Product Differentiation and Public Education

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Abstract

Beginning with Tiebout (1956), numerous studies have argued that we should expect to see differences in public services among localities as a result of people "voting with their feet". Here, we consider differentiation in public services as a way of reducing competition among localities (cities). If cities finance their public services with a property tax that generates "tax competition", we find that adoption of quality differentiation in the public services will change the amount of services provided. If the cities maximize property values, this means a reduction in the level of public services provided for both the city that provides high quality as well as with low quality. The reduction in public services in both cities means that under certain conditions property values in both cities can increase. Thus in a two-stage game of adoption, we can observe quality differentiation in the services when the property tax is used. This is in sharp contrast to the case with a head tax in which we should never observe this type of differentiation. We believe quality differentiation might be particularly relevant to the provision of primary and secondary education. We argue that the extent of the differentiation in the quality and type of educational services provided among school districts might be in part a response to the detrimental effects of tax competition rather than entirely a "Tiebout-like" response to differences in tastes.

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1. *Introduction*

Numerous articles have examined the issue of "tax competition" in the past fifteen years. The majority, though not all, of the literature on tax competition focuses on the inefficiencies that might be created because local governments independently and competitively choose their tax policies. As first argued by Oates (1972), competition for business investment, a large share of the tax base for localities, may lead to localities underproviding public services to keep taxes low and business investment high. Wildasin (1989) refers to this as a "fiscal" externality – local governments, when setting their tax policies, ignore the fact that while an increase in their taxes reduces business capital in their city it will increase it in other cities. Since this is a positive externality that is being ignored, we have an underprovision of public services.

Numerous extensions have been made to the early formal analysis of tax competition made by Zodrow and Mieszkowski (1986) and Wilson (1986) which focused on tax competition among a large number of regions with mobile capital and immobile labor. Some of these extensions include consideration of labor mobility as well as capital mobility (Wilson (1992)), "metropolitan" models in the decision to work is independent of where to reside (Hoyt (1991, 1992, 1993), Krelove (1993)); and models with imperfect mobility (Wildasin and Wilson (1996), Myers (1990), Burbidge and Myers (1994), Wellisch (1994)). In addition, tax competition has been considered in the context of local governments having more than two tax or financing instruments (Wilson (1992); Hoyt (1991); Krelove (1993); Bucovetsky and Wilson (1991); and Jensen and Toma (1991)).

In this paper, we, too, extend the literature on tax competition, but in a very different direction than previous work. We borrow from the industrial organization literature and consider the impact of "product" differentiation on tax competition, where the product is the locally provided public service. As has been well-known within the context of competition among firms since Hotelling (1929), when firms differentiate products, profits can increase. In fact, even in Bertrand price competition, product differentiation can create positive profits in an industry where none would exist without that differentiation (Shaked and Sutton (1982)). We wish to see if this analogy can be applied to the public sector within the context of tax competition. We consider vertical differentiation of public services, where one (or both) cities can offer a high or low quality of the public service. Within the context of this simple model, we are interested in examining whether differentiating public services ameliorate the adverse impacts of tax competition and increase the welfare of constituents of the local government. We then consider the question of "If differentiation does reduce the adverse affects of tax competition, will (and under what conditions) local governments offer different qualities of public services?" within the context of a two stage "adoption" game.

We examine the issue of quality differentiation of public services using a model with capital and residents that is mobile among the cities in the metropolis. Each city provides, and finances through taxation, a single public service. Residents, then, choose where to live based upon the tax/public service mix and housing prices in the cities. For simplicity, we have only two cities, though with appropriate modifications, our results could be extended to models with more than two cities. We assume that local governments, when choosing their tax/public service policies and what type of quality to offer, are maximizing the value of the immobile asset within their city, land (property) value. Later, we discuss how our analysis and the primary implications of our results, that quality differences can both exist and reduce the adverse impacts of tax competition, can be extended to with alternative government objectives including utility and revenue maximization.

We believe quality differentiation might be particularly relevant to the provision of primary and secondary education. In addition to being the 43% of direct general expenditures of local governments[1], numerous studies have found evidence that educational provision influences locational decisions in metropolitan areas.[2] Unlike many other locally-provided public services such as roads, sewers, and waste removal, quality differences across jurisdictions in public education are more apparent, frequent, and likely to be of greater concern to residents.

One interesting implication of our analysis, discussed in more detail later, is how educational quality might be affected by state intervention in the financing and provision of education, particularly the use of tax limits and fiscal equalization plans. While one of the explicit intents of this type of legislation is to reduce educational quality differences related to the income or tax base of the locality, our analysis suggests that this analysis will also reduce quality differences among districts with similar incomes and tax bases.

In *Section 2* we provide an outline of our model, focusing on the equilibrium conditions and the structure of our two-stage game of almost perfect information. To serve as a basis of comparison, in *Section 3* we consider the quality adoption game when the public service is financed using a head tax. *Section 4* considers the impacts of quality differences on property values and quality adoption when the property tax finances public services. In *Section 5* we discuss some possible extensions and policy implications.

2. The Model

The model that we develop is similar to models of "imperfect" mobility in which individuals derive utility from attachment to a region (see DePalma and Papageourgiou (1988), Myers (1990), Burbidge and Myers (1994), and Wellisch (1994)). However, our model differs in two important respects. First, the models in these studies are best interpreted as models of "regions" where residents both work and consume public services. Thus when making a decision about where to live, individuals consider both labor market conditions and the tax/public service mix in the region. Equilibrium in these models requires clearing in each region's labor market. Our model, similar to those of Epple and Zelenitz (1981), Henderson (1985), Hoyt (1991,1992,1993), and Krelove (1993) is best interpreted as a model of a metropolitan area in which the decision about where to reside is independent of the decision of where to work. Residents, when choosing among the alternative providers of public services, "cities", consider the tax/public service mix and housing prices. Equilibrium requires clearing in each city's housing market. Since we are interested in applying our analysis to an understanding of differentiation in public education, a good provided in school districts that are generally much smaller than labor markets in which they are located, a model in which income is independent of locational choice seems appropriate.[3] Our model also allows us to focus on the impact of differences in public services (and quality) in migration without having confounding impacts on migration due to labor market conditions.

A second distinction is that imperfect mobility in our model is the result of the strategic actions of city governments and not exogenously determined. In the absence of any differences in the quality of services mobility is perfect, that is, there is no distinction in the preferences for the two cities among the residents. Only when one city chooses to be high quality will residents differ in their relative valuations of the two cities. Another distinction between our model and those of Burbidge and Myers (1994) and Wellisch (1994) is that, following the industrial organization literature, we have "vertical" differentiation in our cities while the cities in their studies might be best considered to be "horizontally" differentiated. In our model all residents, ceteris paribus, prefer high to low quality but some have a stronger taste for quality than others. In Burbidge and Myers (1994), for example, cities can not be ranked by quality as the "preferred" city differs among the residents. We discuss the implications of this distinction between vertical and horizontal differentiation later.

Two cities in the economy each provide a public service (education) denoted by g of quality (q) of 0 or 1 to the *N* individuals in the economy. All individuals have the same subutility function U(x,h,g) where x is a private traded good and h is housing. Following Burbidge and Myers (1994) and Wellisch (1994), individuals differ in the utility they receive from quality with individual *n* receiving utility of γn if q = 1. Then we let utility for individual *n* be given by the separable function $U^n(x,h,g,q) = x + \alpha(h) + \beta(g) + \gamma n q$. We normalize the population, N, to unity thereby interpreting the population of the cities as the fraction who reside there. We also assume the distribution of types *(n)* is uniform.[4] Then the individual with the highest taste for quality receives utility of γ from quality while the individual with lowest taste receives 0 with the median (and mean) utility from quality equal to $.5\gamma$.

We let p denote the net-of-tax price of housing, τ the ad-valorem (property) tax on housing, and $h(p(1+\tau))$ the demand for housing. While we are primarily interested in examining the provision of services and quality when the cities use a property tax, we also consider financing using a head tax (T) to serve as a basis of comparison.

Capital is used to produce all three goods with K units of capital in the economy. One unit of capital produces one unit of the private good x. Providing g units of education to *N* individuals requires *Ng* units of capital. Providing education quality of q=1 requires $N\delta$ units of capital. Housing is a non-traded good with $F(k_i)$ units produced in city *i* where $\frac{\partial F(k)}{\partial k} > 0$ and $\frac{\partial^2 F(k)}{\partial k^2} < 0$. We assume that the housing production function is same for both cities.[5] Then we can normalize the price of x and capital to unity and let p^i denote the price of housing in city *i*.

To formalize the question of the choice of quality, we model the provision of the education as a two stage game of almost perfect information. In stage one, each city simultaneously chooses the quality of its public good, high (q=1) or low (q=0). Then in stage two, after the outcome of stage one is observed, each city simultaneously chooses its tax. Note that this dynamic structure implicitly assumes the cities must credibly commit to the level of quality they intend to provide before they choose taxes, and thus choose the level of education they provide. Alternatively, this simply assumes that the quality of education is less easily adjusted than its quantity (so quality is chosen first, just as a firm must adopt a production technology before deciding how much to produce with that technology).

As is standard, we focus on the subgame perfect equilibria. The solution technique is, of course, backward induction. We first determine the Nash equilibrium for each of the two-stage sub-games: both cities adopt high quality, both adopt low quality, and the two quality differentiation outcomes in which one city adopts high quality and the other adopts low quality. The equilibrium payoffs from these subgames then are used to define a "reduced-form" single-stage game in which each city can adopt either high quality or low quality. Because these payoffs embody Nash equilibrium behavior in stage two by construction, it follows that a Nash equilibrium in qualities of the reduced-form game (together with the corresponding stage two taxes) is also a subgame perfect equilibrium of the dynamic game of providing education.

3. Provision of Education with Head Taxation

To provide a context in which to better understand the impact of property taxation on the decision to differentiate quality, we first consider as a benchmark the provision of education when the cities can use lump sum taxes.

3.1 Market Clearing Conditions in Stage Two with Head Taxation

With a head tax city *i's* balanced budget constraint is

$$N^i g^i = N^i T^i \text{ , } i = 1, 2. \qquad (1)$$

where N^i is the population of city i. For the moment, we ignore the financing of quality. Equilibrium also requires that at least one individual is indifferent between the two cities or

$$y - T^1 + \alpha(h(p^1)) + \beta(g^1) + \gamma(1 - N^1)q^1 = y - T^2 + \alpha(h(p^2)) + \beta(g^2) + \gamma(1 - N^1)q^2 \text{ .} \qquad (2)$$

We arbitrarily assume that if $q^1 \neq q^2$ then $q^1 > q^2$. If $q^1 > q^2$ then the N^1 individuals with the highest taste for quality will live in city *1*, $n \in [1 - N^1, 1]$. Note that if $q^1 = q^2$ all residents are indifferent between the two cities. Equilibrium in the housing market requires

$$N(p^1) + N(p^2) = 1, \qquad (3)$$

where $N(p^i) = \dfrac{H(p^i)}{h(p^i)}, i = 1,2$ and $H(p^i)$ is the supply of housing in city i with $\dfrac{\partial H}{\partial p^i} > 0$. Finally, if there are differences in the quality between the two cities then it must be the case that

$$N^1 = N(p^1), \qquad (4)$$

where $N(p^1)$ of the population with the strongest taste for quality live in city 1.

3.2 Determination of Housing Prices with Head Taxation

We first consider how balanced budget changes in the public service and tax rate affect housing prices when quality is the same ($q^1=q^2=0$ or $q^1=q^2=1$) in both cities. In this case individuals are not sorted among the two cities by their preference for quality. Since both cities are assumed to have the same housing production function and government objective, we are interested in the price changes in a symmetric equilibrium when $T^1 = T^2$ and therefore $p^1 = p^2$. Then totally differentiating (1) - (3) with respect to T^1 gives

$$dp^1_{T^1}\Big|_{q^1=q^2} = dp^2_{T^2}\Big|_{q^1=q^2} = \frac{(\beta'-1)}{2h}, \quad (5)$$

[6]

where x^i_j denotes $\frac{\partial x^i}{\partial j}, i=1,2; j=p,\tau$ and the superscripts on τ, β, and p are suppressed since we are considering a symmetric equilibrium. Note that $dp^1_{T^1}\Big|_{q^1=q^2} = 0$ when $\beta' = 1$, the first-best level of public service.

We now consider the impact of a balanced-budget increase in T^1 on housing prices when the two cities have different qualities of public services but the same housing prices and populations, $p^1 = p^2$ and $N^1 = N^2$. For these conditions to be satisfied, we assume an additional head tax of $T^q = \frac{\gamma}{2}$ is assessed. Our purpose in maintaining this symmetry with differentiation is so that we can easily compare tax policies with and without differentiation. When $q^1 = 1$ and $q^2 = 0$ we have to also consider the equilibrium condition, (4), the distribution of individuals according to their taste for quality. Then differentiating (1) - (4) gives

$$dp^1_{T^1}\Big|_{q^1>q^2} = dp^2_{T^2}\Big|_{q^1>q^2} = \frac{(\beta'-1)}{2h\left(1+\gamma\frac{(\theta-\varepsilon)}{ph}\right)}. \quad (6)$$

where $\varepsilon < 0$ and $\theta > 0$ are the price elasticities of demand for and supply of housing.[7] Note that again $dp^1_{T^1}\Big|_{q^1>q^2} = 0$ when $\beta' = 1$.

3.3 Property Value Maximization with Head Taxation

Following numerous other studies, we assume each city's objective is to maximize the value of property (land) in the city, or, in our framework, to maximize the price of housing. While with a small number of localities, this objective is not equivalent to maximizing utility for land-owning residents, it enables us to illustrate how quality differentiation affects government policies in a relatively direct and interpretable way. We discuss discuss the impacts of quality differentiation with alternative objectives in *Section 5*.

The first order condition for maximizing the price of housing is simply

$$p^i_{T_i} = 0,\ i = 1,2\,. \tag{7}$$

Comparing the balanced-budget impact of the head tax on the price of housing without quality differences (5) and with quality differences (6) allows us to make some statements regarding housing prices and government policies when cities use head taxes.

For notational convenience, hereafter let p^{U_T} denote the price of housing in each city when both provide education of the same quality (either high or low). Similarly, if they provide education of different quality, denote the price of housing by p^{H_T} in the city with high quality and p^{L_T} in the city with low quality.

Proposition 1: *When the cities use head taxes to finance education, product differentiation in educational quality necessarily results in a higher price of housing in one city, but a lower price of housing in the other. In particular:*

a) *If* $\delta > \frac{\gamma}{2}$ *then* $p^{H_T} < p^{U_T} < p^{L_T}$ *;*

b) *If* $\delta < \frac{\gamma}{2}$ *then* $p^{H_T} > p^{U_T} > p^{L_T}$ *;*

c) *If* $\delta = \frac{\gamma}{2}$ *then* $p^{H_T} = p^{U_T} = p^{L_T}$.

Proof. Let T^H and T^L denote the tax rates in the high and low quality cities. Then since $T^1=T^2$, in equilibrium we must have

$$\alpha(p^{H_T}) + \gamma(1 - N^{H_T}) - \delta = \alpha(p^{L_T}) \tag{2'}$$

as well as (3) and (4) satisfied. For these conditions to be satisfied when $\delta > \frac{\gamma}{2}$ (part a), we must have $p^{H_T} < p^{L_T}$.[8] Further, to maintain equilibrium in housing market it has to be the case that

$p^{H_T} < p^{U} < p^{L_T}$. Part *b* follows analogously.

Figure 1 provides a graphical interpretation of *Proposition 1*. In *Figure 1* the equilibrium conditions, the equal utility condition (2) and clearance in the housing market (3), are depicted. Equilibrium with no quality differentiation is given by p^U, the price (for both cities) that satisfies both (2) and (3). The locus of prices that give equilibrium in the housing market, (3), is unaffected by the quality differentiation. However, quality differentiation will change the locus of

prices that give equal utility condition (V^U *and* V^D refer to the price locus that supports equal utility without quality differentiation (U) and with it (D)). If $\delta < \frac{\gamma}{2}$ the equal utility frontier shifts to the northwest, while if $\delta > \frac{\gamma}{2}$ it shifts to the southeast. Then as the figure clearly shows with $\delta < \frac{\gamma}{2}$, we have $p^H > p^U$ and $p^L < p^U$. But when $\delta > \frac{\gamma}{2}$, we have $p^H < p^U$ and $p^L > p^U$.

3.4 Quality Adoption with Head Taxation

Under the assumption of property value maximization, the reduced-form game of quality adoption can be most easily described by the payoff matrix below.

		2: High	2: Low
1	High	p^U, p^U	p^H, p^L
	Low	p^L, p^H	p^U, p^U

The equilibrium result then follows immediately from *Proposition 1*.

Proposition 2*. If education is financed by a head tax, then both cities adopt the same quality of education (there is no product differentiation in educational quality). In particular:*

(i) If $\delta > \gamma/2$, then the subgame perfect equilibrium is unique and has both cities adopting low quality, $(q^1,q^2)=(0,0)$.

(ii) If $\delta < \gamma/2$, then the subgame perfect equilibrium is unique and has both cities adopting high quality, $(q^1,q^2) = (1,1)$.

Proof: Formally, let $H^i(q^1,q^2)$ be city i's reduced-form payoff (embodying stage two Nash equilibrium behavior) when city 1 adopts quality q^1 and city 2 adopts q^2. Then under the assumption of property value maximization, as shown in the matrix above these payoffs are: $H^i(1,1) = H^i(0,0) = p^{U_T}$ for i=1,2; $H^1(1,0) = H^2(0,1) = p^{H_T}$; and $H^1(0,1) = H^2(1,0) = p^{L_T}$.

If $\delta>\gamma/2$, then $H^1(0,1) > H^1(1,1)$ and $H^1(0,0) > H^1(1,0)$ from *Proposition 1*, which implies that $q^1 = 0$ is a strongly dominant strategy for city 1 in the reduced-form game. By symmetry, $q^2 = 0$ is a strongly dominant strategy for city 2 as well. Hence, $(q^1,q^2) = (0,0)$ is the unique Nash equilibrium of the reduced-form game, and so both provide low quality in the unique subgame perfect equilibrium. Similarly, if $\delta < \gamma/2$, then $H^1(1,0) > H^1(0,0)$ and $H^1(1,1) > H^1(0,1)$ from *Proposition 1*, which implies that $q^1 = 1$ is a strongly dominant strategy for city 1 in the reduced form game. By symmetry, $q^2 = 1$ is also a strongly dominant strategy for city 2. Hence, $(q^1,q^2) = (1,1)$ is the unique Nash equilibrium of the reduced-form game. Finally, for the sake of completeness, we note that anything can be a Nash equilibrium of the reduced-form game in the "razor's-edge" case where $\delta = \gamma/2$, because $H^i(q^1,q^2) = p^{U_T}$ for all (q^1,q^2) and all i.

With head taxation, the efficient level of the public good will be provided in stage two, whatever the level of quality chosen in stage one. The same is true of the choice of quality. Both cities provide low quality when the cost of quality is high, $\delta > \gamma/2$, and high quality when the cost of quality is low, $\delta < \gamma/2$.

4. *Provision of Education with Property Taxation*

We now replicate the analysis of the preceding section for the case where education is financed by property taxation. As expected, the level of education does not maximize property values in the economy when the property tax is used. However, we shall also show that this reduction in the value of land can be, and is in equilibrium, mitigated by quality differentiation in education.

4.1 Market Clearing Conditions in Stage Two with Property Taxation

With a property tax, city *i*'s budget constraint is

$$N^i g^i = N^i \tau^i p^i h(p^i(1+\tau^i)) \text{ , } i = 1, 2. \tag{8}$$

Again, equilibrium also requires one individual be indifferent between the two jurisdictions or

$$y+\alpha(h(p^1(1+\tau^1)))+\beta(g^1)+\gamma(1-N^1)q^1 = y+\alpha(h(p^2(1+\tau^2)))+\beta(g^2)+\gamma(1-N^1)q^2 \tag{9}$$

and clearing in the housing market, or

$$N(p^1,\tau^1)+N(p^2,\tau^2)=1, \tag{10}$$

where the population of city *i*, $N(p^i,\tau^i)$, i=1, 2 is a function of the property tax rate since the property tax affects the housing purchased by a resident. Again, if there are differences in the quality between the two cities, with $q^1= 1$ and $q^2= 0$, then it must be the case that

$$N^1 = N(p^1,\tau^1). \tag{11}$$

4.2 Determination of Housing Prices with Property Taxation

As with head taxation, we first consider how balanced budget changes in the public service and tax rate impact housing prices when quality is the same in both cities. Again, individuals are not sorted among the two cities by their preference for quality. Since the equilibrium we consider is a Nash equilibrium in tax rates, we also are assuming that as a result of changes in the tax/service mix in city 1, city 2 alters its level of public service in response and not its tax rate. Again, we focus on a symmetric equilibrium. Then differentiating (8) - (10) with respect to τ^1 when $q^1 = q^2 = 0$ gives

$$p^1_{\tau_1}\Big|_{q_1=q_2} = p^2_{\tau_2}\Big|_{q_1=q_2} = \frac{\frac{p}{2}\left(\left(\beta'-1\right)+\varepsilon\beta'\frac{\tau}{(1+\tau)}\right)}{\left((1+\tau)-\beta'\tau(1+\varepsilon)\right)}+\frac{1}{2}\frac{\varepsilon}{(\theta-\varepsilon)}\frac{p}{(1+\tau)}. \tag{12}$$

The first term of (12) can be considered the effect on housing prices that arises from the mobility of capital and residents. An increase in the tax rate due to this effect can increase or decrease housing prices depending on whether $\left(\beta'-1\right)+\varepsilon\beta'\frac{\tau}{(1+\tau)}>(<)\,0$ as (1+τ) - β'τ(1+ε) > 0. Intuitively, $\left(\beta'-1\right)+\varepsilon\beta'\frac{\tau}{(1+\tau)}=0$ when the level of public service is chosen to maximize residents' utility and $p^1_{\tau_1}=0$ – there is no capitalization. The second term of (12) is always negative. This is the effect of increasing the tax rate independent of mobility and is due to the reduction in housing demand brought about by the increase in the tax rate. More specifically, this reduction would occur even if both cities raised their tax rates. Note that unlike the head tax, if β' =1, $p^1_{\tau_1}\Big|_{q_1=q_2}\neq 0$.

We now consider the impacts of changes in the public service and tax rate when city 1 has high quality while city 2 has low quality. Again we assume a head tax $T^q=\frac{\gamma}{2}$ to ensure equal prices and populations with quality differentiation. As with the head tax anyone with $n>\frac{1}{2}$ will reside in city 1 while anyone with $n<\frac{1}{2}$ resides in city 2. Totally differentiating (8)-(11) gives

$$p^1_{\tau_1}\Big|_{q_1>q_2} = p^2_{\tau_2}\Big|_{q_1>q_2} = \frac{\frac{p}{2}\left(\left(\beta'-1\right)+\varepsilon\beta'\frac{\tau}{(1+\tau)}\right)}{\left[(1+\tau)-\beta'\tau(1+\varepsilon)+\gamma\frac{(\theta-\varepsilon)}{ph}\right]}+\frac{1}{2}\frac{\varepsilon}{(\theta-\varepsilon)}\frac{p}{(1+\tau)} \tag{13}$$

Inspect of (13) suggests that quality differences reduce the mobility of individuals among the two cities thereby decreasing the magnitude of the price changes that occur as a result of changes in tax and public service policies. More formally, we have:

Proposition 3*: If* $(\beta'-1)+\varepsilon\beta'\frac{\tau}{(1+\tau)}>(<)\,0$ *then* $p^1_{\tau^1}\Big|_{\substack{q^1=q^2\\ \tau^1=\tau^*}}>(<)\,p^1_{\tau^1}\Big|_{\substack{q^1>q^2\\ \tau^1=\tau^*}}$.

This proposition follows from comparing (12) and (13).

4.3 Property Value Maximization with Property Taxation

Consistent with our derivation of $p^i_{\tau^i}$, we consider property value maximization in a Nash equilibrium in tax rates. We assume that while the city uses the property tax to finance the public service it uses a head tax to finance quality. This assumption does not change our qualitative results and keeps our analysis tractable. Then the property value maximizing tax rate for city *i* must satisfy

$$p^i_{\tau_i} = 0. \quad (14)$$

Let the tax rate that satisfies (14) for both cities when $q_1 = q_2 = 0$ be denoted by τ^U and the associated price of housing by p^{U_τ}. Using (12) it is relatively easy to show that β' > 1 – the public service is underprovided. Before considering the policies chosen when quality is different in the two cities, we first present a proposition to help understand our later results.

Proposition 4. *Let $q^1 = q^2 = 0$ or $q^1 = q^2 = 1$ and $\tau^1 = \tau^2 = \tau^U$, the property-value maximizing rate with no quality differences. Then a balanced-budget decrease in the property tax in both cities will increase the price of housing in both cities.*

Proof. *Proposition 4* can be seen immediately by inspection of (3), equilibrium in the housing market. The competition among the two city leads to higher tax rate and public service level than they would have if there were a single city (which would choose $\tau = 0$). Then reducing competition should decrease taxes.

We now can compare the tax rates chosen in the equilibrium without quality differences to those chosen with differences. Since at τ^U it must be the case that $(\beta'-1)+\varepsilon\beta'\frac{\tau}{(1+\tau)}>0$ for $p^i_{\tau_i}=0$ by *Proposition 3* it follows that

$$p^i_{\tau^i}\Big|_{\substack{q^1>q^2 \\ \tau^1=\tau^U}} <0,\ i = 1,2. \quad (15)$$

When the two cities have different levels of quality it is the case that τ^U is above the property value-maximizing rate for both cities. Let the superscripts H and L denote the equilibrium tax rates with high and low quality. Then with well-behaved reaction functions we have the following result.

Proposition 5. $\tau^U > \tau^H$ *and* $\tau^U > \tau^L$.

Both cities will set lower tax rates if they differ in quality of the public service. In addition to having different tax rates, quality differences will lead to differences in property values in the two cities. These differences are summarized next.

Proposition 6. *Assume the cities use property taxes to finance education. If the marginal cost of quality is either high enough or low enough, product differentiation in educational quality necessarily results in a higher price of housing in one city, but a lower price of housing in the other. However, for intermediate values of the marginal cost of quality, product differentiation in educational quality can result in a higher price of housing in both cities. In particular, there exist an* $\varepsilon^H > 0$ *and* $\varepsilon^L > 0$ *such that:*

(i) If $\delta > \frac{\gamma}{2} + \varepsilon^H$, *then* $p^{H_\tau} < p^{U_\tau} < p^{L_\tau}$;

(ii) If $\frac{\gamma}{2} < \delta < \frac{\gamma}{2} + \varepsilon^H$ *then* $p^{U_\tau} < p^{H_\tau} < p^{L_\tau}$;

(iii) If $\frac{\gamma}{2} = \delta$ *then* $p^{H_\tau} = p^{U_\tau} = p^{L_\tau}$;

(iv) If $\frac{\gamma}{2} - \varepsilon^L < \delta < \frac{\gamma}{2}$ *then* $p^{H_\tau} > p^{L_\tau} > p^{U_\tau}$ *; and*

(v) If $\delta < \frac{\gamma}{2} - \varepsilon^L$, *then* $p^{H_\tau} > p^{U_\tau} > p^{L_\tau}$.

Proof. That $p^{H_\tau} < p^{L_\tau}$ when $\delta > \frac{\gamma}{2}$ follows from the fact that if the cost of quality is greater than the marginal quality with equal populations in both cities ($\frac{\gamma}{2}$) then the city that adopts the quality is at a disadvantage relative to the city that does not adopt quality. Therefore its housing prices can not be higher. If $\delta < \frac{\gamma}{2}$, the reverse is true. The more interesting result is *(ii)* which suggests that quality adoption could increase housing prices in both cities even when $\delta > \frac{\gamma}{2}$. To see this, consider the case when $\delta = \frac{\gamma}{2}$. In this case it can easily be shown that in equilibrium the tax rate in the high and low quality cities are the same, $\tau^H = \tau^L < \tau^U$ which implies that $p^{H_\tau} = p^{L_\tau} > p^{U_\tau}$. Now suppose we have a change in δ, $\Delta\delta$, from $\delta = \frac{\gamma}{2}$ to $\delta = \frac{\gamma}{2} + \varepsilon^H$ or $\Delta\delta = \varepsilon^H$. Then, as discussed earlier, $p^{H_\tau} < p^{L_\tau}$. Further assume there exists some k such that $\Delta\tau^H < k\Delta\delta$ and there also exists some c such that $\Delta N^H > c\Delta\delta$.[9] Then since ε is small we can use (4) to obtain the following first-order approximation for the Δp:

$$\Delta p^{H_\tau} = \frac{p}{(\theta-\varepsilon)}\left[\frac{\Delta N^H}{N^H} + \frac{\varepsilon}{(1+\tau)}\Delta\tau^H\right] = \frac{p}{(\theta-\varepsilon)}\left[\frac{c}{N^H} + \frac{\varepsilon}{(1+\tau)}k\right]\varepsilon^H \qquad (16)$$

Since ε^H can be arbitrarily small we can have Δp^{H_τ} also arbitrarily small so that $p^{H_\tau}(\delta=\frac{\gamma}{2}+\varepsilon^H) = p^{H_\tau}(\delta=\frac{\gamma}{2}) - \Delta p^{H_\tau} > p^{U_\tau}$. An analogous argument would apply to part *(iv)* of the proposition.

Figure 2 provides a graphical depiction of *Proposition 6*. In *Figure 2,* analogous to *Figure 1* for the head tax, the equal utility condition (9) and clearance in the housing market (10) are depicted. The equilibrium prices without differentiation are p^U. With the property tax, quality differentiation will decrease the tax rates in the two cities (*Proposition 5*). Since the housing market clearing, $N^1 + N^2 = 1$, is equal to $\frac{H(p^1)}{h(p^1(1+\tau^1))} + \frac{H(p^2)}{h(p^2(1+\tau^2))} = 1$, a reduction in the cities' tax rates will shift the locus of prices that support clearing in the housing market to the northeast. If $\delta = \frac{\gamma}{2}$ the locus of prices that support the equal utility condition is unchanged. Then from *Figure 2* it is clear that housing prices in both cities increase as a result of differentiation when $\delta = \frac{\gamma}{2}$. *Figure 2* also illustrates the conditions that determine ε^H and ε^L.

4.4 Quality Adoption with Property Taxation

The payoff matrix for the reduced-form game with property taxation is also described by the payoff matrix in Section 3.4 above. The difference is that now the relationships between the property values in the various subgames are given by *Proposition 6.* The equilibrium result then follows immediately.

Proposition 7. *If education is financed by a property tax, then both cities adopt the same quality if its marginal cost is either high or low enough. However, for intermediate values of the marginal cost of quality, the cities adopt different qualities of education. In particular:*

(i) If $\delta > \gamma/2 + \varepsilon^H$, then the subgame perfect equilibrium is unique and has both cities adopting low quality, $(q^1,q^2) = (0,0)$.

(ii) If $\gamma/2-\varepsilon^L < \delta < \gamma/2 + \varepsilon^H$, then the subgame perfect equilibria in pure strategies has one city adopting high quality and the other city adopting low quality, either $(q^1,q^2) = (1,0)$ or $(q^1,q^2) = (0,1)$.

(iii) If $\delta < \gamma/2 - \varepsilon^L$, then the subgame perfect equilibrium is unique and has both cities adopting high quality $(q^1,q^2) = (1,1)$.

Proof. Formally define the payoffs to the reduced-form game with property taxation as $P^i(1,1) = P^i(0,0)= p^{U_\tau}$ for each i, $P^1(1,0) = P^2(0,1) = p^{H_\tau}$, and $P^1(0,1) = P^2(1,0)= p^{L_\tau}$.

If $\delta > \gamma/2 + \varepsilon^H$, then $P^1(0,0) > P^1(1,0)$ and $P^1(0,1) > P^1(1,1)$ from *Proposition 5*, which implies that $q^1 = 0$ is a strongly dominant strategy for city 1 in the reduced-form game. By symmetry, $q^2 = 0$ is also a strongly dominant strategy for city 2. Hence, $(q^1,q^2) = (0,0)$ is the unique Nash equilibrium of the reduced-form game.

If $\gamma/2 - \varepsilon^L< \delta<\gamma/2 + \varepsilon^H$, then from *Proposition 5*, $P^1(0,0) < P^1(1,0)$, $P^1(0,1) > P^1(1,1)$, $P^2(0,0) < P^2(0,1)$, and $P^2(1,0) > P^2(1,1)$. It therefore follows that $(q^1,q^2) = (1,0)$ and $(q^1,q^2) = (0,1)$ are the only Nash equilibria of the reduced-form in pure strategies. However, in this case there is also a mixed strategy equilibrium in which each city chooses high quality with probability $\sigma^H= (p^{H_\tau} - p^{U_\tau})/(p^{H_\tau} - p^{U_\tau} + p^{L_\tau} - p^{U_\tau})$.

Finally, if $\delta < \gamma/2 - \varepsilon^L$, then $P^1(1,0) > P^1(0,0)$, $P^1(1,1) > P^1(0,1)$, $P^2(0,1) > P^2(0,0)$, and $P^2(1,1) > P^2(1,0)$. Thus, $q^i = 1$ is a strongly dominant strategy for each i, which implies the Nash equilibrium of the reduced-form game is unique and has both cities adopting high quality, $(q^1,q^2) = (1,1)$.

Therefore, subgame perfect adoption of quality may be very different with property taxation in that it can result in product differentiation in educational quality. For example, suppose the cost of quality is just below the median utility from quality, $\gamma/2 - \varepsilon^L < \delta < \gamma/2$. Then one city adopts high quality and the other adopts low quality, despite the fact that both would adopt high quality under head taxation. The reason is that $p^{H_\tau} > p^{L_\tau} > p^{U_\tau}$ for these costs, so property values are higher in both cities if they provide education of different quality than if they both provide high quality education. The reduced-form game of quality adoption in this case is essentially the same as the well-known game of "Chicken." This game has two Nash equilibria. In each of them one player "chickens out" (adopts low quality) and the other does not chicken out (adopts high quality), and the player who does not chicken out earns a higher payoff ($p^{H_\tau} > p^{L_\tau}$). However, the player that chickens out still earns a higher payoff than if it also had not chickened out ($p^{L_\tau} > p^{U_\tau}$). This game-theoretic analysis cannot, of course, predict which city actually adopts high quality education. Nevertheless, it does predict that the cities provide education of different quality (just as it predicts that the equilibrium outcome of the game of chicken cannot be that neither player chickens out).

When the cost of quality is low, it is not surprising that quality differentiation results in a higher property value in the city with the high quality public good. What is surprising is that it also results in a higher property value in the city with low quality education. Clearly, the cities can diminish the distorting effects of property taxation on property values by differentiating their products.

Similarly, under property taxation cities adopt education of different quality when the cost of quality is just above the median utility from quality, $\gamma/2 + \varepsilon^H > \delta > \gamma/2$, despite the fact that both would adopt low quality under head taxation. Again, for these costs, property values are higher in both cities if they provide education of different qualities than if they both provide low equality. The only difference is that now $p^{L_\tau} > p^{H_\tau} > p^{U_\tau}$, so property value is higher in the city that adopts low quality than the one that adopts high quality, which is not unexpected because these costs are higher than the median utility from quality. In fact, the analogy to the game of Chicken still applies, although now the strategies are chicken out (adopt high quality) and not chicken out (adopt low quality). Quality differentiation in education allows the cities to diminish the distortionary effects of property value taxation.

It is worthwhile to note that these results are somewhat reminiscent of the results from the industrial organization literature on vertical (quality) product differentiation (see, for example, Shaked and Sutton (1982, 1983)). Briefly, this literature notes that, when firms compete in prices, producing goods of the same quality condemns them to zero profit, whereas they could earn positive profit by producing goods of different quality. Analogously, in those cases above where the cities provide education of different quality, they have higher property values than if they had both provided the same quality.

4.5 A Numerical Example

To give some indication of the extent to which quality differences might occur in our two-stage game with property taxation, we provide a simple numerical example. The additively separable sub-utility function for this example is $U(x,h,g) = x + ah^{\alpha} + bg^{\beta}$. The sub-utility function is parameterized so that for an efficient allocation, 25% of spending is on housing and 10% of spending is on the public service, g. Income is normalized to 100 and the model is calibrated so that the price of housing, p, equals unity with a head tax. The elasticity of supply for housing is .5 in all specifications and the parameter b=31.54 and β = .2 in all specifications. We consider two alternative specifications for the elasticity of demand for housing, one of which gives ε = -1.25 and on that gives ε = -2.[10]

The results of these numerical simulations are found in *Table 1*. In addition to considering two alternative demand elasticities for housing we also consider different magnitudes of the taste for quality, γ = 10 and γ = 5. For each specification of (ε,γ) we report simulation results for the case in which both localities have the same quality and both have different qualities with $\delta = \gamma/2$. In addition, we solve for ε^H and ε^L and report the results for these two cases as well. In all four alternative specifications we find, as predicted, that having quality differences when $\delta = \gamma/2$ will indeed increase property values in both cities. This increase in property values is due to the reduction in property taxes that occurs with quality differences. The reduction in property taxes ranges from 3% (ε = -1.25 and γ = 5) to 14.7% (ε=-2 and γ = 10). The increase in property values is more modest, ranging from 0.4% to 1.6%.

What may be the most informative and useful result of the numerical example is a indication of the range $[\gamma/2-\varepsilon_L, \gamma/2-\varepsilon_H]$ in which it is possible to obtain quality differences. To get an indication of the magnitude of this range we report ε^H (or ε^L since they are equal in all specifications) as a percentage of $\gamma/2$, the utility from high quality for the median household. This range, as a percentage of $\gamma/2$, varies from 24% (12% on both sides of $\gamma/2$) when $(\varepsilon,\gamma)= (-1.25,5)$ to 52% when $(\varepsilon,\gamma) = (-2,10)$. One way to interpret this range is context of the distribution of tastes among the households. For example, if $(\varepsilon,\gamma) =(-2,5)$, for quality differences to exist the range for δ must be such that no more than 60% of the population would gain from high quality ($.4\gamma < \delta$) and no more than 40% would lose ($.6\gamma > \delta$). While, logically, quality differences among the cities do not occur with everyone having the same preference for high versus low quality, they can occur even if a significant majority (60% in this case) prefer one level of quality to another. Quality differences in education do not, by any means, require households being evenly divided between those who want higher quality and those who do not want it.

5. Extensions and Policy Implications

Beginning with Tiebout (1956), numerous studies have argued that we should expect to see differences in public services among localities as a result of people "voting with their feet" and selecting the locality that has the most desirable mix of services for them. Here, we consider differentiating in public services across localities that arises in part for a different reason, as a way of reducing competition among these localities. If cities finance their public services through a tax such as the property tax that generates "tax competition", we find that adoption of quality differences in public services will change the amount of services provided. In the case of property value maximization, this means a reduction in the level of public services provided for both the city that provides high quality as well as with low quality. The reduction in public services in both cities means that under certain conditions property values in both cities can increase. Thus in a two-stage game of adoption, we can observe quality differentiation in the services when the property tax is used. This is in sharp contrast to the case with a nondistorting head tax in which we should never observe this type of differentiation.

We have examined quality differentiation in education within the context of property-value maximizing governments. This case was chosen both because of the frequent use of property-value maximization as a determinant of local government policies and because the analysis was straightforward. However, if the objective of the government is to maximize the utility of land-owning residents or government revenues, quality differences may still exist if the property tax is used. Under both of these objectives, however, quality differentiation, by reducing competition, increases property taxes. Because of this increase in taxes it is possible, as it was with property-value maximization, for both cities to be better off. Thus while the observed relationship between quality differentiation and the level of property taxation is reversed from the case with property-value maximization, quality differentiation still increases the value of the government objective.[11]

In the past twenty-five years the role of state governments in primary and secondary education has increased dramatically. One way in which state governments have intervened in primary and secondary education is through the imposition of tax limits such as *Proposition 13* in California and *Proposition 2½* in Massachusetts. Another way is through "fiscal equalization" plans that increase the amount of financing of education that comes from the state government. While the motivation for the tax limits may have primarily been taxpayer resentment and dislike of property taxes, certainly legislation (and court decisions) related to fiscal equalization legislation has been motivated by the desire to reduce "inequities" in education. While the primary focus of this legislation has been to equalize the quality of education between high and low income (and tax base) districts, this legislation and tax limits may also be responsible for reducing quality differences among districts with similar incomes and tax bases. The greater the extent that state funding replaces local funding, the less reliance there is on the property taxation. Then adverse impacts of "tax competition" should also be reduced, thereby reducing the gains to differentiating quality. Thus we might expect that even among districts with very similar income distributions and tax bases, that fiscal equalization will reduce quality differentiation.

A similar argument applies to the enactment of tax limits when governments are setting policies to increase property values. In this case, the limit places a tax ceiling that means that cities can not raise educational spending (and taxes) in an attempt to attract residents and increase property values. The tax limit eliminates competition among localities that leads to overspending relative to the property-maximizing level. Then because the tax limit can effectively eliminate tax competition, there is no reason for districts to differentiate quality to do so.

One interesting implication of our results that we have not discussed is that quality differentiation does not occur in some cases in which it would be socially beneficial for there to be both a high and low quality provider of the public service. Perhaps even more interesting is the fact that this socially beneficial differentiation will never occur when a head tax is used and the socially efficient level of public services is provided but may occur when the distorting property tax is used.

A modification of our model would be to consider the impacts of quality differences when the differentiation in horizontal rather than vertical. In terms of public education, this would mean that the education was not high or low quality in districts, but simply “different” with some residents prefer one type of quality while others prefer another type. We believe that our results on the impact of differentiation on the level of public services should be robust to this modification but the implications for quality adoption might be far different, with differentiation more likely to occur in this setting.

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Notes

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[1] Figure is for 1992 (see *Statistical Abstract of the United States, 1994*, p. 298).

[2] Evidence of the impact of educational quality and spending on locational choice can be seen on their impact on property values in studies including Oates (1969), Brueckner (1979, 1982).

[3] Evidence that each labor market generally, though not always, consists of several alternative school districts can be seen from the fact that in 1992 there were 3,043 counties and 14,566 school districts (*Statistical Abstract of the United States, 1995*, p. 297). When we also consider that a large number of people commute across county lines, these numbers suggest that in most metropolitan areas, the location of employment does not preclude a household from residing in number of alternative school districts.

[4] These assumptions about the distribution of the population and γ have no bearing on the qualitative results we obtain and are made to simplify the analysis. Earlier versions of the paper made no assumptions about the distribution of tastes.

[5] Essentially this is an assumption that both cities are of equal size.

[6] To derive (5) we use some of implications of the indirect utility function, V(y-T,p,g). Thus we have $\alpha'\frac{\partial h}{\partial p}=\frac{\partial V}{\partial p}=-\frac{\partial V}{\partial y}h==-h$ using Roy's identity and the fact that $\frac{\partial V}{\partial y}=1$ for our utility function.

[7] More formally, $\varepsilon=\frac{\partial h}{\partial p}\frac{p}{h}<0$ and θ is the elasticity of supply, $\theta=\frac{\partial H}{\partial p}\frac{p}{H}>0$.

[8] Suppose not. Given $\delta>\frac{\gamma}{2}$ then from (2') for $p^{H_T}>p^{L_T}$ it must be the case that $(1-N^{H_T})\gamma>\frac{\gamma}{2}$. But if $(1-N^{H_T})\gamma>\frac{\gamma}{2}$ then $N(p^{H_T})=N^{H_T}<1/2$ which can only be true if $p^{H_T}<p^{L_T}$ Hence a contradiction.

[9] The sign of $\frac{\partial\tau}{\partial\delta}$ is ambiguous as it depends, in part, on $\frac{\partial n(\gamma)}{\partial\gamma}$.

[10] Given the specification of the utility function the elasticity of demand for housing, ε, is always less than -1.

[11] The results for the cases of utility and revenue maximization are available from the authors.

Figure 1: Equilibrium with the Head Tax

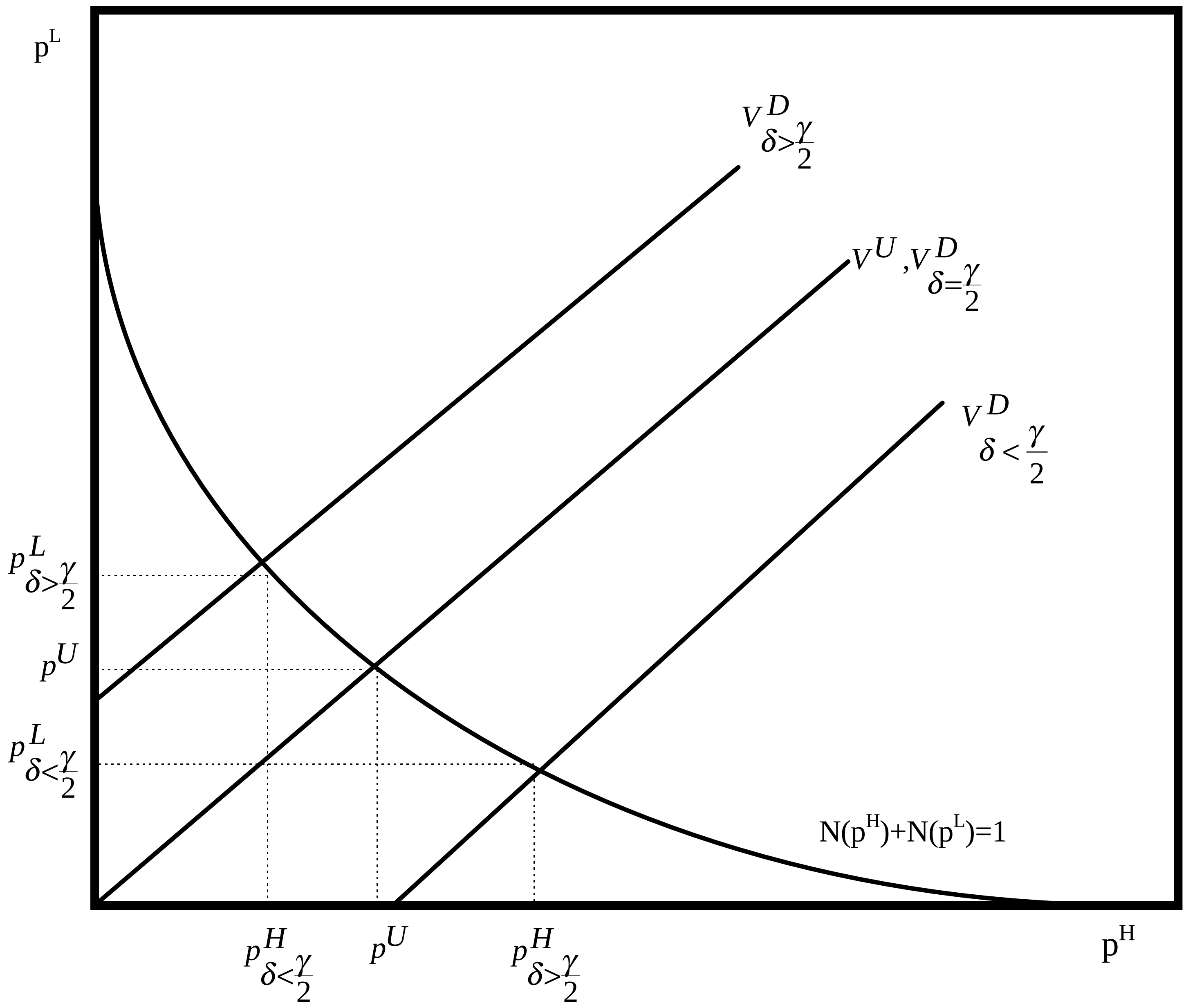

Figure 2: Equilibrium with the Property Tax

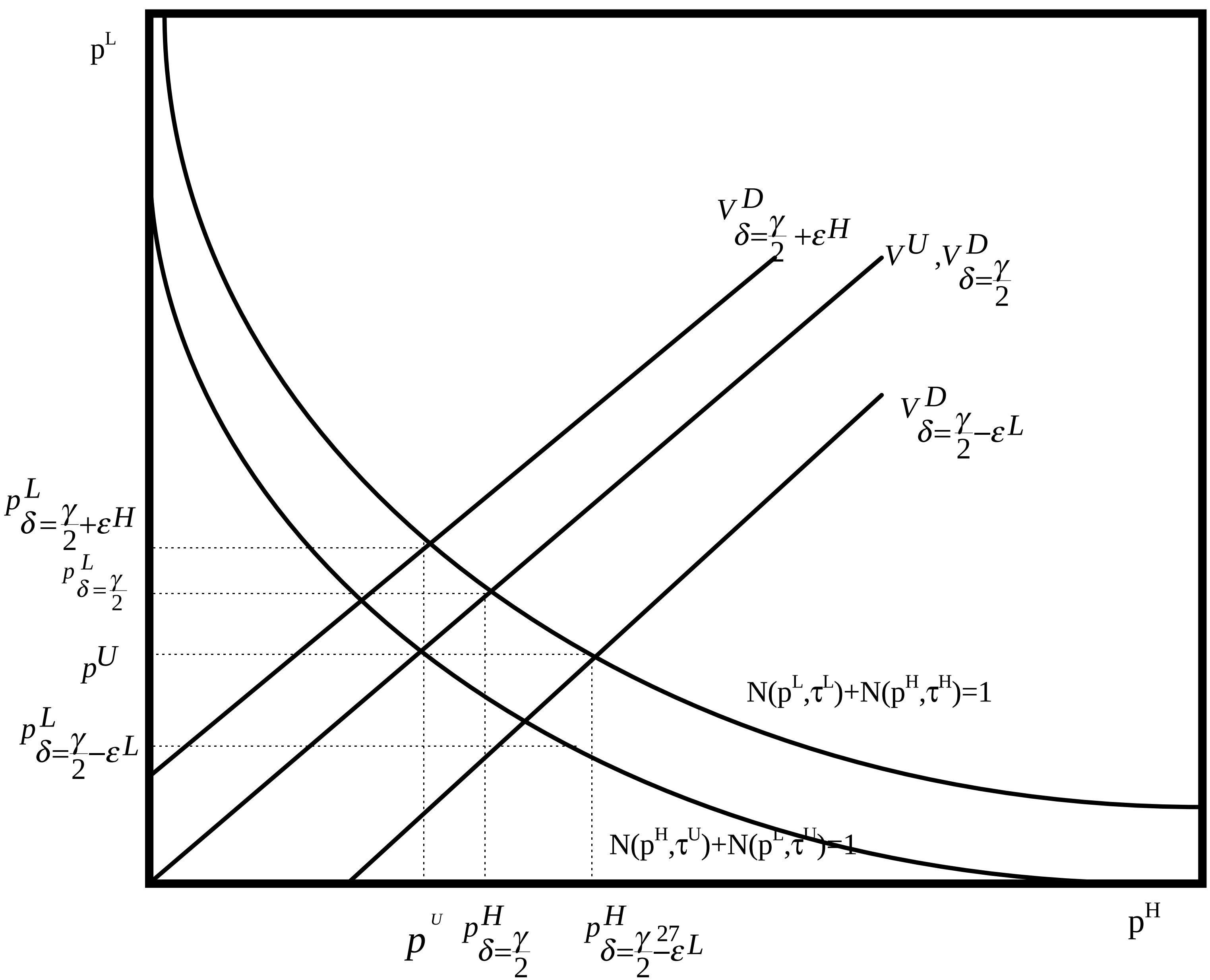

***Table 1: Simulation Results for Property Value Maximization*[1]**

Price Elasticity of Housing	γ	Case	δ	$\frac{\varepsilon}{\gamma/2}$	P^H	P^L	τ^H	τ^L
-1.25	5	Same Quality			0.899	0.899	0.1765	0.1765
		$\delta=\gamma/2$	2.5		0.903	0.903	0.1704	0.1704
		$\delta=\gamma/2+\varepsilon^H$	2.8	12(%)	0.899	0.908	0.1713	0.1700
		$\delta=\gamma/2+\varepsilon^L$	2.2	-12	0.908	0.899	0.1700	0.1710
	10	Same Quality			0.899	0.899	0.1765	0.1765
		$\delta=\gamma/2$	5		0.911	0.911	0.1576	0.1576
		$\delta=\gamma/2+\varepsilon^H$	5.8	16	0.899	0.921	0.1594	0.1557
		$\delta=\gamma/2+\varepsilon^L$	4.2	-16	0.921	0.899	0.1557	0.1595
-2	5	Same Quality			0.903	0.903	0.1347	0.1347
		$\delta=\gamma/2$	2.5		0.909	0.909	0.1264	0.1264
		$\delta=\gamma/2+\varepsilon^H$	3	20	0.903	0.915	0.1277	0.1250
		$\delta=\gamma/2+\varepsilon^L$	2	-20	0.915	0.903	0.1250	0.1277
	10	Same Quality			0.903	0.903	0.1347	0.1347
		$\delta=\gamma/2$	5		0.917	0.917	0.1149	0.1149
		$\delta=\gamma/2+\varepsilon^H$	6.3	26	0.903	0.930	0.1180	0.1117
		$\delta=\gamma/2+\varepsilon^L$	3.7	-26	0.931	0.903	0.1115	0.1183

1. Simulations were based on the utility function $U=x + ah^{\alpha} + bg^{\beta}$ where $b = 31.54$ and $\beta = .2$. The parameters a and α vary to adjust for the price elasticity of housing.