with their diagnostic statistics. To make the test more robust to a wider class of error terms we introduced additional terms Δy_{t-L} on the right hand side of (4). The number of lags (L) was selected optimally using Schwarz's Bayesian criterion.

[Insert Table 4]

[Insert Table 5]

The results in Table 4 indicate that, firstly, μ^* is not significantly different from zero since the null hypothesis $\mu^* = 0$ is not rejected in any of the cases. This finding is consistent with the ADF results in Table 1 which show acceptance of the unit root hypothesis. It is easy to conclude that real GDP per capita does not enter the model linearly. However, as discussed in Section 3, $\mu^* = 0$ is neither a necessary nor sufficient condition for stationarity. A second finding from Table 4 is that the null hypotheses $\mu^* = \varphi_1^* = \varphi_2^* = 0$ and $\varphi_1^* = \varphi_2^* = 0$ are rejected at conventional levels of statistical significance for all countries apart from the UK, thus one can conclude that rejection is most likely due to the part $\varphi_1^* = \varphi_2^* = 0$. Thirdly, the non-linear restriction $\mu^* = \frac{(\varphi_1^*)^2}{4} + \frac{(\varphi_2^*)^2}{4}$ is strongly rejected for all countries apart from Italy, the Netherlands and the UK. This means that for all countries except these three, a non-linear mean reversion process characterizes real GDP per capita movements. Other things being equal, all shocks have temporary effects on output levels, which subsequently revert to their equilibrium values. This finding is consistent with what real business cycle theory predicts. For the three countries in which the non-linear restriction was not rejected, we examined the estimated value of μ^* and r^*. According to Enders and Ludlow (2002) who used power functions, when μ^* is near –0.1 and r^* is near 0.27 the power of the Dickey-Fuller, $\mu^* = 0$ and $\mu^* = \frac{(\varphi_1^*)^2}{4} + \frac{(\varphi_2^*)^2}{4}$ tests are all very low. In these circumstances the F-tests for $\mu^* = \varphi_1^* = \varphi_2^* = 0$ and $\varphi_1^* = \varphi_2^* = 0$ are both quite powerful. In our case the μ^* values are –0.03 for Italy, -0.03 for the Netherlands and –0.07 for the UK. The corresponding r^* values are 0.17, 0.36, and 0.09 respectively. It is obvious that the estimates of μ^* and r^* are close to the values proposed by Enders and Ludlow (2002) only for the Netherlands. This means that for this country our evidence further favors the existence of a non-linear reverting process. For the remaining two countries, that is, Italy and the UK, results based on ADF regression are more powerful compared to those derived from a non-linear unit root model. However, given that $r^* > 0$ and that the μ^* test indicates that we can set $\mu^* = 0$ it is possible to argue that real GDP is reverting. Nevertheless, the inability to reject the more general restriction $\mu^* = (r^*)^2 / 4$ leaves the question somewhat open. Finally, we can derive the various adjustment paths towards the equilibrium of the real GDP per capita based on the values of μ^* and r^*. Given that $|\mu^*| < r^*$ and $|\mu^*| + r^* < 1$ for all countries examined, the real GDP per capita in every country exhibits periods of exploding behavior although the overall process reverts towards the equilibrium level. This means that we have periods with $\hat{a}(t) > 1$ and the number of explosive periods equal k^*, and the rest of the periods are non-explosive such that $0 < \hat{a}(t) < 1$ holds. This behavior is described in Figure 1 within the region ebc. Therefore we can conclude that in the majority of cases we examine we can overturn the non-stationary conclusion of the linear ADF test by applying non-linear unit root statistics.

Finally, comparing our findings shown in Table 4 with those reported in Table 3 we observe that: (a) the Enders and Ludlow unit root test provides additional evidence in favour of the stationary process for three countries where the TAR unit root test failed to reject the unit root hypothesis. These countries are Finland, France and the USA; (b) the Enders and Ludlow statistic provides less strong evidence in favour of the stationarity hypothesis than a TAR test in the case of the Netherlands; (c) for one country, Denmark, both tests produce strong evidence in favour of a non-linear mean reversion process; and (d) the joint use of Enders and Ludlow and TAR statistics produces evidence in favour of stationarity for five out of seven sampled countries. The exceptions are Italy and the UK.

5. Concluding remarks.

Using models that do not assume a linear adjustment, this paper investigates the stationarity of real GDP per capita for seven OECD countries over the period 1900-2000. Standard linear ADF statistics show that the data are basically non-stationary for all these countries apart from the USA. In contrast, when we adopt a non-linear model which has higher power than a standard univariate unit root statistic to reject a false null hypothesis of unit root behavior, the empirical evidence suggests that real GDP per capita is well characterized by a non-linear mean reverting process which exhibits periods of exploding behavior. This might offer an alternative explanation for the difficulty researchers have encountered in rejecting the unit root hypothesis for real GDP per capita.

References

Caner, M., and B.E. Hansen, 2001, “Threshold autoregression with a unit root”, *Econometrica* 69, 1565-1596.

Cheung, Y. -W. and Chinn, M.D., 1996, “Deterministic, stochastic, and segmented trends in aggregate output: A cross-country analysis”, *Oxford Economic Papers* 48, 134-162.

Enders, W. and Granger C.W.J., 1998, “Unit root tests and asymmetric adjustment with an example using the term structure of interest rates”, *Journal of Business of Economic and Statistics* 16, 304-312.

Enders, W. and Ludlow, J., 2002, “Non-linear decay: Tests for an attractor using Fourier approximation”, The University of Alabama, Economics, Finance and Legal Studies, Working Paper Series WP01-02-02.

He, C. and Sandberg, R., 2003, “A Dickey-Fuller type of test against nonlinear globally stationary dynamic models”, Working Paper, Department of Economic Statistics, Stockholm School of Economics.

Kapetanios, G, Shin, Y. and Snell, A., 2003, “Testing for a unit root in the non-linear STAR framework”, *Journal of Econometrics* 112, 359-379.

Ludlow, J. and Enders, W., 2000, “Estimating non linear ARMA models using Fourier coefficients”, *International Journal of Forecasting* 16, 333-347.

Maddison, A., 1995, *Monitoring the world economy 1820-1992*, OECD Development Centre, Paris.

Nelson, C.R. and Plosser, C.I., 1982, “Trends and random walks in macro-economic time series” *Journal of Monetary Economics*, 10, 139-162.

Rapach, D.E., 2002, “Are real GDP levels nonstationary? Evidence from panel data sets”, *Southern Economic Journal* 68, 73-495.

Shin, D.W. and Lee, O., 2001, “Test for asymmetry in possibly nonstationary time series data”, *Journal of Business and Economic Statistics* 19, 233-244.

Wasserfallen, W., 1986, “Non-stationarities in macro-economic-time series- further evidence and implications”, *Canadian Journal of Economics* 19, 498-510.

Table 1. Linear ADF unit root tests

Country	*ADF Statistic*	*L*
Denmark	**-2.18**	1
Finland	**-2.55**	4
France	**-2.38**	1
Italy	**-2.00**	1
Netherlands	**-2.62**	1
UK	**-2.58**	1
USA	-3.90**	1

Notes*: ADF is the augmented Dickey-Fuller t-test for a unit with a constant and trend. The number of lags(L) was selected optimally the using Schwarz criterion. Critical values for the ADF test are -4.05, -3.45 and –3.15 at the 1%, 5% and 10% levels of statistical significance, respectively. Boldface values denote sampling evidence in favour of unit roots. An (**) indicates statistical significance at 5% statistical level.*

Table 2. The Wald tests for a threshold-W_T and unit root- R_T .

Country	W_T	R_T	t_1	t_2
Denmark	22.2 (0.02)	18.6 (0.06)	3.80 (0.05)	2.05 (0.39)
Finland	17.5 (0.07)	5.36 (0.81)	1.27 (0.60)	1.94 (0.40)
France	13.4 (0.16)			
Italy	4.13 (0.94)			
Netherlands	68.0 (0.004)	71.2 (0.004)	-5.02 (0.99)	6.78 (0.006)
UK	6.71 (0.74)			
USA	8.93 (0.52)			

Notes*: Bootstrap p -values are reported in parentheses. For bootstrapping 10000 replications have been used. t_1 is the one sided Wald test for the null of a unit root $H_0 : \gamma_1 = \gamma_2 = 0$ versus the alternative of stationarity only in the first regime ($Z_{t-1} < \lambda$), that is, $H_1 : \gamma_1 < 0$ and $\gamma_2 = 0$ while t_2 tests the null of a unit root against the alternative of stationarity only in the second regime ($Z_{t-1} > \lambda$), that is, $H_1 : \gamma_1 = 0$ and $\gamma_2 < 0$.*

Table 3. Parameter estimates for the TAR unit root model (1).

	$Z_{t-1} < \lambda$				$Z_{t-1} > \lambda$			
	Constant	trend	y_{t-1}	Δy_{t-1}	Constant	trend	y_{t-1}	Δy_{t-1}
Denmark	4.47 [1.15]	0.01 [0.004]	-0.49 [0.13]	1.76 [0.38]	0.94 [0.45]	0.003 [0.001]	-0.10 [0.05]	-0.03 [0.15]
Netherlands	-4.99 [0.99]	- 0.02 [0.003]	0.52 [0.10]	-1.26 [0.28]	2.94 [0.43]	0.009 [0.001]	-0.30 [0.04]	-0.06 [0.11]

Note: Figures in brackets are standard errors.

Table 4. Non-linear unit root tests

Country	*Hypothesis Testing*			
	$\mu^* = 0$ t-statistic	$\varphi_1^* = \varphi_2^* = 0$ F-Statistic	$\mu^* = \varphi_1^* = \varphi_2^* = 0$ F-Statistic	$\mu^* = \frac{(\varphi_1^*)^2}{4} + \frac{(\varphi_2^*)^2}{4}$ F-Statistic
Denmark	-1.59	13.98***	11.37***	11.60*
Finland	-2.59	8.50**	8.83**	15.64**
France	-1.89	12.09***	10.85***	12.35*
Italy	-1.16	13.27***	10.54***	8.80
Netherlands	-0.87	21.46***	17.63***	9.72
UK	-2.25	2.49	3.98	9.21
USA	-3.09	9.46**	11.33**	20.50***
Critical Values				
10%	-3.21	7.14	6.53	11.59
5%	-3.58	8.03	7.33	14.25
1%	-4.27	9.95	9.30	19.55

Notes: *(***), (**) and (*) denote rejection of the null hypothesis at the 1%, 5% and 10% levels of statistical significance, respectively. The source of the critical values is Enders and Ludlow (2002), Table 1.*

Table 5. Estimates of the Fourier model (4).

Country	*Estimated Parameters*						*k*
	φ_0	φ_1	φ_2	Δy_{t-1}	Δy_{t-2}	Δy_{t-3}	
Denmark	-0.05	-0.09*	0.24***	0.08			29
	[1.59]	[1.62]	[4.92]	[0.83]			
Finland	-0.08	0.02	0.19***	0.36***			34
	[2.59]	[0.49]	[4.06]	[3.98]			
France	-0.06	-0.16***	-0.14***	0.22**	-0.19**	0.28***	11
	[1.89]	[3.07]	[3.17]	[2.31]	[2.11]	[2.92]	
Italy	-0.03	-0.11***	0.13***	0.29***			18
	[1.17]	[3.08]	[3.58]	[3.02]]			
Netherlan.	-0.03	0.26***	0.25***	0.02			12
	[0.87]	[4.09]	[4.98]	[0.25]			
UK	-0.07	-0.06	0.07*	0.32***			19
	[2.25]	[1.53]	[1.64]	[3.33]			
USA	-0.16	0.24***	0.16***	0.28***	0.02	0.11***	19
	[3.09]	[3.68]	[2.38]	[2.93]	[0.17]	[1.90]	
Diagnostic Statistics	$adjR^2$	DW					
Denmark	0.24	2.06					
Finland	0.25	1.89					
France	0.32	2.02					
Italy	0.29	1.99					
Netherlands	0.36	1.94					
UK	0.20	2.03					
USA	0.33	2.05					

Notes: The optimal number of lags y_{t-L} was selected using Schwarz's Bayesian criterion. k stands for the optimal frequency in the interval 1 to $T/2$. Figures in brackets indicate absolute t-ratios (***), (**) and (*) indicate statistical significance at the1%, 5% and 10% levels of statistical significance, respectively. D-W is the Durbin and Watson statistic for first order autocorrelation.

Figure 1

The Range of Reversion