DEVELOPMENT, EVALUATION AND ANALYSIS OF A 20-YEAR DEFERRED ANNUITY PRODUCT

by

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Abstract:
This project analyzes an annuity product that suits the needs of many of today's American families under moderate assumptions. It helps in the study of the pricing accuracy in a mutual life insurance company and better understand the extent of the analysis and computations involved in developing a 20-year deferred annuity product designed for a group of 1000 people; ages ranging from 30 - 40 years and having a 5-year old child.

I. INTRODUCTION

In this project, we develop, evaluate and analyze a new annuity product that meets the needs of today's American families under moderate assumptions. This helps study the pricing accuracy in a mutual life insurance company and understand the analysis and computations involved in developing a 20-year deferred annuity product for a group of 1000 people; ages ranging from 30 - 40 years and having a 5-year old child. Questions such as the following will be discussed and answered in this project.

1. Evaluation of the gross premium
2. Balance sheet for the line of business for beginning of year 1 before any policy is sold, middle and end of year 1, middle and end of year 5, middle and end of year 10, and end of year 20 after all premiums are paid
3. Summaries of operations (profit and loss statement) for years 1, 2, 10, and 20
4. Analysis of the sources of any profit or loss made on the overall twenty-year line of business of this form

II. ASSUMPTIONS

For this project, pricing guidelines, actuarial assumptions, premiums and benefits are determined on the basis of:

1. **Pricing guidelines:**
   - Break-even between 5 to 10 years
   - Profit margin is 0.5%
2. **Actuarial assumptions:**
   - Interest rate of 5%
   - Life table assuming that mortality is based on the familiar 5% tables

3. **Premiums:**

   - Level gross premium paid at the beginning of each month during the deferred period

4. **Benefits:**

   - Survivor Annuity payment of \$3,000, payable at the beginning of each month after the deferred period
   - Death \$100,000 payable upon the moment of the death during the deferred period
   - Tuition A 13-year deferred 5-year temporary life annuity of \$15,000 payable continuously
   - Surrender Refund of premiums without interest subject to certain surrender charges

Evaluation of a complete premium analysis of a 20-year deferred annuity product requires additional assumptions. A moderate set of assumptions is sought to evaluate this product and to give a summary of reports and to have discussion of the findings. These assumptions can be easily justified under insurance industries standards. The following assumptions are made to analyze this product according to the given descriptions:

(a). During the entire process of this project, the UDD (Uniform Distribution of Death) assumptions were used in order to simplify the computations to a greater extent.

(b). The familiar 5% Life Table [1] is the source of all information.

(c). A group of 1000 people is distributed randomly according to their ages $30, 31, 32, \cdots, 39, 40$, as $110, 90, 100, 120, 70, 90, 80, 100, 80, 70$, and 90, respectively.

(d). For all amounts that are reflected in this project assumed to have all benefits paid at the time of the evaluation.

(e). It is also assumed that every person of this project group has a 9-year old child, who is qualified to receive the benefit.

(f). An additional amount of expenses has been considered to be a policy maintenance fee of \$50.00 per policy/per year in addition to a surcharge of $0.1 \times$ the gross premium G to be charged.

(g). Loading is 10% of the net premium G, which has been chosen to compensate the Federal Income Taxation (FIT), commissions payable and other expenses that have not been taken into consideration.

(h). Mortality rates for withdrawal $q_x^{(w)}$ are considered to be a multiple of $q_x^{(d)}$ for various age groups, where x is the age of the individual.

$$q_{x+k}^{(w)} = \begin{cases} 2q_{x+k}^{(d)} & \text{for } 0 \le k \le 5, \\ 1q_{x+k}^{(d)} & \text{for } 6 \le k \le 10, \\ \frac{1}{10}q_{x+k}^{(d)} & \text{for } 11 \le k \le 15, \\ 0 & \text{for } 16 \le k \le 20. \end{cases}$$

(i). To ease computational difficulties of integrations, at various stages of the project, we have considered the discrete analogue of the same with appropriate normalizing factor.

(j). We also assumed that at the end of the annuity payable period, namely 20 years, the benefits were all paid thus, making no contribution to the rest of the computations.

(k). Surrender Charges (SC); There is no Surrender Charges during the first year of the coverage, and the return of 5% of the gross premium collected thereafter from the issue of the policy will be made according to the table.

Rate of Return	Year
0	1
5%	2
10%	3
15%	4
⋮	⋮

III. PREMIUM ANALYSIS

The computations were based on the following rule of *Equivalence Principal* [1]. Notations are in accordance with abbreviations found in the literatures [1].

$$
\begin{aligned}
12 \times G\ddot{a}^{(12)}_{x:\overline{20}|} &= \text{DB} + \text{Tui.} + \text{Exp.} + \text{CV} + \text{Ann.} + \text{Pro. Mar., where} \\
\text{DB} &= \bar{A}^{1}_{x:\overline{20}|} \\
&= 100{,}000 \times \frac{i}{\delta}(A_x - {}_{20}E_{x+20}A_{x+20}); \\
\text{Tui.} &= 15{,}000 \times \frac{i}{\delta}(v^{13}\ {}_{13}p_x\ {}_{13}p_9 + v^{14}\ {}_{14}p_x\ {}_{14}p_9 + \\
&\quad v^{15}\ {}_{15}p_x\ {}_{15}p_9 + v^{16}\ {}_{16}p_x\ {}_{16}p_9 + v^{17}\ {}_{17}p_x\ {}_{17}p_9); \\
\text{Ann.} &= 12 \times 3{,}000 \times {}_{20}\left|\ddot{a}^{(12)}_{x}\right. \\
&= 12 \times 3{,}000 \times (\alpha(12)\ {}_{20}E_x\ddot{a}_{x+20} - \beta(12)\ {}_{20}E_x); \\
\text{CV} &= 12 \times G\sum_{k=0}^{19} 0.05 \times v^{k+1}k(k+1)q^{(w)}_{x+k}; \\
\text{Exp.} &= 0.1 \times 12 \times G\ddot{a}^{(12)}_{x:\overline{20}|} + 50 \times \ddot{a}_{x:\overline{20}|}; \\
\ddot{a}^{(12)}_{x:\overline{20}|} &= \ddot{a}^{(12)}_{x} - \ddot{a}^{(12)}_{x+20}\ {}_{20}E_x \\
&= \alpha(12)(\ddot{a}_x - {}_{20}E_x\ \ddot{a}_{x+20}) - \beta(12)(1 - {}_{20}E_x); \text{ and} \\
\text{Profit Margin} &= 12 \times 0.005 \times G\ddot{a}^{(12)}_{x:\overline{20}|}.
\end{aligned}
$$

Break Even analysis is subjected to

$$
\begin{aligned}
&12 \times G\ddot{s}^{(12)}_{x:\overline{10}|} - (\text{ Total Liabilities for first 10 years }) + 12 \times G^{a}\ddot{a}^{(12)}_{x+10:\overline{10}|} \\
= \ &\text{PV[Future Liabilities]},
\end{aligned}
$$

based on the end of the year calculation to solve for G^a, where

Ann. - Annuity

Tui. - Tuition

Exp. - Expenses

DB - Death Benefits

CV - Cash Value

PV - Present Value

Pro. Mar. - Profit Margin.

IV. SUMMARY OF RESULTS

The following discussions and analysis were appropriate for this study.

(a). The initial computation shows that the premium is so expensive beyond someone's imagination, mainly due to the nature and the extent of benefit package that has been designed. This could be a real burden on an average family, because the gross premium is too large (more than \$1500 per month). A substantial reduction could be obtained for a lesser benefit package. For example, we could decrease the annuity payment to \$1500 - 1000.

(b). It is also assumed that the Insurance Industry does not have a starting funds to begin with its operations, and also to meet any maintenance cost, and/or any operational logistics.

(c). An increase of the premium is suggested to off set any reserve, if such reserve is required by the Law.

(d). Since the product is forced to be break-even at the end of 10 years it continued to generate profit in the subsequent years unless an unexpected epidemic caused to have died a larger portion of this group, which in fact could deviate from the figures given in the Life Table.

(e). In our case, it is required $G^a < G$ to break even at the end of year 10, where G^a is the adjusted premium. We could have achieved break even prior to 10 years, if we were to use of G instead of G^a.

(f). The Profit/Loss in usual notations is computed according to

$$\text{Profit(Loss)} =$$

$$\text{(Annu. + Tuit. + Death Bene. + Surr. Charges + Expen. + Profit - G)},$$

where G is the gross premium.

The summary of results is comprising the following.

(i). Year-end worksheets depicting the amounts of premium collected, benefits paid, expenses incurred etc. for year 1 to year 20 - Worksheets (a) are attached.

(ii). Worksheets that depict and summarize the break even analysis - Worksheets (b) are attached.

(iii). Yearly account summary after premium adjustment - Worksheets (c) are attached.

V. WORKSHEETS

Worksheets for (i), (ii) and (iii) of IV are given below in US$.

(1). Evaluation of Gross Premium (Prem.):

Age Group (x)	No. of People	Middle of the year 1				
		Premium	Death Ben.	Annuity	Tuition	Loss/Profit
30	110	975069.79	4259.23	10057764.04	1956742.16	11043695.64
31	80	698246.48	3259.59	7162574.02	1419055.03	7886642.16
32	90	772931.91	3872.03	7881252.90	1591456.67	8703649.69
33	100	844456.03	4530.02	8554680.30	1762227.87	9476982.15
34	70	580815.36	3365.88	5842400.50	1228907.92	6493858.94
35	60	488804.62	3052.07	4879199.72	1048998.98	5442446.14
36	80	639426.43	4332.62	6329389.91	1392321.95	7086618.06
37	110	861926.59	6347.09	8454299.42	1904934.66	9503654.57
38	120	921075.52	7379.63	8944919.69	2066803.00	10098026.79
39	60	450768.36	3947.95	4330192.16	1027231.95	4910603.70
40	120	881704.09	8442.53	8369637.10	2041028.48	9537404.02
Total	1000					

End of the year 1				
Premium	Death Ben.	Annuity	Tuition	Loss/Profit
1972707.83	17247.33	19896370.87	3870847.19	21811757.56
1412597.42	13199.38	14168508.50	2807076.51	15576186.98
1563620.30	15679.39	15589446.08	3147967.49	17189472.66
1708234.58	18343.86	16920750.03	3485602.76	18716462.07
1174855.92	13629.80	11555352.49	2430587.26	12824713.64
988684.81	12359.05	9649760.00	2074641.12	10748075.36
1293256.95	17544.52	12517034.53	2753463.16	13994785.26
1743147.03	25701.90	16718092.47	3766944.15	18767591.49
1862628.70	29883.07	17686946.26	4086725.74	19940926.36
911480.58	15986.83	8561434.24	2030990.42	9696930.90
1782697.30	34187.22	16546522.69	4035052.36	18833064.97

End of the year 2				
Premium	Death Ben.	Annuity	Tuition	Loss/Profit
4043728.09	37148.83	40784292.27	7934600.96	44712313.97
2895564.42	28525.33	29042824.13	5753988.11	31929773.16
3205133.48	33786.89	31955483.32	6452751.57	35236888.30
3501495.91	39844.38	34683715.08	7144698.25	38366761.81
2408189.83	29502.56	23685861.92	4982154.75	26289329.40
2026540.38	26920.48	19779431.17	4252460.29	22032271.55
2650805.48	38237.20	25656320.70	5643807.55	28687559.96
3572915.10	56027.24	34266937.00	7721074.52	38471123.66
3817739.13	65386.72	36252060.91	8376360.07	40876068.57
1868198.47	34950.79	17547772.78	4162778.97	19877304.07
3653797.48	74916.95	33913568.52	8270198.36	38604886.35

Middle of the year 5				
Premium	Death Ben.	Annuity	Tuition	Loss/Profit
9692549.64	52238.89	98964223.69	19253530.71	108577443.65
6892262.78	40176.55	70475696.81	13962702.79	77586313.39
7626259.51	48102.64	77545539.26	15658724.21	85626106.60
8328083.13	56918.84	84169445.55	17338548.91	93236830.17
5725114.33	42443.60	57482189.89	12090975.04	63890494.21
4815458.52	38904.68	48004010.68	10320577.34	53548034.17
6295427.34	55431.32	62270148.05	13698017.56	69728169.60
8480339.21	81648.97	83173047.55	18740668.25	93515025.56
9055618.94	95580.26	87996662.27	20332408.96	99369032.54
4428184.76	51334.73	42597261.76	10105156.23	48325567.97
8653855.82	110634.57	82330650.31	20077238.68	93864667.74

End of the year 5					Adjusted Total Premium	Premium Divisor	Premium Req'd To break-even Adj. Premium
Premium	Death Ben.	Annuity	Tuition	Loss/Profit			
10896428.39	117566.78	109899268.49	21380948.21	120501355.08	131397783.47	7475.93	17576.11
7802373.58	90419.77	78258586.88	15504655.35	86051288.42	93853862.00	5436.44	17263.81
8636190.41	108257.91	86103563.62	17386840.94	94962472.06	103598862.47	6115.20	16941.17
9434451.07	128099.29	93451929.03	19250701.15	103396278.41	112830729.48	6793.71	16608.11
6488380.33	95521.90	63816711.69	13423397.21	70847250.37	77335630.70	4754.88	16264.46
5459883.83	87557.34	53289492.67	11456924.59	59374090.76	64833974.60	4074.92	15910.49
7141485.41	124751.54	69120156.94	15204863.86	77308286.94	84449772.34	5432.25	15546.01
9625240.24	183755.97	92313201.80	20800140.68	103671858.21	113297098.45	7467.84	15171.33
10284266.12	215109.17	97656106.35	22564309.15	110151258.55	120435524.67	8144.94	14786.55
5032269.30	115531.93	47267486.81	11213052.65	53563802.10	58596071.40	4071.49	14391.81
9841351.96	248989.82	91344715.90	22275418.16	104027771.93	113869123.88	8140.77	13987.51

Middle of the year 9				
Premium	Death Ben.	Annuity	Tuition	Loss/Profit
23269099.02	169835.50	237584966.07	46222253.58	260707956.11
16662143.33	131926.36	169186179.15	33519304.43	186175266.61
18443333.43	158908.20	186151736.07	37589508.38	205456819.21
20148695.56	189356.54	202044635.84	41620338.33	223705635.15
13857334.28	142289.53	137976959.93	29022484.74	153284399.92
11661168.70	131332.14	115220304.13	24771681.42	128462148.99
15253254.57	188475.72	149453963.41	32876475.81	167265660.36
20559151.99	278718.06	199611790.15	44976809.77	224308165.98
21967801.41	328078.51	211174487.20	48793737.44	238328501.74
10749766.33	177019.25	102217773.46	24248661.25	115893687.63
21024038.90	382603.42	197548425.59	48174366.11	225081356.22

End of the year 10				
Premium	Death Ben.	Annuity	Tuition	Loss/Profit
24785979.28	362286.60	249986232.06	48634925.00	274197464.38
17746721.57	281420.26	178001135.45	35265730.79	195801564.94
19641880.74	338976.90	195830964.66	39544029.19	216072090.01
21455695.27	403928.14	212526750.74	43779609.56	235254593.17
14754448.31	303526.58	145117729.14	30524495.41	161191302.82
12414419.44	280152.69	121166893.55	26050162.84	135082789.65
16236167.24	402049.20	157144452.66	34568208.68	175878543.30
21880561.25	594550.71	209850529.89	47283817.03	235848336.39
23375630.77	699844.54	221967204.98	51287490.57	250578909.33
11436487.22	377610.69	107421363.93	25483085.54	121845572.94
22362371.20	816154.99	207561066.64	50616059.26	236630909.69

End of the year 20				
Premium	Death Ben.	Annuity	Tuition	Loss/Profit
64973091.45	1927821.28	655303006.51	0.00	592257736.34
46507505.90	1514189.99	466472781.21	0.00	421479465.30
51458437.90	1839362.50	513042700.21	0.00	463423624.81
56191611.84	2208331.86	556597332.81	0.00	502614052.83
38626876.37	1672396.53	379914359.70	0.00	342959879.86
32487911.19	1549429.88	317086658.53	0.00	286148177.22
42470748.92	2232399.21	411058921.70	0.00	370820571.99
57207599.05	3317361.00	548660828.06	0.00	494770590.00
61084295.04	3912347.02	580034280.72	0.00	522862332.71
29867791.44	2115490.91	280543198.56	0.00	252790898.02
58365267.62	4569451.95	541727825.85	0.00	487932010.19

Benefit	Annuity	Tuition	Expenses	Surrender	Profit	Prem.	BR Adj Prem.
3331.98	181153.26	35140.89	26490.41	725.70	14698.28	1714.54	1582.22
3562.49	177391.96	35049.20	26070.31	763.22	14259.69	1688.77	1555.34
3814.09	173511.04	34948.96	25637.93	803.50	13816.36	1662.31	1527.69
4089.17	169510.92	34893.48	25193.55	846.80	13369.15	1635.18	1499.30
4389.16	165390.71	34719.61	24737.07	893.11	12918.83	1607.41	1470.17
4716.48	161153.25	34588.79	24269.13	943.07	12466.40	1579.01	1440.32
5074.73	156798.48	34445.68	23789.94	996.42	12012.96	1550.03	1409.79
5463.33	152329.97	34289.50	23299.72	1052.98	11559.10	1520.47	1378.61
5887.15	147750.25	34119.03	22799.28	1113.49	11106.25	1490.39	1346.81
6348.58	143062.53	33932.98	22289.11	1178.01	10655.52	1459.83	1314.43
6850.76	138272.09	33730.22	21770.11	1246.91	10208.14	1428.86	1281.52

(ii). Break even Analysis:

Age	BR Ben F10 yr	BR Ben S10 yr	BR Sur F10 yr	BR Sur S10 yr	BR Exp F10 yr
30	225101.63	313581.06	59.61	14.09	1669.20
31	195369.51	276030.98	51.89	12.38	1364.01
32	230586.67	330261.00	61.45	14.78	1513.49
33	294483.76	427099.99	78.72	19.08	1813.48
34	183138.57	268718.42	49.08	11.98	1056.12
35	251416.17	372813.38	67.57	16.60	1355.42
36	239038.41	357754.07	64.39	15.91	1202.42
37	319946.03	483025.28	86.33	21.47	1499.75
38	274404.64	417499.50	74.17	18.55	1196.94
39	257716.78	394812.51	69.76	17.54	1044.58
40	356008.61	548626.90	96.51	24.37	1339.17

BR Exp F10 yr	BR Exp S10 yr	BR Exp S10 yr	BR Tuition	BR Annuity
71361.47	1006.32	42932.07	6556268.68	33797957.28
58326.56	821.11	35035.80	5363934.71	27148090.83
64734.03	909.63	38818.76	5959596.16	29587599.85
77585.08	1087.99	46438.43	7151106.89	34793595.67
45196.10	632.38	26997.01	4171262.11	19870267.59
58022.76	809.87	34581.25	5362816.17	24985990.80
51490.87	716.79	30613.23	4766781.99	21698631.82
64247.91	891.76	38095.26	5958176.72	26469007.03
51297.12	709.72	30326.67	4766343.92	20640343.02
44787.81	617.48	26393.00	4170417.26	17582613.53
57447.96	788.98	33733.68	5361873.15	21980210.86

BR Pre F10 yr	BR Pre S10 yr	BR Pro F10 yr	BR Pro F10 yr	BR Adj Prem	Premium
18049.08	10751.50	83.46	50.32	1582.22	1714.54
14816.66	8808.60	68.20	41.06	1555.34	1688.77
16522.86	9801.10	75.67	45.48	1527.69	1662.31
19906.07	11778.45	90.67	54.40	1499.30	1635.18
11662.27	6881.12	52.81	31.62	1470.17	1607.41
15065.45	8860.81	67.77	40.49	1440.32	1579.01
13460.95	7888.64	60.12	35.84	1409.79	1550.03
16920.78	9876.42	74.99	44.59	1378.61	1520.47
13619.68	7913.70	59.85	35.49	1346.81	1490.39
11996.97	6935.49	52.23	30.87	1314.43	1459.83
15537.23	8930.96	66.96	39.45	1281.52	1428.86

(iii). Account Summary After Premium Adjustment:

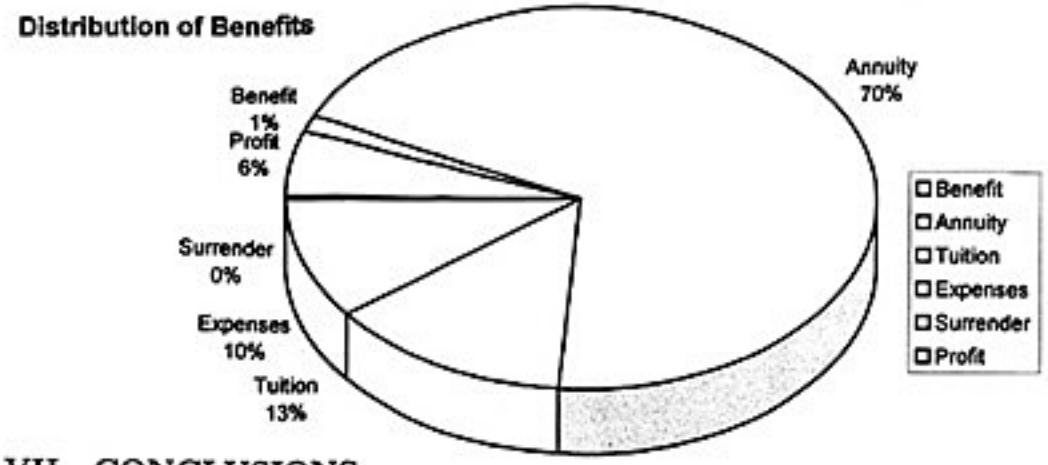
Loss/Profit Summary Statement

Age Group (x)	No. of People	Year 0.5	Year 1	Year 2	Year 4.5	Year 5	Year 9.5	Year 10	Year 20
30	110	11043695.64	21811757.56	44712313.97	108577443.65	120501355.08	260707956.11	274197464.38	592257738.34
31	80	7886642.16	15576186.98	31929773.16	77586313.39	86051288.42	186175266.61	195801564.94	421479465.30
32	90	8703649.69	17189472.66	35236888.30	85626106.60	94962472.06	205456819.21	216072090.01	463423624.81
33	100	9476982.15	18716462.07	38366761.81	93236830.17	103396278.41	223705635.15	235254593.17	502614052.83
34	70	6493858.94	12824713.64	26289329.40	63890494.21	70847250.37	153284399.92	161191302.82	342959879.86
35	60	5442446.14	10748075.36	22032271.55	53548034.17	59374090.76	128462148.99	135082789.65	286148177.22
36	80	7086618.06	13994785.26	28687559.96	69728169.60	77308286.94	167265860.36	175878543.30	370820571.99
37	110	9503654.57	18767591.49	38471123.66	93515025.56	103671858.21	224308165.98	235848336.39	494770590.00
38	120	10098026.79	19940926.36	40876068.57	99369032.54	110151258.55	238328501.74	250578909.33	522862332.71
39	60	4910603.70	9696930.90	19877304.07	48325567.97	53563802.10	115893667.63	121845572.94	252790898.02
40	120	9537404.02	18833064.97	38604886.35	93864667.74	104027771.93	225081356.22	236630909.69	487932010.19
Total	1000								

TUITION	30	31	32	33	34	35	36	37	38	39	40
Year 1	14693.13	14662.12	14628.09	14590.90	14550.23	14505.80	14457.15	14404.01	14345.92	14282.47	14213.25
Year 2	14619.64	14584.41	14545.87	14503.88	14457.82	14407.57	14352.53	14292.42	14226.83	14155.18	14076.94
Year 3	14541.49	14501.78	14458.47	14411.11	14359.26	14302.65	14240.69	14173.11	14099.38	14018.78	13930.95
Year 4	14540.58	14500.09	14455.79	14407.43	14354.46	14296.64	14233.47	14164.57	14089.33	14007.27	13917.95
Year 5	14460.84	14415.51	14365.98	14311.87	14252.66	14188.12	14117.60	14040.64	13956.76	13865.40	13766.01

VI. DISTRIBUTION OF BENEFITS

As we see in the Pie chart, a larger portion of the benefit package is for the Annuity. Death Benefit is about 2%, and rest of the benefits is less than 1%.



VII. CONCLUSIONS

This topic has been presented in a course on life contingencies. The method of analysis gives an accurate way to pricing an insurance product. The motivation is based on use of *equivalence principle*. Generalization of a complicated product can be similarly derived due to familiar steps already found in this analysis. The entire example presented here, and many similar ones, can be done with a minimal amount of calculations. Thus, the method is a source of many ideas and computations. Though granted, this may not provide much motivation for us, but the method is an interesting technique using key ideas from the topics. As such, it is worthy of consideration for actual insurance model used in the industry.

VIII. ACKNOWLEDGEMENT

The author wishes to thank Professor Lija Guo, Department of Mathematical Sciences of Ball State University for presenting this senior team project assignment [2] for further exploration with some pertinent insights to the problem in hand. This helped us to learn the pricing practices and analysis that exists in the mutual life insurance company. We thank her for her patience and quick resolution for our questions during the entire project.

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