THE DEMAND FOR REINSURANCE: THEORY AND EMPIRICAL TESTS

by

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ABSTRACT. This paper investigates the valuation effects of reinsurance purchases in a contingent claims framework. The comparative statics of the model suggest that, other things held constant, the demand for reinsurance will be greater, 1) the higher the firm's leverage, 2) the lower the correlation between the firm's investment returns and claims costs, 3) for firms which write "longer-tail" lines of insurance, and 4) the more the firm concentrates its investments in tax-favored assets. These predictions are tested in an empirical analysis of the reinsurance behavior of U.S. property-liability insurance firms during the 1980's.

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1. INTRODUCTION

Like most business firms, insurance companies issue stock and/or other liabilities in order to raise capital for operations. However, insurance companies are different from other firms because they create explicit liabilities whenever they sell insurance policies. Indeed, most of a typical insurer's liabilities are held by its policyholders. An insurer's success depends not only on charging adequate rates to cover costs, but also on providing credible assurances to policyholders that claim payments will be made. This is a consequence of the nature of the insurance business, since a policy pays off in the joint contingency that the insured loss event occurs and the insurance company is financially solvent. Such assurances can be credibly provided by a number of mechanisms, including the commitment of adequate surplus and the purchase of reinsurance.

When an insurer purchases reinsurance, it effectively reduces its leverage. Therefore, an insurer's reinsurance decision is essentially a capital structure decision that shares a number of conceptual similarities to the capital structure decision of a nonfinancial firm. In spite of these similarities, the problem of optimal reinsurance has only received limited attention in the financial economics literature.[1] Although this problem has been carefully scrutinized in the insurance literature, that particular literature tends to view survival as a primary corporate objective and therefore does not provide an appropriate framework for analyzing the problem.[2]

Modigliani-Miller type propositions have been derived for insurance companies that, quite naturally, closely resemble their nonfinancial counterparts. For example, Garven (1987) shows

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[1]See Doherty and Tinic (1981) and Mayers and Smith (1981, 1990).

[2]Although it is the most common approach used in corporate finance models, value maximization is rarely defined as the corporate objective function in the insurance literature. Instead, insurance theorists typically assign risk averse utility functions to firms as if they were individuals. This particular characterization of the firm has been criticized for failing to provide an adequate basis for modeling the influence of competitive capital markets upon the firm's behavior (see Garven (1987)), and it ignores the "nexus of contracts" nature of the firm (see Jensen and Meckling (1976)).

that, absent taxes and contracting costs, capital structure decisions of insurance companies are irrelevant. In the presence of a corporate income tax, insurers will employ as little surplus as possible. These results obviously imply that either reinsurance will not "matter," or that its purchase will serve only to reduce the value of the firm by reducing the value of its leverage-related tax benefits.

A number of alternative motivations for reinsurance have been previously suggested in the literature. Doherty and Tiniç (1981) show that reinsurance is irrelevant if the pricing of insurance is inelastic with respect to the insurer's ruin probability. However, if insurance prices are sensitive to default risk, then policyholders pay a lower (higher) price for a policy when the probability of default on the part of the insurer is high (low).[3] Mayers and Smith (1981) suggest that agency costs may also be important. Like lenders in the bond market, policyholders face incentive problems in the insurance market. These incentive problems arise because shareholders may be able to effect wealth transfers between policyholders and themselves by altering various aspects of the firm's investment, underwriting or dividend policies after issuing insurance. However, since policyholders recognize the incentives faced by shareholders, the prices they are willing to pay for the policies should reflect unbiased estimates of the expected behavior of stockholders. Furthermore, the greater the firm's leverage, the greater will be the magnitude of these agency costs borne by shareholders. Since the purchase of reinsurance effectively unlevers the firm, it also reduces agency costs that would otherwise be borne by shareholders in the guise of lower insurance premiums. Garven (1987) argues that in order for insurer capital structure decisions (including the decision to reinsure) to "matter" in any meaningful sense, factors such as underutilized tax shields and costs related to financial distress (such as agency and bankruptcy costs) must be present.

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[3]Although Doherty and Tinic don't employ contingent claims analysis, their pricing predictions have been subsequently confirmed by the contingent claims models of Doherty and Garven (1986) and Cummins (1988). Contingent claims models view risky insurance as the economic equivalent of safe insurance minus the value of a "limited liability" put option (see Garven (1992)).

The purpose of this paper will be to set forth a theory of the demand for reinsurance that views reinsurance as both a leverage and risk management mechanism. A contingent claims framework is adopted, since this makes it possible to explicitly model the impact that reinsurance has upon the various leverage-related costs that the firm may incur. Consequently, the paper is organized in the following manner. In Section 2, a valuation model is presented which considers the various costs and benefits of reinsurance purchases in a contingent claims framework. Section 3 outlines various testable hypotheses that are yielded from the comparative statics of the model. Section 4 presents evidence from an empirical study of the reinsurance behavior of U.S. property-liability insurance firms during the 1980's. Section 5 concludes.

2. VALUATION EFFECTS OF REINSURANCE PURCHASE DECISIONS

Next, a single period valuation model is developed which provides the basis for a formal analysis of the demand for reinsurance. The firm is assumed to be formed at the beginning of the period for the purpose of maximizing the market value of its equity, or surplus. A homogeneous set of insurance policies are issued for which the insurer receives premium income P. Some proportion of the resulting liabilities is reinsured at a cost of P_r dollars. Surplus (S_0) and premium income net of reinsurance (P_n) are allocated to an investment portfolio composed of financial assets. At the end of the period, the firm's cash flows from its investment, underwriting, and reinsurance activities are realized.

Since the primary concern of this paper is with the valuation effects of the reinsurance decision *per se*, the underwriting and investment decisions are treated as given. Therefore, the firm's problem is to select the optimal level of reinsurance for the policies it has decided to underwrite. Without loss of generality, the model will only consider the case of quota share reinsurance.

Next, the assumptions and notation that will be needed are presented.

2.1. Model Assumptions

1. Financial markets and insurance markets are perfectly competitive.
2. Insurance (reinsurance) is risky (default-free).
3. Investors' utility functions exhibit constant absolute risk aversion and investment returns, claims costs, and terminal wealths are multivariate normally distributed.

Some explanation of the model assumptions is warranted. Assumptions 1 and 3 are required in order to justify the use of the discrete time contingent claims model.[4] Assumption 2 is made for the sake of analytic simplicity. Essentially, if reinsurance is risky, then the ceding insurer's policies become compound rather than simple options because their payoffs are functionally related to the default risk of both the ceding insurer and its reinsurer.

2.2. Model Notation

The following notation will also be needed:

α = the proportion of the firm's liabilities that are to be reinsured, $\alpha \in [0,1]$ (the firm's decision variable);

$\pi(\alpha)$ = default cost function, $\pi'(\alpha)<0$;

$P(\alpha) = P - \pi(\alpha)$ = gross premium income, $P'(\alpha) > 0$;[5]

P_r = the price of a reinsurance contract that completely insures the firm's liabilities, paid at t=0;

$P_n(\alpha) = P(\alpha) - P_r$ = net premiums written, $P_n(\alpha) < 0$;[6]

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[4]Brennan (1979) notes that the discrete time model is particularly appropriate when valuing non-traded claims such as the firm's taxable income. Also see Doherty and Garven (1986).

[5]In the expression $P(\alpha) = P - \pi(\alpha)$, P is the gross premium income which would obtain in a default-free setting in which all tax shields are fully utilized. Its value is therefore equal to $E(L)/(1-E(r_u))$, where $E(r_u) = -kr_f(1-\theta\tau)/(1-\tau) + \beta_u[E(r_m) - r_f] + (V_e/P_0)r_f(\theta\tau/(1-\tau))$ (see Garven (1987), section 2.2. for the development of this particular formula). $P'(\alpha) > 0$ because higher values of make it possible for the firm to charge higher premiums.

$A(\alpha) = S_0 + kP_n(\alpha)$ = the insurer's beginning-of-period assets, $A'(\alpha) < 0$;

k = the funds generating coefficient, or average claim delay;

θ = the proportion of the firm's investment income that is subject to taxation; $\theta \in [0,1]$;

$f(r_p,L)$ = bivariate normal density function governing the insurer's investment returns (r_p) and claims costs (L);

$\hat{f}(r_p, L)$ = the corresponding risk neutral bivariate density function;[7]

r_f = the riskless rate of interest;

r_m = the rate of return on the market portfolio;

$R_i = 1 + r_i$, $i = f,m,p$;

$n(\cdot)$ = standard normal density function;

$N(\cdot)$ = cumulative standard normal distribution function.

2.3. The Value of the Insurance Firm

Black and Scholes (1973) suggest that the equity of a levered firm represents a call option on the terminal value of the firm, with an exercise price equal to the face value of debt. Galai and Masulis (1976) combine Merton's (1973) continuous time CAPM with the Black-Scholes option pricing model in order to value levered equity and investigate the valuation and risk effects of changes in corporate investment policy. Galai (1983) extends the contingent claim formulation of the firm's capital structure to a valuation of the government's tax claim.[8] The approach followed

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[6] Although the purchase of reinsurance lowers various costs related to agency and tax effects, it also decreases net premiums written and consequently the total amount of capital that can be invested in the financial market.

[7] A "risk neutral" density function is a density function whose location parameter is chosen so that the mean of the distribution is its certainty-equivalent (see Brennan (1979) and Stapleton and Subrahmanyam (1984)). In the case of a multivariate risk neutral density function, the same result holds for the means of the marginal density functions.

[8] A number of other authors have applied contingent claims analysis to valuing the government's corporate tax claim. See the papers by Majd and Myers (1984), Pitts and Franks (1984), and Smith and Stultz (1985).

here is similar to the approach taken by Galai. Specifically, shareholders are viewed as holding a long position in a call option on the pre-tax terminal value of firm and a short position in its taxable income. However, the solution to the problem is somewhat different, because the payoffs on these call options depend upon the outcomes of two random variables rather than just one. Analytically, this implies that these call options have stochastic exercise prices.[9] In the valuation problem at hand, there is one exercise price common to both options, the value of which is determined in part by the realization of claims costs which have not been reinsured.

The value of the pre-tax equity claim will be referred to as $C(AR_p;-U)$, where $AR_p = A(1+r_p)$ is the pre-tax terminal value of the insurer's investment portfolio (i.e., the value of the underlying asset), and $-U = (1-\alpha)L - P_n$ is the negative of the insurer's pre-tax underwriting income (i.e., the exercise price). The value of the government's claim will be subsequently referred to as $\tau C(A\theta r_p;-U)$, where $A\theta r_p$ is the terminal value of taxable investment income.

More formally, the pre-tax value of equity, $C(AR_p;-U)$, can be written as follows:

$$C(AR_p;-U) = R_f^{-1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} MAX\left[\left(AR_p + P_n - (1-\alpha)L\right),0\right] f(r_p, L)dr_p dL. \quad (1)$$

From equation (1), it is readily apparent that if the terminal value of cash flow derived from the firm's investment, underwriting, and reinsurance activities is non-negative, then shareholders will have a valuable claim. However, if cash flow fails to assume a positive value, then shareholders will exercise their "limited liability option" by declaring bankruptcy.

Since R_p and L are normal variates, it is convenient to solve equation (1) by defining a normal variate $Y = AR_p - (1-\alpha)L$, with certainty-equivalent expectation

[9]Fischer (1978) was the first researcher to address the pricing of an option with a stochastic exercise price. However, because his work is a direct extension of Black and Scholes, it relies primarily upon the stochastic calculus. Stapleton and Subrahmanyam (1984, pp. 223-224) present an alternative solution using preference-restricted contingent claims models based upon multivariate normal and lognormal density functions. See Doherty and Garven (1986) for an application of the Stapleton and Subrahmanyam framework to the pricing of property-liability insurance.

$\hat{E}(Y) = AR_f - (1-\alpha)\hat{E}(L)$, variance $\sigma_Y^2 = A^2\sigma_p^2 + (1-\alpha)^2\sigma_L^2 - 2A(1-\alpha)\sigma_{pL}$, and risk neutral density $\hat{f}(Y)$. Then equation (1) reduces to

$$C(AR_p;-U) = R_f^{-1}\int_{-P_n}^{\infty}(Y+P_n)\,\hat{f}(Y)dY. \qquad (2)$$

Changing the random variate Y to a standardized normal variate u and solving yields

$$C(AR_p;-U) = \left\{A + \left[P_n - (1-\alpha)\hat{E}(L)\right]R_f^{-1}\right\}N(X_1) + R_f^{-1}\sigma_Y n(X_1), \qquad (3)$$

where

$X_1 = \left[AR_f + P_n - (1-\alpha)\hat{E}(L)\right]/\sigma_Y$ = the standardized certainty-equivalent terminal value of pre-tax profit;

$N(X_1)$ = the pre-tax certainty-equivalent terminal value of one dollar invested in the firm, provided the firm remains solvent.[10]

Similarly, the value of the government's claim, $\tau C(A\theta r_p;-U)$, can be written as:

$$\tau\, C(A\theta r_p;-U) = \tau\, R_f^{-1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} MAX\left[\left(A\theta r_p + P_n - (1-\alpha)L\right),0\right]\hat{f}(r_p, L)dr_p dL. \qquad (4)$$

From equation (4), it is readily apparent that if the terminal value of taxable income is non-negative, then the government will have a valuable claim. However, if taxable income fails to assume a positive value, then shareholders will exercise their "tax exemption option". Thus the model allows for the existence of a state interval in which tax shields (e.g., investment and/or underwriting losses) are underutilized.

The solution of equation (4) requires that a normal variate Z be defined, where Z = $A\theta r_p$ - $(1-\alpha)L$. The certainty-equivalent expectation of Z is $\hat{E}(Z) = A\theta r_f - (1-\alpha)\hat{E}(L)$, its variance is

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[10] $N(X_1)$ is not the probability of solvency; rather, the probability of solvency is given by $N(X_1^*)$, where $X_1^* = [AE(R_p) + P_n - (1-\alpha)E(L)]/\sigma_y$. Because $N(X_1)$ is in effect a "risk neutral" cumulative distribution function, it understates the solvency probability by the amount of risk bearing costs borne per dollar of income generated in solvent states of nature; i.e., by the difference $N(X_1^*) - N(X_1)$.

$\sigma_z^2 = A^2\theta^2\sigma_p^2 + (1-\alpha)^2\sigma_L^2 - 2A\theta(1-\alpha)\sigma_{pL}$, and its risk neutral density is $\hat{f}(Z)$. Substituting this change of variables into equation (4) yields equation (5):

$$\tau\, C(A\theta r_p;-U) = \tau\, R_f^{-1}\int_{-P_n}^{\infty}(Z+P_n)\,\hat{f}(Z)dZ. \tag{5}$$

Changing the random variable Z to a standardized normal variate v and solving yields

$$\tau\, C(A\theta r_p;-U) = \tau\, R_f^{-1}\left\{A\theta r_f + P_n - (1-\alpha)\hat{E}(L)\right\}N(X_2) + \tau\, R_f^{-1}\sigma_z n(X_2), \tag{6}$$

where

$X_2 = \left[A\theta r_f + P_n - (1-\alpha)\hat{E}(L)\right]/\sigma_z$ = the standardized certainty-equivalent terminal value of taxable profit;

$N(X_2)$ = the certainty-equivalent terminal value of one dollar of taxable profit, provided that tax shields are fully utilized.[11]

Since shareholders hold a long position in $C(AR_p;-U)$ and a short position in $\tau C(A\theta r_p;-U)$, the after-tax value of equity, V_e, is found by subtracting $\tau C(A\theta r_p;-U)$ from $C(AR_p;-U)$:

$$\begin{aligned} V_e = &\left\{A + \left[P_n - (1-\alpha)\hat{E}(L)\right]R_f^{-1}\right\}N(X_1) + R_f^{-1}\sigma_Y n(X_1) \\ &- \tau\, R_f^{-1}\left\{A\theta r_f + P_n - (1-\alpha)\hat{E}(L)\right\}N(X_2) + \tau\, R_f^{-1}\sigma_z n(X_2). \end{aligned} \tag{7}$$

The firm's optimal reinsurance decision maximizes V_e. To see how the purchase of reinsurance affects V_e, the first-order condition is calculated by differentiating equation V_e in equation (7) with respect to α:

$$\begin{aligned} \frac{\partial V_e}{\partial\alpha} = &-P_r\left[N(X_1) - \tau R_f^{-1}N(X_2)\right] + \left[\hat{E}(L)R_f^{-1} - \frac{\partial\pi}{\partial\alpha}\right]\left[N(X_1) + \tau R_f^{-1}N(X_2)\right] \\ &- k\left[\frac{\partial\pi}{\partial\alpha} + P_r\right]\left[N(X_1) - \theta\tau r_f R_f^{-1}N(X_2)\right] + R_f^{-1}\left[\frac{\partial\sigma_Y}{\partial\alpha}n(X_1) - \tau\frac{\partial\sigma_z}{\partial\alpha}n(X_2)\right]. \end{aligned} \tag{8}$$

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[11] $N(X_2)$ has a similar interpretation to the interpretation given $N(X_1)$. That is, $N(X_2)$ has a "risk averse" counterpart $N(X_2^*)$ which represents the probability of taxation, where $X_2^* = X_1^* = [A\theta E(r_p) + P_n - (1-\alpha)E(L)]/\sigma_z$. $N(X_2)$ understates the probability of taxation by the amount of risk bearing costs borne per dollar of taxable income generated in taxable states of nature; i.e., by the difference $N(X_2^*) - N(X_2)$.

The optimal value of α exists at the point at which the firm's after-tax value of equity decreases with any further change in its reinsurance coverage. This occurs at the point at which the expected after-tax marginal costs of reinsurance and risk retention are equal.[12]

The four bracketed terms in equation (8) can interpreted in the following manner:

1. Holding the probabilities of solvency and taxation constant, the first term represents the after-tax marginal cost of reinsurance.

2. Holding the probabilities of solvency and taxation constant, the second term represents the after-tax marginal benefits of lower claims and agency costs obtained from reinsuring.

3. Holding the probabilities of solvency and taxation constant, the third term represents the after-tax marginal cost associated with foregone investment income. Investment income is foregone for the simple reason that the purchase of reinsurance reduces the amount of money that can be invested in the financial market.

4. Holding the expected values of pre-tax profit and taxable income constant, the fourth term represents the effects that changes in the variability of pre-tax income and taxable income have upon the value of the firm. This term essentially provides an indication of how much more or less valuable the limited liability and tax exemption options become as more reinsurance is purchased. As long as investment returns and claims costs are negatively correlated, then the purchase of reinsurance decreases the value of the limited liability option and increases the value of the tax exemption option. However, if positive correlation exists between r_p and L, then the opposite will occur.

3. TESTABLE HYPOTHESES

In the previous section, we showed that the optimal reinsurance decision of a limited liability insurer is influenced by factors such as its investment return and claims cost distributions, the magnitude of tax shields derived from its investment and underwriting activities, agency costs, and default risk. The purpose in this section of the paper will be to briefly outline various testable hypotheses that are yielded by the comparative statics of the model.

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[12]This statement assumes, of course, that the second-order condition for a maximum is also satisfied. Analytically, the second-order condition is not trivially satisfied; however, it obtains for most reasonable parameterizations of the model.

From equation (8), the following set of cross-sectional predictions are derived:[13]

HYPOTHESIS 1: Other things held constant, the demand for reinsurance will be greater the higher the firm's leverage;

HYPOTHESIS 2: Other things held constant, the demand for reinsurance will be greater the lower the correlation between the firm's investment returns and claims costs.

HYPOTHESIS 3: Other things held constant, the demand for reinsurance will be greater for firms that write "longer-tail" lines of insurance;

HYPOTHESIS 4: Other things held constant, the demand for reinsurance will be greater for firms that concentrate their investments in tax-favored assets.

Hypothesis 1 highlights the fact that reinsurance is essentially a substitute for surplus in terms of its leverage effect. The smaller surplus is, the higher the firm's financial leverage. Higher leverage results in a lowering of the probability of solvency and an increase in the probability of tax shield underutilization, which in turn results in an increase in the demand for reinsurance.

The intuition supporting Hypothesis 2 is also quite appealing. With negative correlation, the values of both the pre-tax equity claim and the government's claim will fall due to the fact that the variances of pre-tax income and taxable income will also decline. This is analogous to the well-

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[13]Since no closed form solution exists for the optimal reinsurance decision, the comparative static analysis was accomplished by using the implicit function theorem. The implicit function theorem states that given some function $F(y,x_1,\ldots,x_n) = 0$, if an implicit function $y = f(x_1,\ldots,x_n)$ exists, then the partial derivatives of the implicit function are:

$$\frac{\partial y}{\partial x_i} = -\frac{\partial F / \partial x_i}{\partial F / \partial y},$$

for all i, $i = 1,\ldots,m$. In the model described above, the first-order condition is $V_\alpha^e(\alpha^*,x_1,\ldots,x_m)=0$, where V_α^e corresponds to the partial derivative of equity value with respect to α, the x_i's represent model parameters, and $\alpha^* = f(x_1,\ldots,x_m)$ is the implicit function. Therefore,

$$\frac{\partial \alpha^*}{\partial x_i} = -\frac{\partial V^e / \partial x_i}{\partial V^e / \partial \alpha}$$

for all i, $i = 1,\ldots,m$. Since $\partial^2 V^e / \partial \alpha^2 < 0$ when evaluated at α^*, this means that the sign of $\partial \alpha^* / \partial x_i$ will be the same as the sign of $\partial^2 V^e / \partial a \partial x_i$.

known comparative static relationship between the value of a call and the variance rate of the underlying asset. However, with positive correlation, the firm is provided with a natural hedge if it retains risk. By reinsuring, the natural hedge may be destroyed, thereby increasing the variances of pre-tax income and taxable income and hence the values of both the pre-tax equity claim and the government's claim.

Hypothesis 3 can also be explained in terms of a leverage effect. Although an increase in the average claim delay lowers the firm's premium income and reinsurance premiums, it also causes more investable funds to be generated per dollar of premiums. Overall, this latter effect dominates the former, leading to a net increase in financial leverage. Hence one would expect to observe a greater propensity toward reinsurance purchases by firms that underwrite risks with longer claim delays; specifically, firms that specialize in liability risks should purchase more reinsurance than firms specializing in property risks.[14]

The rationale for Hypothesis 4 can be best explained by first invoking a basic principle of asset pricing; that is, in equilibrium, after-tax certainty-equivalent returns must be equal across all securities. In the case of two financial assets that differ only with respect to taxation, the expected yield on the more fully taxed instrument should be grossed up to completely offset its marginally higher tax burden. Therefore, although the firm may gain valuable tax shields by lowering θ, such an action simultaneously decreases the probabilities of solvency and taxation due to the commensurately lower investment return prospects. Hence, as in the case of an increase in the average claim delay, a decrease in the "tax shield" coefficient should result in an increase in the demand for reinsurance.

Previous empirical research by Mayers and Smith (1990) documents that factors such as ownership structure, firm size, geographic concentration, and line-of-business concentration also

[14]In the empirical tests which follow, this effect will be captured by a variable which measures the proportion of the firm's total premiums accounted for by premiums written in Schedule P lines.

influence the demand for reinsurance.[15] Since these variables are known to be important cross-sectionally, the empirical study that follows will use these factors as control variables. Mayers and Smith note that their analysis is limited by the fact that their data do not allow them to distinguish between firms on the basis of 1) tax status, 2) cash flow volatilities, and 3) within-line policy heterogeneity. They also note that since their data do not classify interfirm reinsurance transactions differently from intrafirm transactions, it is not possible to directly compare the reinsurance behavior of unaffiliated single companies and insurance groups. The present study seeks to address all but the third limitation listed above. Hypothesis 4 addresses the first limitation, since the decision to purchase tax-favored assets is obviously significantly influenced by the firm's tax status. Hypothesis 2 addresses the second limitation, since the calculation of the correlation between investment returns and claims costs involves the estimation of cash flow volatilities. Finally, the fourth limitation can be resolved by either using omitting groups from the analysis or using post-1988 data. Subsequent to 1988, the annual statement forms upon which the A.M. Best data tapes are based lumped reinsurance transactions with group affiliates and unaffiliated insurers under the same categories. However, as of 1988, regulators began to require that groups report intrafirm and interfirm reinsurance transactions separately in the annual statement form. Consequently, two types of experiments are performed. The first experiment involves the use of panel data for unaffiliated (non-group) insurance firms from the period 1980-1989. The purpose of this experiment will be to determine whether the model's cross-sectional predictions hold over time. The second experiment involves the use of cross-sectional data from 1988 and 1989 consisting of unaffiliated single companies and insurance groups.

[15] Mayers and Smith also find that reinsurance demand is negatively related to Best's Ratings. Since firms with low Best's Ratings will, on average, tend to be more highly leveraged than firms with high Best's Ratings, the Best's Rating factor essentially proxies for the effect predicted in Hypothesis 1.

4. DATA AND EMPIRICAL RESULTS

4.1. Panel Data

Ten years of data were obtained from the 1980-1989 Balance Sheet-Income Statement and Premium-Losses-Expenses tapes produced by the A. M. Best Company. The criteria applied in the selection of the sample were as follows:

1. The firm must be an unaffiliated single company.
2. The firm must have been classified as either a stock or mutual company during the entire ten year period. Furthermore, it must not be classified as a specialist reinsurer.
3. Since a number of variables in the regression model involve ratios, only those firms reporting positive (nonzero) values for the denominators of these ratios are included in the sample so as to avoid division by zero.

The application of these criteria resulted in a sample of 54 stock insurers and 94 mutuals. Summary statistics for these firms are presented in Table 1.

4.2. Cross-Sectional Data

The data for the cross-sectional experiments were obtained from the 1988 and 1989 A.M. Best data tapes. The same selection criteria were applied, except insurance groups that met the second and third criteria were also included. The application of these criteria resulted in a 1988 sample consisting of data for 456 stock insurers and 290 mutuals and a 1989 sample consisting of data for 571 stock insurers and 396 mutuals. In the 1988 (1989) sample, 64% (66%) of the stock firms were groups, compared to 61% (68%) of the mutuals. The χ^2 test of independence between ownership structure and group structure reveals that the differences in these percentages are not statistically significant; i.e., there is no significant association between ownership structure and group structure in either of these samples. Summary statistics for the 1988 and 1989 samples are presented in Tables 2 and 3.

4.3. Panel Model

The following regression model was estimated for the sample consisting of panel data:

$$REINS_j = \beta_{0j} + \sum_{i=1}^{10} \beta_{ij} X_{ij} + \varepsilon_{ij,} \qquad (9)$$

where

$REINS_j$ = ratio of reinsurance premiums ceded to total business premiums for firm j;

X_{1j} = $SIZE_j$ = natural logarithm of firm j's size, measured in terms of admitted assets;

X_{2j} = $PSRATIO_j$ = ratio of direct premiums written to surplus for firm j;

X_{3j} = RHO_j = correlation between investment returns and claims for firm j;[16]

X_{4j} = $STDP_j$ = standard deviation of firm j's investment returns;

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[16]RHO_j was measured in the following manner:

$$RHO_j = \sum_{\substack{i=1 \\ i \neq k}}^{3} \sum_{k=1}^{21} X_{ij} W_{kj} COV(r_{ij}, L_{kj}) / \sigma_{pj} \sigma_{L_j},$$

where

X_{ij} = proportion of firm j's assets invested in bonds, stocks, and T-Bills;
r_{ij} = the return on firm j's i^{th} investment;
r_{pj} = the return on firm j's portfolio of investments = $\Sigma X_{ij} r_{ij}$;
σ_{pj} = standard deviation of r_{pj} ;
W_{kj} = proportion of firm j's earned premiums due to line k;
L_j = firm j's claims costs = $\Sigma W_{kj} L_{kj}$;
σ_{Lj} = standard deviation of L_j.

The data for these calculations were obtained for the period from Aggregates and Averages (1976-87 loss ratios), Ibbotson/Sinquefield (1976-87 security returns) and the A.M. Best data tapes (for the calculations of X_{ij} and W_{kj} for 1980-89).

X_{5j} = $STDL_j$ = standard deviation of firm j's claims costs;

X_{6j} = $SCHEDP_j$ = proportion of firm j's premiums written in Schedule P lines;

X_{7j} = $THETA_j$ = firm j's tax shield coefficient θ, which measures the proportion of investment income subject to taxation;[17]

X_{8j} = $HERF_j$= firm j's Herfindahl index $= \sum_{i=1}^{n} \left(\frac{(\text{Direct Premiums Written})_{ij}}{\sum_{i=1}^{n} (\text{Direct Premiums Written})_{ij}} \right)^2$;

X_{9j} = $LICENSE_j$ = the negative of the number of states in which firm j is licensed;

X_{10j} = $MUTUAL_j$ = 1 if firm j is a mutual, 0 if firm j is a stock insurer;

X_{11j}-X_{19j} = T_1-T_9 = year indicators; T_1=1 if *YEAR* = 1981, 0 otherwise, ..., T_9=1 if *YEAR*=1989, 0 otherwise.

The variables $REINS_j$, $SIZE_j$, $HERF_j$, $LICENSE_j$ and $MUTUAL_j$ were measured in the same manner as in the Mayers and Smith (1990) study. Since the model presented in the previous section assumes the purchase of proportional reinsurance, the Mayers and Smith definition for $REINS_j$ most closely fits the theory; viz., $REINS_j$=1 if α_j=1 and $REINS_j$=0 if α_j=0. $SIZE_j$, $HERF_j$, $LICENSE_j$ and $MUTUAL_j$ are controls for cross-sectional variation in size, line-of-business concentration, geographic concentration, and ownership structure. The remaining set of control variables are T_1-T_9, which control for time. When T_1-T_9 and *MUTUAL*j are all turned off, this implies that the data for a stock insurer is being observed for the year 1980. Consequently, the coefficients associated with these variables measure the extent to which the mean value of *REINS* differs over time and between stock and mutual organizations, whereas the associated *t* statistics test whether these differences are statistically significant.

[17]$THETA_j$ was determined by dividing taxable investment income into total investment income. See D'Arcy and Garven's (1990) appendix for further details on the mechanics of this particular calculation.

4.4. Cross-Sectional Model

For the cross-sectional experiments, the regression model described in equation (9) was slightly modified. Since these experiments do not allow time to vary, the dummy variable indicators for time were obviously dropped. Secondly, since the cross-sectional experiments seek to compare the reinsurance behavior of groups with unaffiliated single companies, the dummy variable $GROUP_j$ was included, where $GROUP_j=1$ if the firm j is a group and 0 if it is an unaffiliated single company.

4.5. Empirical Results

Table 4 provides the regression parameter estimates, standard errors, *t* statistics, and two-tail probabilities for the panel experiment and two cross-sectional experiments.

The regression equation obtained from the panel experiment has an adjusted R^2 value of .1694 and *F* statistic of 16.878, the latter of which is statistically significant at the .0001 level. The coefficients associated with the control variables suggested by Mayers and Smith are generally consistent with the findings of their study; specifically, size, line-of-business concentration and geographic concentration have a significant negative impact on the demand for reinsurance. Although the panel experiment failed to find any evidence in support of a difference in the demand for reinsurance between stock and mutual insurers, it should be noted that the differences observed by Mayers and Smith result from their use of a much more detailed stock ownership metric which classifies stock insurers into four subcategories: widely held, closely held, single-owner, and association-owned stocks. I also find that the time controls are all insignificant, which suggests that the cross-sectional relationships are stable over time.

Turning to the model predictions, I find that, as predicted, leverage (*PSRATIO*) and length of tail (*SCHEDP*) have a significant positive impact on the demand for reinsurance. Although the parameter estimate associated with the tax variable *THETA* is of the wrong sign, it does not differ significantly from zero. The parameter estimate associated with *RHO* is of the correct sign, but it too does not differ significantly from zero. Interestingly, the results indicate that the mean value of *REINS* increases as liability risk (*STDL*) increases, and declines as asset risk (*STDP*) increases. Since more than 90% of the firms in the sample have positive values for *RHO*, it is quite plausible that asset risk may act as a substitute for reinsurance in terms of providing a mechanism for lowering the total risk of the firm.

Next, consider the results obtained from the cross-sectional experiments. The regression equations have adjusted R^2 values of .1421 (1988) and .1745 (1989), and *F* statistics of 12.221 (1988) and 19.567 (1989), both of which are statistically significant at the .0001 level. Although these experiments yield similar results concerning the effects of size, line-of-business concentration, geographic concentration, ownership structure, leverage, and length of tail as in the panel experiment, the parameter estimates associated with *SCHEDP* in the 1989 equation and *HERF* in the 1988 and 1989 equations are not significantly different from zero. Possibly the inclusion of insurance groups rendered the *HERF* variable insignificant by suppressing the degree to which it varies cross-sectionally, since groups are likely to record systematically smaller values for this variable. Asset risk (*STDP*) and tax status (*THETA*) have the same effects as before; viz., asset risk substitutes for reinsurance, whereas there is no significant relationship between tax status and the demand for reinsurance. The *MUTUAL* and *GROUP* variables appear to be marginally significant in the 1988 equation, whereas they are not significant in the 1989 equation; consequently, the evidence on organizational structure is mixed. The remaining risk variables, *RHO* and *STDL*, also yield somewhat different results in the cross-sectional experiments. Although *RHO* is insignificant in the 1988 equation, its sign is positive rather than negative as predicted. In the 1989 equation, *RHO*'s coefficient is both positive and significantly different from zero. *STDL*'s sign remains positive in both cross-sectional experiments, but is not significantly different from zero in the 1989 equation.

5. SUMMARY AND CONCLUSION

The purpose of this paper has been to provide a theoretical and empirical analysis of the demand for reinsurance. The comparative statics of the theoretical model suggested that factors such as leverage, the correlation between investment returns and claims costs, length of tail, and tax status influence the demand for reinsurance. Although the empirical analysis focused on these factors, controls were also implemented for other factors that have also been shown to be important, such as ownership structure, firm size, geographic concentration, and line-of-concentration. The evidence strongly supports two of the model's predictions; viz., the demand for reinsurance is positively influenced by leverage and length of tail. Although the evidence on correlation is somewhat mixed, it appears that asset risk negatively influences reinsurance demand, whereas liability risk has a positive influence. Furthermore, reinsurance demand does not appear to be significantly affected by the extent to which the firm purchases tax-favored investments. Finally, in spite of the substantial changes that occurred in insurance tax law and the economics of the U.S. insurance market during the 1980's, the cross-sectional relationships nevertheless appear to be stable over time.

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TABLE 1
Panel Data Summary Statistics

All Firms	N	Minimum	Maximum	Mean	Standard Deviation
REINS	148	0.00	0.91	0.25	0.19
SIZE	148	13.90	20.98	16.81	1.23
PSRATIO	148	0.00	25.51	2.18	1.49
RHO	148	-0.49	0.81	0.55	0.20
STDP	148	2.87	15.08	10.55	2.59
STDL	148	2.15	22.97	7.42	4.04
SCHEDP	148	0.00	1.00	0.65	0.27
THETA	148	0.08	1.00	0.68	0.22
HERF	148	0.14	1.00	0.49	0.26
LICENSE	148	-57.00	0.00	-7.44	12.47
Stock Firms					
REINS	54	0.00	0.91	0.26	0.22
SIZE	54	13.90	20.32	16.95	1.31
PSRATIO	54	0.06	14.32	2.35	1.64
RHO	54	-0.49	0.76	0.50	0.24
STDP	54	2.87	15.07	9.85	2.95
STDL	54	2.15	22.97	8.58	5.01
SCHEDP	54	0.00	1.00	0.57	0.34
THETA	54	0.10	1.00	0.71	0.22
HERF	54	0.14	1.00	0.56	0.26
LICENSE	54	-57.00	0.00	-10.59	15.87
Mutual Firms					
REINS	94	0.00	0.82	0.24	0.17
SIZE	94	14.40	20.98	16.72	1.18
PSRATIO	94	0.00	25.51	2.08	1.38
RHO	94	-0.25	0.81	0.58	0.16
STDP	94	2.88	15.08	10.96	2.26
STDL	94	2.63	22.97	6.75	3.17
SCHEDP	94	0.00	1.00	0.70	0.22
THETA	94	0.08	1.00	0.67	0.23
HERF	94	0.14	1.00	0.44	0.24
LICENSE	94	-52.00	0.00	-5.62	9.57

TABLE 2
Summary Statistics for the 1988 Sample

All Firms	N	Minimum	Maximum	Mean	Standard Deviation
REINS	746	0.00	0.97	0.25	0.21
SIZE	746	13.35	24.32	17.77	1.88
PSRATIO	746	0.00	29.07	2.55	2.35
RHO	746	-0.73	0.78	0.53	0.21
STDP	746	2.85	15.07	10.04	3.16
STDL	746	2.19	236.39	8.57	9.80
SCHEDP	746	0.00	1.00	0.65	0.30
THETA	746	0.05	1.00	0.77	0.21
HERF	746	0.11	1.00	0.48	0.28
LICENSE	746	-57.00	0.00	-16.22	19.60
Stock Firms					
REINS	456	0.00	0.97	0.25	0.22
SIZE	456	13.35	23.78	17.80	1.95
PSRATIO	456	0.00	29.07	2.76	2.68
RHO	456	-0.73	0.78	0.49	0.24
STDP	456	2.85	15.07	9.42	3.43
STDL	456	2.19	78.82	8.61	5.57
SCHEDP	456	0.00	1.00	0.61	0.34
THETA	456	0.05	1.00	0.79	0.23
HERF	456	0.11	1.00	0.54	0.29
LICENSE	456	-57.00	0.00	-19.91	20.80
Mutual Firms					
REINS	290	0.00	0.86	0.24	0.19
SIZE	290	14.43	24.32	17.74	1.77
PSRATIO	290	0.01	20.78	2.22	1.67
RHO	290	-0.24	0.78	0.59	0.13
STDP	290	2.88	14.79	11.02	2.38
STDL	290	2.63	236.39	8.52	14.10
SCHEDP	290	0.00	1.00	0.72	0.21
THETA	290	0.26	1.00	0.76	0.19
HERF	290	0.14	1.00	0.40	0.25
LICENSE	290	-55.00	0.00	-10.41	15.95

TABLE 3
Summary Statistics for the 1989 Sample

All Firms	N	Minimum	Maximum	Mean	Standard Deviation
REINS	967	0.00	0.98	0.26	0.23
SIZE	967	12.09	24.46	17.34	2.06
PSRATIO	967	0.00	27.70	2.27	2.35
RHO	967	-0.73	0.82	0.50	0.24
STDP	967	2.85	15.08	9.70	3.49
STDL	967	2.26	200.08	8.76	7.74
SCHEDP	967	0.00	1.00	0.64	0.32
THETA	967	0.12	1.00	0.81	0.20
HERF	967	0.11	1.00	0.52	0.29
LICENSE	967	-57.00	0.00	-14.52	18.89
Stock Firms					
REINS	571	0.00	0.98	0.26	0.24
SIZE	571	12.97	23.84	17.50	2.05
PSRATIO	571	0.00	27.70	2.46	2.56
RHO	571	-0.73	0.82	0.46	0.27
STDP	571	2.85	15.08	9.21	3.70
STDL	571	2.26	200.08	9.27	9.31
SCHEDP	571	0.00	1.00	0.61	0.36
THETA	571	0.17	1.00	0.83	0.20
HERF	571	0.11	1.00	0.58	0.29
LICENSE	571	-57.00	0.00	-18.53	20.32
Mutual Firms					
REINS	396	0.00	0.95	0.27	0.20
SIZE	396	12.09	24.46	17.12	2.07
PSRATIO	396	0.01	23.32	2.00	1.99
RHO	396	-0.73	0.75	0.56	0.16
STDP	396	2.85	14.91	10.42	3.03
STDL	396	2.63	22.97	8.01	4.55
SCHEDP	396	0.00	1.00	0.70	0.26
THETA	396	0.12	1.00	0.80	0.18
HERF	396	0.14	1.00	0.43	0.26
LICENSE	396	-55.00	0.00	-8.72	14.84

TABLE 4
Regression Parameter Estimates

Panel Data	Parameter Est.	Standard Error	*t* Statistic	Prob > |*t*|
INTERCEPT	0.9829	0.0819	11.9950	0.0001
SIZE	-0.0479	0.0049	-9.7750	0.0001
PSRATIO	0.0232	0.0032	7.2070	0.0001
RHO	-0.0085	0.0301	-0.2810	0.7788
STDP	-0.0102	0.0020	-5.1650	0.0001
STDL	0.0104	0.0014	7.5000	0.0001
SCHEDP	0.0408	0.0199	2.0510	0.0405
THETA	0.0296	0.0222	1.3330	0.1828
HERF	-0.0528	0.0238	-2.2130	0.0271
LICENSE	-0.0030	0.0005	-6.5930	0.0001
MUTUAL	0.0116	0.0107	1.0820	0.2793
T1	-0.0034	0.0205	-0.1670	0.8674
T2	-0.0035	0.0206	-0.1710	0.8639
T3	-0.0044	0.0207	-0.2120	0.8323
T4	-0.0057	0.0207	-0.2760	0.7826
T5	0.0071	0.0208	0.3400	0.7342
T6	0.0193	0.0210	0.9220	0.3565
T7	0.0125	0.0211	0.5900	0.5549
T8	0.0178	0.0212	0.8380	0.4023
T9	0.0182	0.0214	0.8480	0.3964
1988 Cross-Section				
INTERCEPT	0.8574	0.1117	7.6790	0.0001
SIZE	-0.0372	0.0063	-5.8630	0.0001
PSRATIO	0.0171	0.0031	5.4320	0.0001
RHO	0.0534	0.0406	1.3140	0.1892
STDP	-0.0097	0.0027	-3.6300	0.0003
STDL	0.0018	0.0008	2.3150	0.0209
SCHEDP	0.0324	0.0254	1.2750	0.2028
THETA	0.0107	0.0355	0.3020	0.7629
HERF	-0.0310	0.0325	-0.9540	0.3403
LICENSE	-0.0022	0.0005	-4.1510	0.0001
MUTUAL	0.0189	0.0166	1.1340	0.2572
GROUP	0.0140	0.0191	0.7330	0.4636

TABLE 4 (CONTINUED)
Regression Parameter Estimates

1989 Cross-Section	Parameter Est.	Standard Error	*t* Statistic	Prob > |*t*|
INTERCEPT	1.0100	0.0929	10.8710	0.0001
SIZE	-0.0458	0.0054	-8.4940	0.0001
PSRATIO	0.0171	0.0029	5.8480	0.0001
RHO	0.0923	0.0341	2.7070	0.0069
STDP	-0.0122	0.0024	-5.1310	0.0001
STDL	-0.0000	0.0010	-0.0020	0.9985
SCHEDP	0.0751	0.0231	3.2510	0.0012
THETA	-0.0250	0.0369	-0.6770	0.4985
HERF	-0.0169	0.0293	-0.5790	0.5625
LICENSE	-0.0028	0.0005	-5.8310	0.0001
MUTUAL	0.0238	0.0151	1.5710	0.1165
GROUP	0.0327	0.0185	1.7720	0.0768