The Effect of FSD Changes in Multiplicative Background Risk on Risk-taking Attitude

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Abstract

In this paper, we consider the effect of First-degree Stochastic Dominance (FSD) changes in background multiplicative risk on the risk-taking attitude of a decision maker. First, we consider contractive FSD changes in background multiplicative risk and analyze the effect of these changes. Then, we consider general FSD changes in background multiplicative risk. Also, in the context of coinsurance, we determine the effect of simple FSD changes and Monotone Likelihood Ratio (MLR) changes in multiplicative background risk.

key words: FSD changes, Multiplicative background risk, risk-taking attitude, coinsurance

1. Introduction

Usually, background risk is assumed to be additive. But, in this paper, we suppose that background risk is multiplicative. It seems that risk-taking attitude of a decision maker in the presence of multiplicative background risk is first considered systematically by Franke, Schlesinger, and Stapleton [3]. They consider conditions that make the decision maker more risk averse in the case where multiplicative background risk exists. But they don't discuss how changes in multiplicative background risk affect the risk-taking attitude of a decision maker.

In the case where background risk is additive, this problem is considered by Eeckhoudt, Gollier, and Schlesinger [1], and Meyer and Meyer [5]. Eeckhoudt, Gollier, and Schlesinger consider First-degree Stochastic Dominance (FSD) changes and Second-degree Stochastic Dominance (SSD) changes in additive background risk. In particular, they give sufficient and necessary conditions for these changes to imply more risk averse attitude of the decision maker. Meyer and Meyer analyze the effect of changes in additive background risk on the demand for coinsurance. These changes in additive background risk include the strong increase in risk, simple FSD transformations, and simple SSD transformations. The strong increase in risk is a special case of SSD changes. Simple

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FSD transformations and simple SSD transformations are deterministic transformations. Also, they note in the footnote that they consider monotone probability ratio (MPR) changes. MPR changes are included in FSD changes. It is noted that MPR changes include Monotone Likelihood Ratio (MLR) changes.

In this paper, we focus on FSD changes in background multiplicative risk and analyze the effect of these changes on the risk-taking attitude of the decision maker. In section 2.1, we consider contractive FSD changes in background multiplicative risk. In section 2.2, we consider general FSD changes in background multiplicative risk. In section 3, we determine the effect of simple FSD changes and MLR changes in multiplicative background risk in the context of coinsurance.

2. Some FSD changes

2.1. Contractive FSD changes

In this subsection, we consider contractive FSD deteriorations in background multiplicative risk and their effect. The decision maker's utility function $u(w)$ is assumed to be strictly increasing and two times continuously differentiable. We suppose that $w > 0$. Denote the measure of relative risk aversion of $u(w)$ by $R_u(w)$ or R_u. First, we consider the case in which the initial background risk $\tilde{y}_1$ is positive and non-random, that is, $\tilde{y}_1 = y_1 (> 0)$. Let the background risk after change be $\tilde{y_2}$. Suppose that $\tilde{y}_2 = y_1\tilde{\epsilon}$. Here $\tilde{\epsilon}$ has n-point distribution $(n \geq 2)$ such that $P(\tilde{\epsilon} = \epsilon_i) = p_i (i = 1, 2, \cdot, \cdot, \cdot, n)$ and $0 < \epsilon_1 < \epsilon_2 < \cdot, \cdot, \cdot, < \epsilon_n \leq 1$.

Proposition 2.1. If relative risk aversion R_u is nonincreasing, then the decision maker becomes more risk averse after deterioration in the background multiplicative risk.

Proof. Without loss of generality, we assume that $y_1 = 1$. Then, $\nu_1(w) = Eu(wy_1) = u(w)$ and $\nu_2(w) = Eu(wy_1\tilde{y_2}) = Eu(w\tilde{\epsilon})$.

First, suppose that ϵ has two-point distribution, that is, $P(\tilde{\epsilon} = \epsilon_1) = p_1$ and $P(\tilde{\epsilon} = \epsilon_2) = p_2$ where $0 \leq p_i \leq 1 (i = 1, 2)$ and $p_1 + p_2 = 1$. Then, we must prove the following.

$$\begin{aligned} r_{\nu_2}(w) &= \frac{-E[u''(w\tilde{\epsilon})w\tilde{\epsilon}^2]}{E[u'(w\tilde{\epsilon})\tilde{\epsilon}]} \\ &= -\frac{p_1u''(w\epsilon_1)w\epsilon_1^2 + p_2u''(w\epsilon_2)w\epsilon_2^2}{p_1u'(w\epsilon_1)\epsilon_1 + p_2u'(w\epsilon_2)\epsilon_2} \\ &\geq -\frac{wu''(w)}{u'(w)} \\ &= r_{\nu_1}(w). \end{aligned} \tag{1}$$

Here $r_{\nu_i}(w) (i = 1, 2)$ is the measure of absolute risk aversion of $\nu_i(w) (i = 1, 2)$, respectively.

After some transformations, we can rewrite above inequality as

$$p_1u'(w\epsilon_1)\epsilon_1\{R_u(w\epsilon_1) - R_u(w)\} + p_2u'(w\epsilon_2)\epsilon_2\{R_u(w\epsilon_2) - R_u(w)\} \geq 0 \tag{2}$$

Since relative risk aversion R_u is nonincreasing, above inequality holds. It is clear from above proof for the case where ϵ has two-point distribution that $R_{\nu_2}(w) \geq R_{\nu_1}(w)$ holds when $\tilde{\epsilon}$ has n-point distribution. Q.E.D.

Now, we suppose that the initial background risk $\tilde{y}_1$ is random and that it has m-point distribution ($m \geq 2$) such that $P(\tilde{y}_1 = y_{1i}) = p_i (i = 1, 2, \cdot, \cdot, \cdot, m)$ and $0 < y_{11} < y_{12} < \cdot, \cdot, \cdot, < y_{1m}$. Let $\tilde{y}_2$ be written as $\tilde{y}_2 = \tilde{y}_1 \tilde{\epsilon}$. Here $\tilde{y}_1$ and $\tilde{\epsilon}$ are assumed to be stochastically independent. In this case, we need the notion of log-supermodularity(LSPM). Let $h(z, x) \equiv u'(zx)$. Recall that a nonnegative function $h(z, x)$ is LSPM with respect to z and x if $z_1 < z_2$ and $x_1 < x_2$ imply $h(z_1, x_1)h(z_2, x_2) \geq h(z_1, x_2)h(z_2, x_1)$. See, for example, Gollier [4].

Proposition 2.2. If relative risk aversion R_u is nonincreasing, and if $h(z, x)$ is LSPM with respect to z and x, then the decision maker becomes more risk averse after deterioration in the background multiplicative risk.

Proof. $r_{\nu_2}(w) \geq r_{\nu_1}(w)$ is rewritten as

$$\sum_{i,j} p_i q_j y_{1i}^2 \epsilon_j^2 u''(wy_{1i}\epsilon_j) \cdot \sum_l p_l y_{1l} u'(wy_{1l}) \leq \sum_{i,j} p_i q_j y_{1i} \epsilon_j u'(wy_{1i}\epsilon_j) \cdot \sum_l p_l y_{1l}^2 u''(wy_{1l}) \quad (3)$$

where $i, l = 1, 2, , \cdot, \cdot, \cdot, mandj = 1, 2, , \cdot, \cdot, \cdot, n$. Fix j. First, we show that the part of inequality (3) corresponding to $i = l$ is valid. As in the proof of Proposition 2.1, we can rewrite this part as

$$R_u(wy_{1i}\epsilon_j) - R_u(wy_{1l}) \geq 0. \quad (4)$$

Since R_u is nonincreasing, inequality (8) holds. Thus, the part of inequality (3) corresponding to $i = l$ is valid.

Next, consider the part of inequality (3) corresponding to $i \neq l$. To show that this part is valid, it suffices to prove that

$$\begin{aligned} & y_{1i}\epsilon_j u''(wy_{1i}\epsilon_j) \cdot u'(wy_{1l}) + y_{1l}\epsilon_j u''(wy_{1i}\epsilon_j) \cdot u'(wy_{1i}) \\ \leq \quad & u'(wy_{1i}\epsilon_j) \cdot u'(wy_{1i}\epsilon_j) y_{1l} u''(wy_{1l} + u'(wy_{1l}\epsilon_j) y_{1i} u''(wy_{1i}). \end{aligned} \quad (5)$$

Since R_u is nonincreasing, it is easily found that in order to prove inequality (6) it is sufficient to show the following inequality.

$$\{R_u(wy_{1l}) - R_u(wy_{1i})\}\{u'(wy_{1l}\epsilon_j)u'(wy_{1i}) - u'(wy_{1i}\epsilon_j)u'(wy_{1l})\} \geq 0. \quad (6)$$

If $y_{1l} \leq (\geq) y_{1i}$, nonincreasing relative risk aversion R_u implies that the first term of inequality (6) is nonnegative (nonpositive). Also, the assumption that $h(z, x)$ is LSPM with respect to z and x implies that the second term of inequality (6) is nonnegative (nonpositive). Thus, in either case, inequality (6) holds. Above proof holds for any j. This completes the proof. Q.E.D.

2.2. General FSD changes

Now, we consider the case of general FSD changes. The decision maker's utility function $u(w)$ is assumed to be three times continuously differentiable. Denote the measure of relative prudence of $u(w)$ by $P_u(w)$(or P_u). Let the distribution functions of the initial background risk $\tilde{y}_1$ and the background risk after change $\tilde{y}_2$ be $F_1(w), F_2(w)$, respectively. We assume that the support of $\tilde{y}_1$ and the support of $\tilde{y}_2$ are contained in $[a, b]$ where $a > 0$. In this case, we have the following proposition. We prove this proposition by using the method of (**??**)

Proposition 2.3. Suppose that $R_u(wx) > 1$, $P_u(wx) \geq 2R_u$ for all x in $[a, b]$ and that $P_u(wx) \geq R_u(wy)$ for all x, y in $[a, b]$. Then, $\tilde{y}_2 \preceq_{FSD} \tilde{y}_1$ implies $r_{\nu_2}(w) \geq r_{\nu_1}(w)$ at w.

Proof. Since $R_u > 1$, $-yu'(wy)$ is increasing in y. Thus, $\tilde{y}_2 \preceq_{FSD} \tilde{y}_1$ implies

$$-\int_a^b yu'(wy)d(F_2 - F_1) \leq 0. \tag{7}$$

Therefore, $r_{\nu_2}(w) \geq r_{\nu_1}(w)$ is written as

$$\frac{-\int_a^b wy^2u''(wy)d(F_2 - F_1)}{\int_a^b yu'(wy)d(F_2 - F_1)} \geq \frac{-\int_a^b wy^2u''(wy)dF_1}{\int_a^b yu'(wy)dF_1}. \tag{8}$$

Integrating the LHS of (8) by part gives

$$\begin{aligned}\frac{-\int_a^b wy^2u''(wy)d(F_2 - F_1)}{\int_a^b yu'(wy)d(F_2 - F_1)} &= \frac{-\int_a^b[\{2wyu''(wy) + w^2y^2u'''(wy)\}(F_2 - F_1)]dy}{\int_a^b[\{u'(wy) + wyu''(wy)\}(F_2 - F_1)]dy} \\ &= \int_a^b \frac{-[\{2wyu''(wy) + w^2y^2u'''(wy)\}(F_2 - F_1)]}{u'(wy) + wyu''(wy)\}(F_2 - F_1)]} d\eta(y). \\ &= \int_a^b \frac{R_u(wy)(2 - P_u(wy))}{1 - R_u(wy)} d\eta(y), \end{aligned} \tag{9}$$

where η is defined as

$$\eta(y) \equiv \int_a^y \frac{\{u'(wt) + wyu''(wt)\}(F_2 - F_1)}{\int_a^b\{u'(ws) + wyu''(ws)\}(F_2 - F_1)]ds} dt.$$

Now, since $R_u > 1$, $\frac{R_u(wx)(2-P_u(wx))}{1-R_u(wx)} \geq P_u(wx)$ is written as $P_u \geq 2R_u$, which holds from the assumption. Thus,

$$\begin{aligned}\int_a^b \frac{R_u(wy)(2 - P_u(wy))}{1 - R_u(wy)} d\eta(y) &\geq \int_a^b P_u(wy)d\eta(y) \\ &\geq \int_a^b P_u(wy)d\eta(y) \\ &\geq \hat{P} \qquad (10) \\ &\geq \hat{R} \end{aligned}$$

$$\tag{11}$$

Here $\hat{P} \equiv \mathrm{inf}_{x \in [a,b]} \mathrm{P}_u(wx)$, and $\hat{R} \equiv \mathrm{sup}_{x \in [a,b]} \mathrm{R}_u(wx)$. Q.E.D.

Example 2.1. Let $u(w) = (\eta + \frac{w}{\gamma})^{1-\gamma}$, where $\eta > 0 and 0 < \gamma < 1$. We assume that $w > 0$. For this utility function, $R(wx) = wx(\eta + \frac{wx}{\gamma})^{-1}$ and $P(wx) = \frac{1+\gamma}{\gamma} wx(\eta + \frac{wx}{\gamma})^{-1}$. Here we assume that $wx > \frac{\eta\gamma}{1-\gamma}$. Then, $R(wx) > 1$. Also, $P_u > 2R_u$. Let x^* satisfy $P(\frac{\eta\gamma}{1-\gamma}) = R(wx^*)$. Then, for all $x, y \leq x^*$, $P_u(wx) \geq R_u(wy)$. Therefore, if supports of $\tilde{y_1}$ and $\tilde{y_2}$ is contained in $[\frac{\eta\gamma}{w(1-\gamma)}, x^*$, then, $\tilde{y_2} \preceq_{FSD} \tilde{y_1}$ implies $R_{\nu_2}(w) \geq R_{\nu_1}(w)$.

Remark 2.1. Since $R_u(wx) > 1$, $P_u(wx) \geq 2R_u(wx)$ implies $P_u(wx) \geq 1 + R_u(wx)$. Thus, from Lemma 3 in [3], $R'_u(wx) < 0$ for $x \in [a, b]$. Thus, for $k = 1$, the set of utility functions in Proposition 2.3 is the subset of utility functions with nonincreasing relative risk aversion.

Remark 2.2. Suppose that $2 \geq R_u(wx) > 1$ for $x \in [a, b]$ and $P_u(wx) \geq 2R_u(wx)$ for $x \in [a, b]$. Then, from Proposition 2.3, it is easily seen that $\tilde{y}_2 \preceq_{FSD} \tilde{y}_1$ implies $R_{\nu_2}(w) \geq R_{\nu_1}(w)$.

In the next section, we continue the analysis in the coinsurance demand model.

3. Simple FSD changes and MLR changes in the coinsurance model

Meyer and Meyer [5] analyze the effects of changes in additive background risk on the demand for coinsurance. The changes in additive background risk which they refer to include Simple FSD changes in risk and Monotone Likelihood Ratio (MLR) changes in risk. MPR changes include Monotone Likelihood Ratio (MLR) changes. As stated earlier, it is MPR changes that they refer to in their footnote. These are special cases of FSD changes. First, We consider the effects of Simple FSD changes in multiplicative background risk on the demand for coinsurance. We use the same coinsurance model as Meyer and Meyer and use the method of Meyer and Meyer. The decision-maker is assumed to select θ in $[0, 1]$ to maximize $Eu(\tilde{w})$, where

$$\tilde{w} = \{M - \tilde{x} + \theta[I(\tilde{x}) - P]\}\tilde{y}. \tag{12}$$

Here $\tilde{y}$ is a positive multiplicative background risk which is distributed independently from $\tilde{x}$; M is the value of a risky asset; $\tilde{x}$ $(0 \leq \tilde{x} \leq M)$ is the insurable risk; P is the price of one unit of insurance; and $I(\tilde{x})$ is the indemnification of $\tilde{x}$. It is assumed that $I(x)$ is nondecreasing, continuous, and satisfies $0 \leq I(x) \leq x$. The utility function is assumed to be concave and continuously differentiable.

Now, we define the $m(y)$ function by $m(y) = yE_{\tilde{x}}[U'(\tilde{w})[I(\tilde{x}) - P]]$ where $E_{\tilde{x}}$ represents the expectation with respect to $\tilde{x}$. Following Meyer and Meyer, we examine the properties of this $m(y)$ function.

Lemma 3.1. (a) Suppose that $R(w)$ is nonincreasing. Then, at the optimal θ, there exists a y^* such that $m(y) \geq 0$ for all $y \leq y^*$ and $m(y) \leq 0$ for all $y \geq y^*$. Moreover, if we add the assumption that $R(w) \geq (\leq) 1$, then, $m(y) \geq (\leq) 0$ implies $m'(y) \leq 0$.
Proof. After simple calculation, $m'(y)$ is written as

$$m'(y) = E_{\tilde{x}}[u'(\tilde{w})[I(\tilde{x}) - P](1 - R(\tilde{w}))]. \tag{13}$$

As indicated by Meyer and Meyer in the proof of their Lemma 1, there exists a x_0 such that $I(x) - P \leq 0$ for all $x \leq x_0$ and $I(x) - P \geq 0$ for all $x \geq x_0$.

Here we define $z_0 = M - x_0 + \theta[I(x_0) - P]y$ and $z = M - x + \theta[I(x_0) - P]y$. Since z is decreasing with respect to x, $z_0 > z$ for $x > x_0$ and $z_0 < z$ for $x < x_0$. On the other hand, $R(z)$ is nonincreasing with respect to z. Thus, $R(z_0) \leq R(z)$ for $x \geq x_0$ and $R(z_0) \geq R(z)$ for $x < x_0$. Therefore, $[I(x) - P](1 - R(z)) \leq [I(x) - P](1 - R(z_0))$. It follows that

$$\begin{aligned} ym'(y) &= yE_{\tilde{x}}[u'(\tilde{w})[I(x) - P](1 - R(\tilde{w}))] \\ &\leq yE_{\tilde{x}}[u'(\tilde{w})[I(x) - P](1 - R(z_0)) \\ &= m(y)(1 - R(z_0)). \end{aligned} \tag{14}$$

If $R(z_0) \geq 1$, it follows that $m(y) \geq 0$ implies $m'(y) \leq 0$. If $R(z_0) \leq 1$, it follows that $m(y) \leq 0$ implies $m'(y) \leq 0$. In either case, Thus, if $m(y)$ changes sign, it must be true that the change is from positive to negative and that it occurs only one time. Q.E.D.

Similarly, in the case where $R(w)$ is nondecreasing, we can prove the following lemma.

Lemma 3.2. Suppose that $R(w)$ is nondecreasing. Then, at the optimal θ, there exists a y^* such that $m(y) \leq 0$ for all $y \leq y^*$ and $m(y) \geq 0$ for all $y \geq y^*$. Moreover, if we add the assumption that $R(w) \geq (\leq) 1$, then, $m(y) \leq (\geq) 0$ implies $m'(y) \geq 0$.

For the most part, above lemmas correspond to Lemma1 of Meyer and Meyer [5]. [1]

Now, we consider the effect of simple FSD changes in multiplicative background risk on the demand for coinsurance.

We represent multiplicative background risk as $\tilde{y} + \delta k(\tilde{y})$. Here $0 \leq \delta \leq 1$ and $k(\tilde{y}) = t(\tilde{y}) - \tilde{y}$, where $t(y)$ is nondecreasing in y.

Now, we set $\tilde{w} = [M - \tilde{x} + \theta[I(\tilde{x}) - p]][\tilde{y} + \delta k(\tilde{y})]$. The first order condition for the optimal θ is $f(\theta, \delta) = E_{\tilde{y}} l(\tilde{y}) = 0$. Here $l(\tilde{y}) = E_{\tilde{x}} u'(\tilde{w})[I(\tilde{x}) - p][\tilde{y} + \delta k(\tilde{y})]$. The partial derivative of $f(\theta, \delta)$with respect to δ is $f_\delta(\theta, \delta) = E_{\tilde{y}} k(\tilde{y}) E_{\tilde{x}}[I(\tilde{x}) - p]u'(\tilde{w})(1 - R(\tilde{w}))]$. Then, we must sign $f_\delta(\theta, \delta) = E_{\tilde{y}} k(\tilde{y}) n(\tilde{y})$.

Proposition 3.1. Suppose that $k(y) \leq 0, k'(y) \geq 0, R(w) \geq (\leq) 1$ and that $R(w)$ is nonincreasing (nondecreasing). Then, the optimal θ does not become smaller after simple FSD deteriorations.

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[1] We could not prove one property, that is, $E_y m'(y) \leq 0$ which is included in Lemma 1 of Meyer and Meyer [5].

Proof. We give proof for the case where $R(w) \geq 1$ and $R(w)$ is nonincreasing. For other case, similar proof can be given. As follows from the proof of Lemma 3.1,

$$E_{\tilde{y}} k(\tilde{y}) n(\tilde{y}) \geq E_{\tilde{y}} k(\tilde{y}) \frac{l(\tilde{y})}{\tilde{y}} (1 - R(z_0)). \tag{15}$$

Now, $k(y) \leq 0, k'(y) \geq 0$ imply $\frac{k(y)}{y}$ is nondecreasing in y. Thus, $R(w) \geq 1, k(y) \leq 0$ imply $\frac{k(y)}{y}(1 - R(z_0))$ is nonnegative and nonincreasing. It follows that

$$E_{\tilde{y}} k(\tilde{y}) \frac{l(\tilde{y})}{\tilde{y}} (1 - R(z_0)) \geq 0. \tag{16}$$

Therefore,

$$E_{\tilde{y}} k(\tilde{y}) n(\tilde{y}) \geq 0. \tag{17}$$

Hence, the optimal θ after simple FSD deteriorations becomes larger than $\theta(0)$. Q.E.D.

Next, we consider the effect of MLR changes. We suppose that $\tilde{y_1}$ dominates $\tilde{y_2}$ by monotone likelihood ratio ($\tilde{y_1}$ MLR $\tilde{y_1}$), that is, $\frac{f_2(y)}{f_1(y)}$ is nonincreasing in y. Here we denote respective densities of $\tilde{y_1}$ and $\tilde{y_2}$ by $f_1(y)$ and $f_2(y)$.

Proposition 3.2. Suppose that $k(y) \leq 0, k'(y) \geq 0$ and that $R(w)$ is nonincreasing. Then, the optimal θ does not become smaller after MLR deteriorations.

Proof. We need to show that $\int_c^d m(y) f_2(y) dy$ is nonnegative.

We write $\int_c^d m(y) f_2(y) dy$ as $\int_c^d m(y) f_1(y) \frac{f_2(y)}{f_1(y)} dy$. Then,

$$\begin{aligned} \int_c^d m(y) f_1(y) \frac{f_2(y)}{f_1(y)} dy &\geq \int_c^{y^*} m(y) f_1(y) \frac{f_2(y^*)}{f_1(y^*)} dy + \int_{y^*}^d m(y) f_1(y) \frac{f_2(y^*)}{f_1(y^*)} dy \\ &= \frac{f_2(y^*)}{f_1(y^*)} E_{\tilde{y}} m(\tilde{y}) \\ &= 0. \end{aligned} \tag{18}$$

The first inequality follows from the result of Lemma 3.1, that is, $m(y) \geq (\leq) 0$ where $y \leq (\geq) y^*$, and where $\frac{f_2(y)}{f_1(y)}$ is nonincreasing in y. The last equality follows from the fact that at the optimal value θ^*, $E_y m(y) = 0$. Q.E.D.

4. Concluding Remarks

In the first part (section 1 and section 2) we have considered FSD changes in general setting. In the second part(section 3) we have considered FSD changes in coinsurance setting. The results in the first part are useful in the second part because the concept, "more risk averse", would be an important condition in a coinsurance model. Also, the content of FSD changes in background risk is different. The first part contains general FSD changes. The second part treats some special well-known FSD changes. Thus, the conditions for general FSD changes in the first part are weaker than those for special FSD changes in the second part.

This work is a first step for studying changes in multiplicative background risk. Only FSD changes are considered. Much work must be done for understanding the risk taking attitude in face of changes in multiplicative background risk.

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