From Fault Tree to Credit Risk Assessment : An Empirical Attempt

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Abstract

Since 80', fault tree theory has known a great development in industrial systems' sector. Its first goal is to estimate and model the probability and events' combination which could lead a given system to failure. Later static and dynamic studies arise such as Dugan, Venkataraman & Gulati (1997), Gulati & Dugan (1997) and Ngom et al. (1999) for example. Improvements are also proposed by Anand & Somani (1998), Zhu et al. (2001) and Reory & Andrews (2003) among others. Since credit risk valuation attempts to quantify firms' default risk, we propose to apply one alternative approach of fault tree, or equivalently, reliability study to assess firms' default risk. We set a very simple framework and use French firms' bankruptcy statistics to quantify default probabilities. From these empirical default probabilities and under the assumption that the lifetime process follows an exponential law with a constant parameter, we estimate this constant parameter for French sectors. Each parameter's estimation corresponds to the related hazard rate on the time horizon under consideration. Checking for the consistency of our constant parameter's assumption, we compute the monthly implied parameters related to our exponential law setting. Results show a time varying behavior for those parameters. Indeed, each exponential law's parameter is a convex decreasing function of time. Whatever, such an approach may be useful to give a statistical benchmark for common credit risk models' improvement.

Keywords : credit risk, default probability, failure rate, fault tree, reliability, survival.

JEL codes : C1, D8.

1 Introduction

Risk represents a measure of danger which is expressed as a function of both an undesirable event's realization (i.e., probability, frequency) and some measure of this event's effects or consequences. In our study, we define a danger as a situation whose consequences may harm society (i.e., production loss, financial loss). To assess a danger's importance, critical levels are established through a risk scale. Such a risk scale is computed starting from the studied event's realization frequency and effects. Reliability allows to study the required conditions leading to some given undesirable event. Specifically, fault tree theory represents the events' combinations leading to this event's realization through a deduction process. This theory takes place in an operational approach of risk since its goal is to reduce potential risks in case of dangerous or disastrous situations, and to establish or develop compensation and recovery policies in case of default.

Reliability leads to study systems which are subject to some physical process such as (de)fault or repairing among others. Such an analysis assumes the availability of both qualitative (i.e., working state, default triggering) and quantitative (i.e., failure rate, repair rate) informations about the studied system. Moreover, we could consider simple systems (i.e., systems with a single component) or complex systems (i.e., multi-components systems).

In this paper, we consider only single component systems and we assume that only sudden and complete defaults could occur. Two main families of reliability theory could be distinguished : minimal sets models and stochastic process models[1]. Fault tree theory belongs to the first family and is mainly designed to study complex systems. Barlow & Proschan (1975) introduce this approach through the notion of 'event tree'. The study of a system's reliability requires successively a tree building, a qualitative analysis and a quantitative study. The first step's goal is to represent any cause explaining the system's default. The qualitative analysis studies any events' combination corresponding to a minimal set[2], leading to default. Differently, the quantitative analysis first achieves valuation of the tree top-event[3] probability, this undesirable event corresponding to default. Second, this analysis assesses the influence and importance of the events entering the combination which leads to our system's default. Notice that fault tree theory could be also used while dividing our system's state space in working states and non working states for any system modeled through some

[1] For further information, the reader could refer to Limnios (1991) book. This author realizes an exhaustive presentation of fault tree theory.

[2] A minimal set is the smallest events' combination whose simultaneous realization leads to the undesirable event's realization (i.e., default). Namely, minimal sets are significant defaults' combinations. Fault trees' reduction through computation of minimal sets allows to identify critical ways leading to a given system's default.

[3] In a fault tree, the top-event represents the origin (i.e., the top) of the tree which is default state, or equivalently, the worst possible event.

stochastic process. Such an approach is realized, among others, through credit risk structural approach valuation like Merton (1974) or reduced form approach like Jarrow & Turnbull (1995).

Consequently, following our remark, we could apply fault tree theory's alternative approach to assess credit risk. We therefore attempt to apply fault tree theory (mostly used in industry) to assess any firm's default probability in a very simple case. We consider that only one event generates the firm's default, namely our undesirable top-event (without focus on default's severity in our study).

Our paper is organized as follows. Section 2 introduces the theoretical framework and makes a parallel with credit risk valuation. It also gives some key insights to fit reality while modeling a firm's default process. Section 3 attempts to achieve an empirical application of fault tree theory. Finally, section 4 exposes concluding remarks and further required extensions.

2 Fault tree theory

In this section, we make a parallel between the two existing approaches of fault tree theory while assessing a firm's default probability. One of this approach has been commonly used in credit risk literature : structural models and reduced form ones. In such models and therefore one fault tree theory's approach, working state and non working state are modeled through a stochastic process. We propose here to apply the alternative approach.

2.1 Assumptions and framework

In structural models, bankruptcy occurs when a firm assets value crosses a given threshold while decreasing. Such a barrier often corresponds to the book value of the firm's debt[4] (see for example, Black & Cox [1976] or Longstaff & Schwartz [1995]). Indeed, a firm goes bankrupt when it becomes unable to honour its contractual debt commitments and therefore generates losses for its creditors. In reduced form models also called intensity models, the default state is modeled by a bankruptcy jump process which, following a jump, switches from a non default state to a default state[5] (see Jarrow, Lando & Turnbull [1997] for example). Sometimes default is not complete since some given recovery rate is guaranted, depending on each debt contract's terms.

We encompass the two standviews of structural and reduced form approaches while considering the following elementary tree :

[4] This is equivalent to observe if the firm's solvency ratio crosses down a unit barrier.

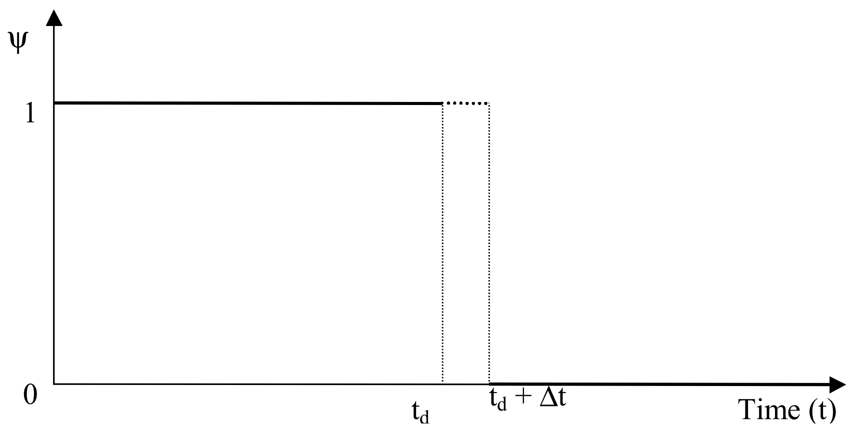
[5] The probability that a jump occurs over a given time set is driven by a default intensity, or equivalently, a hazard rate process. The interested reader could refer to the book of Ammann (2001) for a detailed description of each type of credit risk models.

Elementary default tree.

When a default event occurs, the firm goes bankrupt which means that its lifetime is forced to end. Specifically, we consider one source of economic losses risk which may be harmful for any firm, namely default risk. Our framework requires to make the following assumptions about any firm's default :

- We consider a unique component system corresponding to the firm.
- We are in a continuous time case.
- Any default event is related to firm's debt outstandings.
- Default occurs suddenly and completely at any time[6], which means that our system is non repairable here.
- Before default, the firm is always in a working state.

Indeed, any default event represents a bankruptcy triggering and signals the firm's imminent death. Namely, default is not reversible and occurs immediately after failure so that the default state is an absorbing state. We characterize such a situation underneath through the plot of the related function structure. The function structure Ψ is equal to one as long as the firm is in a working state, and it switches to zero when the firm defaults.



Default process.

[6] Clearly, default may occur at any time between a debt's issue date and its related maturity date. When this event occurs, default is complete which means that the firm ends its activity and is bound to undergo a liquidation. There may be a possible recovery or not for creditors after default. Since we are not interested in recovery conditions here, we make no assumption about a possible recovery rate.

where

- Time 0 is the starting date of observation;
- t_d is the time when a default event occurs;
- $t_d + \Delta t$ corresponds to liquidation date;
- Δt is a negligible time subset.

Therefore, our default study, or equivalently, our fault tree analysis, leads us to concentrate on the firm's lifetime X. Our firm's lifetime is then the heart of our reliability study while considering our firm's default probability in a simple framework.

2.2 Some basic notions

In this subsection, we introduce some basic notions related to our reliability framework and firm's default study.

Let our firm's lifetime X be a random variable corresponding to the time interval during which the firm is non defaulting, or equivalently, in a working state. Specifically, for a firm issuing a debt of maturity T at time 0 or having debt outstandings at this initial time 0, X represents the random time (starting from the observation date 0) at the end of which the firm may default or switch to a non working state[7]. Considering our firm's default probability[8] p_t at time t is then equivalent to consider the probability that its lifetime ends between 0 and t. Namely, we consider :

$$p_t = F(t) = P(X \leq t) \tag{1}$$

where

- t is the current date;
- $F(.)$ is the cumulative distribution function associated to X and supposed to be absolutely continuous;
- with conditions[9] : $F(0) = 0$ and $F(+\infty) = 1$.

[7] Notice that we consider binary systems here since we only have a two states space : default (i.e., working) and non default (i.e., non working). Moreover, we do not focus on the firm's debt structure here since its debt outstandings (i.e., debt's existence) generate a default risk (i.e., firm's death risk).

[8] This is the probability that the firm defaults between times 0 and t, or equivalently, the probability that a default event occurs between times 0 and t.

[9] Initially, the probability of default, for example at the issue date of debt, is zero since the firm is in a working state (i.e., non default state) at this time. And the probability of default when time tends towards infinity is naturally equal to one. Indeed, the longer the time horizon, the riskier the firm's situation and surrounding uncertainty (i.e., the higher default risk).

Therefore, the distribution function or density $f(.)$ of the firm's lifetime may be expressed as follows for each $t \geq 0$:

$$f(t) = \frac{dF(t)}{dt} = \lim_{\Delta t \to 0^+} \frac{P(t < X \leq t + \Delta t)}{\Delta t} \tag{2}$$

Notice that the event $\{t < X \leq t + \Delta t\}$ means that the firm's default event occurs at time t and therefore the firm defaults between dates t and $t + \Delta t$. Moreover, we are able to define the related mean time to failure which corresponds to the expected firm's lifetime. Namely, mean time to failure takes the form :

$$E[X] = \int_0^{+\infty} s\, f(s)\; ds \tag{3}$$

Analogously to its lifetime, we could consider the firm's survival function $R(.)$ which is a complementary notion. The firm's survival or our system's reliability is defined by the next relation for each $t \geq 0$:

$$R(t) = 1 - F(t) = P(X > t) = \int_t^{+\infty} f(s)\; ds \tag{4}$$

with $R(0) = 1$ and $R(+\infty) = 0$. Such a concept[10] leads to the notion of hazard rate also called (de)fault rate in our case. The instantaneous hazard rate $\lambda(t)$ is expressed in the following way for each $t \geq 0$:

$$\lambda(t) = \lim_{\Delta t \to 0^+} \frac{P(t < X \leq t + \Delta t \mid X > t)}{\Delta t} \tag{5}$$

with $\lambda(t) \geq 0$ and $\int_0^{+\infty} \lambda(s)\; ds = +\infty$. Then, the cumulative hazard rate on the time interval $[0, t]$ is $H(t) = \int_0^t \lambda(s)\; ds$ whereas the total hazard rate during the firm's lifetime equals $H = \int_0^X \lambda(s)\; ds$ where H follows an exponential law with a parameter equal to one.

Given the previous basic notions, we could introduce some fundamental relation satisfied by reliability, namely[11] :

$$\frac{dR(t)}{dt} + \lambda(t)\; R(t) = 0 \tag{6}$$

[10] Analogously to relation (3), we could define the mean remaining survival. Refer to appendix for details.

[11] Default process satisfies a Kolmogorov type equation. For further details, the reader is invited to consult the teaching manual of Roussignol & Filipo (2002) for example.

under limit conditions $R(0) = 1$ and $R(+\infty) = 0$. Resolution of relation (6) leads to the next expression for the survival function[12] :

$$R(t) = \exp\left[-\int_0^t \lambda(s)\, ds\right] = e^{-H(t)} \tag{7}$$

From a practical point of view, reliability requires to postulate some law for our firm's survival, or at least to know its distribution[13]. Four probability laws are commonly used, in this context, to characterize our firm's survival time. The most used is the exponential law since it characterizes the markov property of a system's stochastic behavior (i.e., without memory). The lognormal law is also used for two reasons. In one hand, it models repairing times in a good manner and, in an other hand, it characterizes uncertainty's propagation in fault trees. Finally, Weibull and Gamma distributions are also used due to the variety of probability distribution's shapes they encompass through their parameters.

First of all, systems' reliability and therefore firms' reliability requires data related to components or firms. Namely, we have to know default and repairing probabilities or failure and repairing rates for example. Such data are obtained through empirical observations or simulations eventually (i.e., to deduce estimations illustrating the empirical behavior of firms). At least, historical data and observations are required to proceed to our default study along with fault tree theory. Of course, such data could be treated parametrically or non parametrically to get the required risk measures (i.e., default probabilities here).

3 Empirical application

To adapt the alternative approach of fault tree theory in order to study firms' default, we have to analyze some historical data. Since we are interested in firms' default and we consider the simplest framework, we need data allowing us to compute empirical default probabilities. For this purpose, we will focus on French firms' bankruptcy statistics.

[12] We give a complementary basic notion known as maintainability in the appendix.

[13] Indeed, the probability that the top-event occurs is driven by the probability that the basic event occurs in our elementary tree framework. In some simple systems, there may be many basic events. We find systems where one component's default generates the other components' defaults and therefore the system's failure. There are also simple systems where all components' simultaneous defaults generate the system's default. Whatever the considered case, quantifying default requires knowledge about the probability (or probabilities) of basic event's (or events') realization, or equivalently, their statistical laws. Recall that reliability is a static failure analysis in its simplest form and also in our elementary framework.

3.1 Data

We consider monthly data issued by the INSEE[14] (i.e., the French National Institute of Statistics and Economic Studies) and running from january 1990 to december 1999, namely nine years (i.e., 120 observations per series). We consider the total[15] number of French corporate bankruptcy cases and numbers of coporate failures for 16 economic sectors. The sectors[16] we study are private services (SP), food processing sector (IA), hotels, catering and cafés industry (HR), business services (SE), real estate (IM), sport industry (TT), other specialized retail trade (DS), specialized food retail trade (DA), non-specialized retail trade (DN), non-food wholesale trade (GN), food wholesale trade (GA), construction and civil engineering (BP), intermediate goods (BI), capital goods (BE), consumer goods (BC), and finally motor trade and repairing industry (AU). We also consider the total number of existing firms and numbers of existing firms for each sector above-mentioned. Such statistics allow us to compute empirical default probabilities $\hat{p}_t$ (for all firms or for a given sector) as follows for $t \in \{1, ..., 120\}$:

$$\hat{p}_t = \frac{\text{number of failures}}{\text{number of firms}} \tag{8}$$

where the number of firms is the sum of defaulting and non defaulting firms. Recall that failure is envisioned without considering potential losses' severity and firms' debt structure in our study.

However, empirical default probabilities computed here exhibit aberrant (i.e., abnormal) points showing a sharp decrease. This type of decrease takes place mostly in school holiday times depending on the considered year. Those date effects or calendar anomalies come from the fact that, during holiday, French Courts dealing with trade disputes highly slow down their activity, and only deal with urgent receivership proceedings. To solve such anomalies, we realize a smoothing of default probabilities while achieving a linear interpolation on anomalies' occuring times. Namely, abnormal empirical default probabilities are smoothed while replacing each of them with the arithmetic mean of default probabilities of the previous and following months respectively[17].

After our smoothing, we get the following descriptive statistics about default probabilities with 'TOTAL' refering to the general empirical default probability all sectors included. We present our results in the following table.

[14]Although INSEE data consider all French firms, 98% of these data are related to small and medium enterprises (i.e., SMEs). On the other hand, they concern bankruptcy fillings tried by the Court dealing with trade disputes.

[15]All existing sectors included.

[16]Sectors are defined according to the French nomenclature.

[17]Times where linear interpolation takes place are the following ones : april, august, december 1990; may, august, december 1991; may, august, december 1992; may, august, december 1993; april, august, december 1994; april, august, december 1995; may, august, november 1996; april, august, september 1997; may, september, december 1998; and september, november 1999.

Sector	Mean	Standard deviation	Skewness	Excess kurtosis
AU	0.027216	0.003511	-0.045516	-0.234631
BC	0.040650	0.007373	-0.000103	-0.290836
BE	0.035770	0.009847	0.458533	-0.215878
BI	0.031221	0.008797	0.402617	-0.381186
BP	0.035815	0.004825	-0.081663	-0.014400
DA	0.021921	0.003146	-0.166066	-0.332933
DN	0.026944	0.006208	-0.192109	-0.409625
DS	0.022448	0.003530	-0.324960	-0.316261
GA	0.031041	0.005001	0.178000	0.475443
GN	0.035126	0.005170	-0.205242	-0.464660
HR	0.029702	0.004410	-0.071837	-0.393484
IA	0.020638	0.003384	-0.182819	-0.290739
IM	0.040416	0.011411	-0.064042	-0.728700
SE	0.016459	0.002506	-0.082994	-0.414606
SP	0.013838	0.002041	0.134186	-0.286063
Total	0.026314	0.003544	-0.200222	-0.117115
TT	0.026249	0.004252	0.238633	-0.243542

We notice that TOTAL gives some economic trend for failures all sectors included. Among all these average default probabilities on our time horizon, SP sector exhibits the lowest default probability whereas BC sector exhibits the highest one. Moreover, we are able to compute corresponding empirical survival probabilities $\hat{R}(t)$ since $\hat{R}(t) = 1 - \hat{p}_t$. Such survival functions exhibit the characteristics listed in the table underneath.

Sector	Mean	Standard deviation	Skewness	Excess kurtosis
AU	0.972784	0.003511	0.045516	-0.234631
BC	0.959350	0.007373	0.000103	-0.290836
BE	0.964230	0.009847	-0.458533	-0.215878
BI	0.968779	0.008797	-0.402617	-0.381186
BP	0.964185	0.004825	0.081663	-0.014400
DA	0.978079	0.003146	0.166066	-0.332933
DN	0.973056	0.006208	0.192109	-0.409625
DS	0.977552	0.003530	0.324960	-0.316261
GA	0.968959	0.005001	-0.178000	0.475443
GN	0.964874	0.005170	0.205242	-0.464660
HR	0.970298	0.004410	0.071837	-0.393484
IA	0.979362	0.003384	0.182819	-0.290739
IM	0.959584	0.011411	0.064042	-0.728700
SE	0.983541	0.002506	0.082994	-0.414606
SP	0.986162	0.002041	-0.134186	-0.286063
Total	0.973686	0.003544	0.200222	-0.117115
TT	0.973751	0.004252	-0.238633	-0.243542

As expected, BC sector has the lowest survival probability whereas SP sector has the highest average survival probability.

In the following of the paper, we apply fault tree theory (simple framework) to characterize evolutions of default probabilities. Such an application is achieved after choosing a distribution law for each survival.

3.2 Study

Our preliminary analysis of the previous subsection allows us to exhibit some features of French failures. In our elementary setting, the empirical default probabilities computed characterize the evolutions of survival times for each sector and for the whole sectors. Therefore, to realize our simple reliability study, we have to choose some distribution function characterizing survivals' behaviors.

Recall that Jarrow & Turnbull (1995) use a constant parameter exponential law to describe the evolution of any instant of default (i.e., a constant hazard rate). Such a choice leads those authors to get a Markovian survival probability. Therefore, analogously to Jarrow & Turnbull (1995), we choose a constant parameter exponential law to characterize the survival time associated to each default probability as introduced before. Such an assumption implies that the state space process related to any firm's situation (i.e., default or non default) is a Markov chain.

Let the survival time X follow an exponential law with a constant parameter λ. Then, its distribution function f, cumulative distribution function F and related survival probability R have the following expressions[18] :

$$f\left(t\right)=\lambda\,e^{-\lambda t}\mathbf{1}_{\{t\geq 0\}} \tag{9}$$

$$F\left(t\right)=1-e^{-\lambda t} \tag{10}$$

$$R\left(t\right)=e^{-\lambda t} \tag{11}$$

Moreover, relation (6) implies that $\forall t\geq 0,\lambda\left(t\right)=\lambda$. The hazard rate or failure rate is then constant. Finally, the expected survival time, or equivalently, the mean time to failure writes:

$$E\left[X\right]=\frac{1}{\lambda} \tag{12}$$

[18] Computation details are given in the appendix.

And, we therefore have $Var(X) = \frac{1}{\lambda^2}$.

We have a complete characterization of survivals' distribution which has now to be estimated. For this purpose, we introduce the random variable $Z_t = -\ln(1-p_t)$ and consequently $\hat{Z}_t = -\ln(1-\hat{p}_t)$. According to our framework and relation (11), we have $Z_t = \lambda t$ for $t \in \{1, ..., 120\}$. Consequently, to induce an estimation of λ parameter on our time horizon, we achieve the linear regression[19] of Z_t on time t. Unfortunately, our results show the existence of a strong positive autocorrelation between residuals of each regression. Indeed, related Durbin Watson statistics range from 0.389288 to 1.494238, and clearly lie under the critical value of 1.65. Such a stylized fact was expected since we realized a smoothing of our default probabilities to correct for calendar anomalies. Consequently, our previous regressions are biased. In this case, one way to whiten our regressions' residuals is to use the Cochrane-Orcutt methodology. But, such a methodology presents an important drawback since the estimated parameter may correspond to a local minimum of the related regression errors instead of a global minimum[20].

We propose to solve this problem while minimizing the following sum of squared errors relative to λ parameter for each sector under consideration and for all of them:

$$\min_{\lambda} \left\{ \sum_{t=1}^{120} \left(\hat{Z}_t - \lambda t \right)^2 \right\} \tag{13}$$

We achieve this non linear minimization with a quasi-Newton numerical method using a Davidon-Fletcher-Powell type algorithm[21]. On the time horizon ranging from january 1990 to december 1999, the λ parameter estimation's results we get are displayed in the table underneath for each sector and for all of them. Since we work with monthly data, we estimate monthly lambda parameters. To get a global view, annual lambda parameters are also given in the appendix.

[19] We would like to mention that, after achieving a Phillips-Perron test with a constant term, we find that all empirical default probabilities are stationary. Although statistics are not displayed, they remain available upon request. To sum up, default probabilities are stationary at a one percent level except for the four following sectors. For BC and IM sectors, default probabilities are stationary at a five percent level whereas default probabilities of BE and BI sectors are stationary at a ten percent level.

[20] See book of Bourbonnais (2000) for example.

[21] The reader could refer to the book of Press *et al.* (1989) for explanations.

Sector	$\boldsymbol{\lambda}$	**Kolmogorov-Smirnov Statistic**
AU	0.000335	0.362697
BC	0.000489	0.458514
BE	0.000421	0.501945
BI	0.000359	0.428752
BP	0.000438	0.416937
DA	0.000274	0.284780
DN	0.000307	0.445304
DS	0.000268	0.282832
GA	0.000378	0.394610
GN	0.000451	0.344028
HR	0.000376	0.286436
IA	0.000247	0.276315
IM	0.000529	0.423077
SE	0.000207	0.166206
SP	0.000177	0.159042
Total	0.000324	0.275844
TT	0.000318	0.328772

Notice that SP sector exhibits the lowest hazard rate whereas IM sector is characterized by the highest failure rate.

We also display in the previous table the Kolmogorov-Smirnov statistic[22] to test the assumption H_0 of adequacy of survivals' empirical distributions to related theoretical distributions. At a five percent test level, the critical values of Kolmogorov-Smirnov statistic are 1.340000 and 1.346000 for observations numbers of 100 and 200 respectively. After achieving a linear interpolation, we get a critical value of 1.341200 at a five percent level for 120 observations. Since all Kolmogorov-Smirnov statistic values are less than this critical value, we conclude that all survivals' exponential distributions are well specified at a 95% confidence level. Therefore, relation (12) allows to induce estimations about expected survival times[23] for each sector and for all of them. In the same way, we are able to compute easily the probability that a firm survives in the n coming months given that it has not defaulted before december 1999 or equivalently $t = 120$. Such a conditional probability writes $P(X > t + n \mid X > t) = \frac{P(X>t+n)}{P(X>t)} = e^{-\lambda n}$. We compute this conditional survival probability on coming one year, two years and five years horizons for each available sector and for all sectors. Results are displayed underneath.

[22] Explanations about this test are provided in the appendix.

[23] Results are not given but remain available upon request. The expected survival time ranges from approximatively 157 years for IM sector to 470 years for SP sector.

Sector	**1 year**	**2 years**	**5 years**
AU	0.995982	0.991981	0.980073
BC	0.994145	0.988324	0.971065
BE	0.994965	0.989955	0.975075
BI	0.995697	0.991413	0.978672
BP	0.994756	0.989539	0.974053
DA	0.996723	0.993457	0.983722
DN	0.996322	0.992657	0.981743
DS	0.996793	0.993597	0.984069
GA	0.995478	0.990977	0.977594
GN	0.994602	0.989233	0.973300
HR	0.995496	0.991012	0.977682
IA	0.997042	0.994094	0.985299
IM	0.993670	0.987381	0.968749
SE	0.997519	0.995044	0.987656
SP	0.997873	0.995751	0.989412
Total	0.996120	0.992256	0.980752
TT	0.996195	0.992405	0.981120

The longer the time horizon, the smaller the conditional survival probability on this horizon[24]. Furthermore, SP sector exhibits the highest conditional survival probabilities whereas such conditional survival probabilities are the lowest for IM sector.

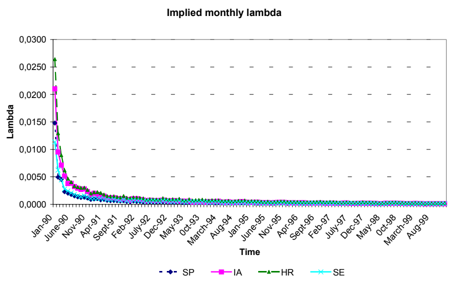
For further investigation, we try to check if assuming a constant λ parameter is a convenient assumption. We investigate our concern while inverting relation (11) according to lambda. Such a process allows us to get implied monthly lambda parameters as follows[25]:

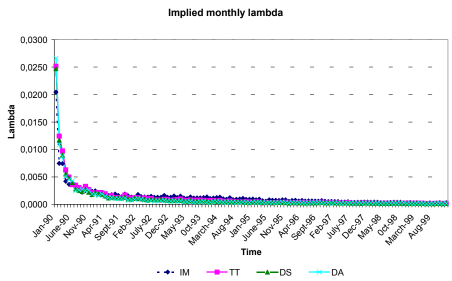
$$\hat{\lambda}_t = \frac{-\ln\left(\hat{R}(t)\right)}{t} \tag{14}$$

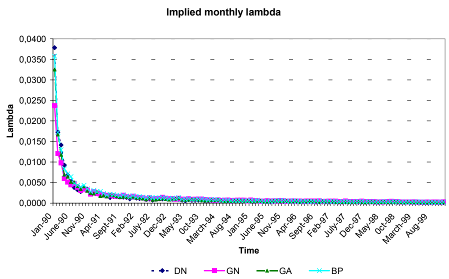
We plot these implied values of lambda parameter in graphs exposed underneath.

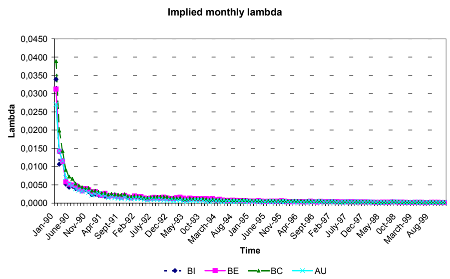
[24] The complementary conditional default probabilities are computed in the appendix.

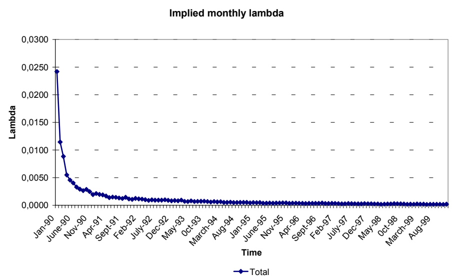
[25] We could have used this methodology to estimate our λ parameter as the arithmetic mean of monthly implied parameters. Namely, we could have set $\lambda = \frac{1}{120}\sum_{t=1}^{120}\hat{\lambda}_t$. But, the Kolmogorov-Smirnov statistics we get in this case are on average 3.804727 times higher than those we get while achieving a 'quadratic' estimation of λ. This suggests that this implied parameter-based methodology is less appropriate for λ estimation.











Roughly speaking, we notice a time varying behavior of the lambda parameter which suggests to model it through some convex decreasing function of time for example. We also could get such a time varying profile while modeling the survival's evolution using at least two constant parameters type distributions. First, we could use a mixture of exponential laws. Second, since an exponential law is equivalent to a gamma law with a shape parameter equal to one, we could charaterize the survival's distribution through some gamma law with a negative shape parameter[26].

[26] For further details, the reader is invited to consult the book of Gouriéroux (1989).

4 Conclusion

In a risk reducing standview, fault tree theory allows to identify causes and failure events leading to an undesirable event's realization. We apply this technique to assess credit risk. Specifically, reliability is used to assess possible corporate death in an elementary framework.

In a first step, we introduce this risk theory and explain the way to fit it to credit risk assessment. We make a brief comparison with current credit risk models which consist of stochastic modeling of default. Namely, data required for fault tree modeling are at least default probabilities in our case, or equivalently, failure rates. In a second step, we apply this theory in a simple framework to French bankruptcy statistics.

Having numbers of failures and existing (i.e., alive at the beginning of each elementary time subset) firms for 16 sectors and for all sectors included, we compute related empirical default probabilities on a monthly horizon ranging from january 1990 to december 1999. Since default probabilities are linked to firms' survival times, we are able to characterize bankruptcy process for each sector or all sectors included. For this purpose, we assume that each survival time follows a constant parameter exponential law. This distribution assumption has some interesting and simple implications for theoretical expressions of default probability, survival function and expected survival time. Mainly, the induced hazard rate process is constant and the state space process characterizing any firm's evolution is a Markov chain. Starting from empirical default probabilities, these simple specifications allow to get estimations of λ parameter for each sector and for all sectors over the studied time horizon. A basic linear regression method is sufficient. Moreover, we compute related conditional survival probabilities for some coming time horizons (i.e., one year, two years and five years). Finally, we investigate the coherency of a constant λ parameter while inverting the survival probabilitiy as a function of this parameter. We induce therefore monthly implied λ parameters which exhibit a time varying behavior. Accordingly, the hazard rate process should be a convex decreasing function of time.

But, such a simple framework exhibits two main shortcomings in so far as fault trees take into account all default causes without taking into account their severity, and allow only to make a static analysis. This last drawback may be solved through graphs theory (i.e., Markov graphs or stochastic Petri networks for example) to take into account systems' dynamics. Indeed, a qualitative risk analysis requires successively to identify any danger, classify risks starting from their severity, and develop some compensation or recovery policy to avoid any disaster (i.e., failure). Namely, this is equivalent to study the impact of a failure on any firm's reliability or lifetime, and establish some default risk scale through time.

Moreover, the simple framework we introduced could be clearly improved to fit reality. First, the firm should be represented by some multicomponents or multistates system whose components could fail separately or exhibit some failure dependence. For example, components could be financial ratios such as solvency ratio in order to characterize the firm's financial state. In such a case, the top-event is driven by basic and intermediate states whose realization probability's knowledge is required. Fault trees would therefore allow to identify the sets of combinations of components' default events leading to corporate bankruptcy and potentially to liquidation. Importance and severity study of each fatal events' combination would then allow to take into account several failure modes. Second, multistates representation would lead to compute transition probabilities from one state to another, and also invariant probabilities analogously to credit rating transitions. Such a representation would lead to another characterization of rating migration risk. Third, we should introduce some failure delay system's representation to encompass the potential delay which could pass between default event's realization and firm's death when significant. Finally, along with fault trees, the study of common cause failures would allow to analyze simultaneous failures of several systems (i.e., firms here) due to a given cause such as economic and financial settings for example. This theoretical feature would allow to set a study framework for default risk at a systemic level. Furthermore, such a process would allow to quantify a risk of multiple or chain coporate failures, or equivalently, systemic credit risk. This concern is highly important in some economic or financial crisis state.

5 Appendix

In this section, we give some details inherent to reliability and explain some of our computations.

5.1 Mean remaining survival

The mean remaining survival $L(t)$ of a system at time t given that this one did not default between times 0 and t has the following expression :

$$L(t) = E[X - t \mid X > t] = \int_t^{+\infty} \frac{R(s)}{R(t)} ds \tag{15}$$

which satisfies the next properties :

$$L(t) \geq 0 \qquad \text{and} \qquad L(0) = E[X] \tag{16}$$

$$\frac{dL(t)}{dt} \geq -1 \qquad \text{and} \qquad \int_0^{+\infty} \frac{ds}{L(s)} = +\infty \tag{17}$$

For further details, the reader could refer to the book of Limnios (1991).

5.2 Maintainability

Maintainability corresponds to the probability that a system is repaired (i.e., our firm goes on working and existing after any failure) between times 0 and t given the system has switched to a non working state (i.e., the firm defaulted) at time 0. In this case, the non working state has to be defined for firms since they lie in a failure state according to the default event but they do not end their activity (i.e., some short term insolvency situation for example).

Let Y be the random variable representing the time during which our firm remains non working (i.e., in a default state) after a default event and let $M(.)$ and $m(.)$ be its continuous cumulative distribution function (supposed to be absolutely continuous) and distribution function respectively. We could write $M(t) = P(Y \leq t)$ and $m(t) = \frac{dM(t)}{dt}$ with condition $M(0) = 0$. This last condition means that we are not able to repair our system instantaneously and a delay is required to make the firm switch to a non default state. Time has to elapse. Therefore, the related repairing rate is expressed as follows for each $t \geq 0$:

$$\delta(t) = \lim_{\Delta t \to 0^+} \frac{P(t < Y \leq t + \Delta t \mid Y > t)}{\Delta t} \tag{18}$$

Notice that when default is complete in so far as it will lead to firm's liquidation, we have $M(t) = 0$ as an extreme scenario, or at least $M(t)$ very low and near to zero. The mean time to repair is then defined as $E[Y] = \int_0^{+\infty} s\, m(s)\, ds$.

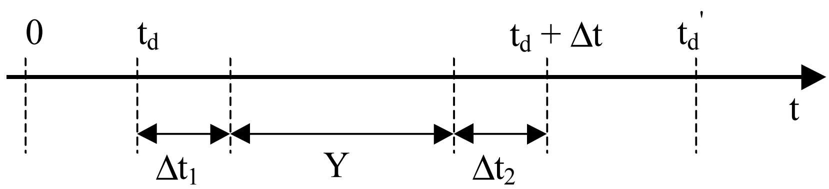
Like the survival function, we have some fundamental relation for maintainability such as :

$$\frac{dM(t)}{dt} + \delta(t)\,[1 - M(t)] = 0 \tag{19}$$

under the condition previously mentioned. This leads to the next expression for maintainability for each $t \geq 0$:

$$M(t) = 1 - \exp\left[-\int_0^t \delta(s)\, ds\right] \tag{20}$$

We represent such a situation through the graph underneath.



Maintainability representation.

where

- Time 0 is the starting date of observation;
- t_d and $t_d^{'}$ are times when a default event occurs;
- $t_d + \Delta t$ corresponds to the time when solvency is recovered;

- Δt_1 and Δt_2 are negligible time subsets.

We are then able to characterize the possibility that a firm survives after undergoing a default, which takes into account the fact that default does not necessarily lead to liquidation. We may then consider some more realistic features of default environment since Y could represent some regulatory observation delay stated by a regulatory entity such as a Federal (Disrict) Court[27] for example. When Y is zero, our firm goes bankrupt 'instantaneously' and dies. When Y is non zero, at the end of Y, whether the firm dies or it goes on doing its business (given some conditions and recovery measures). For example, assuming that our firm's insolvency is the default event, after a default, a firm could undergo a reoganization and then recover solvency. We could therefore encompass some arbitrage between the firm's economic value and its liquidation value while considering creditors and shareholders viewpoints, and also potential economic and social benefits related to the firm's remaining goodwill.

5.3 Exponential distribution features

In this subsection, we explain some implications related to a constant parameter exponential law to describe the survival time's X evolution. Recall that $f(t) = \lambda\, e^{-\lambda t}\mathbf{1}_{\{t\geq 0\}}$ with $\lambda > 0$. Therefore, the related cumulative distribution function is :

$$F(t) = \int_0^t \lambda e^{-\lambda s} ds = \left[-e^{-\lambda s}\right]_0^t = 1 - e^{-\lambda t} \tag{21}$$

Therfore, the related survival function is for each $t \geq 0$:

$$R(t) = 1 - F(t) = e^{-\lambda t} \tag{22}$$

Moreover, according to relation (6), we have $\frac{\frac{dR(t)}{dt}}{R(t)} = -\lambda(t)$ with $\frac{dR(t)}{dt} = -\lambda\, e^{-\lambda t}$ which implies that $\forall t \geq 0,\ \lambda(t) = \lambda$. And the expected survival time writes :

$$E[X] = \int_0^{+\infty} s\lambda e^{-\lambda s} ds = \int_0^{+\infty} u(s)\, v^{'}(s)\, ds \tag{23}$$

with $u(s) = s$ and $v^{'}(s) = \lambda e^{-\lambda s}$. And we have then $u^{'}(s) = 1$ and $v(s) = -e^{-\lambda s}$. Using an integration by part methology, we could write :

$$\begin{aligned} E[X] &= [u(s)\, v(s)]_0^{+\infty} - \int_0^{+\infty} u^{'}(s)\, v(s)\, ds \\ &= \left[-se^{-\lambda s}\right]_0^{+\infty} + \int_0^{+\infty} e^{-\lambda s} ds \end{aligned} \tag{24}$$

[27] In France, the regulatory entity handling legal failure framework is the Court dealing with trade disputes. After filling for bankruptcy, any firm is placed under a receiver's monitoring and given an observation delay at the end of which the firm may be liquidated or not.

We have to underline two points here. First, the exponential function's limit development of first order around zero allows us to write that $\lim\limits_{s \longrightarrow 0^+} e^{-\lambda s} = \lim\limits_{s \longrightarrow 0^+} (1 - \lambda s)$ and therefore $\lim\limits_{s \longrightarrow 0^+} se^{-\lambda s} = \lim\limits_{s \longrightarrow 0^+} s(1 - \lambda s) = 0$. Second, since we know that $\lim\limits_{s \longrightarrow +\infty} \frac{\ln(s)}{s} = 0$ and we have $se^{-\lambda s} = e^{\ln(s) - \lambda s} = e^{s\left(\frac{\ln(s)}{s} - \lambda\right)}$, we deduce the following limit :

$$\lim_{s \longrightarrow +\infty} se^{-\lambda s} = \lim_{s \longrightarrow +\infty} e^{s\left(\frac{\ln(s)}{s} - \lambda\right)} = \lim_{s \longrightarrow +\infty} e^{-\lambda s} = 0 \tag{25}$$

Consequently, the expected survival takes the final form :

$$E[X] = \int_0^{+\infty} e^{-\lambda s} ds = \left[\frac{e^{-\lambda s}}{-\lambda}\right]_0^{+\infty} = \frac{1}{\lambda} \tag{26}$$

In the same way, we could define the probability that a firm survives in the n coming months given that this one did not default during the t previous months. Namely, this conditional survival probability writes :

$$\begin{aligned} P(X > t + n \mid X > t) &= \frac{P(X > t + n)}{P(X > t)} = \frac{R(t + n)}{R(t)} \\ &= \frac{e^{-\lambda(t+n)}}{e^{-\lambda t}} = e^{-\lambda n} \end{aligned} \tag{27}$$

The conditional survival probability is then only depending on the coming running time horizon which is considered. Namely, it depends on the n coming months and corresponds to the failure accounting factor on the length of the time subset $[t, t + n]$.

5.4 Annual lambda parameters

In this subsection, we compute annualized failure rates given estimated monthly failure rates (i.e., estimated lambda parameters) for each sector and for all of them. Extrapolating monthly failure rates to an annual frequency allows to get an average annual trend for failures among sectors. Results are displayed in the table below.

Sector	$\boldsymbol{\lambda}$
AU	0.004033
BC	0.005888
BE	0.005060
BI	0.004320
BP	0.005271
DA	0.003287
DN	0.003691
DS	0.003217
GA	0.004542
GN	0.005426
HR	0.004524
IA	0.002966
IM	0.006368
SE	0.002487
SP	0.002131
Total	0.003894
TT	0.003819

The highest annual failure rate is achieved for IM sector with $\lambda = 0.636836\%$ while the lowest annual failure rate is achieved for SP sector with $\lambda = 0.213093\%$.

5.5 Kolmogorov-Smirnov test

We explain briefly how one distribution's adequacy test[28] called Kolmogorov-Smirnov test runs. Let $\{X_1, \cdots, X_n\}$ be a sample of n observations of some random variable X. Let $\hat{F}(.)$ be the empirical cumulative distribution function of this random variable and assume that $F(.)$ is the theoretical cumulative distribution function chosen to model the random variable's evolution.

The Kolmogorov-Smirnov test studies the maximal distance between theoretical and empirical distributions. Indeed, this test builds the following distance statistic $d_n = \sqrt{n} \max_{i \in \{1,\ldots,n\}} \left| \hat{F}(X_i) - F(X_i) \right|$ to test in a bilateral way the assumption:

$$H_0 : \, adequacy\, of\, \hat{F}(.)\, to\, F(.)\,. \tag{28}$$

Given some level α (i.e., a confidence level of $1 - \alpha$), we accept H_0 if the test statistic d_n is less than its corresponding critical value $d(n, \alpha)$ in the Kolmogorov table. If this is not the case, we reject the adequacy assumption H_0.

[28] We could also have used the independency Khi square test. But such a test requires to define a set of classes which seems to be a tricky task in so far as we do not know what are the optimal classes given the problem under consideration.

5.6 Conditional default probabilities

In this subsection, we compute conditional default probabilities for each sector and for all of them. The probability that a firm defaults in the n coming years given that it has not defaulted before december 1999 (i.e., time t) writes:

$$P(X < t+n \mid X > t) = 1 - \frac{P(X > t+n)}{P(X > t)} = 1 - e^{-\lambda n} \tag{29}$$

Indeed, we have the following relation:

$$\begin{aligned} P(X < t+n \mid X > t) &= \frac{P(t < X < t+n)}{P(X > t)} \\ &= \frac{P(t < X < t+n) + P(X > t+n) - P(X > t+n)}{P(X > t)} \\ &= \frac{P(X > t) - P(X > t+n)}{P(X > t)} \end{aligned} \tag{30}$$

and, consequently:

$$P(X < t+n \mid X > t) = 1 - \frac{P(X > t+n)}{P(X > t)} \tag{31}$$

Related results are displayed in percent in the table below.

Sector	**1 year**	**2 years**	**5 years**
AU	0.401765	0.801915	1.992746
BC	0.585510	1.167592	2.893468
BE	0.503539	1.004542	2.492466
BI	0.430256	0.858661	2.132849
BP	0.524417	1.046085	2.594729
DA	0.327710	0.654346	1.627846
DN	0.367843	0.734332	1.825733
DS	0.320674	0.640320	1.593120
GA	0.452184	0.902323	2.240565
GN	0.539798	1.076682	2.670008
HR	0.450405	0.898781	2.231829
IA	0.295756	0.590636	1.470056
IM	0.632973	1.261939	3.125051
SE	0.248101	0.495587	1.234366
SP	0.212659	0.424866	1.058783
Total	0.387957	0.774409	1.924792
TT	0.380489	0.759531	1.888025

The longer the time horizon, the bigger the conditional default probability on this horizon. Furthermore, IM sector exhibits the highest conditional default probabilities whereas such conditional default probabilities are the lowest for SP sector.

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