Taxing sales to tourists over time

Leon Taylor
Dillard University
New Orleans, La.
e-mail: ltaylor@hotmail.com
tel. (504) 865-7731
Running head: Taxing tourist sales

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Contents

Abstract

An optimal control model shows how a jurisdiction can tax tourists in a way that maximizes its revenues net of its costs in serving tourists: By relating its tax rate to its popularity with tourists. When its popularity waxes, it should raise the tax rate; when its popularity wanes, it should lower the tax rate. Extensions consider the effects on the tax of the discount rate, tourist prices, tourist congestion, and of the rise in the relative cost of services that is due to rising productivity in manufacturing. Computer simulations generate a concave tax path for a small city launching a tourism program.

1 Introduction

Taxes generated by travel are a source of revenues that is modest for many governments but that is growing more important. In many cases, the taxation of travel may redistribute income to poor areas that have trouble procuring foreign aid or loans for financing investment in physical and human capital. The moment seems right for preliminary analyses that yield guidelines for the planning of travel taxes.

After all, travel spending has become a large potential target for tax exporting. Over three decades, international tourism grew more rapidly than any other service sector; in receipts, it became by 1991 the second largest service sector (after banking) in world trade [19].[1] Worldwide, international arrivals of tourists almost tripled over the period from 1970 to 1990, from 160 million to 430 million [30]. The World Tourism Organization estimates that international tourism receipts, in current U.S. dollars, were $305.7 billion in 1993. For many countries, international travel is a relatively large source of export earnings and of hard currency. In the OECD, travel receipts in 1989 came to more than a tenth of exports by Spain (22.5%), Austria (18.6%), Greece (17.7%), Portugal (15.5%), and Turkey (13.5%) [20]. Spending by domestic travelers is several times larger than spending by international travelers. In 1993, domestic travelers spent an estimated $323.3 billion on overnight trips and on day trips of 100 miles or more in the United States; travel receipts by foreigners in the U.S. were estimated at $57.62 billion.[2]

[1] The measure of tourism here is the travel account in the Balance of Payments statistics, which excludes international passenger transport.

[2] Tables 428 and 431 in [25].

Governments are beginning to take advantage of the opportunity to export taxes. Even in the United States, where travel taxes have historically been small, tax revenues accounted for an estimated 1.7% of total tax revenues in 1992, more than triple the share in 1967, for 18 states that levied sales taxes on lodgings, hotel rooms and meals as well as on other functions related to travel in 1991.[3] In almost every state, governments created or raised taxes on hotel and motel rentals, amusement and entertainment attractions, or on meals and alcoholic beverages at bars and restaurants [16]. For a few areas in the United States, travel taxes already are major sources of revenue. In 1990, total state and local tax revenues generated by travel came to 23.3% of total state and local tax revenues for Nevada; 17% for Hawaii; and 10.6% for the District of Columbia [1].

An analysis of travel taxes should account for the wish of many jurisdictions to reap revenues from travel without suffering unduly from its effects. For instance, tourism is the economic mainstay for the Aegean island of Mykonos; but as arrivals almost quintupled from 1981 to 1991, islanders deplored congestion and pollution [9]. Jurisdictions such as Costa Rica explore ecotourism as an alternative to industries that may deplete natural resources more rapidly, such as farming, ranching, mining and logging. How can jurisdictions maximize their tax revenues from tourism, given that tourism itself will have both immediate and cumulative effects on their environments?

That travel taxation is a dynamic process is also suggested by fluctuations over time in the sensitivity of how much states and localities collect by taxing travel to how heavily they tax travel – that is, to travel tax effort. In the U.S. over the late 1970s, a drop in effort – travel tax revenues as a share of total travel spending – was associated with a proportionally greater rise in travel tax revenues. Since 1980, however, a rise in travel tax effort has been associated with a proportionally greater rise in travel tax revenues (Figure 1).[4] The changing sensitivity of revenue to effort suggests a need for dynamic models that link travel tax revenues to travelers' behavior. This paper develops such a model.

Rules-of-thumb are derived for taxing tourist sales in a way that maximizes tax revenues, net of the costs of services that the jurisdiction provides to tourists. The setup is flexible. For instance, it permits the jurisdiction to use the revenues in a separate problem to maximize the income of residents; or to earmark the revenues to pay off the bonds for a civic center already built, putting any excess into general funds; or to increase its holdings of foreign exchange.

Although the model accommodates other settings, I will mainly discuss the

[3]The states are Delaware, Massachusetts, New Hampshire, New York, Rhode Island, Vermont, Alabama, Florida, Louisiana, Oklahoma, South Carolina, Texas, Colorado, Hawaii, Idaho, Montana, Illinois and Michigan.

Hotel room taxes, including general and special taxes, were levied as a percentage of room sales in 47 states [1]; the exceptions were Alaska, California and Oregon. I have presented data for 18 states for which long time series on travel-related sales taxes are available.

[4]Data for Figure 1 were calculated from annual editions of [24].

case in which the demand of tourists to visit a site resembles a life cycle. This is the classic case in the geographical literature on tourism, reaching back at least to the work by Christaller in 1963 [8] and particularly developed by Butler in 1980 [7]. At least a dozen empirical studies in geography have applied the life cycle model to tourism [11]. Cooper notes a need for mathematical modeling of the tourist life cycle, which this paper tries to provide [11].[5]

Here is the basic story: The current demand to visit a tourist area initially rises with the cumulative number of visits there, then levels off or falls [12]. An adventurous few – for instance, artists seeking an untouched area to paint – discover a remote region. They spread the word to their friends. More travelers come to the region; entrepreneurs build the first hotels. Word continues to spread. The site's fame attracts many conventional tourists, catered to by hotels, camp sites and artificial attractions. A unique area has become "everybody's tourist haunt" [8]. The commercialization of the region, and the saturation of its tourist market, reduce the flow of tourists. Without rejuvenation – such as a venture into winter sports – the area will stagnate and die [7]. "Destination areas carry with them the potential seeds of their own destruction, as they allow themselves to become more commercialized and lose their qualities which originally attracted tourists" [21].

The jurisdiction can make money from tourism, without prematurely surrendering its own charms, by pursuing a judicious tax policy.[6] Taxes affect tourist prices. Under certain conditions that the paper will discuss, the jurisdiction can maximize its revenues by raising its tax until the tourist site peaks in popularity – then lowering its tax thereafter, persuading additional tourists to visit the site though its novelty has worn off.

My main premise is that the current demand to visit a site may depend on past demands to visit that site. Several empirical studies have addressed potential links between current demand and past demand. Witt & Witt [29] compared seven models for forecasting tourist flows, using 1965-1980 data for flows from France, Germany, the United Kingdom, and the United States to two destination countries for each origin country. They concluded that, for two-year forecasts, a simple autoregressive model was most accurate; for one-year forecasts, the autoregressive model and a random walk model were most accurate.[7] A trend term in [29] implied a 14% annual drop in the demand for tourist trips from France to Switzerland due to a wane in popularity. On the

[5] The notion of a life cycle in tourism is widely but not universally accepted. For instance, Getz is skeptical of the life cycle [13]. One can analyze the absence of a life cycle as a special case of the model in the present paper.

[6] Combs and Elledge assert that the jurisdiction can export much of a hotel tax, because lodging demand is price-inelastic [10]. The assertion received some support from a study of the *ex post* effects of a 5.25% hotel room tax that Hawaii imposed in 1987. Modeling real net rental receipts of hotels as a time series interrupted by introduction of the tax, Bonham *et al.* found that the tax reduced receipts by about 1% [5].

[7] The measures of inaccuracy were the square root of the mean square percentage error and the mean absolute percentage error.

other hand, Artus concluded that a trend variable "was not significant" when inserted into regressions of German spending on foreign travel for the periods 1955-1969 and 1960-1969 [2].

Despite some evidence of intertemporal effects, I do not know of a statistical study of a nonmonotonic link between the current demand to visit a site and cumulative visits to that site. My model incorporates a nonmonotonic link but also extends easily to cases with a monotonic link or with no link. The model may thus facilitate empirical work on the nature of links.

The analysis will characterize the number of current visits as a flow – and the number of cumulative visits as a stock. Economic models typically permit the flow to influence the stock, but I also require the converse. Here I have drawn upon the industrial organization literature [22], [15].

In most of the analysis, the jurisdiction is not a price-taker. It can raise its taxes without losing all its tourists to competing areas. The reader might associate the model with a town that is blessed with unusual, and desirable, traits of location: A spectacular view of the mountains, a natural hot spring, or a quarter of the city drenched in history.

Or the reader might associate the model with a nation. There is evidence that international tourism is not extremely sensitive to price. Bird concludes that the elasticity of tourist spending, with respect to prices in receiving countries, is often in the neighborhood of -1 when one excludes spending on transport [4].[8] Tourism may respond little to price in countries that are far from the origins of visitors (so that local costs are only a small part of total travel spending) and that have unique attributes such as pyramids. In contrast to other exports, "many tourist countries seem to undertax their tourist exports," Bird writes. While nations often try to attract as many tourists as possible by holding down taxes, this policy may not in fact maximize receipts of foreign exchange, since crowding and deterioration may repel some tourists [14]. The analysis will consider the cost of controlling these repellants.

2 Analysis

2.1 Setup

Consider the jurisdiction of Host. The cost of traveling to Host is $p_2 \geq 0$; the untaxed price to the tourist of "doing" the place – e.g., the price of lodging, entertainment and meals – is $p_1 > 0$. To keep the initial exposition simple, let p_1 and p_2 be constants for all tourists.

Host taxes tourist expenditures at rate $\tau(t)$ at time t. In all, the tourist pays $[1+\tau(t)]p_1 + p_2$ to visit Host. The demand to visit Host at time t depends partly on this price. The function $f[(1+\tau(t))p_1 + p_2] \geq 0$ represents this aspect

[8] Tisdell [26] analyzes the impact of supply and demand elasticities on foreign exchange receipts from a tourism tax.

of demand. A price of zero will induce a finite number of visits demanded, because the price does not reflect time costs. Raising the price will reduce the number of visits demanded at perhaps an increasing rate, since a vacation is a discretionary purchase by the household which is most likely to be delayed when it would otherwise consume a large share of income. These considerations suggest that[9]

$$f' < 0, \ f'' \leq 0. \tag{1}$$

Here, f' denotes df/dx and f'' denotes d^2f/dx^2, where $x = (1+\tau(t))p_1 + p_2$.

Demand also depends partly on the total number of visits that have been made to Host by time t, $Q(t)$. In the beginning, as more people visit Host, they spread the word about its wonders to more of their neighbors, increasing the current quantity of visits demanded. But after enough people have seen Host – when cumulative visits exceed Q^* – the current quantity of visits demanded falls. For instance, those who already have visited Host may decide not to come back. The function $g(Q(t)) \geq 0$ represents this life-cycle aspect of demand:

$$\begin{array}{lcl} Q < Q^* & \rightarrow & \partial g/\partial Q > 0 \\ Q = Q^* & \rightarrow & \partial g/\partial Q = 0 \\ Q > Q^* & \rightarrow & \partial g/\partial Q < 0 \end{array} \tag{2}$$

where $g(0) = 0$.

Each aspect of demand magnifies the other. Suppose that Q rises slightly while Host's prices remain low. A few more people tell their neighbors about Host and its low prices. The result is a large increase in current quantity demanded. Had Host's prices been high, the increase in current quantity demanded would have been much smaller. Thus the number of visits to Host at time t is

$$f[(1+\tau(t))p_1 + p_2]g(Q) \geq 0. \tag{3}$$

In sum, the functions f and g represent aspects of demand, and their values should be interpreted as simply numbers. Their product fg represents the demand to visit Host at a given moment.

Now consider what tourism costs Host. Congestion costs – such as the cost of alleviating weekend traffic jams – arise from the current number of visitors. I will consider them later in the paper. For now, I will focus on depreciation costs, which arise from the cumulative number of visitors. They include the costs of maintaining the infrastructure that supports tourists – the airport, the main roads into the shopping districts, and the utilities serving the hotels. They

[9]The necessary conditions for a maximum, which will be described, also hold when $f'' > 0$ as long as one also assumes that $p_1 > 1$.

also include the costs of controlling litter as well as of controlling crime that stems from Host's growing reputation for tourism. For instance, the Hawaiian government has a fund that reimburses visitors for losses from theft [17].

The depreciation cost to Host of one tourist visit is $c(Q) > 0$. The sign of $dc/dQ > 0$ is not immediately evident. Suppose that, as more tourists tromp through Host, its public works and environmental assets will wear out. Then its maintenance cost will rise: $dc/dQ > 0$. On the other hand, suppose instead that experience with tourists lowers the cost to Host of dealing with them: Then $dc/dQ < 0$. For the moment, I will defer signing dc/dQ. But, to establish that a function satisfying the conditions necessary for a solution is also sufficient for a solution, I will assume that $dc^2/d^2Q \geq 0$. If wear and tear force up maintenance costs, then they will rise at an increasing rate; if learning by doing lowers maintenance costs, then they will fall at a diminishing rate, if only because they are bounded by zero.

2.2 Basic results

Host's problem is to maximize discounted revenues from the tourist tax, net of the costs of serving tourists, over the time horizon T, $0 \leq t \leq T$. Host will pick T and the tax path, $\tau(t)$, to maximize[10]

$$\int_0^T e^{-rt}[\tau(t)p_1 - c(Q)]f[(1+\tau(t))p_1 + p_2]g(Q)\,dt \tag{4}$$

subject to

$$\frac{\partial Q}{\partial t} = f[(1+\tau(t))p_1 + p_2]g(Q), \tag{5}$$

$$Q(0) = Q_0 \geq 0. \tag{6}$$

On a point in the tax path, I place upper and lower bounds, τ_U and τ_L, that are politically infeasible for Host to consider. Thus, for all $t \in [0, T]$, $\tau(t) \in [\tau_L, \tau_U]$, where only interior choices of τ are feasible.[11] Host may consider subsidies ($\tau_L << 0$) as well as extractions ($\tau_U >> 0$) in its quest to maximize tax revenues.

The current-value Hamiltonian is

[10] This expression of the problem assumes that every tourist pays the same amount, p_2, to travel to Host. Alternatively, one can assume that p_2 varies with travel distance. Host may then draw up a schedule of taxes that vary according to the tourist's zone of origin. Host could discriminate among tourists at the hotel desk by inspecting their drivers' licenses. Appendix B presents results.

[11] As Appendix B discusses, the existence of a solution to the maximization problem can be ensured by applying the Filippov-Cesari theorem. Bounding and closing the control set that defines admissible values for $\tau(t)$ will satisfy the theorem.

$$H = [\tau(t)p_1 - c(Q) + \mu(t)]f[(1+\tau(t))p_1 + p_2]g(Q). \tag{7}$$

The necessary conditions imply that, for every t satisfying $0 \le t \le T$,[12]

$$r\mu + \frac{f^2\left(\frac{\partial g}{\partial Q}\right)}{f'} = -p_1\left(\frac{\partial \tau}{\partial t}\right)\left[2 - \frac{ff''}{(f')^2}\right]. \tag{8}$$

Appendix B derives (8).

What does (8) tell us? From (1), the bracketed term is positive. From (1), $f^2/f' < 0$ when $f > 0$. So if the discount rate is 0, and if taxes are not so high that they choke off all demand, then $\partial\tau/\partial t$ has the same sign as $\partial g/\partial Q$. Host should hold a finger to the wind: It should raise taxes when its popularity with tourists grows, and it should lower taxes when its popularity wanes. The intuition is that Host should cash in on its celebrity as long as it can – but not longer.

2.2.1 Varying tourist prices

Suppose that tourist prices change over time. For instance, in the market for tour operators, a certain group may acquire market power and raise its prices. If $\dot{p_1} \neq 0$, then replace (8) with

$$r\mu + \frac{f^2\left(\frac{\partial g}{\partial Q}\right)}{f'} = -\left[p_1\frac{\partial \tau}{\partial t} + \tau\frac{\partial p_1}{\partial t}\right]\left[2 - \frac{ff''}{(f')^2}\right]. \tag{9}$$

Again set $r = 0$.[13] If $sgn[\partial g/\partial Q] = -sgn[\dot{p_1}]$, then $sgn[\partial g/\partial Q] = sgn[\dot{\tau}]$. Host should raise taxes when its popularity with tourists grows if tourist prices simultaneously drop; Host should cut taxes when its popularity with tourists wanes if tourist prices simultaneously rise – for instance, when its currency appreciates.

Those results accord with intuition. If the tourist industry is foolish enough to settle for a small piece of a growing pie, then Host should claim a big piece for itself. But what if tourist prices rise as Host grows more popular – or fall as Host loses popularity? Then it remains true that Host should raise taxes when its popularity with tourists grows, as long as tourist prices rise less rapidly than taxes; and Host should cut taxes when its popularity with tourists wanes, as long as tourist prices fall less rapidly than taxes. More precisely, if $sgn[\partial g/\partial Q] = sgn[\dot{p_1}]$, then $sgn[\partial g/\partial Q] = sgn[\dot{\tau}]$ if $|\dot{p_1}/p_1| < |\dot{\tau}/\tau|$.

[12] The necessary conditions are also sufficient when the second derivatives of f or g are large enough in absolute value and when g' is not too negative. Intuitively, the necessary conditions suffice when the rate of the number of visits demanded changes sharply in response to changes in either the price of a visit or in the cumulative number of visitors – but it does not fall too sharply when the cumulative number of visitors rises. Appendix B has details.

[13] I continue to assume that $p_1(t) > 1$ for $0 \le t \le T$.

2.2.2 Positive discount rate

So far, the analysis has supposed that Host just wanted to collect as many tax dollars as it could, net of costs; that it did not care when it received a tax dollar. In reality, governments often prefer to receive money sooner rather than later. How, then, would Host change its tax policy to satisfy its preference to collect money quickly? To focus on the question, let us hold tourist prices constant over time. Set $\dot{p_1} = 0$. If $r > 0$, then an optimal tax rate will begin to fall before $g(Q)$ does. To see this, define the parameter $a > 0$ to satisfy

$$a = -p_1 \frac{f'}{f^2} \left[2 - \frac{ff''}{f'^2} \right], \tag{10}$$

given t. Use a to rewrite (8):

$$r\mu = -\frac{f^2}{f'} \left[\frac{\partial g}{\partial Q} - a \frac{\partial \tau}{\partial t} \right]. \tag{11}$$

Thus, when $f > 0$,

$$r\mu > 0 \rightarrow \frac{\partial g}{\partial Q} > a \frac{\partial \tau}{\partial t}. \tag{12}$$

From (12), if $\partial g / \partial Q = 0$, then $\partial \tau / \partial t < 0$: Host should begin lowering its tax rate at least an instant before its popularity wanes. Moreover, from (11), if a is large enough, then $\partial g / \partial Q > 0$ implies that $\partial \tau / \partial t < 0$: Host should cut taxes even while its popularity grows.[14] The size of a depends indirectly on f and directly on p_1. This means that, given a positive discount rate, a tax cut during a tourist boom is more likely to be optimal for Host if the jurisdiction is expensive for tourists (p_1 is large and f is small) than if it is cheap.

The higher the discount rate, the less that Host should raise the tax rate at a given time.[15] One may loosely infer that when the interest rate is high, Host may gain by cutting the tax rate while the jurisdiction is still popular with tourists; putting the tax revenues into an account that pays interest; and later using the proceeds to pay off the depreciation costs entailed by the early surge in tourists.

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[14]In (11), the bracketed expression must be positive. Given $\partial g / \partial Q$, if a is large, then the bracketed expression is positive only if $a \partial \tau \ \partial t$ is negative. That requires $\partial \tau / \partial t$ to be negative.

[15]Both r and $\dot{\tau}$ enter (8) explicitly, and neither enters it implicitly. Rewriting (8) as the function $\dot{\tau}()$ and taking a derivative give us

$$\frac{\partial \dot{\tau}}{\partial r} = -\frac{\mu}{p_1 \left[2 - \frac{ff''}{(f')^2} \right]} < 0. \tag{13}$$

2.2.3 No word-of-mouth effect

If the growth in Host's fame – or infamy – alone stirs no traveler from his armchair, then $\partial g/\partial Q = 0$ for all Q. From (8), any optimal tax rate is then constant throughout the tourism program ($\dot{\tau} = 0$) if there is no discount rate ($r = 0$). If the discount rate is positive ($r > 0$), then from (12) any optimal tax rate falls over time ($\dot{\tau} < 0$). In either case, if depreciation costs rise over time, then they will dictate when the tourism program ends (see (18)).[16]

2.2.4 No wear-and-tear effect

Curiously, the wear-and-tear of Host's infrastructure due to its continued use by visitors does not affect the shape of the tax path that it picks. Suppose that the public-sector cost of a visit stays the same ($c' = 0$ and $c'' = 0$) as the cumulative number of visits grows. From (8), Host will pick the same tax path as when it faces wear-and-tear costs; and setting $p_1 = 1$ ensures that this tax path will uniquely maximize net revenues. Now suppose that learning by doing dominates costs so that the public-sector cost of a visit falls ($c' < 0$). Again, Host will pick the same tax path as before; and, if learning by doing is not too dominant (i.e., if c' is not too large of a negative number), then picking units such that $p_1 = 1$ ensures that the tax path will uniquely maximize net revenues. If learning by doing is quite dominant, however, then one cannot guarantee sufficiency.[17]

2.2.5 Marginal value of tourist stock

Evaluated at optimal values, the multiplier μ gives the current value to Host of adding another visit to the stock Q. To obtain $\mu(t)$, solve (51) in Appendix B as a differential equation, using (52) and (55). When the discount rate is zero, and when at least some tourists visit Host ($Q(T) > Q(0)$), then

$$\mu(t) = \int_t^T f\left(\frac{\partial g}{\partial Q}\right)\tau p_1 - f\left(g\frac{\partial c}{\partial Q} + c\frac{\partial g}{\partial Q}\right) ds. \tag{14}$$

The current marginal value of $Q(t)$ is the change in tax receipts minus the change in costs, summed over the future. The marginal visitor affects the volume of future tourism (the integral of $f\,\partial g/\partial Q$) and of tourist revenue. Cumulative visits affect costs in two ways. First, they affect costs directly, by imposing wear and tear on facilities or by conferring the benefits of learning by doing (the integral of $fg\,\partial c/\partial Q$). Second, they affect costs indirectly, via an effect on the number of current visits (the integral of $fc\,\partial g/\partial Q$). Cumulative visits hasten tourism through the life cycle.

[16]The necessary conditions suffice for a unique maximum when the public-sector cost of a visit rises steeply with the cumulative number of tourists (c'' is large) or when the quantity of visits demanded falls steeply as the price of a visit rises (f'' is large in absolute value). Appendix B has the derivations.

[17]Appendix B has derivations.

As Appendix B shows, one can also express the current marginal value of $Q(t)$ as

$$\mu = p_1 \tau \left(\frac{1}{\epsilon} - 1 \right) + c \tag{15}$$

where ϵ is the absolute value of the tax elasticity of tourist demand to visit Host. The equation in (15) yields an expression for the tax – call it τ_c – that prevails when perfectly competitive jurisdictions vie for tourists. In this case, ϵ approaches infinity, so Host must choose τ to be τ_c. After all, a higher tax would drive away all tourists; a lower tax would not maximize net revenues. In a long-run competitive equilibrium, jurisdictions have entered the tourism market to the point of driving the net revenue of an additional tourist to zero. So μ is zero. From (15), the competitive tax is

$$\tau_c = \frac{c}{p_1}. \tag{16}$$

Given a constant tourist price, the competitive tax moves in lockstep over time with the public-sector cost of tourism services. Thus (16) may provide a benchmark for determining empirically whether a tourism market is competitive.

2.2.6 Comparative statics

Using necessary conditions, one may express an optimal tax path as the implicit function $\tau(\mu(t), Q(t))$.[18] It turns out that, under normal conditions of demand, when there is an increase in the value to Host of adding another tourist visit, at a given time, then the optimal tax decreases. More precisely, we have that $\partial\tau/\partial\mu < 0$, given t, when $f' < 0$.

One may think about the result in this way. Host can "buy" a tourist at the price $-\tau p_1$ in lieu of selling him services. When the marginal tourist becomes more valuable to Host – for instance, the tourist may be a travel writer with a large audience for his vacation sagas – the jurisdiction will raise its purchase price. Also, it turns out that $\partial\tau/\partial Q$ has the same sign as $\partial c/\partial Q$, given t, when $f' < 0$. If a rise in the stock of tourists would incur higher wear-and-tear costs ($\partial c/\partial Q > 0$), then that will induce Host to raise its tax in order to cover costs. If a rise in the stock of tourists would help Host learn how to parlay the effects of tourism ($\partial c/\partial Q < 0$), then that will induce Host to cut its tax rate in order to boost the number of tourists and the level of tax revenues. If $f' = 0$, then both derivatives of τ are zero: Host will not change its tax rate, in response to changes in the stock or value of tourists, if tax changes do not affect the flow of tourists.

[18] Appendix B has derivations.

2.2.7 Endtime conditions

Suppose that Host does encourage tourism for a while. How will the jurisdiction know when it's time to roll up the welcome mat? One necessary condition for the optimal choice of T is[19]

$$e^{-rt}\mu(T) = 0. \tag{17}$$

The value of adding another tourist to Host's "stock" is zero. Perhaps, upon his return home, this tourist is so tepid over Host that his vacation sagas fail to inspire anyone else to visit the jurisdiction.

The second condition is

$$(\tau p_1 - c)fg = 0,\ t = T. \tag{18}$$

At the optimal endtime, tax revenues just cover the costs that the tourist imposes upon the jurisdiction ($\tau p_1 = c$); or Host has exhausted one of the two aspects of demand ($f = 0$ or $g = 0$). Thus, if tourist flow is positive ($fg > 0$) at time T, then $\tau(T) > 0$. If tourists are still coming as the tourism program closes, then Host should tax them.

Sometimes jurisdictions such as communist Albania have at times banned all tourism. Such bans maximize tax revenues, net of public-sector costs, when the jurisdiction would lose money in its optimal moment T for ending a tourism program.[20] Here is an intuition into the result. Conceivably, it may be optimal for a jurisdiction to lose money on tourists who visit at a particular moment in the middle of its program; after all, through word of mouth, those tourists may drum up business later that compensates for the loss. But it cannot be optimal for a jurisdiction to lose money at the last moment of its program, since it cannot hope to offset the loss with later business. If it can only lose money at the endtime, then it should pull down its tourism shingle.

If the tourist market is perfectly competitive, then any t will satisfy the two necessary conditions for the optimal choice of T. When hotly contested by its neighbors, Host makes no net tax revenues from tourism at any moment, so it may always be agreeable to ending the program.

2.3 Extending the cost function

In addition to wear and tear, two forces can raise the cost of serving tourists. One is congestion. Particularly in small, rural communities, costs may rise sharply once the flow of tourists exceeds a fairly low threshold level.

The second force is the debilitating effect on services of productivity improvements in manufacturing. Tourism is the most labor-intensive of the service sectors [19]. Many tourist services require a certain amount of labor time. If

[19] Appendix B derives (17).

[20] Appendix B has derivations.

a guided tour of a historic home requires 40 minutes, we cannot hope for an innovation that will enable the guide to speed through his spiel in 15 minutes.[21] As technological innovations boost labor productivity in manufacturing, the opportunity cost of labor devoted instead to services will rise [3].

To analyze this Baumol effect, add the argument t to the cost function: $C = C(Q(t), t)$. The Baumol effect will increase the relative cost of tourism over time: $\partial C/\partial t > 0$.

To analyze the congestion effect, add the argument fg to the cost function:

$$C = C\left[f(\tau, p_1, p_2)g(Q), Q, t\right]. \tag{19}$$

I set $\partial C/\partial(fg) \geq 0$. For simplicity, I rewrite the cost function as

$$C = C\left[\tau, Q, t; p_1, p_2\right]. \tag{20}$$

Set $\partial C/\partial \tau \leq 0$ and $\partial^2 C/\partial \tau^2 \geq 0$. For tractability, I set cross-partials of the cost function to zero.[22]

The sign that one should assume for $\partial C/\partial Q$ is murky. For $Q \leq Q^*$, congestion and deterioration are on the rise, suggesting that $\partial C/\partial Q > 0$. Learning by doing, however, will mitigate this increase. For $Q > Q^*$, deterioration continues, but congestion eases and learning by doing continues. I shall not restrict the sign of $\partial C/\partial Q$ *a priori*.

The current-value Hamiltonian is

$$H = [\tau(t)p_1 - C(\tau, Q, t; p_1, p_2) + \mu] f\left([1 + \tau(t)]p_1 + p_2\right) g(Q). \tag{21}$$

2.3.1 Congestion effects

I will set the Baumol effect ($\partial C/\partial t$) to zero. The necessary conditions yield

$$\begin{aligned} & r\mu + \left[\frac{f^2}{f' p_1}(p_1 - C_\tau)\right]\frac{\partial g}{\partial Q} \\ &= \dot{\tau}\left[\frac{f}{f' p_1}C_{\tau\tau} - (p_1 - C_\tau)\left(2 - \frac{f f''}{(f')^2}\right)\right]. \end{aligned} \tag{22}$$

Congestion effects (C_τ and $C_{\tau\tau}$) appear in the bracketed expressions on both sides of the equation. The bracketed expressions are always nonpositive, but the discount rate may reshape the path of any optimal tax rate. Consider two cases:

First, suppose that the discount rate is zero ($r = 0$). Then Host should raise taxes if its popularity grows; and it should cut taxes if its popularity wanes

[21] Few tourists regard a tape recorder as a perfect substitute for a guide.

[22] Appendix B presents more general – but less tractable – results with nonzero cross-partials.

(i.e., $\dot{\tau}$ will have the same sign as $\partial g/\partial Q$).[23] In this case, congestion does not influence when the jurisdiction will begin cutting the tax rate.

Now consider a positive discount rate ($r > 0$). Then Host should raise taxes when its popularity burgeons ($\dot{\tau} > 0$ when $\partial g/\partial Q$ is a large positive number); and Host should cut taxes when its gains in popularity are modest ($\dot{\tau} < 0$ when $\partial g/\partial Q$ is a small positive number). Host should also cut taxes when its popularity wanes ($\dot{\tau} < 0$ whenever $\partial g/\partial Q < 0$). These considerations suggest that τ will begin to fall earlier in the tourist cycle than in the case of a zero discount rate. This resembles the result obtained in the case of a positive discount rate and no congestion costs.

In summary, congestion costs do not dramatically affect the basic path of an optimal tax rate.

2.3.2 Baumol effect

To focus on the Baumol effect, set congestion effects to zero:

$$r\mu + \frac{f^2}{f'}\frac{\partial g}{\partial Q} - C_t = -\dot{\tau}p_1\left(2 - \frac{ff''}{(f')^2}\right). \tag{23}$$

Set the discount rate to zero ($r = 0$). When Host gains popularity, then it should raise its tax. (That is, when $\partial g/\partial Q > 0$, then $\dot{\tau} > 0$). Host should also raise its tax when it loses just a little popularity – a little, that is, relative to the Baumol effect. (More precisely, when $\partial g/\partial Q < 0$ – but small in absolute value, relative to C_t – then $\dot{\tau} > 0$.) Host should cut its tax when it loses a lot of popularity, relative to the Baumol effect (when $\partial g/\partial Q < 0$ – but large in absolute value, relative to C_t – then $\dot{\tau} < 0$). An optimal tax rate may thus fall later in the tourist cycle than in the case of a zero discount rate and of no Baumol effect. Here is an intuition into these results: A sustained rise in the tax will dampen the flow of tourists into Host – and thus hold down the rise in service costs that is fueled by the Baumol effect. But if Host's loss in popularity dwarfs the Baumol effect, then Host should cut the tax to revive its fortunes.

Suppose that the discount rate is large relative to the Baumol effect ($r\mu > C_t$). Then Host should raise its tax if it is gaining a lot in popularity (when $\partial g/\partial Q$ is a large positive number, $\dot{\tau} > 0$). Host should cut its tax if it loses popularity or gains only a little (when $\partial g/\partial Q < 0$, then $\dot{\tau} < 0$; when $\partial g/\partial Q$ is a small positive number, then $\dot{\tau} < 0$). Any optimal tax rate may begin falling earlier in the tourist cycle than in the case of a zero discount rate and of no Baumol effect. Quite simply, if the discount rate dominates the Baumol effect, then it will also dominate the shape of the tax path.

Finally, suppose that the discount rate is small relative to the Baumol effect ($r\mu < C_t$). Host should raise its tax if it is gaining popularity or losing little

[23] If one of the bracketed terms is zero, then $\dot{\tau}$ and $\partial g/\partial Q$ may differ in sign. Since a similar remark would attach to the event of a zero bracketed term in the other cases considered by the text, I will address instead the cases in which both bracketed terms are nonzero.

popularity (when $\partial g/\partial Q > 0$, then $\dot{\tau} > 0$; when $\partial g/\partial Q < 0$ but small in absolute value, then $\dot{\tau} > 0$). Host should cut its tax if it is losing a lot of popularity (when $\partial g/\partial Q < 0$ but large in absolute value, then $\dot{\tau} < 0$). An optimal tax rate may begin falling later in the tourist cycle than in the case of a zero discount rate and of no Baumol effect. If the Baumol effect dominates the discount rate, then it will also dominate the shape of the tax path.

One can also interpret these results in light of the gains to Host of attracting another tourist. Consider what happens after a short time. Host earns $r\mu$ in interest from taxing that tourist; and it pays C_t more to serve another tourist than it would have had to pay earlier. Suppose that Host will have a little left over after paying this cost; that its tourism coffers are accumulating. More precisely, $r\mu > C_t$. Then Host should cut its tax when it gains or loses popularity mildly among tourists. Since costs are rising slowly over time, Host can risk a tax cut, especially since the exogenous change in the number of tourists is small. On the other hand, suppose that Host will lose a little after paying the cost; that its tourism coffers are draining. More precisely, $r\mu < C_t$. Then it should raise its tax when it gains or loses popularity mildly among tourists. Since costs are rising rapidly over time, Host should raise taxes to cover them.

3 Conclusions

How can a jurisdiction extract the maximum amount of money from tourists, net of costs? In the simplest case, the jurisdiction may be able to apply three rules of thumb:

• Every tourist should provide enough in taxes, directly or indirectly, to compensate for at least the public-sector costs of his trip. By "indirectly," I mean that, by word of mouth, the tourist could drum up more visits and tax revenues for Host.

• If tourism grows more popular (in the sense that $\partial g/\partial Q > 0$), raise the tax.

• If tourism becomes less popular (in the sense that $\partial g/\partial Q < 0$), cut the tax.

These rules apply to the case of no discount rate and no Baumol effect.[24] This case might approximate the situation of a rural jurisdiction with little manufacturing and with poor access to financial markets.

In the real world, the jurisdiction observes the current number of tourists (fg) and not simply the current number of tourists who were prompted to come by word-of-mouth (g). Even so, the jurisdiction can use the first two rules of thumb in circumscribed cases. Over a time interval dt, the jurisdiction observes $d(fg)$, where

[24] The rules characterize an optimal tax path for an arbitrary initial time and initial stock of tourists. It is perhaps evident that the optimality of the tax path does not require an optimal initial tax.

$$d(fg) = \left[g f' p_1 \dot{\tau} + f^2 g \frac{\partial g}{\partial Q} \right] dt. \tag{24}$$

Thus, under some circumstances, the jurisdiction can sign $\partial g/\partial Q$ by observing $d(fg)$. If $d(fg) > 0$ and $\dot{\tau} > 0$, then $\partial g/\partial Q > 0$. If $d(fg) < 0$ and $\dot{\tau} < 0$, then $\partial g/\partial Q < 0$. The jurisdiction may thus surmise that if the current number of tourists rises while the tax rate rises, then its popularity with tourists is growing, so it should keep raising the tax rate; and if the current number of tourists falls while the tax rate falls, then its popularity with tourists is waning, so it should keep cutting the tax rate. But, without more information, the jurisdiction cannot infer the sign of $\partial g/\partial Q$ if $d(fg) > 0$ while $\dot{\tau} < 0$ or if $d(fg) < 0$ while $\dot{\tau} > 0$.

Not all variants on the basic case require variants of the rules. Congestion costs do not affect the basic properties of an optimal tax path; neither do moderate changes in tourist prices. On the other hand, if tourist prices rise dramatically as the jurisdiction grows more popular, then cutting the tax rate may be optimal; if tourist prices fall dramatically as the jurisdiction loses popularity, raising the tax rate may be optimal. If the discount rate is positive, then the jurisdiction may wish to lower taxes earlier in the tourist cycle than it would otherwise. If the Baumol effect is larger than the rents that the jurisdiction can collect on the marginal tourist, then the jurisdiction may wish to raise taxes later in the tourist cycle than it would otherwise.

4 Appendix A: Numerical analysis

A simple simulation of the model graphically illustrates the impact of changes in external factors on the tax rate. The paucity of data on the public-sector costs of tourism, and the restrictions inherent in identifying a numerical solution to the problem, compel one to use hypothetical figures. Still, one might think of the following runs as describing a small city in the United States, with two motor hotels, that is modestly attractive to tourists. A visitor might expect to spend \$100 on a day trip to the town; thus I set p_1 to \$100. I initially set $p_2 = 0$.

I assumed the demand functions

$$f(t) = 1100 - 5[(1 + \tau(t))p_1 + p_2] \tag{25}$$

and

$$g(Q(t)) = \frac{10e^{-.0025Q(t)}}{(1 + 4e^{-.0025Q(t)})^2}. \tag{26}$$

The linearity of the demand function in (25) helps ensure the existence of a solution to the problem. The demand function in (26) builds upon the idea that the city would attract no more than 1,000 tourists by word-of-mouth alone; and that the word-of-mouth effect would diminish ($\partial g/\partial Q$ would turn negative) after 555 tourists had visited the town.[25]

To help ensure a solution, I bound the control variable. I assume that the jurisdiction wants to provide tax revenues from tourism at every moment; perhaps local politics rule out subsidies and moratoria. So I constrain the tax rate, τ, to the closed interval $[0, ((220 - p_2)/p_1) - 1]$.[26] Finally, I seek an interior solution, since a boundary choice of the tax rate would produce zero revenues for that moment.

I assumed the depreciation cost function

$$c(Q(t)) = .002Q(t). \tag{27}$$

I estimated the basic model of Section 2, using the Runge-Kutta method of the fourth order, in steps of 1/16, to compute $\tau(t)$ and $Q(t)$ [6].[27] To identify an interior candidate, I estimate (5) and a rearrangement of (8) as a system of simultaneous differential equations for $\tau(t)$ and $Q(t)$; check the transversality conditions; and, if the conditions are not met, proceed to time $t+1$, using the estimated values for $\tau(t)$ and $Q(t)$ in the new iteration.[28] Initial values are 0 for $\tau(t)$ and $Q(t)$. The simulations, written in Turbo Pascal, begin with $t = 1$.[29] Because the necessary conditions cannot always be shown to be sufficient, one should interpret the simulated path as simply characteristic of a solution.

The basic case assumes a zero discount rate and zero travel costs ($r = 0$ and $p_2 = 0$). In this run, the tax rate peaks above 22% in the third decade, then falls nearly to zero by the sixth decade, when the town ends its tourism program (Figure 2).

[25] Appendix B has details.

[26] I thus rule out subsidies ($\tau < 0$) and negative demand.

[27] The Runge-Kutta method deploys Simpson's rule for numerical integration. By the fundamental theorem of calculus,

$$\tau(t_n + h) - \tau(t_n) = \int_{t_n}^{t_n + h} \dot{\tau}(t)\, dt \tag{28}$$

where h is the iteration step. Using Simpson's rule,

$$\tau(t_n + h) \sim \tau(t_n) + \frac{h}{6}\left[\dot{\tau}(t_n) + 2\dot{\tau}(t_n + \frac{h}{2}) + 2\dot{\tau}(t_n + \frac{h}{2}) + \dot{\tau}(t_n + h)\right]. \tag{29}$$

The Runge-Kutta method estimates the derivatives in brackets. I split $4\dot{\tau}(t_n + \frac{h}{2})$ into two terms, because the method uses the estimate of the first term to devise an improved estimate of the second term.

For $h = 1/16$, the cumulative error on an interval is of the order h^4, about $1.53E^{-5}$.

[28] In most runs, the simulation terminates at period T_1 when each period nearing T_1 produces a diminishing gain for Host and each period moving away from T_1 produces a growing loss, indicating that $\mu(T_1) \sim 0$. This typically happens when tax rates turn negative within two periods after T_1. Negative values are not feasible in the constrained problem; and, as the text shows, paying a subsidy is not optimal for Host at the endtime in any event.

[29] Copies of the program are available upon request from the author.

Raising the discount rate r causes the tax path to shift to the southwest. For higher discount rates, the jurisdiction begins cutting the tax rate sooner and at a lower level. When the discount rate is 20%, the tax path peaks at about 8% in the second decade, and the tourism program ends in the third decade.

Higher costs to the tourist (p_1) lower the tax path and extend the tourism program. When the discount rate is 0, increasing p_1 from \$100 to \$150 lowers the peak tax rate from roughly 23% to below 10%; the tourism program lengthens by almost two-thirds. Higher travel costs (p_2) also lower the tax path and extend the tourism program. When the discount rate is 5%, doubling travel costs from \$25 to \$50 will lower the peak tax rate from about 15% to 10%.

Here is an intuition behind the results that concern costs. Host begins with a lower tax rate to offset the rise in costs to tourists. Because the offset is partial, total costs to the tourist rise. Thus fewer tourists come to Host in the early periods. Since Q is lower for every value of t, the word-of-mouth effect does not diminish (in the sense that $\partial g/\partial Q$ turns negative) until later in the program, so Host does not begin cutting taxes until later. The program elongates. As a consequence of this effect, Host will charge higher tax rates in the late periods of the program than it would otherwise.

A higher capacity for attracting tourists (Qb) raises the tax path and shortens the tourism program. When the discount rate is 5%, doubling Qb to 2000 raises the peak tax rate slightly, from 20% to 22%; and it shortens the tourism period dramatically, from about five decades to three. An intuition into this result is that wear-and-tear costs rise rapidly as Host exploits its heightened power in drawing tourists.

5 Appendix B: Derivations

5.1 Deriving a necessary condition for an optimal tax path

To ensure that the problem is interesting, assume that $g(Q) > 0$. Let $x = (1+\tau(t))p_1 + p_2$. Define f' to be df/dx and f'' to be d^2f/dx^2.

The optimality condition is

$$gp_1 \left[f + f'(\tau p_1 - c + \mu)\right] = 0. \tag{30}$$

Solve for μ:

$$\mu = -\frac{f}{f'} + c - \tau p_1. \tag{31}$$

Totally differentiate with respect to t:

$$\frac{\partial \mu}{\partial t} = -p_1 \frac{\partial \tau}{\partial t}\left[2 - \frac{ff''}{(f')^2}\right] + \frac{\partial c}{\partial Q}\frac{\partial Q}{\partial t}. \tag{32}$$

The equation of motion for the costate variable is

$$\frac{\partial \mu}{\partial t} = r\mu + f\left[\frac{\partial c}{\partial Q} g - (\tau p_1 - c + \mu)\frac{\partial g}{\partial Q}\right]. \tag{33}$$

Substitute (31) into (33):

$$\frac{\partial \mu}{\partial t} = r\mu + f\left[\frac{\partial c}{\partial Q} g + \frac{f\frac{\partial g}{\partial Q}}{f'}\right]. \tag{34}$$

To obtain (8), equate (32) to (34), using (5).

5.2 Sufficiency

Manipulating the current-value Hamiltonian H in (7) yields

$$H_{\tau\tau} = 2p_1 f' g + [\tau p_1 - c + \mu] f'' g < 0 \tag{35}$$

and

$$H_{QQ} = f[g''(\tau p_1 - c + \mu) - 2c'g' - gc''] \tag{36}$$

which is negative unless g' is strongly negative – that is, unless a rise in the cumulative number of visitors to Host sharply diminishes the number of tourists who now want to visit Host.

We also have that

$$H_{\tau Q} = p_1 f g' + [\tau p_1 - c + \mu] f' g' - c' f' g \tag{37}$$

so that the value of the discriminant of the quadratic form for H is

$$\begin{aligned} H_{QQ} H_{\tau\tau} - {H_{Q\tau}}^2 \quad = & \\ & fg[(\tau p_1 - c + \mu)g'' - 2c'g' - c''g][2p_1 f' + (\tau p_1 - c + \mu) f''] \\ - & \quad [p_1 f g' + (\tau p_1 - c + \mu) f' g' - c' f' g]^2. \end{aligned} \tag{38}$$

This expression is positive if the absolute value of f'' or of g'' is large enough – that is, if either demand function f or g changes sharply in response to changes in the price of a visit or in the cumulative number of visitors.

When there is no word-of-mouth effect, then $g' = 0$ and $g'' = 0$, and (38) reduces to

$$\begin{aligned} H_{QQ} H_{\tau\tau} - {H_{Q\tau}}^2 = \\ -fgc''[2p_1 f' g + (\tau p_1 - c + \mu) f'' g] - [c' f' g]^2. \end{aligned} \tag{39}$$

This expression is positive if the absolute value of c'' or of f'' is large enough.

5.3 Comparative statics

I will obtain comparative statics for an implicit function of the optimal tax path, $\tau(\mu, Q)$. Again I assume that $g(Q) > 0$. Expanding (30) gives us

$$f\left([1+\tau(\mu,Q)]\,p_1+p_2\right)+\left[f'\left([1+\tau(\mu,Q)]\,p_1+p_2\right)\right]\left[\tau(\mu,Q)p_1-c+\mu\right]=0. \tag{40}$$

Differentiate (40) with respect to μ:

$$f'\frac{\partial\tau}{\partial\mu}p_1+\left(f''\frac{\partial\tau}{\partial\mu}p_1\right)(\tau p_1-c+\mu)+f'\frac{\partial\tau}{\partial\mu}p_1+f'=0. \tag{41}$$

Solving for $\partial\tau/\partial\mu$ yields

$$\frac{\partial\tau}{\partial\mu}=-\frac{f'}{p_1\left[2f'+f''\left(\tau p_1-c+\mu\right)\right]}. \tag{42}$$

If $f'=0$, then $\partial\tau/\partial\mu=0$. If $f'\neq 0$, then divide the numerator and the denominator of (42) by f' to obtain

$$\frac{\partial\tau}{\partial\mu}=-\frac{1}{p_1\left[2+\frac{f''}{f'}\left(\tau p_1-c+\mu\right)\right]}. \tag{43}$$

Substitute (31) into (43):

$$\frac{\partial\tau}{\partial\mu}=-\frac{1}{p_1}\left[\frac{1}{2-\frac{f''f}{(f')^2}}\right]<0. \tag{44}$$

Now differentiate (40) with respect to Q to obtain

$$f'\frac{\partial\tau}{\partial Q}p_1+f''(\tau p_1-c+\mu)\frac{\partial\tau}{\partial Q}p_1+f'\frac{\partial\tau}{\partial Q}p_1-f'\frac{\partial c}{\partial Q}=0. \tag{45}$$

Solve for $\partial\tau/\partial Q$:

$$\frac{\partial\tau}{\partial Q}=\frac{f'\frac{\partial c}{\partial Q}}{p_1\left[2f'+f''(\tau p_1-c+\mu)\right]}. \tag{46}$$

Again use (31) to obtain

$$\frac{\partial\tau}{\partial Q}=\frac{\frac{\partial c}{\partial Q}}{p_1\left[2-\frac{f''f}{(f')^2}\right]} \tag{47}$$

which takes the same sign as $\partial c/\partial Q$.

5.4 Deriving necessary conditions for the optimal endpoint T

The current-value Hamiltonian is

$$H = [\tau(t)p_1 - c(Q) + \mu]\, f\left([1+\tau(t)]p_1 + p_2\right) g(Q). \tag{48}$$

The solution must satisfy, for all t such that $0 \leq t \leq T$,

$$gp_1 \left[f + (\tau p_1 - c + \mu)\frac{\partial f}{\partial \tau} \right] = 0, \tag{49}$$

$$g(p_1)^2 \left[2\frac{\partial f}{\partial \tau} + \frac{\partial^2 f}{\partial \tau^2}(\tau p_1 - c + \mu) \right] < 0, \tag{50}$$

and

$$\frac{\partial \mu}{\partial t} = r\mu + f \left[\frac{\partial c}{\partial Q} g - (\tau p_1 - c + \mu)\frac{\partial g}{\partial Q} \right]. \tag{51}$$

The solution must also satisfy, at $t = T$,

$$e^{-rT}\mu(T) = \phi, \tag{52}$$

$$\left[\tau p_1 - c + e^{-rT}\mu(T)\right] fg = 0, \tag{53}$$

$$\phi \geq 0, \tag{54}$$

and

$$\phi\,[Q(T) - Q_0] = 0. \tag{55}$$

If Host finds it optimal to encourage tourism, $Q(T) > Q_0$. Use this condition, plus (55), (52) and (53), to obtain (17) and (18).

5.5 When is it optimal to ban tourism?

Use (52) to substitute for ϕ in (55), obtaining

$$e^{-rT}\mu(T)\,[Q(T) - Q(0)] = 0. \tag{56}$$

Now suppose that, for the optimal choice of T, Host would lose money from the tourist flow at that moment: $(\tau p_1 - c)fg < 0$. Then, from (53), $e^{-rT}\mu(T) > 0$. From (56), $Q(T) = Q(0)$. Host should ban tourism if it would lose money at the best moment T for ending the program.

These results are bolstered by straightforward interpretations. Suppose that the discounted value to Host of adding a visitor to the stock of tourists at time

T is negative: $e^{-rT}\mu(T) < 0$. For instance, the marginal visitor may discourage future visitors from flocking to Host. Then $Q(T) = Q(0)$ is optimal for Host. On the other hand, suppose that the discounted value to Host of adding to the tourist stock is positive: $e^{-rT}\mu(T) > 0$. Then $Q(T) = Q(0)$ is optimal if the addition to the tourist flow at T is negative: $(\tau p_1 - c)fg < 0$. If the addition to the tourist flow at T is not negative, then – by (53) – T itself is not optimal: For if the marginal visitor adds to the value of the stock without subtracting from the value of the flow, then Host should continue its program of tourism rather than end it at T.

Thus, in the model at hand, Host should not embark on tourism if it can only lose money. The main text assumes that some tourism is optimal.

5.6 Incorporating travel distance into the model

Let i index the tourist's zone of origin, $i = 1, 2, ..., n$. I assume that when the tourist returns home, he spreads word of his visit among others who live in his zone of origin. Thus the stimulus to visit Host due to word-of-mouth depends upon the zone of origin: $g = g(Q_i)$. But the wear-and-tear costs of tourism – or the benefits of learning-by-doing – depend on the total number of cumulative visits, so $c = c(Q)$, where $Q = \sum Q_i$.

Host would seek the tax schedule $\tau_i(t)$ and the endtimes T_i, $i = 1, 2, ..., n$, that maximize

$$\sum_{i=1}^{n} \int_0^{T_i} e^{-rt} \left[\tau_i(t)p_1 - c(Q)\right] f\left(\left[1 + \tau_i(t)p_1\right] + p_{i2}\right) g(Q_i)\, dt \tag{57}$$

subject to

$$\frac{\partial Q_i}{\partial t} = f\left(\left[1 + \tau_i(t)\right] p_1 + p_{i2}\right) g(Q_i), \tag{58}$$

$$Q_i(0) = Q_{i0} \tag{59}$$

and

$$Q = \sum_{i=1}^{n} Q_i. \tag{60}$$

One may view this problem most simply as a system of n Hamiltonians. Obtain the n optimality conditions

$$\frac{\partial H_i}{\partial \tau_i} = p_1 f\left(\left[1 + \tau_i(t)\right] p_1 + p_{i2}\right) g(Q_i) + \left[\tau_i p_1 - c(Q) + \mu_i\right] f' g(Q_i) p_1 = 0. \tag{61}$$

Note that $f\left(\left[1 + \tau_i(t)\right] p_1 + p_{i2}\right)$ may vary with i, so f' may vary with i, too.

Also obtain the n equations of motion for the costate variable

$$\frac{\partial \mu_i}{\partial t} = r\mu + f\left[\frac{\partial c}{\partial t} g(Q_i) - (\tau_i p_1 - c + \mu_i)\frac{\partial g}{\partial Q_i}\right]. \tag{62}$$

By a method like that in the first section of this appendix, one can show that when the discount rate is zero, Host will pick $\dot{\tau}_i$ to have the same sign as $\partial g / \partial Q_i$.

A terminal condition that characterizes the optimal choice of T_i is

$$(\tau_i p_1 - c) fg = 0 \tag{63}$$

for $i = 1, 2, ..., n$.

5.7 Deriving an expression to yield the competitive tax

Using (31) and the result $f' = f_\tau / p_1$, one can write

$$\mu = \left(-\frac{f}{f_\tau} - \tau\right) p_1 + c \tag{64}$$

or

$$\mu = \left(-\frac{f}{f_\tau \tau} - 1\right) p_1 \tau + c. \tag{65}$$

But the tax elasticity of demand for tourism is $-f_\tau \tau / f$, so write

$$\mu = \left(\frac{1}{\epsilon} - 1\right) p_1 \tau + c \tag{66}$$

which gives us (15).

5.8 Evaluating the congestion and Baumol effects

A necessary condition for maximizing (21) is

$$\frac{\partial H}{\partial \tau} = fgp_1 + [\tau p_1 - C + \mu] f' p_1 g - C_\tau fg = 0. \tag{67}$$

Solving for μ,

$$\mu = -\frac{f}{f' p_1}[p_1 - C_\tau] - \tau p_1 + C. \tag{68}$$

Differentiate totally with respect to t:

$$\frac{\partial \mu}{\partial t} = \frac{f}{f' p_1}\left[C_{\tau\tau}\dot{\tau} + C_{\tau Q}\dot{Q} + C_{\tau t}\right] - \dot{\tau}[p_1 - C_\tau]\left[2 - \frac{ff''}{(f')^2}\right] + C_Q \dot{Q} + C_t. \tag{69}$$

Any solution to the intertemporal problem requires that, for every t,

$$\frac{d^2H}{d\tau^2} = p_1 g\,(p_1 - C_\tau)\left(2 - \frac{ff''}{(f')^2}\right) f' - C_{\tau\tau} f g \leq 0. \tag{70}$$

Recall that $C_{\tau\tau} \geq 0$, so $-C_{\tau\tau} f g \leq 0$. Also, $C_\tau < 0$, so $p_1 g(p_1 - C_\tau) f' < 0$. To ensure concavity, then, impose the condition that

$$2 - \frac{ff''}{(f')^2} \geq 0 \tag{71}$$

on (70).

The equation of motion for the costate variable gives us

$$\frac{\partial \mu}{\partial t} = r\mu + C_Q f g - [\tau p_1 - C + \mu]\, f \frac{\partial g}{\partial Q}. \tag{72}$$

Substitute (68) into (72):

$$\frac{\partial \mu}{\partial t} = r\mu + C_Q f g \frac{f}{f' p_1}\,(p_1 - C_\tau)\,\frac{\partial g}{\partial Q} f. \tag{73}$$

Equate the two expressions for $\partial\mu/\partial t$, (68) and (73):

$$\begin{aligned} r\mu + \frac{f}{f'p_1}\,(p_1 - C_\tau)\,\frac{\partial g}{\partial Q} f \\ = \dot{\tau}\left[\frac{f}{f'p_1} C_{\tau\tau} - (p_1 - C_\tau)\left(2 - \frac{ff''}{(f')^2}\right)\right] \\ + \frac{f}{f'p_1}\left[C_{\tau Q}\dot{Q} + C_{\tau t}\right] + C_t. \end{aligned} \tag{74}$$

A separable cost function gives us $C_{\tau Q} = C_{\tau t} = 0$. The equation (74) then reduces to

$$\begin{aligned} r\mu + \frac{f}{f'p_1}(p_1 - C_\tau)\frac{\partial g}{\partial Q} f \\ = \dot{\tau}\left[\frac{f}{f'p_1} C_{\tau\tau} - (p_1 - C_\tau)\left(2 - \frac{ff''}{(f')^2}\right)\right] + C_t. \end{aligned} \tag{75}$$

Setting the Baumol effect to zero ($C_t = 0$) gives us

$$r\mu + \frac{f}{f'p_1}(p_1 - C_\tau)\frac{\partial g}{\partial Q} f = \dot{\tau}\left[\frac{f}{f'p_1} C_{\tau\tau} - (p_1 - C_\tau)\left(2 - \frac{ff''}{(f')^2}\right)\right]. \tag{76}$$

Setting instead the congestion effect to zero ($C_\tau = C_{\tau\tau} = 0$), we get

$$r\mu + \frac{f^2}{f'}\frac{\partial g}{\partial Q} - C_t = -\dot{\tau} p_1 \left(2 - \frac{ff''}{(f')^2}\right). \tag{77}$$

5.9 Existence of a solution

We have that:

(a) There exists an admissible pair $(Q(t), \tau(t))$;

(b) The set $N(Q,\tau,t) = \{e^{-rt}(\tau p_1 - c)fg + \gamma, fg\}$ is convex for each (Q,t), $\gamma \leq 0$;

(c) The control set $\tau \in [\tau_L, \tau_U]$ is closed and bounded;

(d) There is a number of tourists, Q_m, such that $Q \leq Q_m$ for all $t \in [0,T]$ and for all admissible pairs $(Q(t), \tau(t))$; and

(e) T is in $[T_1, T_2]$, $0 \leq T_1 < T_2$.

I choose T_2 so that

(f) conditions *(a)* through *(d)* are satisfied on $[0, T_2]$.

Given *(a)* through *(f)*, the Filippov-Cesari theorem implies that an optimal pair $(Q^{**}(t), \tau^{*}(t))$ exists.[30]

Here is an interpretation for the requirement in (b) of a convex set.[31] Suppose that the stock of tourists in the jurisdiction at time t is $Q(t)$. Suppose that we can change the stock at the rate $\dot{Q}_1$ or at the rate $\dot{Q}_2$. Then we can instead change the stock at any rate $\dot{Q}$ that is a convex combination of $\dot{Q}_1$ and $\dot{Q}_2$. Further, $\dot{Q}$ will increase the net revenues of tourism at time t, $(\tau p_1 - c)fg$, by an amount at least as great as the convex combination of the increases related to $\dot{Q}_1$ and $\dot{Q}_2$.

I will outline a proof that $N[Q,\tau,t]$ is a convex set when $f'' = 0$. The proof is similar for the case in which $f'' < 0$.

Let us fix Q, t and normalize τ to $[0,1]$. Consider two arbitrary points y_1, y_2 in $N[Q,\tau,t]$, where

$$y_1 = (e^{-rt}[\tau_1 p_1 - c]f(\tau_1)g, f(\tau_1)g) \tag{78}$$

and

$$y_2 = (e^{-rt}[\tau_2 p_1 - c]f(\tau_2)g, f(\tau_2)g), \tag{79}$$

where $\tau_1 \leq \tau_2$. Let $\lambda \in [0,1]$. Let $y_3 = \lambda y_1 + (1-\lambda)y_2$. We want to show that $y_3 \in N[Q,\tau,t]$.

Toward that end, let

$$\lambda y_1 + (1-\lambda)y_2 = (z_1, z_2) \tag{80}$$

and consider the components z_1, z_2 separately. We have that

$$z_1 = \lambda e^{-rt}[\tau_1 p_1 - c]f(\tau_1)g + (1-\lambda)e^{-rt}[\tau_2 p_1 - c]f(\tau_2)g. \tag{81}$$

Now, let $U = [(\tau p_1 - c)f(\tau)g]$, where $U' > 0$ and $U'' \leq 0$. Since U is concave,

[30] I rely here on the restatement of a modified version of the theorem in [23].

[31] I draw here upon [23].

$$\begin{aligned}
&\lambda U[(\tau p_1 - c)f(\tau_1)g] + (1-\lambda)U[(\tau_2 p_1 - c)f(\tau_2)g] \\
\leq\;& U[\lambda(\tau_1 p_1 - c)f(\tau_1)g + (1-\lambda)(\tau_2 p_1 - c)f(\tau_2)g]. \qquad (82)
\end{aligned}$$

In the simulations, we also have that $f'(\tau) < 0$ and $f''(\tau) = 0$. Let $\tau_3 = \lambda\tau_1 + (1-\lambda)\tau_2$, $\tau_3 \in [0,1]$. Thus

$$\lambda f(\tau_1) + (1-\lambda)f(\tau_2) = f(\lambda\tau_1 + (1-\lambda)\tau_2) = f(\tau_3). \qquad (83)$$

Bearing in mind (83), note that

$$\begin{aligned}
&U[\lambda(\tau_1 p_1 - c)f(\tau_1)g + (1-\lambda)(\tau_2 p_1 - c)f(\tau_2)g] \\
=\;& U[\lambda\tau_1 p_1 f(\tau_1)g + (1-\lambda)\tau_2 p_1 f(\tau_2)g - c(\lambda f(\tau_1) + (1-\lambda)f(\tau_2))g] \\
=\;& U[p_1(\lambda\tau_1 f(\tau_1)g + (1-\lambda)\tau_2 f(\tau_2)g) - cf(\tau_3)g]. \qquad (84)
\end{aligned}$$

Consider now the expression

$$\lambda\tau_1 f(\tau_1) + (1-\lambda)\tau_2 f(\tau_2). \qquad (85)$$

Manipulations yield

$$\begin{aligned}
&\lambda\tau_1 f(\tau_1) + (1-\lambda)\tau_2 f(\tau_2) \\
\leq\;& \lambda\tau_1 f(\tau_1) + (1-\lambda)\tau_2 f(\tau_1) \\
=\;& \tau_3 f(\tau_1) \\
\leq\;& \tau_3 f(\lambda\tau_1) \\
\leq\;& \tau_3 f(\lambda\tau_1) + \tau_3 f((1-\lambda)\tau_2) \\
\leq\;& \tau_3 f(\tau_3). \qquad (86)
\end{aligned}$$

We thus have that

$$\begin{aligned}
&U[p_1(\lambda\tau_1 f(\tau_1)g + (1-\lambda)\tau_2 f(\tau_2))g - cf(\tau_3)g] \\
\leq\;& U[p_1\tau_3 f(\tau_3)g - cf(\tau_3)g] \\
=\;& U[(p_1\tau_3 - c)f(\tau_3)g]. \qquad (87)
\end{aligned}$$

It follows that

$$\begin{aligned}
&e^{-rt}U[p_1(\lambda\tau_1 f(\tau_1)g + (1-\lambda)f(\tau_2)\tau_2)g - cf(\tau_3)g] \\
\leq\;& e^{-rt}U[(p_1\tau_3 - c)f(\tau_3)g]. \qquad (88)
\end{aligned}$$

Turning to the second component z_2, we have that

$$\begin{aligned} z_2 &= \lambda f(\tau_1)g + (1-\lambda)f(\tau_2)g \\ &= f(\tau_3)g. \end{aligned} \tag{89}$$

Putting together (z_1, z_2), we have $\tau_3 \in [0,1]$ such that

$$\lambda y_1 + (1-\lambda)y_2 = (e^{-rt}(\tau_3 p_1 - c)f(\tau_3)g + \gamma_3, f(\tau_3)g) \tag{90}$$

where $\gamma_3 \leq 0$. So $\lambda y_1 + (1-\lambda)y_2 \in N[Q, \tau, t]$.

5.10 Deriving g(Q) for simulations

Let $\bar{Q}$ give the maximum number of tourists that would come to Host by word-of-mouth. Let the function

$$h(Q(t)) = \frac{\bar{Q}(t)}{1 + ae^{-bQ(t)}} \tag{91}$$

give the number of tourists that visit Host by time t through word-of-mouth. So

$$g(Q) = h'(Q) = \frac{\bar{Q}abe^{-bQ}}{(1+ae^{-bQ})^2}. \tag{92}$$

Note that

$$g'(Q) = \frac{\bar{Q}ab^2e^{-bQ}\left[ae^{-bQ} - 1\right]}{\left(1+ae^{-bQ}\right)^3}. \tag{93}$$

Thus $g'(Q) > 0$ for $Q < Q^*$, $g'(Q^*) = 0$, and $g'(Q) < 0$ for $Q > Q^*$, where $Q^* = -ln(1/a)/b$. I assume that $a > 1$ and $b > 0$.

In the simulations, I wish to set $Q^* = 555$. So I set $a = 4$ and $b = .0025000001$.

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