Some Recent Developments in Futures Hedging

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Abstract: The use of futures contracts as a hedging instrument has been the focus of much research. At the theoretical level, an optimal hedge strategy is traditionally based on the expected-utility maximization paradigm. A simplification of this paradigm leads to the minimum-variance criterion. Although this paradigm is quite well accepted, alternative approaches have been sought. At the empirical level, research on futures hedging has benefited from the recent developments in the econometrics literature. Much research has been done on improving the estimation of the optimal hedge ratio. As more is known about the statistical properties of financial time series, more sophisticated estimation methods are proposed. In this survey we review some recent developments in futures hedging. We delineate the theoretical underpinning of various methods and discuss the econometric implementation of the methods.

Keywords: Futures; Hedge Ratio; Lower Partial Moments; Minimum-variance Hedge

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1 Introduction

A forward or futures contract is a promise (and obligation) to deliver a specific amount of a commodity (or asset) at a future time. Forward contracts usually contain detailed specifications of the underlying commodity (or asset) and the delivery process. Thus, the grade (or quality) of the commodity, the delivery date, and the delivery location are specified. Futures contracts, on the other hand, assume a standard form with some allowance for flexibility. There are usually choices of deliverable grades and delivery locations. The delivery dates may also be allowed to vary within a month or so. The standardization facilitates futures to be traded on organized exchanges. Due to the benefits and costs of specificity versus flexibility, individuals choose between forward and futures contracts when both are available. In general, one engages in forward contracts with the expectation of delivering and receiving the commodity. On the other hand, futures positions are often offset prior to expiration. An individual assumes a short position if he sells futures contracts and a long position if he purchases futures contracts.

While futures contracts are popular among investors as a class of speculative assets, they are important in the financial markets due to their use as a hedging instrument. The latter role of futures has been the focus of much research, especially on the formulation of an optimal hedge strategy and the implementation of the strategy. Recently, much progress has seen made in the theoretical and empirical aspects of futures hedging. At the theoretical level, an optimal hedge strategy is traditionally based on the expected-utility maximization paradigm. A simplification of this paradigm leads to the minimum-variance criterion. Although this paradigm is quite well accepted, alternative approaches have been sought. Firstly, the use of variance as a measure of risk is questioned. It is argued that as far as hedging is concerned, a one-sided measure as such as the downside risk is more relevant. Secondly, the application of the theories of stochastic dominance has resolved some of the restrictions in the expected-utility maximization framework. Developments in the stochastic dominance literature, such as the mean-Gini approach, facilitates the implementation of a hedge strategy.

At the empirical level, research on futures hedging has benefited from the recent developments in the econometrics literature. Much research has been done on improving the estimation of the optimal hedge ratio. As more is known about the statistical properties of financial time series, more sophisticated estimation methods are proposed. Firstly, the classical regression method, which assumes a time-invariant hedge ratio, has been replaced by time-varying estimates. Secondly, the cointegration literature suggests that futures and spot prices are cointegrated and better estimates are obtainable by exploring this relationship. Thirdly, the conditional volatility literature has provided many models, whether univariate or multivariate, that capture the time-varying variance and covariance of the spot and futures. Fourthly, the use of nonparametric methods has freed researchers from making assumptions (such as normality for asset returns) that may not be justifiable.

In this survey we review some recent developments in the literature of futures hedging. Our approach is to delineate the theoretical underpinning of various methods, and then discuss the econometric implementation of the methods. It is noted that alternative instruments are available for hedging. The most important alternative is perhaps option. While we shall make some comparisons between futures and option, we shall leave out works that are specifically targeted on option hedging. Also, we shall discuss futures hedging in general. While empirical results on specific futures contracts are covered, no efforts will be made to distinguish between hedging for specific assets such as bonds, equities, commodities or currencies. Finally, we shall assume that the hedger is faced with a given spot position to hedge. The problem of hedging quantity uncertainty will not be discussed. We should note that the rationale for excluding these topics is to narrow the scope of this survey to a manageable scale. It is not a reflection of the (lack of) importance of these issues.

In Section 2 we discuss the conventional hedging framework of expected-utility maximization and minimization of portfolio variance. Section 3 extends the traditional framework to time-varying hedge ratios. Various estimation methods are discussed, including the stochastic volatility models and the conditional heteroscedasticity models. In Section 4 we review some issues in the implementation of the traditional approach. We discuss the extended mean-Gini approach in Section 5. The lower-partial-moment criterion is introduced in Section 6. In Section 7 we review the problems with rollover hedge in futures. Finally, the paper is concluded in Section 8.

2 Conventional Hedging Analysis

Conventional wisdom suggests that to hedge a unit of a spot position one should assume a unit of the opposite position in the futures market. Thus, the optimal hedge ratio, that is, the amount of the futures position divided by the amount of the spot position, is 1 for both long (long in futures and short in spot) and short (short in futures and long in spot) hedgers. This hedging strategy is incorporated into the Commodity Futures Trading Commissions (CFTC) guidelines which define a *bona fide hedger* as a hedger who has equal but opposite spot and futures positions. Recognizing that the spot and futures prices have parallel but not exactly identical movements, Johnson (1960) and Stein (1961) adopted a portfolio approach to determine the optimal hedge strategy via expected-utility maximization. Mean-variance analysis then follows as a special case. Ederington (1979) applied the method to the stock index futures markets and proposed a measure for hedging effectiveness. In this section we shall focus on this conventional approach and some extensions of it in the time-invariant context. Time-varying hedging strategies will be discussed in Section 3.

2.1 The Analytical Framework

We illustrate hedging decisions with a one-period model. At the beginning of the period, that is, $t = 0$, an individual is committed to a given spot position, Q, on a specific asset.[1] A futures market for the security is available with different maturities. To reduce the risk exposure, the individual may choose to go short in the futures market. Due to liquidity

[1]The assumption that the spot position is fixed will be maintained throughout this survey. For the case where the spot position is uncertain, see Rolfo (1980), Honda (1983), Paroush and Wolf (1980) and Lapan, Moschini and Hanson (1991).

and other concerns, we assume that he trades only in the "nearby" futures contract (that is, the contract the maturity of which is closest to the current date). With the futures trading, the individual becomes a short hedger. Let X denote the futures position. At the end of the period, say, $t = 1$, the hedger's return, r, is calculated as follows:

$$r = (r_p Q - r_f X)/Q, \tag{1}$$

where r_p is the return of the spot position and r_f is the return of the futures position.[2] As both spot and futures returns are unknown at $t = 0$, r is a random variable. The hedger will choose X to minimize the risk (or uncertainty) associated with the random return.

In the finance literature, the risk of a random variable is usually measured by the variance (or standard deviation) conditional on the available information. Let Φ denote the information set at $t = 0$. Then the hedger's risk is summarized by the conditional variance of r, $\mathrm{Var}(r|\Phi)$. From equation (1),

$$\mathrm{Var}(r|\Phi) = [\mathrm{Var}(r_p|\Phi)Q^2 - 2\mathrm{Cov}(r_p, r_f|\Phi)XQ + \mathrm{Var}(r_f|\Phi)X^2]/Q^2. \tag{2}$$

The optimal futures position X^* is chosen to minimize $\mathrm{Var}(r|\Phi)$. Thus,

$$X^* = [\mathrm{Cov}(r_p, r_f|\Phi)/\mathrm{Var}(r_f|\Phi)]Q = hQ, \tag{3}$$

where $h = \mathrm{Cov}(r_p, r_f|\Phi)/\mathrm{Var}(r_f|\Phi)$ is the minimum-variance hedge ratio.

A more general approach to the hedging problem relies upon the expected-utility framework. Suppose that the hedger is endowed with a von-Neumann Morgenstern utility function $U(.)$ such that $U'(.) > 0$ and $U''(.) < 0$. Let $\mathrm{E}\{.\}$ denote the expectation operator with respect to the joint distribution of r_p and r_f. The optimal futures position, X^e, is chosen to maximize the (conditional) expected utility $\mathrm{E}\{U(r|\Phi)\}$. That is, X^e must satisfy the following condition:

$$\mathrm{E}\{U'(\frac{r_p Q - r_f X^e}{Q})\frac{r_f}{Q}|\Phi\} = 0, \tag{4}$$

or alternatively,

$$\mathrm{Cov}(U'(\frac{r_p Q - r_f X^e}{Q}), \frac{r_f}{Q}|\Phi) + \mathrm{E}\{U'(\frac{r_p Q - r_f X^e}{Q})|\Phi\}\mathrm{E}\{\frac{r_f}{Q}|\Phi\} = 0. \tag{5}$$

Assume that $r_p = \alpha(\Phi) + \beta(\Phi)r_f + \epsilon$, where r_f and ϵ are stochastically independent.[3] Now suppose that $\mathrm{E}\{r_f|\Phi\} = 0$ (that is, the futures price is unbiased), the above equation leads to $X^e = \beta(\Phi)Q$, which is in turn equal to $\mathrm{Cov}(r_p, r_f|\Phi)Q/\mathrm{Var}(r_f|\Phi)$. Thus, the optimal hedge ratio as defined by X^e/Q and derived from a general utility function is equal to the minimum-variance hedge ratio. This result was first discussed in Benninga,

[2]Except for the initial margin and subsequent margin calls, no investment is required for a futures position. Thus, the term *return of the futures position* should not be interpreted in the conventional sense. Notwithstanding this difficulty, however, we shall continue to use this term for convenience.

[3]This assumption is satisfied when r_p and r_f are jointly normally distributed conditional on the information set Φ.

Eldor and Zilcha (1983) and later extended by Lence (1995a) and Rao (2000). Lien (2000a) validated the result under Knightian uncertainty.

If the futures price is biased such that $\mathrm{E}\{r_f|\Phi\} \neq 0$ (due to transaction cost, for example), then the optimal hedge ratio diverges from the minimum-variance hedge ratio. In this case, however, there is a speculative motivation to trade so as to take advantage of the bias in the futures market. Consequently, X^e contains both hedging and speculative components. The former is characterized by the condition $\mathrm{E}\{r_f|\Phi\} = 0$. Thus, the hedging component of the optimal futures position is equal to the minimum-variance futures position. Assuming the hedger has a mean-variance utility function given by $\mathrm{E}\{r|\Phi\} - (A/2)\mathrm{Var}(r|\Phi)$, where $(A/2)$ is the Arrow-Pratt risk aversion coefficient, the optimal futures position X^* is $\mathrm{E}\{-r_f|\Phi\}/A + [\mathrm{Cov}(r_p, r_f|\Phi)/\mathrm{Var}(r_f|\Phi)]Q$. The first componenet represents the speculative trading whereas the second is the usual optimal hedge position.

Note that in the above derivation both minimum-variance and optimal hedge ratios are functions of the information set Φ. As Φ changes, both hedge ratios change. Typical information sets include the historical spot and futures returns, the contract maturity and the hedge horizon. Whenever the spot and futures return distributions depend on the information variables, both optimal hedge and minimum-variance hedge strategies depend on the time-varying dynamic hedge ratios. We now turn to the issue of estimating the minimum-variance hedge ratio in this conventional framework.

2.2 Estimating Hedge Ratio by Regression Method

To estimate the minimum-variance hedge ratio, a conventional method involves estimating the following linear regression model:

$$r_{pt} = \alpha + \beta r_{ft} + \varepsilon_t, \tag{6}$$

where r_{pt} and r_{ft} are the spot and futures returns for period t. The ordinary least squares (OLS) estimator of β provides an estimate for the minimum-variance hedge ratio. This approach has been extensively applied in the literature.

A major problem with the OLS hedge ratio is its dependence on the unconditional second moments, whereas the (true) minimum-variance hedge ratio is based on conditional second moments. Bell and Krasker (1986) argued that the correct regression model should allow the regression coefficients to be functions of the information available, that is,

$$r_{pt} = \alpha(\Phi) + \beta(\Phi) r_{ft} + \varepsilon_t. \tag{7}$$

Note the similarity between the above equation and the pre-condition for the minimum-variance hedge ratio to be optimal.[4] Unfortunately, in empirical works the functional forms of $\alpha(\Phi)$ and $\beta(\Phi)$ are unknown and researchers have to decide on the model specification. Cita and Lien (1992) applied this method to the wheat futures market allowing both the intercept and the slope to be linear functions of the historical spot and futures returns. They found that the modified regression model outperforms the conventional method in describing the spot price behavior. The Cita-Lien specification prescribes

[4] The information set Φ here includes information up to but not including t.

the minimum-variance hedge ratio to be a function of the conditioning information, and hence is time-varying. We will return to this issue in the next section. Upon allowing the intercept to be a linear function of the conditioning information variables while retaining a constant slope, Myers and Thompson (1989) proposed a generalized hedge ratio via a regression model with a large number of lagged price changes as information variables. Specifically, they assumed that the spot and futures returns can be described by the following equations:

$$r_{pt} = Z_t'\theta_p + \varepsilon_{pt}, \tag{8}$$

$$r_{ft} = Z_t'\theta_f + \varepsilon_{ft}, \tag{9}$$

where Z_t is a (column) vector of (exogenous) information variables. Then the minimum-variance hedge ratio can be calculated as an estimate of β from the following regression:

$$r_{pt} = \beta r_{ft} + Z_t'\theta_p + \varepsilon_{pt}. \tag{10}$$

Myers and Thompson (1989) suggested that Z_t should consist of a large number of lagged spot and futures returns. Fama and French (1987), however, argued that the basis (defined as the difference between the futures and spot prices) has predictive power for the spot returns. They specified the intercept term as a linear function of the lagged basis while the slope term remains constant. Let p_t and f_t denote the spot and futures prices (in logarithms), respectively, at time t. The regression equation for the estimation of the hedge ratio is given by:

$$r_{pt} = \beta r_{ft} + \delta_p(f_{t-1} - p_{t-1}) + \varepsilon_{pt}. \tag{11}$$

Viswanath (1993) adopted the above approach with a different rationale. Specifically, it was argued that, due to the convergence between the spot and futures prices at the maturity date, the spot returns would adjust to the basis. Moreover, different regressions were run for different durations and different timing when the hedge is lifted. This approach can be seen as a reduced form of the Garbade and Silber (1983) equations:

$$r_{pt} = \alpha_p + \beta_p(f_{t-1} - p_{t-1} - c_{t-1}) + \varepsilon_{pt}, \tag{12}$$

$$r_{ft} = \alpha_f + \beta_f(f_{t-1} - p_{t-1} - c_{t-1}) + \varepsilon_{ft}, \tag{13}$$

where c_t is the carrying cost from time t to contract maturity. Using a market microstructure approach, Garbade and Silber (1983) derived a restriction on the coefficients, namely, $\beta_p + \beta_f = 1$. Castelino (1992) assumed that the futures price follows a random walk whereas the spot price adjusts to the lagged basis so that

$$p_t = p_{t-1} + (1/\tau_t)(f_{t-1} - p_{t-1}) + \varepsilon_{pt}, \tag{14}$$

where τ_t is the time to maturity. The minimum-variance hedge ratio is again the ratio of the (conditional) second moments. Because of the specific adjustment coefficient specification, $1/\tau_t$, the hedge ratio is a function of τ_t.

2.3 The Cointegration Approach

While Fama and French (1987), Castelino (1992) and Viswanath (1993) argued that the basis reflects the cash-futures price convergence and, therefore, is an important information variable, this approach turns out to be fully justified by the recent statistical findings in the cointegration literature. Specifically, it is now well known that spot and futures price series typically contain a unit root. Thus, random-walk type of price behavior is prevalent. The unit-root property leads to the possible existence of a cointegration relationship (Engle and Granger, 1987). In spite of some skepticism (Quan, 1992; Brenner and Kroner, 1995), the cointegration relationship plays an important role in the statistical modelling of many spot and futures prices (Lien and Luo, 1993; Ghosh, 1993; Wahab and Lashgari, 1993; Tse, 1995).

Let Δ denote the difference operator such that $\Delta p_t = p_t - p_{t-1}$ and $\Delta f_t = f_t - f_{t-1}$. Thus, Δp_t and Δf_t are, respectively, the continuously compounded return (or log return) of the spot and futures.[5] Suppose that both (logarithmic) price series contain a unit root and are, furthermore, cointegrated. Then we have the following Engle-Granger specification for an error-correction (EC) model:

$$\Delta p_t = \alpha_p + \sum_{i=1}^{m} \beta_{pi} \Delta p_{t-i} + \sum_{j=1}^{n} \gamma_{pj} \Delta f_{t-j} + \theta_p z_{t-1} + \varepsilon_{pt}, \tag{15}$$

$$\Delta f_t = \alpha_f + \sum_{i=1}^{m'} \beta_{fi} \Delta p_{t-i} + \sum_{j=1}^{n'} \gamma_{fj} \Delta f_{t-j} + \theta_f z_{t-1} + \varepsilon_{ft}, \tag{16}$$

where z_t is a stationary linear combination of p_t and f_t. The Engle-Granger representation specifies that at least one of θ_p or θ_f is nonzero. In many empirical studies, it is found that z_t can be well approximated by the basis $f_t - p_t$. On the other hand, the no-arbitrage principle concludes that there is a cost-of-carry relationship between the spot and futures prices depending on the time to maturity. Thus, z_t should assume the following form: $z_t = f_t - p_t - k\tau_t$, where k is the per-period cost of carry. As mentioned above, Garbade and Silber (1983) adopted this specification. However, each futures contract is usually active for only three months or so (when its maturity is the closest to the current date). Furthermore, the price series during the delivery month tend to be noisy and unreliable. To construct a long time series for a meaningful time-series study, a futures price series corresponding to the "nearby" contract is usually constructed. The differences in the time to maturity across data points in the "nearby" futures price series are rather small (usually ranging from two or three weeks to less than four months). Thus, the time-varying nature of the cointegration relationship is usually not identifiable from the data set (see Tse, 1995).

Following Myers and Thompson (1989), we consider the following regression model:

$$\Delta p_t = \alpha_p + \lambda \Delta f_t + \sum_{i=1}^{m} \beta_{pi} \Delta p_{t-i} + \sum_{j=1}^{n} \gamma_{pj} \Delta f_{t-j} + \theta_p z_{t-1} + \varepsilon_{pt}. \tag{17}$$

[5] See Campbell, Lo, and MacKinlay (1997) for this terminology and Footnote 2 for the interpretation for futures return.

The OLS estimate of λ is the estimated minimum-variance hedge ratio. Chou, Denis and Lee (1996) applied this approach to the Hang Seng Index and found that the hedging performance improved over the conventional OLS method. Lien and Luo (1993), Ghosh (1993), and Wahab and Lashgari (1993) all demonstrated the superior hedging performance when the hedge ratio was estimated taking into account the cointegration relationship. Lien (1996) provided a theoretical analysis to characterize the effects of the cointegration relationship on the minimum-variance hedge ratio and the hedging effectiveness.

2.4 Fractional and Threshold Cointegration

The above approach relies upon the assumption that z_t is integrated of order zero. In admitting the broader framework of fractional difference order for z_t, Sowell (1992), Cheung and Lai (1993) and Dueker and Startz (1998) considered fractional cointegration relationships between two variables of the same integration order. Lien and Tse (1999) applied this approach to determine the minimum-variance hedge ratio. Suppose that both p_t and f_t are integrated of order one whereas z_t is integrated of order $d < 1$. Let B denote the backshifting operator such that $Bz_t = z_{t-1}$. Then z_t follows a fractionally integrated autoregressive moving average (ARFIMA) model if it is generated from the following equation:

$$\Phi(B)(1-B)^d z_t = \Theta(B)\varepsilon_t, \tag{18}$$

where $\Phi(B)$ and $\Theta(B)$ are finite-order polynomials in B and ε_t is a white noise. Here the differencing operation on z_t is given by $\Delta^d = (1-B)^d$ with

$$(1-B)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)B^k}{\Gamma(-d)\Gamma(k+1)} \tag{19}$$

where $\Gamma(.)$ is the gamma function. We assume that all roots of $\Phi(B) = 0$ and $\Theta(B) = 0$ are outside the unit circle. If $-0.5 < d < 0.5$, z_t is stationary. When $0 < d < 0.5$, z_t exhibits long-memory characteristics.

Given the above assumptions, a fractional cointegration relationship is established between the spot and futures prices. Following Granger (1986), we have the following fractionally integrated error-correction (FIEC) model:

$$\Delta p_t = \omega_p + \sum_{i=1}^{m} \omega_{pi}\Delta p_{t-i} + \sum_{j=1}^{n} \tau_{pj}\Delta f_{t-j} + \delta_p[(1-B)^d - (1-B)]z_t + \varepsilon_{pt}, \tag{20}$$

$$\Delta f_t = \omega_f + \sum_{i=1}^{m'} \omega_{fi}\Delta p_{t-i} + \sum_{j=1}^{n'} \tau_{fj}\Delta f_{t-j} + \delta_f[(1-B)^d - (1-B)]z_t + \varepsilon_{ft}. \tag{21}$$

The minimum-variance hedge ratio can be estimated based on estimates of $\mathrm{Cov}(\varepsilon_{pt}, \varepsilon_{ft})/\mathrm{Var}(\varepsilon_{ft})$ calculated from the residuals of the two equations. Lien and Tse (1999) estimated the minimum-variance hedge ratios for the Nikkei Stock Average (NSA) futures contract under the alternative assumptions of error correction and fractionally integrated error correction. It was found that while fractional cointegration is supported by the data, it does not lead to any improvement in the hedging performance.

To generalize the above analysis further, we may allow the spot and futures prices to be of different (fractional) integration order. Vinod (1997) demonstrated that in this case there will be no fractional cointegration. He proposed a new concept, the so-called *tie integration*, to examine the relationship among these variables. Because the spot and futures prices are expected to have similar statistical properties, tie cointegration may not be useful in futures hedging analysis.

Several recent papers addressed the possible non-linear effects of the basis on the spot and futures prices. Balke and Fomby (1997) considered a threshold-cointegration model in which the spot and futures prices follow different regimes depending upon whether the basis exceeds or falls below a threshold level. While cointegration is driven by the cost-of-carry considerations, the existence of transaction costs (in any form) validates the prevalence of threshold cointegration. Indeed, Dwyer, Locke and Yu (1996) found that the basis of the S&P 500 can be well described by a threshold autoregressive model. Gao and Wang (1999) established a similar result for the S&P 500 futures price. While the literature on threshold cointegration has grown in the last few years, the implications of threshold cointegration on the minimum-variance hedge ratio and on the hedging performance have not been investigated. Based upon the transaction-cost interpretation, we expect spot and futures prices to lie within the threshold most of the time. Any data points lying outside the threshold reflect temporary imbalance and should be short lived. Thus, threshold cointegration is only useful for high-frequency data. On the other hand, many authors found threshold in daily data. It is difficult to apply transaction-cost arguments to validate the results. One possible interpretation is that, the model simply captures parts of the underlying non-linearity contained in the data.

Broll, Chow and Wong (2000) introduced a quadratic futures term into the cointegration equation and derived the optimal hedge strategy therefrom. The results were applied to six currency markets: Australian dollar, British pound, Canadian dollar, Deutsche mark, French franc and Japanese yen. They found empirical support in every market except the Australian dollar.

3 Time-Varying Hedge Ratios

The above section assumes that the minimum-variance hedge ratio is constant over time. Bera, Garcia and Roh (1997) considered the hedge ratio to be time-varying and, more specifically, following a random walk. They adopted a random coefficient (RC) regression model in which

$$\Delta p_t = \alpha + \beta_t \Delta f_t + \varepsilon_t, \tag{22}$$

$$\beta_t = \beta_{t-1} + u_t, \tag{23}$$

where $\{\varepsilon_t, u_t\}$ is a bivariate white noise. However, they found that the hedge strategy obtained from this method failed to improve the hedging performance. As the minimum-variance hedge ratio depends on the conditional moments of the spot and futures returns, when the conditional moments vary over time so does the hedge ratio. Recent empirical works strongly support that time-varying volatility prevails in many economic and financial time series. While deterministic volatility functions are sometimes considered (see

Dumas, Fleming and Whaley, 1998), most researchers adopt the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) framework. Specifically, the bivariate GARCH models are widely adopted to examine the behavior of the spot and futures prices and the dynamic hedge strategy (Baillie and Myers (1991); Myers (1991); Lien and Luo (1994); Pirrong (1997)).

3.1 The GARCH Framework

Consider the error-correction model described by equations (15) and (16). Let σ^2_{pt} denote $\mathrm{Var}(\varepsilon_{pt})$, σ^2_{ft} denote $\mathrm{Var}(\varepsilon_{ft})$ and σ_{pft} denote $\mathrm{Cov}(\varepsilon_{pt}, \varepsilon_{ft})$. The general VECH-GARCH model suggested by Bollerslev, Engle and Wooldridge (1988) consists of the following conditional second-moment equation:[6]

$$\begin{bmatrix} \sigma^2_{pt} \\ \sigma_{pft} \\ \sigma^2_{ft} \end{bmatrix} = A + \sum_{j=1}^{m} B_j \begin{bmatrix} \sigma^2_{p,t-j} \\ \sigma_{pf,t-j} \\ \sigma^2_{f,t-j} \end{bmatrix} + \sum_{k=1}^{n} C_k \begin{bmatrix} \varepsilon^2_{p,t-k} \\ \varepsilon_{p,t-k}\varepsilon_{f,t-k} \\ \varepsilon^2_{f,t-k} \end{bmatrix}, \tag{24}$$

where A is a 3×1 column vector, and B_j and C_k are 3×3 matrices. The model contains $3 + 9m + 9n$ parameters and renders some estimation problems. To achieve parsimony, a diagonal version is sometimes considered. This model assumes that all B_j and C_k matrices are diagonal. In other words, the conditional variance of the spot returns are affected by its own history and the history of the squared innovations in the spot returns. Similar structures apply to the conditional variance of the futures returns and the conditional covariance between the spot and futures returns. While very general, the VECH-GARCH model (or the diagonal version) fails to ensure the conditional variance-covariance matrix of the spot and futures returns to be positive semi-definite. Indeed, Lien and Luo (1994) applied the model to the foreign currency markets and found in their empirical estimation that the positive semi-definiteness condition failed frequently. To overcome this difficulty, Engle and Kroner (1995) suggested the so-called BEKK specification (named after Baba, Engle, Kraft and Kroner). Bera, Garcia and Roh (1997), however, showed that this specification produced the worst hedging performance when compared to the OLS and the RC hedge ratios.

The constant-correlation GARCH (CC-GARCH) model suggested by Bollerslev (1990) is an alternative specification to resolve the problem of a possible non-positive semi-definite conditional variance-covariance matrix. In this model the conditional second-moment equations are specified as follows:

$$\sigma^2_{pt} = \gamma_p + \sum_{j=1}^{m} \alpha_{pj}\sigma^2_{p,t-j} + \sum_{k=1}^{n} \beta_{pk}\varepsilon^2_{p,t-k}; \tag{25}$$

$$\sigma^2_{ft} = \gamma_f + \sum_{j=1}^{m'} \alpha_{fj}\sigma^2_{f,t-j} + \sum_{k=1}^{n'} \beta_{fk}\varepsilon^2_{f,t-k}; \tag{26}$$

[6]Note that the GARCH models assume that the conditional variance and covariance depend only on the past conditional second moments and residuals. Extraneous information is ignored. The conditional moments can be specified more generally using transfer function models as in Chiang and Chiang (1996).

$$\sigma_{pft} = \rho\sigma_{pt}\sigma_{ft}. \tag{27}$$

Note that equations (25) and (26) maintain the assumptions of a diagonal GARCH model, while equation (27) states that the conditional correlation coefficient between the spot and futures returns, ρ, is time invariant. Thus, a CC-GARCH model imposes restrictions on the general vector GARCH (VGARCH) model to achieve parameter parsimony while maintaining the positive semi-definiteness property. Kroner and Sultan (1993) applied this model to obtain the minimum-variance hedge ratios of currency futures. Park and Switzer (1995) adopted it to estimate the minimum-variance hedge ratios of stock index futures.

Empirical results concerning the performance of GARCH hedge ratios are generally mixed. Within-sample comparisons show that, in some cases, dynamic hedging generates much better performance in terms of risk reduction but in others the benefits seem too minimal to warrant the efforts. Post-sample comparisons are mostly in favor of the conventional hedge strategy. However, these studies usually do not revise the second-moment forecasting equations (as new data arrived). Lien, Tse, and Tsui (1999) applied the CC-GARCH models to a variety of commodity, financial, and currency markets. They updated the forecasting models as new data arrived to construct the dynamic hedge ratios. The results indicate that conventional hedge strategies perform as well as or better than the GARCH strategies.

3.2 Stochastic Volatility Analysis

Stochastic volatility (SV) model is an alternative specification to capture time-varying second moments. The model is akin to the mixture-distribution hypothesis and is congruent with the concept that price changes are driven by information arrival (Andersen, 1996). Andersen and Sorensen (1996) provided rationales for an alternative specification to the GARCH models to be sought after. Based upon Bayesian model selection criteria, Geweke (1994) chose a stochastic volatility model over a GARCH specification for the dollar/pound exchange rates. Heynen and Kat (1994) found that the stochastic volatility model has better forecasting performance for the stock indices than either the GARCH or the exponential GARCH (EGARCH) models. For the foreign exchange markets, the EGARCH model outperforms the stochastic volatility model marginally. Andersen and Bollerslev (1998) suggested that both models should perform well for high frequency data. Andersen (1994) provided a general framework that incorporates both models as special cases.

To obtain the hedge ratio, a multivariate stochastic volatility model as suggested by Harvey, Ruiz, and Shephard (1994) is required. Danielsson (1998) reported an empirical study on the comparison between the multivariate stochastic volatility model and some VGARCH models. Uhlig (1997) provided a Bayesian analysis for the multivariate stochastic volatility models. Consider again the error-correction model of equations (15) and (16). The stochastic volatility model describes the variances of p_t and f_t and their covariance as follows:

$$\varepsilon_{it} = \exp(h_{it})u_{it}, \qquad i = p, f, \tag{28}$$

$$h_{it} = a_i + b_i h_{i,t-1} + v_{it}, \qquad i = p, f, \tag{29}$$

where both u_{pt} and u_{ft} are standard normal random variables and the correlation between them is ρ. In addition, $\{v_{pt}, v_{ft}\}$ is a bivariate normal vector, independent of $\{u_{pt}, u_{ft}\}$, such that $\mathrm{E}\{v_{it}\} = 0$ and $\mathrm{Cov}(v_{it}, v_{jt}) = \theta_{ij}$, for $i, j = p, f$. Equations (28) and (29) imply that the variance of ε_{it} and the covariance of ε_{pt} and ε_{ft} are all random variables. A general specification would allow h_{pt} (or h_{ft}) to be a function of $h_{p,t-1}$ and $h_{f,t-1}$. The simplifying assumption adopted in equation (29) is similar to that for the diagonal VECH-GARCH models. Indeed, the stochastic volatility model is similar to the EGARCH model, with the latter containing a conditional-variance equation of the following form:

$$h_{it} = a_i + b_i h_{i,t-1} + c_i \varepsilon_{i,t-1}^2, \qquad i = p, f. \tag{30}$$

That is, the random term in equation (29), v_{it}, is replaced by a lagged ε_{it}^2 term.

Lien (1999a) provided a theoretical analysis of the minimum-variance hedge strategy based upon the stochastic volatility model. He found that the mean of the stochastic-volatility minimum-variance hedge ratio tends to be larger than the conventional hedge ratio. Using the Ederington (1979) hedging effectiveness measure, he concluded that the stochastic volatility hedge strategy is most beneficial when the conditional variances of the spot and futures prices are highly volatile and nearly uncorrelated. Otherwise, the conventional hedge ratios will do fine even if the true data generation process follows a multivariate stochastic volatility model. To the best knowledge of the authors, to date there is no empirical result on stochastic volatility hedge strategy.

4 Implementation of the Minimum-Variance Hedge

In the empirical implementation of the minimum-variance hedge strategy the hedge ratio must be estimated based on some statistical models. As the hedge ratios are determined by the second moments of the spot and futures prices, we may conclude that different statistical models will give rise to different hedge ratios to the extent that they produce different estimated second moments. Obviously, the differences in the estimates of the second moments with respect to different hedge horizons have impacts on the estimated hedge ratios. In general, as the hedge horizon increases, the trading noises are smoothed out, resulting in a larger hedge ratio.

Lien and Tse (2000c) pointed out that under certain model specifications, the theoretical minimum-variance hedge ratios are stable under aggregation. That is, the same theoretical hedge ratio is applicable irrespective of the hedge horizon. In general, to estimate the optimal hedge ratio it is natural to use a statistical model in which the sampling interval coincides with the hedge horizon. However, if the stable-under-aggregation property holds, it may be desirable to use a shorter sampling interval for a more effective use of the sample data.[7] Thus, the choice of the sampling interval depends upon the tradeoff between accepting possibly some model specification errors (when the hedge ratios are in fact not stable under aggregation) versus improving the estimation efficiency through more effective use of the sample data. It is possible that, in terms of out-of-sample

[7]Lien and Tse (2000c) pointed out that the stable-under-aggregation property holds for both the regression and the vector autoregression models. In contrast, the error-correction model and the GARCH models do not satisfy the stability property.

hedging effectiveness, the hedge ratio estimated from 1-day return data outperforms the hedge ratio estimated from 5-day return data, even when the hedge horizon is 5 days.

Lence and Hayes (1994a, 1994b) criticised the usual practice of substituting the estimated hedge ratio for the unknown optimal hedge ratio (Lence and Hayes called this the Parameter Certainty Equivalent (PCE) strategy) in implementing the hedge strategy. They argued that this decision rule ignores the estimation risk and is not commensurate with the expected-utility maximisation paradigm. In other words, when estimation risk exists, the sample estimate of the minimum-variance hedge ratio may not lead to the optimal decision. They advocated a decision rule based on Bayes' criterion. Thus, the optimal hedge strategy is selected by maximising the expected utiltiy based on the posterior probability density of the random hedged portfolio return.

Let $\mathbf{Y}$ denote the sample observations (historical futures and spot returns) based on which the parameter vector θ of the probability density function of the hedged portfolio return $f(r|\theta)$ is estimated. Denoting D as the feasible decision set, Bayes' optimal hedge ratio h_B is given by:[8]

$$h_B = \max_{h \in D} \ \mathrm{E}_\theta\{\mathrm{E}_{r|\theta}\{U(r)\}\}. \tag{31}$$

Applying Bayes' theorem to evaluate the expectation, Lence and Hayes (1994b) obtained the result:

$$h_B = \max_{h \in D} \int_{\mathbf{R}} U(r) f(r|\mathbf{Y})\, dr, \tag{32}$$

where $\mathbf{R}$ is the domain of r and $f(r|\mathbf{Y})$ is the predictive probability density function of r given $\mathbf{Y}$. To implement Bayes' strategy, the researcher has to specify the utility function of the hedger and the prior probability density function of θ. Assuming $U(r) = -\mathrm{Var}(r|\Phi)$, Lence and Hayes (1994a) derived an algorithm to compute h_B. In their numerical application to the soybean data, Lence and Hayes (1994a) showed that there are substantial differences between the the PCE and the Bayesian hedge ratios. They also reported that slight changes in the data may induce major impacts on the PCE hedge ratio.

A main attraction of the minimum-variance hedge strategy is that, under certain conditions, it is consistent with the expected-utility paradigm regardless of the utility function chosen. Such is the case if (i) the hedger is not allowed to borrow or lend, (ii) there are no transaction costs such as trading fees and margin accounts, and (iii) current futures prices are unbiased for future futures prices. While much has been done to improve the statistical estimation of the minimum-variance hedge ratio, not much attention has been paid to the impacts of the above restrictions on the performance of the hedge strategy. Lence (1995b) argued that the benefits of sophisticated estimation techniques of the hedge ratio is small. He advocated that hedgers may do better by focusing on simpler and more intuitive hedge models. His concerns appear to be supported by some empirical studies (see, for example, Lien, Tse and Tsui, 1999).

[8] Note that the hedged portfolio return is a function of the hedge ratio h.

5 Hedging in the Mean-Gini Framework

It is well-known that the mean-variance portfolio theory is based on the assumptions that either the asset returns are normally distributed or the utility functions of decision makers are quadratic.[9] While the first assumption has often been refuted empirically, the second assumption leads to the implausible conclusion that decision makers exhibit increasing absolute risk aversion. Based on the weak assumptions of nonsatiability and risk aversion, stochastic-dominance rules in investment decision making are not subject to these faults of the mean-variance theory. The mean-Gini approach as developed by Yitzhaki (1982, 1983) and Shalit and Yitzhaki (1984) provides a method of implementing investment decisions that are coherent with the stochastic-dominance theories. Although consistent with the principle of maximizing expected utility, the stochastic-dominance approach is difficult to implement in practice. The mean-Gini approach provides a method of constructing portfolios that are efficient in the stochastic-dominance framework.

5.1 Gini's Mean Difference and the Extended Gini Coefficients

Gini's mean difference measures the variability of a random variable. We shall discuss the concept of Gini's mean difference and describe its applications to futures hedging. Deviating slightly from the notation used in Sections 2 and 4, we denote R as the random return of a portfolio. We assume R falls inside the range $[a, b]$, such that $F(a) = 0$ and $F(b) = 1$, where $F(.)$ is the distribution function of R. Denoting the density function of R by $f(.)$, Gini's mean difference Γ is defined as:

$$\begin{aligned} \Gamma &= \frac{1}{2}\,\mathrm{E}\{|R_1 - R_2|\} \\ &= \frac{1}{2}\int_a^b \int_a^b |r_1 - r_2|\, f(r_1)\, f(r_2)\, dr_1 dr_2, \end{aligned} \tag{33}$$

where R_1 and R_2 are independent and have the same distribution as R. In practice, Γ can be evaluated using the following formula:

$$\Gamma = \int_a^b [1 - F(r)]\, dr - \int_a^b [1 - F(r)]^2\, dr. \tag{34}$$

In the special case when a is finite, the formula can be rewritten as:

$$\Gamma = \mu - a - \int_a^b [1 - F(r)]^2\, dr, \tag{35}$$

where $\mu = \mathrm{E}\{R\}$. Alternatively, Γ can be calculated as:

$$\Gamma = 2\int_a^b \left[F(r) - \frac{1}{2}\right] f(r)\, dr. \tag{36}$$

[9] The normality assumption may be relaxed to allow the returns to be ellyptically distributed. See Ingersoll (1987) for the details.

It is noted that the variance σ^2 of R can be defined as:

$$\begin{aligned} \Gamma &= \frac{1}{2}\,\mathrm{E}[(R_1 - R_2)^2] \\ &= \frac{1}{2}\int_a^b\int_a^b (r_1 - r_2)^2\, f(r_1)\, f(r_2)\, dr_1 dr_2, \end{aligned} \tag{37}$$

so that the similarity between Γ and σ^2 is obvious.

As a measure of income inequality, the Gini index is often defined as $\mathrm{E}\{|R_1 - R_2|\}/(2\,\mathrm{E}\{R\})$ (here R represents the wealth or income). The higher the Gini index is, the more uneven wealth is distributed. In equation (33) above, we follow the definition given by Shalit and Yitzhaki (1984), which has been adopted in the literature of options and futures hedging.

Shalit and Yitzhaki (1984) extended Gini's mean difference to a family of coefficients of variability differing from each other in a parameter denoted by ν, where $1 \leq \nu < \infty$. The extended Gini coefficients $\Gamma(\nu)$ are defined as:

$$\Gamma(\nu) = \int_a^b [1 - F(r)]\, dr - \int_a^b [1 - F(r)]^\nu\, dr, \tag{38}$$

which is reduced to:

$$\Gamma(\nu) = \mu - a - \int_a^b [1 - F(r)]^\nu\, dr \tag{39}$$

when a is finite. Shalit and Yitzhaki (1984) pointed out that $\Gamma(\nu)$ is nonnegative, bounded from above and nondecreasing in ν. Furthermore, it can be viewed as the risk premium that should be subtracted from the expected value of the portfolio. The case of $\nu = 1$ represents a risk-neutral investor as $\Gamma(1) = 0$. On the other hand, $\Gamma(\nu) \to \mu - a$ as $\nu \to \infty$. In this case the risk-adjusted return is the minimum value of the return distribution. Thus, $\Gamma(\nu)$ can be used as a representative measure of risk, assuming the role of the variance in the mean-variance analysis. The decision maker may vary the criterion of measuring the *risk* of a portfolio by changing the value of ν. A more risk-averse decision maker will choose a larger value of ν. Gini's mean difference is a special member of the family of extended Gini coefficients with $\nu = 2$, that is, $\Gamma = \Gamma(2)$. The approach of formulating a hedging strategy based on the extended Gini coefficients is called the extended mean-Gini (EMG) approach.

It is not easy to evaluate $\Gamma(\nu)$ using equation (33). However, Shalit and Yitzhaki (1984) proved the following alternative formula:

$$\Gamma(\nu) = -\nu\,\mathrm{Cov}(R, [1 - F(R)]^{\nu-1}), \tag{40}$$

which provides a convenient method of calculating the extended Gini coefficients. This formula has been adopted in the literature for the practical implementation of the extended mean-Gini hedging strategy. We now proceed to discuss how the mean-Gini framework can be applied to futures hedging.

5.2 Hedging Using the Extended Mean-Gini Approach

We consider two portfolios A and B, and let the distribution functions of the return of these portfolios be denoted by $F(.)$ and $G(.)$, respectively. A decision maker may use the decision rules based on the stochastic-dominance theories to construct the set of efficient portfolios. Thus, A remains in the efficient set until a portfolio can be found to dominate A stochastically. Now, consider λ_n defined as follows:

$$\lambda_n = \int_a^b [1 - F(r)]^n \, dr - \int_a^b [1 - G(r)]^n \, dr, \qquad n = 1, 2, 3... \tag{41}$$

The following result was proved by Yitzhaki (1982): The condition $\lambda_n \geq 0$ for $n = 1, 2, ...,$ is necessary for A to dominate B under the first-degree stochastic dominance (FSD) and the second-degree stochastic dominance (SSD).

The above result can be used to derive necessary conditions for stochastic dominance based on the extended Gini coefficients. Using integration by part, we have:

$$\begin{aligned} \lambda_1 &= \int_a^b [1 - F(r)] \, dr - \int_a^b [1 - G(r)] \, dr \\ &= \mu_A - \mu_B, \end{aligned} \tag{42}$$

where μ_A and μ_B denote the mean return of portfolios A and B, respectively. Denoting $\Gamma_i(\nu)$ as the extended Gini coefficients for $i = A, B$, we have:

$$\begin{aligned} \lambda_2 &= \int_a^b [1 - F(r)]^2 \, dr - \int_a^b [1 - G(r)]^2 \, dr \\ &= (\mu_A - \Gamma_A(2)) - (\mu_B - \Gamma_B(2)). \end{aligned} \tag{43}$$

Thus, the following are necessary conditions for A to FSD/SSD B:

$$\mu_A \geq \mu_B \tag{44}$$

and

$$\mu_A - \Gamma_A(2) \geq \mu_B - \Gamma_B(2). \tag{45}$$

Note that these conditions form only a subset of the conditions $\lambda_n \geq 0$. Additional conditions will require:

$$\mu_A - \Gamma_A(\nu) \geq \mu_B - \Gamma_B(\nu), \qquad \nu = 3, 4, ... \tag{46}$$

Because $\Gamma(1) = 0$, equation (46) represents a sequence of necessary conditions with $\nu = 1, 2, ...,$ for FSD and SSD. There are two notions as to how the extended Gini coefficients can be applied to select hedging strategies. First, the hedger can obtain an *efficient* set based on each value of ν. The efficient set is progressively reduced when the hedger performs the EMG analyses for different values of ν and retains only the intersection of the efficient sets. Second, $\Gamma(\nu)$ can be used as a representative measure of risk. It is in this context that ν is interpreted as the risk-aversion parameter. Conditional on ν, an optimal hedging strategy can be obtained by minimizing $\Gamma(\nu)$. This is analogous to the minimum-variance strategy obtained by minimizing the variance. We shall come back to these points when we discuss the empirical applications below.

5.3 Empirical Applications of the Extended Mean-Gini Approach

Cheung, Kwan and Yip (1990) presented the first empirical application of the mean-Gini approach to hedging. They considered hedging Japanese yen using futures and options. Portfolios consisting of spot and option as well as spot and futures were formed. Conditional on a desired portfolio return they calculated the required hedge ratios of the spot-option and spot-futures portfolios that would give rise to the preset return target. Subsequently, they compared the option and futures strategies based on the variance σ^2 and Gini's mean difference Γ of these portfolios. At each preset level of return, equation (43) is satisfied when the spot-option and spot-futures portfolios are compared. Also, the portfolio with a positive λ_2 has a smaller Γ. Thus, an efficient set that is consistent with FSD and SSD can be obtained by retaining the portfolio with the smaller Γ. On the other hand, the portfolio based on minimizing the variance need not be efficient under the stochastic dominance theory. The two risk criteria would be consistent with each other if the differences in the risk measures (whether σ^2 or Γ) between the hedged portfolios involving option and futures agree in sign. However, Cheung *et al* found that the mean-variance criterion favours the use of futures as a hedging instrument, whereas the mean-Gini approach supports the use of option.

Kolb and Okunev (1992) extended the work of Cheung *et al* and considered the hedging strategies based on the EMG approach. Instead of tracing the frontier of the extended Gini coefficients corresponding to various portfolio returns, Kolb and Okunev treated the extended Gini coefficients as measures of risk with different risk-aversion parameters and examined the hedging strategies that minimize these risks. To implement this procedure a sample estimate of $\Gamma(\nu)$ based on equation (40) was used. Denoting r_{pi} and r_{di} as the return of the spot and the derivative (option or futures), respectively, and r_i as the return of the portfolio consisting of the spot and derivative, we have:

$$r_i = r_{pi} - hr_{di}, \qquad i = 1, 2, ..., N, \tag{47}$$

where h is the hedge ratio and N is the sample size. Suppose $F(.)$ is estimated by the empirical distribution function $\hat{F}(.)$, then $\Gamma(\nu)$ can be estimated by:[10]

$$\hat{\Gamma}(\nu) = -\frac{\nu}{N}\left\{\sum_{i=1}^{N} r_i(1-\hat{F}(r_i))^{\nu-1} - \left(\sum_{i=1}^{N}\frac{r_i}{N}\right)\left(\sum_{i=1}^{N}(1-\hat{F}(r_i))^{\nu-1}\right)\right\}. \tag{48}$$

The optimal hedge ratio is obtained by minimizing $\hat{\Gamma}(\nu)$ with respect to h. As $\hat{\Gamma}(\nu)$ is a rather complicated function of h, evaluating the derivative of $\hat{\Gamma}(\nu)$ with respect to h is difficult. Kolb and Okunev (1992) adopted the search method for finding the minimum. This method was applied to five spot-futures series (one stock index, one currency and three commodities). They found that the optimal hedge ratios based on the EMG approach (the EMG hedge ratios) for low level of risk aversion (that is, ν from 2 to 5) are similar to the MV hedge ratios. For higher levels of risk aversion, however, the EMG hedge ratios generally differ from the MV hedge ratio, with no regularity in

[10] This presentation follows Lien and Luo (1993).

their relative size. They also found that the EMG hedge ratios follow a step function. Thus, investors with similar values of ν may still have significantly different optimal futures positions. Furthermore, changes in the hedge ratio sometimes lead to significant changes in the return of the hedged portfolio.

In addition to the static analysis in which the optimal hedge ratios are assumed to be constant, Kolb and Okunev (1992) considered the situation when investors vary their positions based on recent information. Using moving windows of 50 most recent trading days, they estimated the hedge ratios. It was found that the MV hedge ratios are quite stable through time. In contrast, for moderately risk-averse ($\nu = 5$) or highly risk-averse ($\nu = 50, 100$) investors, the EMG hedge ratios are very volatile. Thus, a strongly risk-averse investor who follows a hedge-and-forget strategy would run the risk of being significantly mishedged through the contract's life.

Lien and Luo (1993) argued that the Kolb-Okunev findings were due to an inherent numerical instability in their search method for the optimal hedge ratio. One defect of this approach is that the distribution function is estimated by the empirical distribution function, which is a step function. Consequently, the estimated extended Gini coefficient is not differentiable, rendering the conventional first-order condition inapplicable. As a remedy for this, the smooth kernel method was suggested. Lien and Luo (1993) found that generally the estimated Gini coefficient has two local minima. When one local minimum overtakes the other, the resulting shift in the hedge ratio is sharp. Their analyses also showed that the hedge ratio is a smooth and monotonic function of the risk-aversion parameter. Indeed, the monotonicity of the hedge ratio could be found in Kolb and Okunev (1992), although this point had not been explicitly pointed out.

While Kolb and Okunev (1992) examined the use of the strategy of globally minimizing the extended Gini coefficients, it would be interesting to see how this strategy compares against one of maximizing the expected utility. Using the extended Gini coefficients as a measure of risk, Kolb and Okunev (1993) defined a class of utility functions in terms of the return and the extended Gini coefficients.[11] The hedging strategies corresponding to utility maximization and risk minimization were then compared. Specifically, the expected utility $\mathrm{E}\{U[R]\}$ is assumed to be given by:

$$\mathrm{E}\{U[R]\} = \mu - \Gamma(\nu). \tag{49}$$

Expected utility is maximized at the point when

$$\frac{\partial\,\mathrm{E}\{U[R]\}}{\partial\,h} = \frac{\partial\,\mu}{\partial\,h} - \frac{\partial\,\Gamma(\nu)}{\partial\,h} = 0, \tag{50}$$

which implies

$$\frac{\partial\,\mu}{\partial\,\Gamma(\nu)} = 1. \tag{51}$$

Thus, the utility-maximizing hedge ratio corresponds to the point where the derivative in the mean-Gini coefficient space is 1.

[11]Kolb and Okunev (1993) considered the utility function as a function of wealth rather than return. For a given spot position the assumptions based on wealth and return are equivalent. To follow the notations used in this paper, we present our discussions in terms of returns.

Equation (46) can be compared to the quadratic utility function:

$$\mathrm{E}\{U[R]\} = \mu - m\sigma^2, \tag{52}$$

where m is the risk-aversion parameter. Hedge ratios that minimize the expected utility in the mean-variance space correspond to different points on the mean-variance efficient frontier. The utility-maximizing portfolio moves along the mean-variance efficient frontier as m varies. The optimal hedged portfolio moves toward the global minimum-variance portfolio as m tends to infinity.

Kolb and Okunev (1993) examined the hedging strategies of four cocoa-producing countries. Various scenarios of risk aversion were considered, and both utility-maximization and risk-minimization strategies were examined. They concluded that for strongly risk-averse investors, risk-minimizing and utility-maximizing hedge ratios are generally similar. For weakly risk-averse investors, however, these hedge ratios differ significantly.

Shalit (1995) argued that the EMG hedge ratio based on risk minimization cannot be compared with the MV hedge ratio unless an explicit form of the EMG hedge ratio is available. He suggested an instrumental-variable method for an explicit formula of the estimate of the EMG hedge ratio. Suppose f_0 and f_1 denote the futures prices at time 0 (initialization of the hedge) and time 1 (close of the hedge), respectively. Similarly, let p_1 denote the spot price at time 1. The time-1 value W of the hedged portfolio (based on a unit spot position) is:

$$W = p_1 + h(f_0 - f_1). \tag{53}$$

Shalit (1995) showed that the optimal EMG hedge ratio h^* is given by:

$$h^* = \frac{\mathrm{Cov}(s_1, \, [1 - H(W)]^{\nu-1})}{\mathrm{Cov}(f_1, \, [1 - H(W)]^{\nu-1})}, \tag{54}$$

where $H(.)$ is the distribution function of W. To obtain an explicit estimate for h^*, Shalit assumed that the distribution function of f_1 is similar to that of W in the sense that the empirical rankings of the two variables are the same. Thus, in equation (50) $H(W)$ is replaced by the distribution function of f_1. This results in the following estimate of h^*:

$$\hat{h}^* = \frac{\sum_{i=1}^{N}(p_{1i} - \bar{p}_1)(y_i - \bar{y})}{\sum_{i=1}^{N}(f_{1i} - \bar{f}_1)(y_i - \bar{y})}, \tag{55}$$

where p_{1i} and f_{1i} denote the sample values of p_1 and f_1, respectively, for $i = 1, ..., N$ and

$$y_i = [1 - \hat{H}(f_{1i})]^{\nu-1}, \tag{56}$$

with $\hat{H}(.)$ being the empirical distribution function of f_1.

In addition to providing an analytic formula for the estimate of the optimal EMG hedge ratio, Shalit (1995) suggested a method for testing the statistical significance of the difference between the EMG and the MV hedge ratios using Hausman's specification test. The validity of Shalit's approach, however, critically depends on the assumption that at the close of the hedge the rankings of the futures prices are the same as the rankings of the hedged portfolio values. Lien and Shaffer (1999) pointed out that a necessary condition for this assumption to hold is that the hedge ratio is less than one. Indeed, their empirical study showed that Shalit's hedge ratios performed inconsistently, both economically and statistically, as estimates of the true EMG hedge ratios. Following Lien and Luo (1993), Lien and Shaffer (1999) adopted a smooth kernel estimate to calculate the extended Gini coefficient. They examined six stock indexes and concluded that Shalit's instrumental-variable method does not provide a reliable estimate of the EMG hedge ratio.

6 The Lower-Partial-Moment Approach

Under the mean-variance asset pricing framework risk is measured in a two-sided notion. The survey by Adams and Montesi (1995), however, suggested that corporate managers are mostly concerned with one-sided risk, in which case only shortfall of a target is regarded as *risk*.[12] The notion that people treat gains and losses differently is commensurate with the prospect theory proposed by Kahneman and Tversky (1979) and Tversky and Kahneman (1992) in which utility is defined over gains and losses rather than levels of wealth. Benartzi and Thaler (1995) argued that, based on the psychology of decision making, individuals are more sensitive to reductions in their levels of wealth than to increases. They proposed the myopic loss aversion theory as an explanation for the equity premium puzzle pointed out by Mehra and Prescott (1985). Thus, the behaviour of loss aversion plays a central role in investment decisions.

Under the notion of loss aversion, risk may be more appropriately measured asymmetrically. Lien (2000b) analyzed the effect of loss aversion on optimal futures hedging. Previously, Kang, Brorsen, and Adam (1996) suggested a similar concept to evaluate futures hedging programs. Bawa (1975, 1978) proposed the lower partial moment as a measure of risk. Bawa and Lindenberg (1977) developed a capital asset pricing model using a mean-lower partial moment framework and showed that many of the results in the mean-variance framework can be carried over. Fishburn (1977) developed a mean-risk analysis independently, where risk is only associated with a below-target return. He proposed the α-t model and showed how different values of α are related to the efficient sets resulting from the first-, second- and third-order stochastic dominance.[13]

6.1 The Lower Partial Moment

Consider an individual with a given portfolio that generates a random return R. If $F(.)$ denotes the distribution function of R, the nth order lower partial moment of R (n is a nonnegative integer) is defined as follows:

$$\mathrm{LPM}_n(c; F) = \int_{-\infty}^{c} (c - r)^n \, dF(r), \tag{57}$$

[12] An earlier survey by Petty and Scott (1981) also found that many managers identified risk as returns falling below a certain target.

[13] In Fishburn's model, α corresponds to n (the order) and t corresponds to c (the target) of the lower partial moment.

where c is the target rate of return.[14] Thus, only returns of an amount lower than c contribute to the integral, which is regarded as the relevant risk measure. Note that this measure is the same as Fishburn's risk measure, which, however, allows n to be a non-integer positive number. The parameter n is supposed to reflect the decision maker's assessment about the consequences of falling short of the target c. If the size of the shortfall is of no serious concern, a small value of n is appropriate. On the other hand, if large shortfalls are of serious concern, a large value of n is applicable. The following results, given by Bawa (1978), relate the lower partial moment criteria to the stochastic-dominance theories:

1. F is preferred to G for all strictly increasing utility functions if and only if $\mathrm{LPM}_0(c; F) \leq \mathrm{LPM}_0(c; G)$ for all c, with strict inequality for some c.[15]

2. F is preferred to G for all strictly increasing concave utility functions if and only if $\mathrm{LPM}_1(c; F) \leq \mathrm{LPM}_1(c; G)$ for all c, with strict inequality for some c.

3. F is preferred to G for all strictly increasing concave utility functions with positive third derivatives if and only if $\mu_F \geq \mu_G$ and $\mathrm{LPM}_2(c; F) \leq \mathrm{LPM}_2(c; G)$ for all c, with strict inequality for some c, where μ_F and μ_G are the means of the distributions F and G, respectively.

Note that when $n = 0$, the LPM is equal to the probability of shortfall. For $n = 1$, the LPM is the expected shortfall. Bigman (1996) considered the criterion $\mathrm{LPM}_1(c)$.[16] Setting $c = 0$ and $n = 2$, we obtain $\mathrm{LPM}_2(0)$, which is the semivariance.

6.2 The Optimal Lower-Partial-Moment Hedge Ratio

Bawa's (1978) result states that for stochastic dominance to occur there has to be a consistent relationship between the lower partial moments of two return distributions for an infinite set of target returns. The existence of a dominant strategy is, of course, not guaranteed. Given a safety-first target c, however, a hedging strategy can be formulated by minimizing the lower partial moment. This strategy is commensurate with the one-sided risk measure.[17] We shall call this strategy the minimum LPM strategy (conditional on a given c) and the corresponding hedge ratio the minimum LPM hedge ratio.

Consider a hedger with wealth W_0 and a given nontradable spot position Q at time 0. To reduce the risk exposure, the hedger establishes hQ futures positions. Let Δp and Δf denote the changes in the spot and futures prices from time 0 to time 1, respectively, The end-of-period wealth is then given by:

$$\begin{aligned} W_1 &= W_0 + (\Delta p - h\Delta f)Q \\ &= W_0 + (r_p + \theta r_f)p_0 Q, \end{aligned} \tag{58}$$

[14] Note that c may be regarded as a desired or critical Roy (1952) safety-first target rate of return.

[15] We say "F is preferred to G" to mean that the portfolio with return distribution given by F is preferred to the one with return distribution given by G.

[16] We shall drop the argument representing the distribution function when it is generic.

[17] The existence of a dominant strategy can then be checked over a wide range of target returns.

where p_0 and f_0 are, respectively, the spot and futures prices at time 0, and $r_p = \Delta p/p_0$ and $r_f = \Delta f/f_0$ are, respectively, the returns from the spot and futures positions. Here $-\theta = hf_0/p_0$ is the *adjusted* hedge ratio.[18] We shall denote $r = r_p + \theta r_f$, which is the rate of return of the hedged portfolio.

The minimum LPM hedge ratio minimizes the nth order LPM of the portfolio return. When $n \geq 1$, we may rewrite $\mathrm{LPM}_n(c)$ as $\mathrm{E}\{(\max\{0, c - r\})^n\}$.[19] We denote the optimal hedge ratio under the LPM criterion, which is a function of c and n, to be $-\theta^* = -\theta^*(c, n)$. Thus, θ^* satisfies the following first order condition:

$$-n\mathrm{E}\{(\max\{0, c - r_p - \theta^* r_f\})^{n-1} r_f\} = 0, \tag{59}$$

while the second order condition that θ^* maximizes $\mathrm{LPM}_n(c)$ can be easily shown to be always satisfied. To examine the variation of θ^* with respect to n and c under the LPM criterion we revert to some comparative static analysis. From equation (55), we obtain:

$$\frac{d\theta^*}{dn} = \frac{\mathrm{E}\{(\max\{0, c - r_p - \theta^* r_f\})^{n-1} \log(\max\{0, c - r_p - \theta^* r_f\} r_f)\}}{\mathrm{E}\{(\max\{0, c - r_p - \theta^* r_f\})^{n-2} r_f^2\}}. \tag{60}$$

While the denominator is positive, the sign of the numerator is undetermined. This shows that when the order of the LPM increases, the optimal hedge ratio may increase or decrease.

To evaluate the effect of the target return on the optimal hedge ratio, we consider the following derivative:

$$\frac{d\theta^*}{dc} = \frac{\mathrm{E}\{(\max\{0, c - r_p - \theta^* r_f\})^{n-2} r_f\}}{\mathrm{E}\{(\max\{0, c - r_p - \theta^* r_f\})^{n-2} r_f^2\}}. \tag{61}$$

As the sign of this derivative is again ambiguous, the optimal hedge ratio may increase or decrease with respect to the target return c.

It would be interesting to compare the optimal hedge ratio under the criteria of minimum variance and minimum LPM. To reduce the scope of comparison we consider the semivariance in the latter family. Thus, we assume $n = 2$ and $c = 0$. From equation (58), the first-order condition for θ^* is:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{-\theta^* r_f} (r_p + \theta^* r_f) r_f \, dF(r_p, r_f) = 0, \tag{62}$$

where, as stated before, the second order condition:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{-\theta^* r_f} r_f^2 \, dF(r_p, r_f) > 0 \tag{63}$$

is always satisfied.

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[18] We shall use the term hedge ratio for both $-\theta$ and h. Whether it refers to the adjusted or unadjusted ratio should be clear from the context.

[19] Strictly speaking, the LPM requires the parameter n to be an integer. Here we adopt an approach closer to Fishburn's measure and allow n to be any positive number.

We denote the minimum-variance hedge ratio by $\tilde{\theta}$ and substitute θ^* in the left hand side of equation (62) by $\tilde{\theta}$. If this results in a negative number, then $\tilde{\theta} < \theta^*$. Otherwise, $\tilde{\theta} > \theta^*$. In other words, the sign of $\tilde{\theta} - \theta^*$ is the same as that of D defined as:

$$\begin{aligned} D &= \int_{-\infty}^{\infty}\int_{-\infty}^{-\tilde{\theta} r_f} r_p r_f \, dF(r_p, r_f) + \int_{-\infty}^{\infty}\int_{-\infty}^{-\tilde{\theta} r_f} \tilde{\theta}\, r_f^2 \, dF(r_p, r_f) \\ &= \mathrm{E}\{(r_p + \tilde{\theta} r_f) r_f \mid r_p + \tilde{\theta} r_f \leq 0\} \\ &= \mathrm{E}\{\tilde{r}\, r_f \mid \tilde{r} \leq 0\}, \end{aligned} \tag{64}$$

where $\tilde{r} = r_p + \tilde{\theta} r_f$ is the portfolio return for the minimum-variance hedged portfolio. A further decomposition of the above leads to:

$$D = \mathrm{Cov}(\tilde{r},\, r_f \mid \tilde{r} \leq 0) + \mathrm{E}\{\tilde{r} \mid \tilde{r} \leq 0\}\mathrm{E}\{r_f \mid \tilde{r} \leq 0\}. \tag{65}$$

Now suppose both the spot and futures markets are unbiased, that is, $\mathrm{E}\{r_p\} = \mathrm{E}\{r_f\} = 0$. If r_f is mean independent of $\tilde{r}$, it can be easily shown that $D = 0$, so that $\tilde{\theta} = \theta^*$.[20] Thus, when the portfolio return provides no information about the mean of the futures return, the optimal hedging strategy that minimizes the downside risk coincides with that which minimizes the variance.

A special case of mean independence between the portfolio and futures returns under the assumption of unbiased spot and futures markets is the case when r_p and r_f are jointly symmetrically distributed. That is, the joint density function $f(r_p, r_f)$ of r_p and r_f satisfies $f(r_p, r_f) = f(-r_p, r_f) = f(r_p, -r_f) = f(-r_p, -r_f)$. In this case, $\tilde{r}$ and r_f are also symmetrically distributed, implying mean independence. Note that the mean independence condition is stronger than the condition of no correlation, but weaker than the condition of stochastic independence. It follows that if the portfolio return and the futures return are stochastically independent, the minimum-variance hedge ratio is the same as the minimum-semivariance hedge ratio.

6.3 Nonparametric Time-Invariant Hedge Ratio

Unlike the mean-variance criterion, there is no explicit analytic solution for the hedge ratio that minimizes the LPM. Lien and Tse (2000a) proposed a nonparametric method for the calculation of the optimal LPM hedge ratio. This method is summarized below. For a given θ, we calculate the sample of portfolio returns denoted by $r_1, r_2, ..., r_N$, where $r_i = r_{pi} + \theta r_{fi}$. The density function of r is then estimated by the kernel method given by:

$$\hat{f}(r) = \frac{1}{Nb}\sum_{i=1}^{N} g\left(\frac{r - r_i}{b}\right), \tag{66}$$

where b (> 0) is the bandwidth and $g(.)$ is the kernel function.[21] Substituting $\hat{f}(.)$ for the unknown density function in equation (53) we obtain an estimate of $\mathrm{LPM}_n(c)$. Thus,

[20] r_f is said to be mean independent of $\tilde{r}$ if $\mathrm{E}\{r_f|\tilde{r}\} = \mathrm{E}\{r_f\}$. Chew and Herk (1990) showed that if r_f is mean independent of $\tilde{r}$, $\mathrm{E}\{q(\tilde{r})r_f\} = \mathrm{E}\{q(\tilde{r})\}\mathrm{E}\{r_f\}$ for any measurable function $q(\tilde{r})$. The fact that $D = 0$ can be proved by defining $q(\tilde{r}) = \tilde{r}$ when $\tilde{r} \leq 0$, and zero otherwise.

[21] A commonly used kernel function is the density function of the standard normal variate $\phi(x) = (1/\sqrt{2\pi})\exp(-x^2/2)$.

$\text{LPM}_n(c)$ can be estimated by:

$$\hat{\ell}_n(c) = \int_{-\infty}^{c} (c-r)^n \left(\frac{1}{Nb}\right) \sum_{i=1}^{N} g(\frac{r-r_i}{b})\, dr, \tag{67}$$

which, after a change of variable, can be written as:

$$\hat{\ell}_n(c) = \frac{1}{N} \sum_{i=1}^{N} Q_n(c,\, r_i), \tag{68}$$

with

$$Q_n(c,\, r_i) = \int_{-\infty}^{(c-r_i)/h} (c - bz - r_i)^n g(z)\, dz. \tag{69}$$

The evaluation of Q_n can be facilitated using recursive formulas.[22] To estimate the optimal band width Lien and Tse (2000a) followed the data-driven cross validation method as given in Silverman (1986).

Apart from the kernel estimation method, Lien and Tse (2000a) also considered the empirical distribution function method. Using the empirical distribution function method, the LPM is estimated by:

$$\tilde{\ell}_n(c) = \sum_{r_i<c} \frac{1}{N}(c - r_i)^n. \tag{70}$$

It should be noted that when the bandwidth b is very small, the kernel estimation method is similar to the empirical distribution function method. Thus, the kernel method encompasses the empirical distribution function method as a special case.

The above methods provide algorithms for the calculation of the LPM. The optimal minimum-LPM hedge ratios can be estimated by minimizing $\hat{\ell}_n(c)$ (or $\tilde{\ell}_n(c)$) as a function of θ. Lien and Tse (2000a) reported some results on comparing the minimum-LPM hedge strategy versus the minimum-variance hedge strategy using the Nikkei Stock Average futures traded on the Singapore International Monetary Exchange (SIMEX). The data set consists of weekly prices from January 1988 through August 1996. Lien and Tse (2000a) considered $n = 1$, 2 and 3, and c ranging from -1.5% to 1.5%. The following results were found. First, when $n = 3$, the minimum-variance strategy overhedges against the minimum-LPM strategy. Second, for $n = 1$ or 2, both overhedging or underhedging are possible. However, for $c = 0$, the optimal hedge ratios under the two strategies are about the same. This result agrees with the analysis above as the spot and futures returns were found to be approximately symmetrically distributed. Third, the minimum-variance hedge strategy incurs significantly more downside risk than the minimum-LPM strategy when the target return c is small and the order of the LPM n is large. Overall, a hedger who is willing to absorb small losses but otherwise extremely cautious about large losses, the optimal hedge strategy that minimizes the LPM may be sharply different from the minimum-variance hedge strategy. Thus, if a hedger cares for the downside risk only, the conventional minimum-variance hedge strategy is inappropriate.

[22] See Patel and Read (1982) for the details.

6.4 Parametric Time-Varying Hedge ratio

The nonparametric estimation methods for the LPM assume that the returns are independently and identically distributed across time. This assumption results in a constant optimal hedge ratio. As asset prices are often found to have time-varying volatility, the optimal hedge ratio ought to be time-varying as well. under the time-varying framework, nonparametric estimation appears to be infeasible. Lien and Tse (1998) examined the consequences of time-varying conditional heteroscedasticity on the optimal hedge ratios. For analytical tractability they assumed that the conditional distribution of the spot and futures returns follow a bivariate normal distribution. Conditional heteroscedasticity models were then fitted to historical data. The time-varying minimum-LPM hedge ratios were then estimated from the parameters of the fitted model.

Suppose r_p and r_f follow a bivariate normal distribution with $\mathrm{E}\{r_i\} = \mu_i$ for $i = p,\, f$, and with variance and covariance given by $\mathrm{Cov}(r_i,\, r_j) = \sigma_{ij}$ for $i,\, j = p,\, f$. The first-order condition for θ^* to be the minimum-LPM hedge ratio is:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{c-\theta^* r_f} (c - r_p - \theta^* r_f)^{n-1} r_f \; dF(r_p,\, r_f) = 0. \tag{71}$$

Let $\mu = (\mu_p + \theta^* \mu_f - c)\gamma$ where $\gamma = (\theta^{*2}\sigma_{ff} + 2\theta^* \sigma_{pf} + \sigma_{pp})^{-2}$, Lien and Tse (1998) showed that the condition above can be written as:

$$\begin{aligned} & \int_{\mu}^{\infty} \gamma^{-1}(w-\mu)^n (\theta^* \sigma_{ff} + \sigma_{pf}) \exp(-\frac{w^2}{2}) dw \qquad (72) \\ = & \int_{\mu}^{\infty} (w-\mu)^{n-1} [(c-\mu_p)(\theta^* \sigma_{ff} + \sigma_{pf}) + \mu_f (\theta^* \sigma_{pf} + \sigma_{pp})] \exp(-\frac{w^2}{2})\, dw. \end{aligned}$$

Thus, when $\mu_f = 0$, the minimum-LPM hedge ratio is given by $-\theta^* = \sigma_{pf}/\sigma_{ff}$, which is the minimum-variance hedge ratio.

To compute the LPM, Lien and Tse (1998) showed that the following equation can be established:

$$\mathrm{LPM}_n(c,\, n) = \int_{\mu}^{\infty} \gamma^{-n} (w-\mu)^n \phi(w)\, dw. \tag{73}$$

This equation involves the evaluation of:

$$I_n(\mu) = \int_{\mu}^{\infty} w^n \phi(w)\, dw, \tag{74}$$

which can be calculated using the recursive formulas given in Patel and Read (1982).

Lien and Tse (1998) fitted a bivariate conditional heteroscedasticity model to the Nikkei Stock Average futures. The model is one with conditional correlation, but the individual conditional-variance equations are postulated to follow the asymmetric power GARCH (APARCH) model suggested by Ding, Granger and Engle (1993). This model allows for the leverage effect commonly found in the equity market.[23] Furthermore, the conditional variance is allowed to enter into the conditional-mean equation so that it

[23] The leverage effect refers to the empirical finding that negative shock causes a bigger change in future volatilities compared to a positive shock of the same magnitude.

forms a system with APARCH in mean. It was found that the conditional mean of the futures return μ_f is nonzero, which implies the minimum-LPM hedge ratio differs from that of the minimum-variance hedge ratio.

Lien and Tse (1998) examined the cases of $n = 1$, 2, 3 and 4, with c varying from -1.5% to 1.5%. When $c = -1.5\%$ or -1.0%, the optimal LPM hedge ratios are on average smaller than the minimum-variance hedge ratios, regardless of the order n. Also, the difference is larger for larger values of n. For larger values of c, the LPM hedge ratios are on average larger than the minimum-variance hedge ratios, and the gap shrinks as n increases. Thus, the largest difference between the LPM and the MV hedge strategies occurs when the LPM order n is large and the target return c is negative with a large absolute value.

6.5 Options versus Futures

Because options (call or put) can eliminate downside risk associated with certain (in particular, negative) target returns, it is often argued that option would be a better derivative to use as a hedging instrument compared to futures. This conjecture was, however, rejected by Ahmadi, Sharp and Walther (1986). Using data from the Philadelphia Stock Exchange, Ahmadi *et al* showed that for a nontradable spot position, futures provide significantly more effective hedging than options for the British pound, the Deutschemark and the Japanese yen when the target return is zero. Based upon simulations from hypothetical distributions, Korsvold (1994) demonstrated that the incorporation of the quantity risk, that is, the uncertainty in the spot position, leads to the dominance of options over futures. This result is, however, not convincing as it is based upon hypothetical distributions and simulated data. On the other hand, Adams and Montesi (1995) found that corporate managers prefer to hedge the downside risk using futures rather than options, citing the large transaction costs in options trading as the main reason.

An alternative argument that favours options hedging stems from the case of contingent exposures. Ware and Winter (1988), however, challenged this conventional wisdom. Moreover, within an analytical framework, Steil (1993) rejected this argument. Both papers concluded that options play no role in the hedging of transaction risk exposures, although Ware and Winter (1988) supported the role of options in hedging economic risk exposures. Recently, Battermann, Braulke, Broll and Schimmelpfennig (2000) showed that, based on expected-utility maximization within the production and hedging framework, futures is a better instrument than option. Given a general utility function, Bowden (1994) compared option spreads (combining calls and puts) to futures and found that there exist parameter ranges such that one dominates the other. After correcting a mistake in Bowden's analysis, Lien (1997) showed that option spreads always outperform futures.

Empirical results are mostly in favour of futures. Based upon the mean-variance type of criteria, Chang and Shanker (1986) concluded that currency futures are better hedging instruments than currency options. Hancock and Weise (1994) showed that the optimal hedge positions for the S&P 500 index options (and spreads) and that for the S&P 500 futures lead to similar mean returns. They argued that other factors are responsible for the choice of the instruments. Benet and Luft (1995) extended the earlier work of Chang and Shanker (1986) to stock index instruments. They showed that the S&P 500 futures outperform the S&P 500 options in variance reduction. Moreover, without taking into account the transaction costs, options lead to a larger excess return per unit risk than futures. Transaction costs reverse the above conclusion, however. With the exceptions of Ahmadi, Sharp and Walther (1986) and Korsvold (1994), the comparisons are always based upon mean and variance, which requires restrictive assumptions.

We now consider how the lower-partial-moment criterion may be applied to option hedging. Suppose a hedger purchases qQ ($q \geq 0$) units of put options, the strike price of which is K, at a premium d. The end-of-period wealth becomes:

$$W_1 = W_0 + \{\Delta p + q\,[\max(K - p_1,\, 0)] - qd\}Q. \tag{75}$$

The market profit π is given by $\Delta p + q\,[\max(K - p_1, 0)] - qd$. If a unit hedge ratio is adopted (that is, $q = 1$), then $\pi \geq K - p_0 - d$. There will be no downside risk if the target "profit" is below the lower bound. Otherwise, downside risk exists. Because $K - p_0 - d$ is usually a negative number, downside risk occurs when the target return is set to zero. For any other hedge ratio, π remains a function of p_1 and, therefore, downside risk exists. To determine the optimal hedge ratio (when the target profit is larger than $K - p_0 - d$), the methods discussed above can be applied.

A similar analysis applies to call options. Suppose the hedger sells q^*Q ($q^* \geq 0$) units of call options with strike price K at a premium d^*. The end-of-period wealth is then given by:

$$W_1 = W_0 + \{\Delta p + q^*\,[\min(K - p_1,\, 0)] + q^* d^*\}Q. \tag{76}$$

Let π_c denote the market profit $\Delta p + q^*[\,\min(K - p_1,\, 0)] + q^* c$. Then $\pi_c \leq K - p_0 + dq^*$. Clearly call options do not eliminate downside risk for the hedger.

The above analysis assumes the options are held until expiry. If a call option is held over a fixed hedging period, the end-of-period wealth is:

$$W_1 = W_0 + (r_p + \theta\, r_c)\, p_0 Q, \tag{77}$$

where r_c is the return of the call option and $-\theta = -q^* d^*/p_0$. The analysis is similar to the case of futures hedging .

Lien and Tse (2000b) examined the hedging effectiveness of the futures and options for three major currencies, namely, the British pound, the Deutschemark and the Japanese yen. They considered a hedging horizon of one week. Two estimation methods were applied to estimate the optimal LPM hedge ratio: the empirical distribution function method and the kernel density estimation method. They considered various values of target returns c and degree of risk aversion n. The results showed that the currency futures are almost always a better hedging instrument than the currency options. The only situation in which options outperform futures occurs when the hedger is optimistic (c is large) and not too concerned about large losses (n is small).

7 Multi-Period Hedging and Rollover Hedging

So far our analysis has assumed that the hedger faces a one-period decision problem. Alternatively, we may consider the problem when the hedger holds the same futures position within a multi-period framework. Because a futures position is marked to the market on a daily basis (i.e., the losses or gains from a futures position are calculated and accounted for daily), a multi-period consideration seems appropriate. Let $t = 0$ denote the current time. Suppose a futures position will be held in each period until the final period T. At any time t such that $0 \leq t < T$ a futures position X_t is assumed. This will be lifted at $t+1$. For the most general case, the hedger chooses the optimal sequence of futures positions to maximize his end-of-period expected utility, $\mathrm{E}\{U(W_T)\}$ subject to the constraints:

$$W_t = W_{t-1} + (\Delta p_t)Q - (\Delta f_t)X_{t-1}, \qquad 1 \leq t \leq T. \tag{78}$$

Dynamic programming can be applied to solve the maximization problem. Specifically, at the final decision point $t = T-1$ the hedger chooses the optimal futures position, X^*_{T-1}, by the following condition:

$$\mathrm{E}\{U'(W_{T-1} + (\Delta p_T)Q - (\Delta f_T)X^*_{t-1})\Delta f_T\} = 0. \tag{79}$$

Clearly X^*_{T-1} depends upon the utility function and the joint density function of Δp_T and Δf_T. Given X^*_{T-1}, at $t = T-2$ the hedger perceives the end-of-period wealth as follows:

$$W_T = W_{T-2} + (\Delta p_{T-1})Q - (\Delta f_{T-1})X_{T-2} + (\Delta p_T)Q - (\Delta f_T)X^*_{T-1}. \tag{80}$$

The optimal futures position X^*_{T-2} satisfies the equation

$$\mathrm{E}\{U'(W_{T-2} + (\Delta p_{T-1})Q - (\Delta f_{T-1})X^*_{T-2} + (\Delta p_T)Q - (\Delta f_T)X^*_{T-1})\Delta f_{T-1}\} = 0. \tag{81}$$

Consequently, X^*_{T-2} depends upon the utility function and the joint density function of Δp_T, Δf_T, Δp_{T-1}, and Δf_{T-1}. Similar procedures can be applied to derive X^*_t for $0 \leq t \leq T-3$.

To proceed further, assumptions for the utility function or the joint density function of $\{\Delta p_t, \Delta f_t\}$ for $t = 1, ..., T$, must be imposed. Suppose that $\Delta p_t = \beta_t \Delta f_t + \varepsilon_t$ such that Δf_s and ε_t are stochastically independent for any t and s. Then the optimal futures position is $X^*_t = \beta_t Q$. This is the simplest case in which the multi-period hedge ratio is identical to the one-period hedge ratio. Myers and Hanson (1996) provided similar results under alternative and less restrictive assumptions. Within a two-period model, Anderson and Danthine (1983) adopted a mean-variance utility function to derive the optimal futures positions. Arguing that a hedger should be concerned only with risk, Howard and D'Antonio (1991), Mathews and Holthausen (1991), and Lien (1992) assumed the hedger attempts to minimize the variance of the end-of-period wealth. Under this assumption, X^*_{T-1} is chosen to minimize $\mathrm{Var}(W_T)$, where $W_T = W_{T-1} + (\Delta p_T)Q - (\Delta f_T)X_{t-1}$. Thus, $X^*_{T-1} = Q[\mathrm{Cov}(\Delta p_T, \Delta f_T)/\mathrm{Var}(\Delta f_T)]$, which is the one-period solution. At time $t = k$, the hedger chooses X^*_k to minimize

$$\mathrm{Var}(W_T) = \mathrm{Var}[(\Delta p_{k+1})Q - (\Delta f_{k+1})X_k + \sum_{t=k+1}^{T-1} (\Delta p_{t+1})Q - (\Delta f_{t+1})X^*_t]. \tag{82}$$

Note that the hedger knows X_t^*, $k+1 \le t \le T-1$ at $t = k$. Upon taking the partial derivative with respect to X_k, the following recursive relationship can be derived:

$$B_k^* = \frac{\text{Cov}(\Delta p_{k+1}, \Delta f_{k+1})}{\text{Var}(\Delta f_{k+1})} + \sum_{t=k+1}^{T-1} \frac{\text{Cov}(\Delta f_{k+1}, \Delta p_{t+1} + \Delta f_{t+1} B_t^*)}{\text{Var}(\Delta f_{k+1})}, \tag{83}$$

where $B_k^* = X_k^*/Q$. Imputing the second moments of $\{\Delta p_t, \Delta f_t\}$ for $t = 1, ..., T$, helps solving for B_k^*.

Lien (1992) considered the following simple cointegration model:

$$\begin{aligned} \Delta p_t &= \alpha(f_{t-1} - p_{t-1}) + \varepsilon_{pt}, \\ \Delta f_t &= -\beta(f_{t-1} - p_{t-1}) + \varepsilon_{ft}, \end{aligned} \tag{84}$$

where $\{\varepsilon_{pt},\ \varepsilon_{ft}\}$ are independently and identically distributed over time. To simplify the notation let σ denote $\text{Cov}(\varepsilon_{pt}, \varepsilon_{ft})/\text{Var}(\varepsilon_{ft})$. It can be shown that

$$B_k^* = \sigma + \sum_{j=1}^{T-k-1} (1 - \alpha - \beta\sigma)^{j-1}(\alpha + \beta\sigma)(1 - \sigma) \tag{85}$$

provided $\alpha + \beta\sigma \neq 1$. Otherwise, $B_k^* = 1$. Consequently the optimal multi-period hedge ratios collapse to the one-period hedge ratio if and only if $\alpha + \beta\sigma = 0$ or $\sigma = 1$. Following Garbade and Silber (1983), $0 < \alpha, -\beta < 1$. It can be shown that B_k^* is a decreasing function of k (provided $\alpha + \beta\sigma \neq 1$). That is, the hedger begins with a large hedge ratio at $t = 0$ and gradually reduces the hedge ratio until the final decision point $t = T - 1$ when the hedge ratio is set at σ. A similar result was established by Howard and D'Antonio (1991).

In Lien and Luo (1993), lagged spot and futures prices are added into the cointegration equations:

$$\Delta p_t = \alpha_1(f_{t-1} - p_{t-1}) + \beta_1 \Delta p_{t-1} + \gamma_1 \Delta f_{t-1} + \varepsilon_{pt}, \tag{86}$$

$$\Delta f_t = \alpha_2(f_{t-1} - p_{t-1}) + \beta_2 \Delta p_{t-1} + \gamma_2 \Delta f_{t-1} + \varepsilon_{ft}. \tag{87}$$

Let $\varepsilon_t = (\varepsilon_{pt}, \varepsilon_{ft}, \varepsilon_{pt} - \varepsilon_{ft})'$ and let

$$A = \begin{bmatrix} \beta_1 & \gamma_1 & \alpha_1 \\ \beta_2 & \gamma_2 & \alpha_2 \\ \beta_2 - \beta_1 & \gamma_2 - \gamma_1 & 1 + \alpha_2 - \alpha_1 \end{bmatrix} \tag{88}$$

Then

$$B_k^* = \sigma + \sum_{t=k+1}^{T-1} (0,1,0)\text{Var}(\varepsilon_t)(A')^{t-k}(1, B_t^*, 0)'[\text{Var}(\varepsilon_{2t})]^{-1}. \tag{89}$$

Lien and Luo (1993) applied the above methods to five currency markets and three stock index markets. They found that the optimal hedge ratios first increases and then decreases as the maturity date becomes closer. Recall that the hedge ratio always decreases when the lagged price terms are not incorporated into the model.

The problem becomes much more complicated once conditional heteroscedasticity is incorporated into the cointegration system. To illustrate, consider a three-date ($t = 1, 2, 3$) two-period model. As usual, $B_2^* = \mathrm{Cov}(\varepsilon_{p3}, \varepsilon_{f3})/\mathrm{Var}(\varepsilon_{f3})$. Let $\delta_1(X) = X - \mathrm{E}_1\{X\}$, where E_1 denotes the expectation taken at time 1. Lien and Luo (1994) adopted the decomposition of Bohrnstedt and Goldberger (1969) to derive the following formula for B_1^*:

$$B_1^* = \sigma + B_{11} + B_{12} + B_{13}, \tag{90}$$

where

$$B_{11} = \alpha_1 + \gamma_1 + (\beta_1 - \alpha_1)[\mathrm{Var}_1(\varepsilon_{f2})]^{-1}\mathrm{Cov}_1(\varepsilon_{p2}, \varepsilon_{f2}), \tag{91}$$

$$B_{12} = \mathrm{E}_1\{B_2^*\}[\alpha_2 + \gamma_2 + (\beta_2 - \alpha_2)[\mathrm{Var}_1(\varepsilon_{f2})]^{-1}\mathrm{Cov}_1(\varepsilon_{p2}, \varepsilon_{f2})], \tag{92}$$

$$B_{13} = [\mathrm{Var}_1(\varepsilon_{f2})]^{-1}[(\alpha_2 + \gamma_2)\mathrm{E}_1\{\delta_1(B_2^*)\varepsilon_{f2}^2\} + (\beta_2 - \alpha_2)\mathrm{E}_1\{\delta_1(B_2^*)\varepsilon_{p2}\varepsilon_{f2}\}]. \tag{93}$$

When examining five currency markets (British pound, Canadian dollar, Deutsche mark, Japanese yen and Swiss franc), they found that B_1^* was far different from B_2^* in the British pound and the Swiss franc markets. Nonetheless, the improvement in hedging effectiveness is minimal.

Multi-period hedge ratios for the above problem arise from the consideration of rebalancing (due to mark-to-market). The hedger changes positions on a given futures contract periodically. An alternative multi-period problem occurs when the long-term futures markets are missing. For example, a hedger may want to hedge a spot position that will be lifted two years from now. However, there is no futures trading available for two-year maturity. The hedger therefore continuously trades on the nearby futures contracts and upon its maturity rollover the position to the next nearby futures contract. Thus, we have a "rollover hedging" problem. In fact, even if the long-term contract is available for trading, the market may be thin such that the hedger is better off adopting a rollover hedge strategy.

Let $f_{i,j}$ denote the price of a futures contract with maturity i at time j. Suppose there are k futures contracts with maturity t_n, for $n = 1, ..., k$, such that $1 < t_1 < ... < t_{k-1} < T \leq t_k$. Adopting rollover hedge on the nearby futures contracts, the end-of-period wealth is:

$$\begin{aligned} W_T \;=\; & (p_{t_1} - p_1)Q - (f_{t_1,t_1} - f_{t_1,1})X_1 + (p_T - p_{t_{k-1}})Q - (f_{t_k,T} - f_{t_k,t_{k-1}})X_{t_{k-1}} \\ & + \sum_{n=1}^{k-2}(p_{t_{n+1}} - p_{t_n})Q - (f_{t_{n+1},t_{n+1}} - f_{t_{n+1},t_n})X_{t_n}, \end{aligned} \tag{94}$$

where X_j is the position on the nearby futures contract at time j. Assume the hedger is an expected-utility maximizer. Applying dynamic programming technique, the optimal futures position, $X_{t_{k-1}}^*$, is found to satisfy the following equation:

$$\mathrm{E}\{U'(W_T)(f_{t_k,T} - f_{t_k,t_{k-1}})\} = 0. \tag{95}$$

Also, the optimal futures position, $X_{t_n}^*$, can be solved from the following equation:

$$\mathrm{E}\{U'(W_T)(f_{t_{n+1},t_{n+1}} - f_{t_{n+1},t_n})\} = 0. \tag{96}$$

These equations are similar to (and more complicated than) the case of multi-period hedging discussed above. Additional structure on the utility function and the data generation processes must be imposed to obtain explicit solutions.

Earlier studies on rollover hedging include Baesel and Grant (1982), McCabe and Franckle (1983), Grant (1984), and Gardner (1989). In these studies, the mean-variance approach is adopted. Moreover, to eliminate speculative motivation, it is assumed that the hedger attempts to minimize $\mathrm{Var}(W_T)$. Thus,

$$B^*_{t_{k-1}} = \mathrm{Cov}(p_T - p_{t_{k-1}}, f_{t_k,T} - f_{t_k,t_{k-1}})/\mathrm{Var}(f_{t_k,T} - f_{t_k,t_{k-1}}). \tag{97}$$

where $B^*_{t_{k-1}} = X^*_{t_{k-1}}/Q$. Moreover, at t_{k-2},

$$\begin{aligned} B^*_{t_{k-2}} &= \frac{\mathrm{Cov}(p_{t_{k-1}} - p_{t_{k-2}}, f_{t_{k-1},t_{k-1}} - f_{t_{k-1},t_{k-2}})}{\mathrm{Var}(f_{t_{k-1},t_{k-1}} - f_{t_{k-1},t_{k-2}})} + \\ & \frac{\mathrm{Cov}((p_T - p_{t_{k-1}})Q - (f_{t_k,T} - f_{t_k,t_{k-1}})B^*_{t_{k-1}}, f_{t_{k-1},t_{k-1}} - f_{t_{k-1},t_{k-2}})}{\mathrm{Var}(f_{t_{k-1},t_{k-1}} - f_{t_{k-1},t_{k-2}})}. \end{aligned} \tag{98}$$

The first term is the usual one-period hedge ratio whereas the second term is due to multi-period considerations. Upon repeating the procedure, a general recursive relationship for $B^*_{t_n}$ can be derived. It is similar to that applied to the multi-period hedging problem. Using empirical data, numerical values for the optimal rollover hedge ratios can be estimated and the hedging effectiveness can be assessed. Indeed, Gardner (1989) found that rollover hedging is a good substitute for (missing) long-term commodity futures contracts.

The case of Metallgesellschaft (MG) and its controversial rollover hedging strategy has increased academic scrutiny of using rollover strategies.[24] Specifically, MG adopted a hedge ratio of near 1. Culp and Miller (1994, 1995a, 1995b) defended the program as conceptually sound but faulted supervisory board members for abandoning the program when prices moved against MG. Mello and Parsons (1995) and Pirrong (1997) suggested the strategy as conceptually flawed. Edwards and Canter (1995) highlighted the potential risks and benefits of the program but were reserved in their judgement. Within the continuous-time framework, Hilliard (1997) and Brennan and Crew (1997) derived the optimal hedging strategy assuming different underlying stochastic process. Ross (1997) adopted a mean-reverting process for the spot price and derived an optimal hedge ratio that is robust to model misspecifications. All the previous works restrict the hedger to a stack strategy, that is, the hedger holds a position only on a given contract (i.e., the nearby contract). Neuberger (1995) considered the possibilty of multiple futures contracts. The optimal futures positions were derived under a "linear spanning" condition for futures prices. Using crude oil data, it was shown that the so-called strip strategy outperforms the stack strategy. However, the strategy requires a transaction volume and therefore a large transaction cost. While validating Neuberger's results in crude

[24] In the early 90s MG sold a huge volume of five- to ten-year heating oil and gasoline fixed price contracts. It hedged its exposure with long positions in futures contracts that were rolled over. It turned out that the price of oil fell and there were margin calls. As a result, MG closed out all the hedge positions and suffered a loss of $1.33 billion.

oil, Lien and Shaffer (2000) also found contrasting results in heating oil and unleaded gasoline. Lence and Hayenga (2000) considered multi-year rollover hedge-to-arrive contracts. They demonstrated that it is theoretically infeasible to lock in high current prices.

8 Conclusions

The importance of risk management in the financial markets cannot be overemphasized. As a widely used instrument in hedging, futures plays a significant role in risk management. Recent research has made significant contributions to the understanding of futures hedging, both in terms of the theoretical underpinning and the implementation of the strategies. In this survey we have reviewed some of the recent findings. There are, of course, many more questions that remain to be answered. For example, although nonlinearity in spot and futures returns has been incorporated in recent works, most researchers continue to adopt linear processes to model the second moments. It is likely that nonlinear processes may perform better for hedging purposes. Also, the effects of taxes on futures hedging requires a different modelling strategy. Pirrong (1995), Lien (1999b) and Lien and Metz (2000) provided some preliminary results on this issue. Assessment of hedging performance that explicitly incorporates these factors will provide better understanding of hedging price risks.

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