Designing Electricity Auctions: Uniform, Discriminatory and Vickrey

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Abstract

Motivated by the new auction format introduced in the England and Wales electricity market and the recent debate in California, we characterize bidding behavior and market outcomes in uniform, discriminatory and Vickrey electricity auctions. The aim is to gain an improved understanding of how different auction formats affect the degree of competition and overall welfare in decentralized electricity markets. We find that the uniform auction is (weakly) outperformed in consumer surplus terms by the discriminatory auction, but that uniform auctions are (weakly) more efficient. Vickrey auctions guarantee productive efficiency, but at the expense of large payments to firms. The overall welfare ranking of the auctions is thus ambiguous. The paper also clarifies some methodological issues in the analysis of electricity auctions. In particular we show that analogies with continuous share auctions are misplaced so long as firms are restricted to a finite number of bids. We also provide a characterization of multi-unit Vickrey auctions with reserve pricing.

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1 Introduction

Electricity wholesale markets differ in numerous dimensions, but until recently all have been organized as uniform first-price auctions. Recent experience - and the perceived poor performance - of some decentralized electricity markets however, has led certain regulatory authorities to consider adopting new auction designs. In England and Wales a major overhaul of the electricity trading arrangements introduced in 1990 has recently taken place, and among the reforms implemented in March 2001, a discriminatory or 'pay-your-bid' auction format has been adopted. The British regulatory authorities believed that uniform auctions are more subject to strategic manipulation by large traders than are discriminatory auctions, and expected the new market design to yield substantial reductions in wholesale electricity prices.[1] Similarly, the California Power Exchange recently commissioned a report by leading auction theorists on the advisability of a switch to a discriminatory auction format for the Exhange's day ahead market, due to the increasing incidence of price spikes in both on- and off-peak periods (see Kahn et al., 2001).

It is well-known that discriminatory auctions are not generally superior to uniform auctions. Both types of auction are commonly used in financial and other markets, and there is now a voluminous economic literature devoted to their study.[2] In multi-unit settings the comparison between these two auction forms is particularly complex. Neither theory nor empirical evidence tell us that discriminatory auctions perform better than uniform auctions in markets such as those for electricity, although this has now become controversial.

Wolfram (1999), for instance, argues in favor of uniform auctions for electricity, and Rassenti, Smith and Wilson (2002) cite experimental evidence which suggests that discriminatory auctions may reduce volatility (i.e. price spikes), but at the expense of higher average prices. Other authors have come to opposite conclusions. Federico and Rahman (2001) find theoretical evidence in favor of discriminatory auctions, at least for the polar cases of perfect competition and monopoly, while Klemperer (2001, 2002) suggests that discriminatory auctions might be less subject to 'implicit collusion'. Kahn et al. (2001), on the other hand, reject outright the idea that switching to a discriminatory auction will result in greater competition or lower prices.

While the debate in the UK and California has been focused on the ad-

[1] See Ofgem (1999). Harbord and McCoy (2000) discuss of the auction reforms in England and Wales, and are critical of the reasoning of the British regulatory authorities. Ofgem (2001) nevertheless claims that its objectives have been achieved. Frontier Economics (2001) takes a different view of the evidence.

[2] See Ausubel and Cramton (1998) and Binmore and Swierzbinski (2000) for the theory and empirical evidence. Archibald and Malvey (1998) and Belzer and Reinhart (1996) discuss the US Treasury's experiments with these auction formats in more detail. See also Nyborg (2002).

vantages or disadvantages of uniform versus discriminatory auctions, among economists Vickrey auctions are often favored. Vickrey auctions make sincere bidding, e.g. bidding at marginal cost, a (weakly) dominant strategy and hence result in least cost production, or despatch efficiency. This comes at a cost, since traders with market power need to be paid the 'opportunity costs' of their bids, and these payments can be large (see Wilson, 2002, for a discussion). Vickrey auctions in markets such as those for electricity have to date received relatively little detailed analysis however.[3]

The purpose of this paper is to address this electricity market design issue in a series of models which represent some of the key features of decentralized electricity markets, albeit within a simplified framework. We characterize equilibrium market outcomes in a discrete, multi-unit auction model for uniform, discriminatory and Vickrey electricity auctions under a variety of assumptions concerning demand elasticities, bid formats and the number of suppliers in the market. Our purpose is to gain an improved understanding of how these different auction formats affect suppliers' bidding incentives, the degree of competition and overall welfare in decentralized electricity markets.

Not surprisingly, we find that the welfare ranking of the auction types is inherently ambiguous. If the regulator is solely concerned with productive efficiency (equivalent to the maximization of total surplus when demand is perfectly inelastic), then the Vickrey auction should always be chosen, as it guarantees efficiency independently of industry and market data. If, on the other hand, the regulator is solely concerned with the maximization of consumer surplus, then a uniform auction should probably never be chosen, as it is typically outperformed by the discriminatory auction, and in some cases by the Vickrey auction. For more general regulatory preferences, for example a weighted average of consumer and producer surplus, the ranking is uncertain. For some specifications of demand, costs, and suppliers' capacities the discriminatory auction dominates the uniform auction on both efficiency and consumer surplus criteria. In other scenarios the reverse ranking can be shown to (weakly) hold. Hence if the regulator is restricted to a choice between discriminatory and uniform auctions, this should be viewed as an empirical question which depends upon the nature of demand, market structure and the relative efficiencies of suppliers. Our analysis, however, provides no support for the presumption of some regulatory authorities that by changing the auction format from uniform to discriminatory a significant improvement in market performance can be achieved.

Our analysis proceeds by first considering a 'basic duopoly model', similar

[3] See Section 2 below. von der Fehr and Harbord (1993) studied Vickrey auctions with reserve prices in electricity markets for some extremely simple cases. Ausubel and Cramton (1999) provide a general framework.

to that described in von der Fehr and Harbord (1993), which is then varied in several directions. In the basic duopoly model, two 'single-unit' suppliers with asymmetric capacities and (marginal) costs face a market demand curve which is assumed to be both perfectly inelastic and known with certainty when suppliers submit their offer prices. By 'single-unit' we mean that each supplier must submit a single price offer for its entire capacity (i.e. its bid function is horizontal). This assumption simplifies the analysis considerably, but in Section 6.1 we show that it is largely inessential. The assumption that suppliers have perfect information concerning market demand is descriptively reasonable when applied to markets in which offers are 'short-lived', such as in Spain where there are 24 daily markets. In such markets suppliers can be assumed to know the demand they face in any period with a high degree of certainty. In markets in which offer prices remain valid for longer periods, e.g. a whole day, such as in Australia and in the original market design in England and Wales, on the other hand, it is more accurate to assume that suppliers face some degree of demand uncertainty or volatility at the time they submit their offers. Hence we allow for this type of uncertainty in Section 6.2. The assumption of price-inelastic demand can be justified by the fact that the vast majority of consumers purchase electricity under regulated tariffs which are independent of the prices negotiated in the wholesale market, at least in the short run.[4] However, in order to evaluate some of the possible effects of demand-side bidding, we consider downward-sloping demand functions in Section 6.3. Lastly, in Section 6.4 we consider the case of a symmetric oligopoly in order to assess the effects of changes in the number of suppliers. The equilibria in the three auction formats considered are then compared in each case in terms of their implications for total welfare and consumer surplus.

The paper is structured as follows. Section 2 describes some of the key features of decentralized electricity markets which have influenced our modelling approach, and discusses related literature. Section 3 describes the basic duopoly model and the auction formats to be considered. Section 4 characterizes equilibria in this model for the Vickrey, uniform and discriminatory auctions. Section 5 compares these equilibria in terms of total welfare and consumer surplus. Section 6 treats the extensions discussed immediately above: multiple-unit suppliers, downward-sloping demand, demand volatility and oligopoly, and Section 7 concludes. Proofs of results are relegated to the Appendix.

[4] See Wolak and Patrick (1997) and Wilson (2002) on this. In most countries, larger industrial consumers may buy electricity directly from the pool, but their demand comprises only a small fraction of the total volume traded.

2 Modeling Electricity Auctions

As Klemperer (2001) has recently noted, although 'it was not initially well-understood that deregulated electricity markets are best described and analyzed as auctions,' this is now uncontroversial. With the exception of the new discriminatory auction format adopted in England and Wales, all electricity markets to date have been organized as first-price, multi-unit auctions. Competition in these markets occurs by suppliers submitting offer prices which specify the minimum prices at which they are willing to supply energy, and the amount of capacity available at each price. On the basis of these offer prices an industry supply curve is constructed, which together with a forecast of demand, determines which generating units will be despatched in any particular period. In uniform, or first-price, electricity auctions, market prices are determined in each period by the offer price of the marginal accepted unit. In discriminatory auctions, such as the England and Wales balancing market, suppliers are paid their offer prices while consumers pay a (weighted) average of the accepted offer prices.

Within this general framework there is huge variation in auction designs, most of which we ignore in this paper.[5] Three features of electricity auctions are crucial to our analysis however. First, *all* electricity auctions limit the number of offer prices that may be submitted by any supplier to a small number. For example, in the original UK market design generators were permitted only three incremental bid prices per unit of capacity, while in the Spanish electricity market generators may submit up to twenty-five price-quantity pairs per production unit. This means that all electricity auctions are *discrete* multi-unit auctions rather than continuous 'share auctions' or auctions for perfectly divisible goods. This distinction is important because the analysis of auctions with discrete (i.e. step) offer or bid functions differs in significant ways from that for auctions with continuous supply or demand schedules. We expand on this subject immediately below.

Secondly, electricity auctions differ in the duration of suppliers' bids. In Australia and Argentina (and in the original UK market), generator bids are 'long-lived' so a single step-bid function remains valid for an extended period during which demand varies. In contrast, in the Spanish, Nordic and (now defunct) California markets, bids are 'short-lived', and last for a single market period only. In Australia and Argentina, although the value of demand in any period may be known with a high degree of certainty, suppliers' bids are constant over numerous periods during which demand fluctuates. Thus from the point of

[5] See Wilson (2002) for discussion of 'market architecture' issues.

view of bidders, demand may be viewed as being either stochastic or variable.[6] Conversely, when bids are short-lived, demand in any period will be known with certainty, i.e. fixed, before offer prices are submitted.

Finally, suppliers in electricity markets operate under binding capacity constraints, which means that in many periods there will be no excess supply when the capacity of a single firm is taken out of the system. While much has been made of this fact in recent regulatory inquiries and policy discussions (c.f. Ofgem 2000a, 2000b), it has been ignored in some theoretical discussions, despite its strong implications for the analysis of equilibria in any auction format.

These three characteristics of electricity auctions have been important in determining our choice of models, and they are also key to understanding the related literature. Most analyses of electricity auctions have tended to adopt one of two possible modelling approaches: the continuous 'share auction' or 'supply function' approach pioneered by Wilson (1979), Back and Zender (1993) and Klemperer and Meyer (1989), and applied to the British electricity market by Green and Newbery (1992),[7] or the discrete, multi-unit auction approach, first applied to electricity markets by von der Fehr and Harbord (1993).[8] The predictions of these models differ significantly however, and where they do careful interpretation of the results is required. In the remainder of this section we discuss the comparison of uniform and discriminatory auctions based on these (and related) modelling approaches, and then discuss the much less extensive literature on Vickrey electricity auctions.

2.1 Uniform or discriminatory?

2.1.1 Markets with short-lived bids

For electricity auctions with short-lived bids, to a first approximation, demand in each period will be known with certainty by all market participants before offer prices are submitted. Hence these markets are best analyzed as auctions with fixed, or deterministic, demand. As noted by Klemperer (2001), if such markets are treated as auctions for 'infinitely-divisible quantities of homogeneous units,' then 'collusive-like' equilibria can arise, resulting in very high profits and prices. Such outcomes can be supported in a uniform auction, *even in the absence of binding capacity constraints*, because suppliers are concerned with only a single point on their (continuous) supply curves, the point corresponding to the market-clearing price. The rest of the supply curve can then be used to inhibit

[6] As pointed out by Green and Newbery (1992), these two representations are mathematically equivalent.

[7] See also Green (1996), Federico and Rahman (2000), Baldick and Hogan (2001) and Krishna and Tranaes (2001).

[8] See also Garcia-Diaz (2000), Garcia-Diaz and Marin (2000), Brunekreeft (2001), Crampes and Creti (2001), Fabra (2001) and Parisio and Bosco (2002).

competition from the other suppliers. For example, if a supplier submits a very steep supply curve, the residual demand curve facing his rivals will also be steep. A steep residual demand curve implies that the opportunity cost of capturing an increment in supply beyond the supplier's collusive allocation is high. In this way artificially high prices can be supported in equilibrium. These strategies are costless in a uniform auction because the low inframarginal offer prices used to support the equilibria are payoff-irrelevant, and never received by the supplier.[9]

In a discriminatory auction, on the other hand, any offer price below the market-clearing price will be paid to the supplier. This means suppliers will care about their entire supply curves, rather than just a single point on it, effectively restricting the set of strategies that may be played in equilibrium. In the absence of payoff irrelevant offer prices, the 'collusive-like' equilibria of the uniform auction cannot be implemented.

Klemperer (2002) has recently suggested that the collusive equilibria of the continuous uniform auction are one reason that the regulatory authorities in the UK decided to adopt a discriminatory auction format:

> "*...Uniform-price auctions are more vulnerable than...discriminatory auctions to collusion.... In a uniform-price auction ... bidders can tacitly agree to divide up the market at a very favorable price for themselves by each bidding extremely aggressively for smaller quantities than its collusive share, thus deterring other bidders from bidding for more.... The U.K. electricity regulator believes this market has fallen prey to exactly this kind of implicit collusion.... By contrast, implicit collusion is harder in a discriminatory auction. Partly for this reason the U.K. regulator has proposed a set of New Electricity Trading Arrangements (NETA) that will replace the uniform-price auction by an exchange market followed by a discriminatory auction....*"

Electricity auctions are not continuous share auctions however, and the equilibrium outcomes of the continuous model differ significantly from those of the discrete, multi-unit model. They cannot, therefore, safely be used to diagnose competition problems in existing electricity markets. In particular, as we show in Section 6 below, where the uniform auction with continuous offer-price functions yields a continuum of pure-strategy equilibria, some of which are 'collusive' in the sense described above, the discrete multi-unit auction model predicts a unique, Bertrand-like equilibrium. This is because in the continuous auction, as noted, suppliers can bid in very steep supply functions which eliminate a rival's

[9]See Klemperer and Meyer (1989), Section 2, Back and Zender (1993) (2000) and Wang and Zender (2002) for further discussion.

incentive to bid more aggressively. Discreteness in the bid functions rules this out however. When suppliers are limited to a finite number of price-quantity bids, a positive increment in output can always be obtained by just slightly undercutting the price of a rival's unit. Since this 'quantity effect' outweighs the 'price effect', the collusive-like equilibria found in the continuous auction cannot be implemented.

Because this remains true in the limit, as we allow the bid-step size to become infinitesimal, it cannot even be argued that the continuous share auction model is a valid approximation to the discrete model for small enough bid-steps. This means that the collusive-like equilibria of the share auction model are probably irrelevant for policy prescription in electricity markets, and should not be used to diagnose competition problems. They are derived from an auction model which simply does not apply.[10]

2.1.2 Markets with long-lived bids

When suppliers' bids are long-lived, i.e. stay constant over many separate market periods, then demand is best treated as being variable, or uncertain, from a supplier's point of view, rather than fixed and known with certainty at the time bids are submitted. The relevant version of the continuous auction is then Klemperer and Meyer's (1989) 'supply function' equilibrium model. The addition of demand variability or uncertainty can reduce the set of equilibria in the auction significantly, since there are fewer payoff-irrelevant bids that can be used to support 'collusive-like' equilibria. As shown by Klemperer and Meyer (1989), equilibria in the supply function model will typically lie between the perfectly competitive and Cournot market outcomes.

The reduction in the number of equilibria in the continuous model also reduces the extent to which the continuous and discrete multi-unit auction models disagree. Nevertheless the two models still diverge in significant ways. In the first place, where the supply function model yields a continuum of pure-strategy equilibria, some of which involve prices well above marginal costs of any firm, the discrete multi-unit model again predicts a unique Bertrand-like market outcome. And secondly, in the discrete multi-unit auction with capacity constraints, there will frequently be no pure-strategy equilibria at all. The models thus differ both in their description of equilibrium bidding behavior, and in the predicted market outcomes. Again, since the equilibria of the discrete model do not converge to

[10] This point was first alluded to in von der Fehr and Harbord (1993), and has now been made independently, and particularly clearly, by Nyborg (2002). He shows that the collusive equilibria of the Wilson and Back and Zender models are eliminated when bidders can only make a finite number of bids (or there is a quantity multiple), and instead Bertrand-like price competition is induced. Indeed, Nyborg suggests that this may explain the prevalence of uniform auctions, despite the theoretical warnings of severe underpricing, as well as the ambiguous conclusions reached by the US Treasury experiments.

the equilibria of the continuous supply function model as we let the size of the bid-step become small, use of the supply function model cannot be justified by arguing that it approximates the discrete multi-unit model in the limit.

2.1.3 Other literature

Kahn et al. (2001) have also compared uniform and discriminatory auctions for electricity, based on results from the general auction literature. In their report for the California Power Exchange they concluded that the proposed shift from a uniform to a discriminatory auction was ill-advised, and unlikely to result in lower electricity prices. In particular:

> "*The immediate consequence would be a radical change in bidding behavior that would: (i) forestall the anticipated savings; (ii) introduce unmeasurable inefficiencies in the dispatch of power and impose new costs on generating companies, which would inevitably tend to increase rather than decrease average prices; (iii) tend to weaken the competition in generation ...; and (iv) impede...the expansion of capacity that, along with intensified demand-side response, is the only fundamental remedy for the recent poor performance of electricity markets in California.*"

Kahn et al. (2001) argued that in a competitive electricity market, in a uniform auction, each generating company would have strong incentives to bid at marginal (avoidable) cost, hence ensuring both productive and allocative efficiency. In a discriminatory auction, on the other hand, although this may theoretically still be the case, bidders' attempts to predict the marginal accepted bid will inevitably lead to forecasting errors and hence to dispatch inefficiency, as well as to inefficient investments in market forecasting. They also argued, based on results from Maskin and Riley (2000), that:

> "*Inefficiencies will not be a consequence only of forecasting errors if bidders differ substantially and consistently in their relative marginal costs. In that case, occasional inefficient outcomes are a consequence of rational strategic bidding. For example, if there are two bidders with uncertain costs...and one is known to have lower costs than the other on average, the bidder likely to have higher costs will rationally bid less aggressively...than the bidder with lower costs.... The consequence will be that the disadvantaged bidder will be called on to supply too often, because it will have submitted a lower bid in some instances in which it has higher costs than its more efficient rival.*"

Since Kahn et al. (2001) do not specify a model it is difficult to evaluate all of their claims. In particular, it is not always made clear what is meant by a 'competitive electricity market'. The argument taken from Maskin and Riley (2000) is slightly easier to place in context however. Maskin and Riley show that in a single-unit auction with two bidders and cost or valuation asymmetries, in a discriminatory auction low-valuation types might be induced to bid more aggressively than high-valuation types, and hence win the auction even when it is inefficient for them to do so. Nevertheless, the discriminatory auction still yields higher (expected) revenues for the seller - or lower prices for the buyer in a procurement auction - and so might be preferred to the uniform auction on those grounds. This is somewhat similar in spirit to our demonstration that the discriminatory auction may result in higher-cost firms producing too often when mixed strategies are played, but yield lower average prices than the uniform auction.

Maskin and Riley's result, however, comes from a single-unit auction with two competing buyers, in which the source of the inefficiency is incomplete information concerning the buyers' valuations. Our result, on the other hand, comes from a multi-unit auction model in which information is complete and the source of the suppliers' market power is a tight demand/capacity balance. The Maskin and Riley result might be viewed as providing an efficiency rationale for preferring the uniform to the discriminatory auction even in the absence of capacity constraints, given the right kind of cost asymmetries and incomplete information. However if the discriminatory auction results in lower expected prices, the welfare ranking of the two auctions will remain ambiguous. Indeed, the Vickrey auction would appear to dominate the two other auction formats in this setting, which leads us to our final topic in this paper.

2.2 Vickrey electricity auctions

Although much recommended by economists, Vickrey auctions have rarely been applied in practice, at least in multi-unit settings.[11] The fundamental insight of Vickrey was that by making the price received by a bidder independent of its own offer price, marginal cost bidding can be induced as a weakly dominant strategy. von der Fehr and Harbord (1993) considered a version of a Vickrey auction in which each supplier is paid a price for each unit accepted by the auctioneer determined by the intersection of the demand curve with the 'residual' supply curve obtained by subtracting the higher-priced units of that supplier. A supplier can then influence its own payoff only to the extent that its bids affect the probability of being dispatched. Since a supplier will prefer to be operating

[11] See Rothkopf, Teisberg and Kahn (1990).

for all realizations of demand when its payoff is positive, and will prefer not to operate whenever its payoff is negative, offering to supply at a price equal to marginal cost becomes a weakly dominant strategy.

An important feature of Vickrey auctions for electricity is that there may not exist any excess supply when we remove units of a given supplier, i.e. the intersection of the residual supply curve with the demand curve may be empty. When this occurs a 'reserve price' must be defined. If demand is perfectly inelastic, the reserve price is given by consumers' common maximum willingness to pay. For downward-sloping demand curves, the reserve price is the point on the demand curve corresponding to consumers' marginal willingness to pay for that unit. This is a simple version of the Vickrey auctions with reserve pricing considered by Ausubel and Cramton (1999).

Krishna and Tranaes (2001) and Hobbs, Rothkopf and Hyde (2000) have analyzed Clark-Groves-Vickrey mechanisms for electricity markets in which each supplier is paid its own bid for each unit of capacity accepted, plus the improvement in social welfare that results from its bid (i.e. the cost savings to the auctioneer). This formulation is essentially equivalent to the multi-unit Vickrey auction. However in both of these analyses it is assumed that there is always sufficient excess supply, when the capacity of single supplier is removed from the system, to define this cost saving from the rejected bids of the other suppliers. Hence neither paper considers what happens when bidders are large and reserve pricing is required. [12]

The key difficulty with a Vickrey auction, with or without reserve pricing, is that the auctioneer's revenues and payments will typically not balance, i.e. the auctioneer will run a deficit. This feature of the Vickrey auction is well-known, and a general characteristic of optimal incentive-compatible revelation mechanisms. In order to induce the truthful revelation of private information, agents must be offered a positive informational rent. Payment of this rent must come from alternative (non-distorting) sources if the efficiency of the market allocation is to be maintained. When demand is price-inelastic this problem is easily solved, as the market price can include a mark-up to cover informational rents without distorting efficiency. With downward-sloping demand on the other hand, non-distorting payments from other sources, e.g. lump-sum taxes levied on market participants, may be required.

Another problem with a Vickrey auction (c.f. Hobbs, Rothkopf and Hyde, 2000) is that, like the discriminatory auction, it does not define a market-clearing price, and such a price may be required to reconcile deviations from agreed upon

[12] Krishna and Tranaes (2001) assume continuous supply functions, following Klemperer and Meyer (1989) and Green and Newbery (1992). Hobbs, Rothkopf and Hyde (2000) are not explicit on this point. However the continuity assumption is of no importance when considering Vickrey auctions.

forward transactions.[13] Determining a price for such purposes has proved both difficult, and controversial, for the UK regulatory authorities.[14]

3 The Basic Duopoly Model

We now turn to the welfare analysis of the three auction formats in our basic duopoly model, variations on which are considered in Section 6. In the basic duopoly model two independent suppliers compete to supply the market with productive capacities given by $k_i > 0$, $i = 1, 2$. The suppliers are indexed such that $k_1 \leq k_2$ and capacity is assumed to be perfectly divisible. Supplier i's marginal cost of production is $c_i \geq 0$ for production levels less than its capacity, while production above capacity is impossible (i.e. infinitely costly). Without further loss of generality we may normalize suppliers' marginal costs so that $\min\{c_1, c_2\} = 0 \leq \max\{c_1, c_2\} = c$. Demand θ is determined each period as a random variable independent of the market price, i.e. it is perfectly price inelastic. In particular, $\theta \in \left[\underline{\theta}, \overline{\theta}\right] \subseteq (0, k_1 + k_2)$ is distributed according to some known distribution function $G(\theta)$.

The two suppliers compete on the basis of bids made to the auctioneer. Both the timing of the game, and the quantities supplied by each supplier given their offer prices, are independent of the auction format. The timing of the game is as follows. Having observed the realization of demand, each supplier simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply the whole of its capacity, $b_i \in [0, P]$, $i = 1, 2$, where P denotes the 'market reserve price,' possibly determined by regulation.[15] We use $\mathbf{b} \equiv (b_1, b_2)$ to denote the bid profile. On the basis of this profile the auctioneer calls suppliers into operation. If suppliers submit different bids, the lower-bidding supplier's capacity is despatched first. If this capacity is not sufficient to satisfy the total demand θ, the higher-bidding supplier's capacity is then despatched to serve the residual demand, i.e. total demand minus the capacity of the lower-bidding supplier. If the two suppliers submit equal bids, then supplier i is ranked first with probability ρ_i, where $\rho_1 + \rho_2 = 1$, $\rho_i = 1$ if $c_i < c_j$ and $\rho_i = \frac{1}{2}$ if $c_i = c_j$, $i = 1, 2, i \neq j$.

Hence the output allocated to supplier i, $i = 1, 2$, denoted by $q_i(\theta; \mathbf{b})$, is

[13] Cramton and Wilson (1998) emphasize the importance of this feature of uniform-price electricity auctions.

[14] See Harbord and McCoy (2000) for a discussion. A third much-discussed problem with Vickrey auctions, or mechanisms, is their vulnerability to collusion, especially between the sellers and buyers, c.f. Rothkopf, Teisberg and Kahn (1990) and Krishna and Tranaes (2001).

[15] P can be interpreted as the price at which all consumers are indifferent between consuming and not consuming, or a price cap imposed by the regulatory authorities. In the British and Australian electricity markets, for example, P is the administratively determined 'value of lost load'. See von der Fehr and Harbord (1993, 1998).

given by

$$q_i(\theta;\mathbf{b}) = \begin{cases} \min\{\theta, k_i\} & \text{if} \quad b_i < b_j \\ \rho_i \min\{\theta, k_i\} + [1-\rho_i]\max\{0, \theta - k_j\} & \text{if} \quad b_i = b_j \\ \max\{0, \theta - k_j\} & \text{if} \quad b_i > b_j, \end{cases} \tag{1}$$

and is solely a function of demand and the bid profile (and their costs when equal price bids are submitted). The *payments* made by the auctioneer to the suppliers do depend upon the auction format however, and these are described immediately below. Both suppliers are assumed to be risk neutral, and hence aim to maximize their expected payoffs in the game[16].

3.1 Uniform, discriminatory and Vickrey auctions

Three auction formats are considered throughout the paper. In the *uniform auction*, the price received by a supplier for any positive quantity despatched by the auctioneer is equal to the highest accepted bid in the auction. Hence, for a given value of θ, and a bid profile $\mathbf{b} = (b_i, b_j)$, supplier i's profits, $i = 1, 2$, $i \neq j$, can be expressed as

$$\pi_i^u(\theta;\mathbf{b}) = \begin{cases} [b_j - c_i]\, q_i(\theta;\mathbf{b}) & \text{if } b_i \leq b_j \text{ and } \theta > k_i \\ [b_i - c_i]\, q_i(\theta;\mathbf{b}) & \text{otherwise,} \end{cases} \tag{2}$$

where $q_i(\theta;\mathbf{b})$ is determined by (1).

In the *discriminatory auction*, the price received by supplier i for its output is equal to its own offer price, whenever a bid is wholly or partly accepted. Hence for a given value of θ, and a bid profile $\mathbf{b}$, supplier i's, $i = 1, 2$, profits can be expressed as

$$\pi_i^d(\theta;\mathbf{b}) = [b_i - c_i]\, q_i(\theta;\mathbf{b}),$$

where again $q_i(\theta;\mathbf{b})$ is determined by (1).

In the *Vickrey auction,* by design, a supplier's payment is independent of its own bid. Instead, each supplier receives a payment equal to the 'opportunity cost' of each unit of output supplied, i.e. the additional cost of clearing the market that the auctioneer would have incurred without it. This means that each supplier is paid a price equal to its rival's rejected offer price for an amount of output corresponding to the rival's excess supply, so long as this is positive, and the 'market reserve price' P for any remaining output, i.e. when the supplier's output exceeds its rival's excess supply.[17] Hence, for a given value of θ and a bid profile $\mathbf{b}$, supplier i's, $i = 1, 2$, $i \neq j$, profits can be expressed as,

$$\pi_i^v(\theta;\mathbf{b}) = \begin{cases} [b_j - c_i]\, q_i(\theta,\mathbf{b}) & \text{if } b_i \leq b_j \text{ and } \theta \leq k_j \\ [b_j - c_i]\,[k_j - q_j(\theta,\mathbf{b})] + [P - c_i]\,[\theta - k_j] & \text{if } b_i \leq b_j \text{ and } \theta > k_j \\ [P - c_i]\, q_i(\theta;\mathbf{b}) & \text{if } b_i > b_j. \end{cases}$$

[16] We make the standard assumption that all aspects of the game, including the auction format, are common knowledge.

[17] This is a (trivial) implementation of Ausubel and Cramton (1997)'s Vickrey auction with reserve pricing, foreshadowed in Vickrey (1961).

4 Equilibrium Analysis

We now characterize the Nash equilibria of the basic duopoly model described in the previous section for each of the three auction types.

4.1 Vickrey auctions

In the Vickrey auction, for any realization of demand, a supplier's payment for each unit of output is independent of its own bid. Hence its bid enters into its profit function only in so far as it affects its probability of being despatched. A supplier will always prefer to produce whenever its payoff from doing so is positive, and prefer not to produce otherwise. Thus pricing at marginal cost maximizes its chances of operating whenever its payoff is non-negative, while eliminating any chance of producing whenever its payoff would be negative. Indeed, it is a well-known property of Vickrey auctions that, with private values, 'truthful' or 'sincere' pricing is a weakly dominant strategy, and a single round of elimination of weakly dominated strategies removes all strategies except for sincere pricing.[18] This standard result is stated in Lemma 1 below.

Lemma 1 *In the Vickrey auction, for any realization of demand, there exists a unique equilibrium in weakly dominant strategies in which suppliers offer prices at marginal cost, i.e.* $b_i = c_i$, $i = 1, 2$.

Because in the equilibrium of the Vickrey auction the less efficient supplier produces only when its rival's capacity is exhausted, the Vickrey auction guarantees productive efficiency independently of the industry capacity and cost configuration. This feature is not shared by other standard auction formats.

4.2 Uniform and discriminatory auctions

The types of equilibria induced by the uniform and discriminatory auctions depend upon the realization of demand relative to the suppliers' individual and aggregate capacities. It will be useful to distinguish three cases: *low demand realizations*, in which either supplier has sufficient capacity to supply the entire market (i.e. $\theta \in (0, k_1]$); *intermediate demand realizations,* in which demand exceeds the capacity of the smaller supplier but not the capacity of the larger supplier (i.e. $\theta \in (k_1, k_2]$); and *high demand realizations,* in which the capacity of both suppliers is needed to satisfy demand but there is excess capacity overall (i.e. $\theta \in (k_2, k_1 + k_2)$). We consider these three cases repeatedly below. For expository reasons we first characterize the equilibria for low and high demand realizations, leaving intermediate demand realizations for last.

[18] See, for instance, Fudenberg and Tirole (1995, ch 7) and Ausubel and Cramton (1999).

4.2.1 Low demand realizations

In a low demand realization, the total demand θ can be satisfied by the capacity of either supplier. This allows for a particularly simple characterization of equilibrium strategies in both the uniform and discriminatory auctions.

Lemma 2 *Assume $\theta \in (0, k_1]$. In both the uniform and discriminatory auctions, the unique equilibrium is given by $b_i = c$, $i = 1, 2$, and market demand is served by the lower-cost supplier.*

The uniform and the discriminatory auctions are thus strategically equivalent here. The market outcomes in this case correspond to that of a standard Bertrand price-setting game in which the unique pure-strategy equilibrium has both suppliers submitting offer prices equal to the marginal cost of the less efficient supplier, and only the most efficient supplier producing.[19] Given that all demand is served by the low-cost supplier, the equilibria in both auctions are efficient.

4.2.2 High demand realizations

In a high demand realization the capacity of the larger supplier is insufficient to satisfy total demand, so both suppliers must produce in equilibrium. In the uniform auction there now exists a continuum of pure-strategy equilibria in which the market price is equal to the market reserve price.[20] In the discriminatory auction, on the other hand, an equilibrium in pure strategies fails to exist. In the unique mixed-strategy equilibrium suppliers' offer prices lie strictly above the marginal costs of the less efficient supplier and weakly below the market reserve price

It is convenient to distinguish the case in which the smaller supplier is at least as efficient as the larger supplier, from that in which the larger supplier is more efficient, as this affects the equilibria. Pricing behavior in each of these two cases respectively is characterized in Lemmas 3 and 4 below.

Lemma 3 *Assume $\theta \in (k_2, k_1 + k_2)$ and suppose that the small supplier is at least as efficient as the large supplier, i.e. $c_1 = 0 \leq c_2 = c$.*

(i) In the uniform auction, if $\theta \in \left(k_2, k_2 + \frac{c}{P} k_1\right)$, all pure-strategy equilibria are given by offer-price profiles satisfying $b_1 \leq [P - c] \frac{\theta - k_1}{k_2} + c$ and $b_2 = P$; if $\theta \in \left[k_2 + \frac{c}{P} k_1, k_1 + k_2\right)$, all pure-strategy equilibria are given by offer-price profiles satisfying $b_i \leq \frac{\theta - k_i}{k_j} [P - c_j] + c_j$ and $b_j = P$, $i, j = 1, 2$, $i \neq j$.

[19] Indeed, the discriminatory auction is just such a game.

[20] See von der Fehr and Harbord (1993, 1998).

(ii) In the discriminatory auction, there exists a unique equilibrium in which supplier $i = 1, 2$ offer prices $b_i \in [\underline{b}, P]$, $\underline{b} \in (c, P)$, according to the probability distribution $F_i^d(b)$, with $F_1^d(b) \geq F_2^d(b)\ \forall\ b \in [\underline{b}, P]$.

Lemma 4 *Assume $\theta \in (k_2, k_1 + k_2)$ and suppose that the large supplier is more efficient than the small supplier, i.e. $c_2 = 0 < c_1 = c$.*

I. Assume $\frac{k_1}{k_2} \geq \frac{P-c}{P}$.

(i) In the uniform auction, if $\theta \in \left(k_2, k_1 + \frac{c}{P}k_2\right)$, all pure-strategy equilibria are given by offer-price profiles satisfying $b_2 \leq \frac{\theta - k_2}{k_1}[P - c] + c$ and $b_1 = P$; if $\theta \in \left[k_1 + \frac{c}{P}k_2, k_1 + k_2\right)$ all pure-strategy equilibria are given by offer-price profiles satisfying $b_i \leq \frac{\theta - k_i}{k_j}[P - c_j] + c_j$ and $b_j = P$, $i, j = 1, 2$, $i \neq j$.

(ii) In the discriminatory auction, there exists a unique equilibrium in which supplier $i = 1, 2$ offer prices $b_i \in [\underline{b}, P]$, $\underline{b} \in (c, P)$, according to the probability distribution $F_i^d(b)$, with $F_1^d(b) \leq F_2^d(b)\ \forall b \in [\underline{b}, P]$.

II. Assume $\frac{k_1}{k_2} \leq \frac{P-c}{P}$.

(i) In the uniform auction, all pure-strategy equilibria are given by offer-price profiles satisfying $b_i \leq \frac{\theta - k_i}{k_j}[P - c_j] + c_j$ and $b_j = P$, $i, j = 1, 2$, $i \neq j$.

(ii) In the discriminatory auction, there exists a unique equilibrium in which supplier $i = 1, 2$ offer prices $b_i \in [\underline{b}, P]$, $\underline{b} \in (c, P)$, according to the probability distributions $F_i^d(b)$, with $F_1^d(b) \leq F_2^d(b)$ for $b \in \left[\underline{b}, \frac{k_2}{k_2 - k_1}c\right]$ and $F_1^d(b) \geq F_2^d(b)$ for $b \in \left[\frac{k_2}{k_2 - k_1}c, P\right]$.

Remark 1 *Note that the ratio $\frac{k_1}{k_2}$ decreases as the degree of capacity asymmetry increases, whereas the ratio $\frac{P-c}{P}$ decreases as the degree of cost asymmetry increases. Hence, the case $\frac{k_1}{k_2} \geq \frac{P-c}{P}$ is more likely the more symmetric are the suppliers' capacities and the more asymmetric are the suppliers' costs.*

Remark 2 *In the uniform auction, there also exists a continuum of mixed-strategy equilibria. For $c > 0$, all of these involve the inefficient supplier playing (weakly) dominated strategies (i.e. offering to supply at prices below cost). Moreover, all of these equilibria are payoff dominated from the point of view of the suppliers by some pure-strategy equilibrium. See the proof of Lemma 3 for details.*[21]

In a pure-strategy equilibrium of the uniform auction, the lower-bidding supplier will produce at capacity while the higher-bidding supplier serves the residual demand. All supply is paid the market reserve price (i.e. the highest accepted bid). For some cost and capacity configurations, in all pure-strategy equilibria the low-cost supplier offers its capacity at a low price and the high-cost supplier serves the residual demand, hence the equilibria are efficient. For

[21] For this reason we focus on pure-strategy equilibria in the uniform auction in the following.

other cost and capacity configurations, inefficient pure-strategy equilibria, in which the low-cost supplier serves the residual demand, also exist.

In the discriminatory auction suppliers randomize their price bids over a range of prices that lie strictly above the marginal cost of the less efficient supplier, and weakly below the market reserve price. This equilibrium is not *ex ante* efficient, as with strictly positive probability the low-cost supplier will be undercut by its higher-cost competitor.

4.2.3 Intermediate demand realizations

Lastly, we turn to the case in which demand exceeds the capacity of the smaller supplier but does not exceed the capacity of the larger supplier. In this case, the types of equilibria that arise in the uniform and discriminatory auctions lie between those identified immediately above for low and high demand realizations respectively. In particular, for some demand realizations and cost-capacity configurations, in both auction formats the market price equals the marginal cost of the less efficient supplier, as in the low demand case. For other demand realizations and cost-capacity configurations, as in the high demand case, price equals the market reserve price in the uniform auction, and an equilibrium in pure strategies fails to exist in the discriminatory auction. We again distinguish between the case in which the smaller supplier is at least as efficient as the large supplier, from the case in which the large supplier is more efficient than its rival.

Lemma 5 *Assume $\theta \in (k_1, k_2]$ and suppose that the small supplier is at least as efficient as the large supplier, i.e. $c_1 = 0 \leq c_2 = c$.*

(i) In the uniform auction, all pure-strategy equilibria are given by offer-price profiles satisfying $b_1 \leq [P - c] \frac{\theta - k_1}{\theta} + c$ and $b_2 = P$.

(ii) In the discriminatory auction, there exists a unique equilibrium in which supplier $i = 1, 2$ offer prices $b_i \in [\underline{b}, P]$, $\underline{b} \in (c, P)$ according to the probability distribution $F_i^d(b)$, with $F_1^d(b) \geq F_2^d(b)$.

If the small supplier is at least as efficient as the large supplier, the unique pure-strategy equilibrium in the uniform auction has the lower-cost, smaller supplier producing up to capacity, and the higher-cost, larger supplier serving the residual demand, with all supply paid the market reserve price P. Given that the capacity of the larger supplier exceeds demand, we cannot have an equilibrium in which this supplier offers prices lower than its rival. In such a case the smaller supplier would produce nothing and would therefore be better off undercutting the larger supplier's offer price (which in equilibrium must exceed its marginal cost). Clearly, the uniform auction guarantees productive efficiency in this case.

In the discriminatory auction an equilibrium in pure strategies fails to exist. In the unique mixed-strategy equilibrium, the high-cost, larger supplier's strategy profile first-order stochastically dominates the strategy profile of the low-cost, smaller supplier, for the same reasons as those given above. In this equilibrium, productive efficiency is not guaranteed, as there is a strictly positive probability that the low-cost supplier will be undercut by the high-cost supplier.

Lemma 6 *Assume $\theta \in (k_1, k_2]$ and suppose that the large supplier is more efficient than the small supplier, i.e. $c_1 = c > c_2 = 0$.*

I. Assume $\frac{k_1}{k_2} > \frac{P-c}{P}$. In both the uniform and discriminatory auctions, there exists a unique equilibrium in which $b_i = c$, $i = 1, 2$, and all demand is served by Supplier 2.

II. Assume $\frac{k_1}{k_2} \leq \frac{P-c}{P}$.

(i) In the uniform auction, if $\theta \in \left(k_1, \frac{P}{P-c}k_1\right)$, there exists a unique equilibrium in which $b_i = c$, $i = 1, 2$, and all demand is served by Supplier 2; if $\theta \in \left[\frac{P}{P-c}k_1, k_2\right]$, all pure-strategy equilibria are given by offer-price profiles satisfying $b_1 \leq P\frac{\theta - k_1}{\theta}$ and $b_2 = P$.

(ii) In the discriminatory auction, if $\theta \in \left(k_1, \frac{P}{P-c}k_1\right)$, there exists a unique equilibrium in which $b_i = c$, $i = 1, 2$, and all demand is served by Supplier 2; if $\theta \in \left[\frac{P}{P-c}k_1, k_2\right]$, there exists a unique equilibrium in which supplier $i = 1, 2$ offer prices $b_i \in [\underline{b}, P]$, $\underline{b} \in (c, P)$ according to the probability distribution $F_i^d(b)$, with $F_1^d(b) \geq F_2^d(b)$.

If the larger supplier is more efficient than the small supplier, the equivalence between the uniform and the discriminatory auctions is restored when either one of the following two conditions is satisfied: either $\frac{k_1}{k_2} \geq \frac{P-c}{P}$, i.e. the degree of capacity (cost) asymmetry is not too large (small), or $\frac{k_1}{k_2} \leq \frac{P-c}{P}$ for demand realizations $\theta \in \left(k_1, \frac{P}{P-c}k_1\right)$. For these cases, in both auctions, the larger, low-cost supplier is better off serving the entire demand at the marginal cost of its rival than supplying the residual demand at the market reserve price, and obtaining profits of $c\theta$ rather than $P[\theta - k_1]$. Given that it would be unprofitable for the high-cost supplier to undercut its rival's offer price, this constitutes the unique equilibrium in both auction formats.

However, for a larger (smaller) degree of capacity (cost) asymmetry and higher demand realizations, the equivalence breaks down. When $P[\theta - k_1]$ exceeds $c\theta$, in the uniform auction the larger supplier is better off pricing at the market reserve price P. Hence we cannot have an equilibrium in which both suppliers price at the marginal cost of the less efficient supplier. Neither can there be an equilibrium in which the less efficient supplier offers prices at or above its rival's offer price, since it would then produce nothing. Therefore the equilibrium in which the smaller, high-cost supplier sells all of its capacity at the high price set by the efficient supplier constitutes the unique equilibrium in the uniform auction. This equilibrium results in the largest possible degree of productive inefficiency and maximum market prices.

The unique equilibrium in the discriminatory auction again involves mixed-strategy pricing. In this equilibrium the larger supplier must play the market reserve price with strictly positive probability because otherwise the smaller supplier would earn zero profits almost surely whenever it played a price sufficiently close to this price. This equilibrium is not efficient, since with some positive probability the low-cost supplier will be undercut by its higher-cost rival.

5 Welfare Analysis

Having characterized the equilibria for the Vickrey, uniform and discriminatory auctions, we may now turn to a comparison of their welfare properties. Unlike standard auction theory, in which the auctioneer's objective would be to minimize purchase costs, we assume that the regulatory authority (the auction designer) will also be concerned with the level of social welfare. One possibility would be to assume that the regulator is solely concerned with the level of total surplus, and hence indifferent between all auctions which achieve the same level of efficiency. Another possibility would be to allow for 'lexicographic' preferences, so that, for instance, whenever two auction formats are equally efficient, the regulator will prefer the auction with the lowest prices. A third possibility would be to assume that the regulator assigns unequal weights to producer and consumer surplus, e.g. wishes to maximize consumer surplus subject to suppliers earning nonnegative profits, say. Rather than assuming that the regulator has these nor any other specific preferences, in the following subsections we compare the equilibria in the Vickrey, uniform and discriminatory auctions in terms of both total surplus and consumer surplus respectively.

5.1 Total surplus comparison

Given the assumption of price-inelastic demand, total surplus in any auction is determined solely by the degree of productive efficiency, since it is not affected by the level of prices. From the characterization of the equilibria in the preceding section, we readily obtain the following results, where we have denoted total surplus by S^v, S^u and S^d for the Vickrey, uniform and discriminatory auctions respectively.[22]

[22] We have excluded the possibility that $c = 0$, given that in this case all three auctions would be trivially equivalent in terms of cost efficiency.

Proposition 7 *Assume that the small is supplier at least as efficient as the large supplier, i.e.* $c_1 = 0 < c_2 = c$.

(i) When $\theta \in (0, k_1]$, $S^v = S^u = S^d$.

(ii) When $\theta \in \left(k_1, k_2 + \frac{c}{P}k_1\right]$, $S^v = S^u > S^d$.

(iii) When $\theta \in \left(k_2 + \frac{c}{P}k_1, k_1 + k_2\right)$, $S^v = S^u > S^d$ *if in the uniform auction the pure-strategy equilibrium with* $b_1 < b_2$ *is played, and* $S^v > S^d > S^u$ *otherwise.*

Proposition 8 *Assume that the large supplier is more efficient than the small supplier, i.e.* $c_1 = c > c_2 = 0$.

I. Assume $\frac{k_1}{k_2} \geq \frac{P-c}{P}$.

(i) When $\theta \in (0, k_2]$, $S^v = S^u = S^d$.

(ii) When $\theta \in \left(k_2, k_1 + \frac{c}{P}k_2\right]$, $S^v = S^u > S^d$.

(iii) When $\left(k_1 + \frac{c}{P}k_2, k_1 + k_2\right]$, $S^v = S^u > S^d$ *if in the uniform auction the pure-strategy equilibrium with* $b_1 > b_2$ *is played, and* $S^v > S^d > S^u$ *otherwise.*

II. Assume $\frac{k_1}{k_2} \leq \frac{P-c}{P}$.

(i) When $\theta \in \left(0, \frac{P}{P-c}k_1\right]$, $S^v = S^u = S^d$.

(ii) When $\theta \in \left(\frac{P}{P-c}k_1, k_2\right]$, $S^v > S^d > S^u$.

(iii) When $(k_2, k_1 + k_2]$, $S^v = S^u > S^d$ *if in the uniform auction the pure-strategy equilibrium with* $b_1 > b_2$ *is played, and* $S^v > S^d > S^u$ *otherwise.*

As a general property (see Lemma 1), the Vickrey auction is efficient. It therefore (weakly) outperforms both the uniform and discriminatory auctions in efficiency terms for all possible cost-capacity configurations. For low demand realizations however, all three auctions are efficient.

The comparison of the uniform and the discriminatory auctions for intermediate demand realizations depends on whether the small supplier is more or less efficient than its larger competitor. If the small supplier is at least as efficient, the discriminatory auction is not *ex ante* efficient, since with some positive probability the low-cost supplier will be under-priced by the higher-cost supplier. The uniform auction is efficient however, since in equilibrium the low-cost supplier produces up to capacity. In this case, the uniform auction outperforms the discriminatory in total surplus terms.

In the alternative case in which the large supplier is more efficient than its smaller competitor, both the uniform and discriminatory auctions guarantee efficiency for some demand realizations, i.e. for $\theta \in \left(k_1, \frac{P}{P-c}k_1\right)$. For demand realizations exceeding this level however, the uniform auction results in larger efficiency losses than the discriminatory auction. This is because in the unique pure-strategy equilibrium of the uniform auction, the smaller, high-cost supplier produces up to capacity, whereas in the mixed-strategy equilibrium of the discriminatory auction such an event occurs with probability less than one. Hence the discriminatory auction results in higher expected (i.e. *ex ante*) total surplus in this case.

For high demand realizations the welfare comparison between the uniform and the discriminatory auctions depends upon the equilibrium played in the uniform auction. When, in a pure-strategy equilibrium, the low-cost supplier is also the lower-pricing supplier, the uniform auction is efficient and outperforms the discriminatory auction. However, for some high demand realizations there also exist equilibria of the uniform auction in which the high-cost supplier submits the lowest offer price, while the probability that this occurs in the mixed-strategy equilibrium of the discriminatory auction is strictly less than one. In the absence of an equilibrium selection device therefore, the comparison between the two auction types is ambiguous.

5.2 Consumer surplus comparison

Propositions 9 and 10 below compare the three auction types on the basis of consumer surplus.

Proposition 9 *Assume that the small supplier is at least as efficient as the large supplier, i.e.* $c_1 = 0 \leq c_2 = c$.

(i) When $\theta \in [0, k_1]$, $CS^v = CS^u = CS^d$.

(ii) When $\theta \in (k_1, k_2)$, $CS^u \leq CS^d \leq CS^v$, *where the inequalities are strict for* $P > c$.

(iii) When $\theta \in (k_2, k_1 + k_2)$, $CS^u \leq CS^d \leq CS^v$, *where the first inequality is strict for* $P > c$ *and the second inequality is strict for* $c > 0$ *or* $k_1 < k_2$ *and* $c = 0$.

Proposition 10 *Assume that the large supplier is more efficient than the small supplier, i.e.* $c_1 = c > c_2 = 0$ *for* $k_1 < k_2$.

I. Assume $\frac{k_1}{k_2} \geq \frac{P-c}{P}$.

(i) When $\theta \in (0, k_1]$, $CS^v = CS^u = CS^d$.

(ii) When $\theta \in (k_1, k_2]$, $CS^v \leq CS^u = CS^d$, *where the inequality is strict for* $P > c$.

(iii) When $\theta \in (k_2, k_1 + k_2)$, $CS^u \leq CS^v \leq CS^d$, *where the inequalities are strict for* $P > c$.

II. Assume $\frac{k_1}{k_2} \leq \frac{P-c}{P}$.

(i) When $\theta \in (0, k_1]$, $CS^v = CS^u = CS^d$.

(ii) When $\theta \in \left(k_1, k_1 \frac{P}{P-c}\right]$, $CS^v < CS^u = CS^d$.

(iii) When $\theta \in \left(k_1 \frac{P}{P-c}, k_2\right)$, $CS^u < CS^d < CS^v$.

(iv) When $\theta \in (k_2, k_1 + k_2)$, $CS^u < CS^d$, CS^v. There exists a value $\widehat{c} \in \left(0, P\frac{k_2 - k_1}{k_2}\right)$ such that $CS^d \leq CS^v$ for $c \leq \widehat{c}$ and $CS^d > CS^v$ for $c > \widehat{c}m$.

Independently of the capacity-cost configuration, the uniform auction is (weakly) outperformed by the discriminatory auction in terms of consumer surplus, and also by the Vickrey auction when the small supplier is at least as efficient as its rival. The uniform auction leads to higher consumer surplus than the Vickrey auction only when the larger supplier is more efficient than its rival. The more asymmetric the capacities and/or costs of the suppliers, the more likely this case is to occur.

When suppliers have equal costs and capacities, the discriminatory and Vickrey auctions yield equal levels of consumer surplus. With cost asymmetries but symmetric capacities on the other hand, the discriminatory auction is outperformed by the Vickrey auction: even if suppliers' earn equal revenues in the two auction formats, the fact that the discriminatory auction potentially leads to efficiency losses implies that consumers will pay higher prices in the discriminatory auction than in the Vickrey auction. With both cost *and* capacity asymmetries however, the ranking of the two auctions is ambiguous. For sufficiently asymmetric costs (or capacities), i.e. for $\frac{k_1}{k_2} \geq \frac{P-c}{P}$, the discriminatory auction (weakly) outperforms the Vickrey auction. However, for other cost-capacity configurations the reverse ranking may hold.[23]

5.3 Regulatory preferences

We now compare the three auction formats both in terms of total surplus (cost efficiency) and consumer surplus, given different possible preferences for the regulator.

If the regulator is solely concerned with the maximization of total surplus, then the Vickrey auction should always be chosen, as it guarantees productive efficiency independently of the industry and market data. If, among the auction formats that guarantee productive efficiency, the regulator prefers the one that leads to lower prices, then the choice of auction format is no longer unambiguous. For some levels of demand and the industry cost-capacity configuration, the uniform and discriminatory auctions also guarantee productive efficiency, but lead to lower prices than the Vickrey auction.

If the regulator is only concerned about the minimization of prices, then the only unambiguous conclusion is that he should never choose the uniform auction, as it is always (weakly) outperformed by the discriminatory auction, and in some

[23] This occurs, for instance, when the larger supplier is more efficient than its rival and the degree of capacity (cost) asymmetries is sufficiently large (small), for demand realizations $\theta \in \left(k_1 \frac{P}{P-c}, k_2\right)$ or $\theta \in [k_2, k_1 + k_2)$ for $c \leq \widehat{c}$.

cases, also by the Vickrey auction. The discriminatory and the Vickrey auctions result in equal prices when the suppliers are symmetric. Otherwise the ranking can go in either direction.

Finally, if the regulator assigns unequal, but positive, weights to both productive efficiency and consumer surplus, the auction ranking will then depend both on the specific weights assigned to each, and on the industry data. In some cases the choice between the uniform and the discriminatory auction involves a trade-off between greater efficiency and higher prices in the uniform auction, versus reduced efficiency and lower prices in the discriminatory auction. Such a trade-off also arises in the choice between the Vickrey auction and either the uniform or the discriminatory auctions.

6 Variations on the Basic Duopoly Model

The preceding sections analyzed electricity auctions for an asymmetric duopoly under the assumptions that each supplier could submit only a single offer price for its entire capacity, and that demand was both known with certainty at the time offer prices were submitted and perfectly inelastic. The following subsections relax each of these assumptions in turn.

6.1 Multiple-unit suppliers

We first extend the analysis by allowing suppliers to submit (upward-sloping) step offer-price functions instead of constraining them to submit a single bid for their entire capacity. An offer-price function for supplier i, $i = 1, 2$, is then a set of price-quantity pairs (b_{in}, k_{in}), $n = 1, ..., N_i$, $N_i < \infty$. For each pair the offer price b_{in} specifies the minimum price for the corresponding capacity increment k_{in}, where $b_{in} \in [0, P]$ and $\sum_{n=1}^{N_i} k_{in} = k_i$, $i = 1, 2$.

The following lemma states that the *equilibrium outcomes* - but not the equilibrium *pricing strategies* - are essentially independent of the number of admissible steps in each supplier's bid function. This implies that our comparisons of total welfare and consumer surplus across auction types remain valid in this setting.

Lemma 11 *(Multiple-unit suppliers)*

(i) Vickrey auction: There exists a unique equilibrium in (weakly) undominated strategies in which suppliers offer all units at marginal costs.

(ii) Uniform auction: The set of (pure-strategy) equilibrium outcomes is independent of the number of units per supplier (in particular, whether $N_i = 1$ or $N_i > 1$).

(iii) Discriminatory auction: For low demand realizations, there is a unique equilibrium outcome independent of the number of units per supplier. For high demand realizations, there exists a set of strategies that constitute an equilibrium independently of the number of units per supplier; when $N_1 = N_2 = 1$ these strategies constitute the unique equilibrium.

The proof of this result for the Vickrey auction is trivial. The same argument used in the proof of Lemma 1 implies that it is a weakly dominant strategy in the Vickrey auction to submit offer prices equal to marginal cost for all payoff relevant units: pricing some of these units at prices above marginal cost reduces their chances of being despatched but (conditional upon being despatched) does not affect the suppliers' payoff. The proof is also trivial for both the uniform and discriminatory auctions for low demand realizations. An argument corresponding to that in the proof of Lemma 2 implies that in equilibrium, all payoff-relevant units are offered at the marginal cost of the inefficient supplier in the uniform and the discriminatory auctions. This equilibrium outcome is therefore unique.[24]

The existence of a unique zero-profit equilibrium in the uniform auction in which all (payoff-relevant) units are offered at marginal cost is in stark contrast to analyses which assume continuously differentiable supply functions, i.e. $N_i = \infty$.[25] As first shown by Wilson (1979), and further developed by Back and Zender (1993), Wang and Zender (2002) and Nyborg (2002), in the uniform auction with continuous supply functions there exists a continuum of pure-strategy equilibria, some of which result in very low revenues for the auctioneer (or high payments to suppliers in procurement auctions). The latter are characterized by participants offering very steep supply functions which inhibit competition at the margin: faced with a rival's steep supply function, a supplier's incentive to price more aggressively is offset by the large decrease in price (price effect) that is required to capture an infinitesimal increment in output (quantity effect). Since the price effect always outweighs the quantity effect for units of *infinitesimal size*, extremely collusive-like equilibria can be supported in the continuous uniform auction, even in a one-shot game.[26]

Discreteness of the bid functions rules out such equilibria however. When suppliers are limited to a finite number of price-quantity bids, a positive incre-

[24] The equilibrium offer price functions, however, do depend upon the number of units or admissible bids. For instance, there can be payoff-irrelevant units which are offered at a price above marginal cost as long as there are sufficiently many units priced at marginal cost. That is, $b_{in} = 0 \; \forall \; n \in \{1, ..., m\}$ and $b_{in} > 0 \; \forall \; n \in \{m+1, ..., N_i\}$, $i = 1, 2$ for m such that $\sum_{n=1}^{2m} k_{in} \geq \theta$, and $b_{in} = 0 \; \forall \; n \in \{1, ..., N_i\}$ $i = 1, 2$, are both equilibrium offer price functions that support the zero-profit equilibrium.

[25] See the discussion in Section 2 above.

[26] This type of equilibrium cannot be supported in a discriminatory auction for reasons cited in Section 2. Klemperer (2002) provides a particularly clear discussion.

ment in output can always be obtained by just slightly undercutting the price of a rival's unit. Since the price effect no longer outweighs the quantity effect, the collusive-like equilibria found in the continuous auction cannot be implemented.

As pointed out in Section 2 above (see also Nyborg, 2002), this observation casts serious doubt on the relevance of applying the continuous share auction model to electricity markets, in which participants are limited to a small number of bid prices per generating unit. The collusive-like equilibria obtained under the assumption that bid functions are continuous do not generalize to models in which offer increments are of positive size, no matter how small these are.

We conclude that the equilibrium outcomes for the three types of auction are independent of the number of admissible steps in the offer-price functions, so as long as this number is finite. Hence the characterization of the equilibrium outcomes provided in Section 4 would remain unchanged if we had instead assumed that suppliers submit offer-price functions rather than a single offer price for their whole capacity.

6.2 Uncertain demand

We now consider the case in which suppliers face time-varying, or stochastic, demand. This is of particular relevance to electricity markets in which suppliers submit offers that remain fixed for twenty-four or forty-eight market periods, such as in Australia and the original market in England and Wales.

We assume now that offers must be made before the realization of demand (i.e. θ) is known. It is easy to verify that our previous analysis is robust to this change in the timing of decisions so long as the largest possible demand realization is low enough or the lowest possible demand realization is large enough. When demand never exceeds the capacity of a single supplier, for instance, the equilibria correspond to those analyzed for low demand realizations in Section 4.2.1.[27] The introduction of demand variability adds a new dimension to the problem only when *both* low and high demand realizations occur with positive probability. We therefore assume that demand θ takes values in the support $\left[\underline{\theta}, \overline{\theta}\right] \subseteq (0, k_1 + k_2)$, *with* $\underline{\theta} \leq k_1 \leq k_2 \leq \overline{\theta}$, according to some (commonly known) distribution function $G(\theta)$.

6.2.1 Equilibrium analysis

From Lemma 1, we know that in the Vickrey auction with deterministic demand, pricing at cost is a (weakly) dominant strategy. It is straightforward to verify

[27] Likewise, when demand is such that both suppliers must always be producing, then the equilibria correspond to those analysed above for high demand realisations; the only difference is that expected demand replaces the deterministic level of demand in all expressions. See von der Fehr and Harbord (1993).

that this property is preserved in the case of stochastic demand. The equilibria of the uniform and discriminatory auctions differ significantly, however, from the case in which demand is known with certainty before bids are submitted. Demand uncertainty, or variability, upsets all candidate pure-strategy equilibria in both types of auction.[28]

We therefore consider equilibria in mixed strategies. For both the uniform and discriminatory auctions there exist unique mixed-strategy equilibria, and it is possible to derive explicit formulae for the suppliers' strategies. In the Appendix we prove the following result:

Lemma 12 *Assume* $\left[\underline{\theta},\overline{\theta}\right] \subseteq (0, k_1 + k_2)$*, with* $\underline{\theta} \leq k_1 \leq k_2 \leq \overline{\theta}$*. (i) In the uniform auction, there exists a unique equilibrium in which supplier* $i = 1, 2$ *offer prices* $b_i \in \left[\underline{b}^u, P\right]$, $\underline{b^u} > c$*, according to the probability distribution* $F_i^u(b)$*. (ii) In the discriminatory auction, there exists a unique equilibrium in which supplier* $i = 1, 2$ *offer prices* $b_i \in \left[\underline{b}^d, P\right]$, $\underline{b^d} > c$*, according to the probability distribution* $F_i^d(b)$*.*

In a mixed-strategy equilibrium in either type of auction, suppliers must strike a balance between two opposing effects: on the one hand, a higher offer price tends to result in higher equilibrium prices; on the other hand, pricing high reduces each suppliers' expected output, *ceteris paribus*. The first effect is less pronounced in the uniform auction than in the discriminatory auction. In the uniform auction, a higher offer price translates into a higher market price only in the event that the offer price is marginal, while in the discriminatory auction pricing higher always results in the supplier increasing the expected price it receives, conditional on being despatched. Consequently, there is a tendency for suppliers to price less aggressively in the discriminatory auction compared to a uniform auction. This intuition is confirmed in the symmetric case (i.e., when $k_1 = k_2 = k$ and $c_1 = c_2 = 0$), in which the equilibrium mixed-strategy distribution function for each supplier in the discriminatory auction first-order stochastically dominates the corresponding distribution function in the uniform auction, i.e. $F_i^u(b) \geq F_i^d(b)$.[29]

We have not been able to characterize in detail the relationship between the model parameters and suppliers' equilibrium strategies in the general case. In the case of symmetric capacities, however, we can show that in the limit, as $\overline{\theta} \to k$ (or $k \to \overline{\theta}$), so that demand is always less than the capacity of a single supplier, the mixed-strategy equilibrium outcome in either auction approaches the equilibrium outcome for a low demand realization, with price equal to the

[28] This is proved in the appendix. See also von der Fehr and Harbord (1993) and Garcia-Diaz (2000).

[29] The result follows from the observation that $F_i^u(b) < F_i^d(b) \Longrightarrow \pi_i^u > \pi_i^d$, whereas in the symmetric case $\pi_i^u = \pi_i^d$.

marginal cost of the higher-cost supplier. Similarly, as $\underline{\theta} \to k$ (or $k \to \underline{\theta}$), so that demand always exceeds the capacity of a single supplier, the equilibrium outcomes approach those for a high demand realization. Further, in the uniform auction the limiting equilibrium outcome is efficient. That is, the more efficient supplier produces at capacity and the less efficient supplier supplies the residual demand. This is in contrast to the model with nonstochastic demand, in which there exist both efficient and inefficient equilibria in high demand realizations in the uniform auction.[30]

6.2.2 Welfare comparisons

As in the previous section, we now consider the welfare properties of the equilibria in each auction format. In the purely symmetric case, suppliers have equal costs of production, so productive efficiency is not an issue. Further, since demand is perfectly price inelastic, the level of prices will not affect the level of total surplus. Total surplus is therefore equal in the three auction types. The comparison across auctions hence depends only upon how the total surplus is shared between suppliers and consumers. Whereas when demand is high with probability one, the uniform auction leads to higher producer surplus and lower consumer surplus than the discriminatory and Vickrey auctions, with stochastic demand and symmetric suppliers, the three auctions are equivalent.[31]

Proposition 13 *In a duopoly model with symmetric suppliers and uncertain demand $\theta \in [\underline{\theta}, \overline{\theta}] \subseteq (0, 2k)$, (i) when $\underline{\theta} \leq k < \overline{\theta}$, $CS^u = CS^d = CS^v$. (ii) otherwise, Proposition 9 applies.*

The auctions are no longer equivalent with asymmetric costs or capacities, however. Since a direct comparison of the auction outcomes in these cases is intractable, we restrict ourselves to considering limit results. In particular, we have the following result:

Proposition 14 *In a duopoly model with asymmetric costs and uncertain demand $\theta \in [\underline{\theta}, \overline{\theta}] \subseteq (0, 2k)$, there exists $\varepsilon > 0$ such that if $k - \varepsilon < \underline{\theta} \leq k < \overline{\theta}$, $CS^u \leq CS^d \leq CS^v$ (where the first inequality is strict if $P > c$ and the second if $c > 0$) and $S^d \leq S^u \leq S^v$ (where the first inequality is strict if $c > 0$ and the second if $\underline{\theta} < k$).*

In the uniform auction, in the limit, the lower-cost supplier supplies its entire capacity and earns profits of Pk. In the discriminatory auction, with positive

[30] The fact that with uncertain demand the efficient outcome is unique might be viewed as a justification for treating this as a natural 'focal point' in the certain demand case also.

[31] Garcia-Diaz (2000) obtains the same result for uniform and discriminatory auctions in a similar setting.

probability, the low-cost supplier submits a higher price offer than the higher-cost supplier, resulting in inefficiency. Profits of the lower-cost supplier are greater in the uniform auction however, resulting in the usual trade-off between the auction types. By continuity, this remains true over a range of values for $\underline{\theta}$ in a neighborhood of k, i.e. in a half-open interval $(k-\epsilon, k]$.

6.3 Symmetric duopoly with downward-sloping demand

Our next variation on the basic model considers the case of price-elastic demand. For this purpose we let the market demand function be represented by $D(p)$, which is assumed to satisfy the following standard assumptions: $D(p)$ is a continuous, bounded function; there exists a price $P > 0$ such that $D(p) = 0$ if and only if $p \geq P$; $D(p)$ is decreasing in p, $\forall p \in [0, P]$; and $pD(p)$ is strictly quasi-concave in p, $\forall p \in [0, P]$. From these assumptions it follows that market demand is a continuous and decreasing function of price and that, whenever $D(0) > k$, there exists a unique price P^r which maximizes a supplier's profits from serving the residual demand, i.e. $P^r = \arg\max_p \{p[D(p) - k]\}$. P^r will be referred to as the 'residual monopoly price'. For simplicity we assume that suppliers have identical marginal costs, normalized to zero, and identical capacities given by $k > 0$.

Given a downward-sloping demand function, in any auction the output allocated to supplier i, $q_i(\mathbf{b})$, as a function of the offer-price profile $\mathbf{b} = (b_i, b_j)$, becomes:

$$q_i(\mathbf{b}) = \begin{cases} \min\{D(b_i), k\} & \text{if} \quad b_i < b_j \\ \frac{1}{2}\min\{D(b_i), k\} + \frac{1}{2}\min\{\max\{0, D(b_i) - k\}, k\} & \text{if} \quad b_i = b_j \\ \min\{\max\{0, D(b_i) - k\}, k\} & \text{if} \quad b_i > b_j, \end{cases}$$

for $i = 1, 2$.

We must also redefine the suppliers' profit functions as a function of the offer-price profiles in the Vickrey auction. Following the logic that in a Vickrey auction a supplier receives the opportunity cost of its output, with an elastic demand function a supplier receives the cost of buying the rival's excess supply, so long as this is positive and the rival's offer price does not exceed consumer marginal willingness to pay. For any remaining output, the supplier is paid the corresponding point on the demand curve (i.e. consumers' marginal willingness to pay). This implies that the higher-pricing supplier (if one exists) becomes a monopolist over the residual demand and obtains the same profits as a perfectly price-discriminating monopolist. Formally, supplier i's profits in the Vickrey auction can be expressed as a function of the offer-price profile $\mathbf{b}$ as follows:

$$\pi_i^v(\mathbf{b}) = \begin{cases} b_j D(b_j) + \int_{D(b_j)}^{D(b_i)} D^{-1}(q)dq & \text{if } b_i \leq b_j \text{ and } D(b_j) \leq k \\ b_j [k - q_j(\mathbf{b})] + \int_k^{D(b_j)} D^{-1}(q)dq & \text{if } b_i \leq b_j \text{ and } k < D(b_j) \leq 2k \\ \int_k^{D(b_i)} D^{-1}(q)dq & \text{if } b_i > b_j \text{ and } D(b_i) \leq 2k \\ \int_k^{2k} D^{-1}(q)dq & \text{otherwise.} \end{cases}$$

Independently of the payments made to suppliers in any auction format, we will assume that consumers are charged the market-clearing price, i.e. the highest accepted offer price. The possibility that this leads to the auctioneer running deficits or surpluses in the Vickrey and discriminatory auctions is discussed below. We now characterize suppliers' pricing strategies in the three auction formats.

6.3.1 Equilibrium analysis

As in the case of inelastic demand, in the Vickrey auction sincere bidding remains the unique equilibrium that survives the elimination of weakly dominated strategies.

Lemma 15 *In the Vickrey auction, for all realizations of demand, there exists a unique equilibrium in (weakly) undominated strategies in which all suppliers price at marginal cost, i.e.* $b_i = 0$, $i = 1, 2$.

As noted above, we assume that the market price is set equal to the highest accepted offer price whenever total capacity is sufficient to cover demand at this price, and set so as to clear the market at full capacity utilization otherwise. This means that the auctioneer's revenues and payments will typically not balance in a Vickrey auction, i.e. the auctioneer will run a deficit.[32] This feature of the Vickrey auction is well-known, and a general characteristic of optimal incentive-compatible revelation mechanisms (see Section 2.2 above).

The existence, multiplicity and the types of equilibria in the uniform and discriminatory auctions depend as always upon the magnitude of demand relative to suppliers' capacities. We distinguish the usual three cases, redefined as follows: *low demand realizations,* $k \geq D(0)$, in which the capacity of a single supplier is enough to supply the whole market at a price equal to marginal cost; *high demand realizations,* $[D(P^r) - k] < k < D(0)$, for which a single supplier cannot supply the entire market at a price equal to marginal costs but the high-pricing supplier has excess capacity when serving the residual demand at its best response; and *very high demand realizations,* $[D(P^r) - k] > k$, in which either

[32] Only when demand is extremely low will the auctioneer's revenues equal payments to suppliers; that is when the market price is equal to marginal cost. For higher levels of demand payments to suppliers will exceed the revenues received from consumers.

supplier will be capacity-constrained at its best response to its rival selling at capacity. The following lemma provides a characterization of the equilibria in the uniform and discriminatory auctions with downward-sloping demand.

Lemma 16 . *(i) Low demand realizations. In both the uniform and discriminatory auctions there exists a unique pure-strategy equilibrium in which both suppliers offer prices equal marginal cost, i.e.* $b_i = 0$*,* $i = 1, 2$*.*

(ii) High demand realizations. In the uniform auction, all pure-strategy equilibria are given by offer-price profiles satisfying $b_i \leq \frac{1}{k}\left[D(P^r) - k\right] P^r$ *and* $b_j = P^r$*,* $i = 1, 2$*,* $i \neq j$*. In the discriminatory auction there exists a unique equilibrium in which supplier* i*,* $i = 1, 2$*, offer prices* $b_i \in \left[\frac{1}{k}\left[D(P^r) - k\right] P^r, P^r\right]$ *according to the probability distribution* $F^d(b) = \frac{k}{2k - D(b)}\left[1 - \frac{\underline{b}}{b}\right]$*.*

(iii) Very high demand realizations. In the uniform auction any offer-price profile satisfying $\max\{b_1, b_2\} = D^{-1}(2k)$ *constitutes a pure-strategy equilibrium. These are the only pure-strategy equilibria. In the discriminatory auction there exists a unique pure-strategy equilibrium in which both suppliers offer the market-clearing price* $b_i = D^{-1}(2k)$*,* $i = 1, 2$*.*

When the capacity of either supplier is sufficient to supply the market at a price equal to marginal cost, i.e. in a low demand realization, both suppliers offer their capacities at a marginal cost. Hence - as in the inelastic demand case - suppliers' pricing behavior in the two auctions coincides. For high demand realizations, in the uniform auction there are multiple pure-strategy equilibria in each of which the market price equals the residual monopoly price P^r; in the discriminatory auction the unique equilibrium entails suppliers randomizing their price offers over an interval bounded below by marginal cost and above by the residual monopoly price.[33] In very high demand realizations, the equilibrium outcomes (but not necessarily the equilibrium strategies) in the two auctions are equivalent. Each supplier sells its entire capacity at the market clearing price $P = D^{-1}(2k)$.

6.3.2 Welfare comparisons

A price-elastic demand function allows for somewhat richer welfare comparisons since the level of prices now affects the level of total surplus (i.e. allocative efficiency). Productive efficiency, however, is not an issue given the assumption of symmetric suppliers. Assuming that the auctioneer's surpluses or deficits are dealt with via lump-sum transfers, total surplus in equilibrium will then be solely determined by the market clearing price, *i.e.* the value of the highest accepted

[33] Note that in the discriminatory auction the auctioneer will frequently run a surplus since consumers pay the bid price of the marginal supplier, while the inframarginal supplier receives its (typically lower) bid price.

offer price in equilibrium. From the characterization of the equilibria above we readily obtain the following results, where p^d, p^u and p^v denote the price consumers pay in the discriminatory, uniform and Vickrey auctions, respectively:

Proposition 17 *In the duopoly model with symmetric suppliers and downward-sloping demand: (i) In low demand realizations, $p^d = p^u = p^d = 0$, and so $S^d = S^u = S^v$ and $CS^d = CS^u = CS^v$; (ii) In high demand realizations, $p^v = \min\left\{0, D^{-1}(2k)\right\} \leq \frac{1}{k}\left[D(P^r) - k\right] P^r < p^d \leq p^u = P^r$ and so $S^v > S^d \geq S^u$ and $CS^v > CS^d \geq CS^u$; (iii) In very high demand realizations, $p^d = p^u = p^v = D^{-1}(2k)$ and so $S^d = S^u = S^v$ and $CS^d = CS^u = CS^v$.*

The uniform auction leads to weakly higher prices than the other two auction types in high demand realizations, resulting in lower total and consumer surplus, while the Vickrey auction results in the (weakly) highest level of total surplus and consumer surplus. The three auctions types otherwise result in equal market prices.

6.4 Symmetric oligopoly and market structure

Our final variation on the basic duopoly model considers the case of a symmetric oligopoly. This allows us to analyze the impact of changes in the number of suppliers on profits and pricing behavior in the three auction formats.

Accordingly we now consider N suppliers with identical marginal costs c, normalized to zero, and identical capacities $k > 0$. As before, the types of equilibria which arise in the different auction formats depend upon the value of the market demand θ relative to suppliers' individual and aggregate capacities. We again distinguish between low and high demand realizations, redefined for oligopoly suppliers as: *low demand realizations,* in which the capacity of any $N-1$ suppliers is sufficient to supply the market (i.e. $\theta \in (0, [N-1]\,k])$); and *high demand realizations,* in which the capacity of all N suppliers is needed to satisfy demand, but there is excess capacity overall (i.e. $\theta \in ([N-1]\,k, Nk)$).

6.4.1 Equilibrium analysis

From Lemma 1, we know that in the two-player Vickrey auction pricing at marginal cost is a weakly dominant strategy for each supplier. It is straightforward to show that this property extends to oligopoly. We therefore need only consider oligopoly' pricing behavior in the uniform and discriminatory auctions.

When total demand θ can be satisfied by the capacities of any $N-1$ suppliers, in the unique pure-strategy equilibrium all suppliers price at marginal costs and earn zero profits.

Lemma 18 *Assume $\theta \in (0, [N-1]\,k]$. In both the uniform-price and discriminatory auctions there exists a unique equilibrium in which all suppliers price at (zero) marginal costs, i.e., $b_i = 0$, $i = 1, 2, ..., N$.*

The next result describes equilibria for the case in which the capacity of all suppliers is needed to satisfy demand but there is excess capacity overall.

Lemma 19 *Assume $\theta \in ([N-1]\,k, Nk)$. (i) In the uniform auction, any offer-price profile satisfying $b_i \le \frac{\{\theta - [N-1]k\}}{k} P$ and $b_j = P$, $i \in \{1, ..., N\} \setminus \{j\}$, $j \in \{1, ..., N\}$, constitutes a pure-strategy equilibrium. (ii) In the discriminatory auction, there exists a unique equilibrium in which supplier i, $i = 1, ..., N$, offer prices $b_i \in \left[\frac{\theta - [N-1]k}{k} P, P\right]$, according to the probability distribution $F^d(b) = \left\{\frac{k}{Nk - \theta}\left[1 - \frac{\theta - [N-1]k}{k}\frac{P}{b}\right]\right\}^{\frac{1}{N-1}}$.*

As in the duopoly model, in the uniform auction there is a continuum of outcome-equivalent, pure-strategy equilibria in which one supplier offers its capacity at the market reserve price P, and serves the residual demand, while all other suppliers submit offer prices low enough to prevent profitable undercutting by the high-pricing supplier.

In the discriminatory auction suppliers play symmetric mixed strategies. As explained above, in equilibrium the suppliers' bidding strategies strike a balance between a 'price' and a 'quantity' effect. Lowering the bid reduces the price received, but increases the likelihood of undercutting rivals and hence gaining a larger market share. For a given level of demand, the 'quantity effect' is more important the larger the number of competitors. Hence, in the discriminatory auction, price competition will be more intense the less concentrated is the market structure.

6.4.2 Welfare comparisons

Given our assumptions on demand and costs, total surplus is constant in equilibrium, hence the welfare comparison depends only on how this total surplus is shared between suppliers and consumers, i.e. on the level of prices. From the characterization of the equilibria above we readily obtain the following result, which corresponds directly to the results obtained in the duopoly case:

Proposition 20 *In the oligopoly model with symmetric suppliers: (i) In a low demand realization suppliers earn zero profits in all three auctions; that is, $\Pi^d = \Pi^v = \Pi^u = 0$ and, hence, $CS^u = CS^d = CS^v$. (ii) In a high demand realization, joint profits in the discriminatory and the Vickrey auctions are equal to $\Pi^d = \Pi^v = PN\{\theta - [N-1]\,k\}$. In a pure-strategy equilibrium of the uniform auction joint profits are $\Pi^u = P\theta$. Hence $CS^u < CS^d = CS^v$.*

In low demand realizations the three auction types are again equivalent in welfare terms. In high demand realizations the uniform auction is out-performed in consumer surplus terms by both the discriminatory and Vickrey auctions.

Note that market structure affects the equilibrium outcomes differently in the three auction formats. In all auction formats, the threshold that determines whether demand is 'low' or 'high' is increasing in the number of suppliers; in other words, pricing at marginal cost is more likely in a more fragmented industry. However, in the discriminatory auction, as opposed to the other two auction formats, the market structure also affects strategies in high demand realizations. Consequently, a more fragmented market structure will improve market performance in all three cases, but the effect will tend to be greater for a discriminatory auction.

7 Conclusions

We have characterized suppliers' equilibrium pricing behavior in Vickrey, uniform and discriminatory auctions in a series of multi-unit auction models which reflect some of the key features of electricity auctions. The properties of the equilibria have been compared in terms of the resulting levels of overall welfare, consumer surplus and profits.

We can only derive two unambiguous conclusions from this analysis: first, the uniform auction is always (weakly) outperformed by the discriminatory auction in terms of consumer surplus, and second, the Vickrey auction results in the highest level of productive efficiency. The ranking across auctions otherwise remains ambiguous. There are capacity and cost configurations for which the uniform and the discriminatory auctions guarantee productive efficiency but lead to lower prices than the Vickrey auction. However the reverse ranking can also hold. In some cases the choice between a uniform and the discriminatory auction is a trade-off between productive efficiency and higher prices. In other cases the discriminatory auction dominates the uniform auction. The choice of an auction format ought therefore to be viewed as an empirical issue which depends upon demand, the cost-capacity configuration of the industry under consideration, and the particular preferences of the regulator.

From a methodological point of view, this paper has also contributed to the analysis of multi-unit electricity auctions in a number of ways. First, we have shown that the set of equilibrium outcomes in Vickrey, uniform and discriminatory auctions is essentially independent of the number of admissible steps in suppliers' offer-price functions, so as long as this number is finite. This reduces much of the complexity involved in the analysis of multi-unit auctions as it allows us to focus on the single-unit case with no significant loss in generality.

Secondly, we have demonstrated that the 'implicitly collusive' equilibria found in the uniform auction when offer prices are infinitely divisible are unique to this formulation of the auction (i.e. to share auctions), and do not arise when offer-price functions are discrete. Hence the concerns expressed in the literature that uniform auctions may lead to collusive-like outcomes, even in potentially competitive periods when there is considerable excess capacity, are likely misplaced. This point has recently been made independently by Nyborg (2002). Finally, we have provided a characterization of the multi-unit Vickrey auction with reserve pricing and capacity constraints, with downward-sloping demand functions.

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