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Public-Good Productivity Differentials and Non-Cooperative Public-Good Provision

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Abstract. We explore the generality of Konrad and Lommerud (1995)'s *Rotten Spouse Theorem*. While the result holds for an arbitrary number of agents, it fails to hold for general technologies. We discuss some of the implications for CO_2-emissions models.

Keywords. Rotten Spouse, CO_2 abatement, climate change

JEL Classification System. Q20, H4, D62

1. Introduction

Private provision of public goods is an important economic phenomenon. In the U.S. annual reported donations to charity amount to approximately 2% of its GDP. Much of the activity that takes place within the household can be explained as the outcome of voluntary contributions —see, *e.g.*, Becker (1981), and Lommerud (1997). Bergstrom, Blume and Varian (1986) (BBV henceforth) is a classic reference for models of private provision of public goods (see Ley (1996) for a graphical exposition). Hoel (1991) and Chichilinsky and Heal (1994) have used variants of this model to tackle global environmental issues; in particular, CO_2 emissions. This will also be a main motivation of this paper. The results, however, apply to more general models. A key characteristic of the greenhouse problem is that it is a global problem. Each individual country has some control over the level of its own CO_2 emission and abatement activities but, in an unregulated World, it cannot control other countries' activities. It is the total accumulation of greenhouse gases in the atmosphere (a pure public bad) that could lead to climate change over the next century —see, *e.g.*, Schmalensee (1993).

In a CO_2-emissions context, a crucial distinguishing feature is that each country has access to a possibly different abatement technology. The agents in BBV's model, however, all share the same (linear) technology for producing the public good. Hoel (1991) has studied cooperative and non-cooperative outcomes in a global-emissions game using a model essentially similar to BBV. Sandler (1996) has also used an analogous framework to study various issues related to global carbon emissions. While these models use a common production process for the public good, recent contributions have allowed for agent-specific technologies. Buchholz and Konrad (1994) study the strategic choice of (a linear) technology with different emission-reduction costs in a two-stage game. Ihori (1996) investigates the welfare effects of changes in the public-good productivity differentials in an international public-good context that can easily be applied to

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CO_2 emission. Chichilnisky and Heal (1994) depart from linear technologies and instead endow each agent (country) with a specific concave technology to study international emission-permit markets.

Konrad and Lommerud (1995) develop a non-cooperative model of the family where a spouse has comparative advantage in the production of a household public good. They establish (in their proposition 3) a remarkable *Rotten Spouse Theorem*[1] which states that income transfers to the spouse with comparative advantage in producing the household public good are Pareto improving. Even a rotten egoistic spouse (who has comparative advantage in market activities) will want to transfer income to the other spouse raising the welfare of both in the process. In this paper we study the generality of this result, an issue which is of particular relevance to CO_2-emissions games. While the result generalizes to n agents —*i.e.*, strategic interactions do not get in its way when more agents come into play—, it fails to hold when the technologies for producing the public good are strictly concave instead of linear.

2. Non-Cooperative Public-Good Provision

Consider n agents, indexed by i, who are each endowed with w_i, which can be allocated to the *private* production of a pure public good, G, or to their private consumption, x_i. By *private* production we mean that each agent has available an individual and, possibly unique, technology that transforms private resources, $y_i = w_i - x_i$, into a public good $g_i = f_i(y_i)$, with $f_i(0) = 0$, $0 < f_i'(\cdot) < \infty$, and $f_i''(\cdot) \leq 0$. The total amount of the public good is given by $G = \sum_i g_i$. This formulation is most relevant in a CO_2-emission context, where agents (countries) are endowed with wealth w_i which can be either consumed, x_i, or used to abate emission, y_i; G being the "atmosphere's quality." The interpretation then is that w_i is what country i can obtain (consume) by maximizing output when there are no climate-change concerns ($g_i = y_i = 0$, and $x_i = w_i$). If the theorem applied there, cost-effectiveness problems in carbon abatement would not be an issue since cost-efficiency would always be achieved through voluntary wealth transfers among countries.

We assume that agents have preferences over (x_i, G) that can be represented by a twice-differentiable strictly quasi-concave utility function, $U_i(x_i, G)$. It is assumed that both goods are strictly normal. To avoid dealing with corner solutions on the private-good axis, we shall assume that $S_i(x_i, G) \equiv \frac{\partial U_i(x_i,G)}{\partial x_i} / \frac{\partial U_i(x_i,G)}{\partial G} \to \infty$ as $x_i \to 0$.

Agents are modeled as participants in a one-shot non-cooperative game. Agent i's problem is given by

$$\max_{x_i} \qquad U_i(x_i, f_i(w_i - x_i) + G_{\sim i}) \quad \text{s.t.} \quad 0 \leq x_i \leq w_i. \tag{1}$$

where $G_{\sim i} = \sum_{j \neq i} g_j$. That is, each agent takes the other agent's contributions to the public good as given and, furthermore, assumes they will be unaffected by their own choices.

A Nash equilibrium (NE) is a consumption vector $(x_1^e, \ldots, x_n^e)$ which solves (1) for all i when $G_{\sim i}$ is replaced by $G_{\sim i}^e = \sum_{j \neq i} f_j(w_j - x_j^e)$. The first-order conditions of problem (1) imply that, in a NE, we must have (i) $[S_i(x_i^e, G^e) - f_i'(w_i - x_i^e)](w_i - x_i^e) = 0$, for all i; where $G^e = \sum_i f_i(w_i - x_i^e)$; together with (ii) $S_i(x_i^e, G^e) \geq f_i'(w_i - x_i^e)$, and (iii) $w_i \geq x_i^e$. In an interior solution, if $x_i^e < w_i$, we then have $S_i(x_i^e, G^e) = f_i'(w_i - x_i^e)$.

The existence, uniqueness, and inefficiency of NE follow from proofs similar to those used to establish the corresponding results in the literature of private provision of public goods —see, *e.g.*, BBV. A few words regarding inefficiency. Provided that an agent is contributing towards the public good, the NE will be Pareto inefficient. Nevertheless it is possible to have a Pareto

[1] This terminology is due to Ted Bergstrom.

efficient NE where no agent is devoting any resources to the production of the public good. (However, not all NE with $y_i^e = 0$ for all i are necessarily Pareto efficient.) This inefficiency of the NE is partly just the well-known result on public good underprovision. Here, however, there is an additional source of inefficiency since, at the NE, the marginal costs of producing the public good might differ among agents.

2.1. Konrad-Lommerud's Rotten Spouse Theorem

In the literature of private provision of public goods, Warr-type redistributions are 'small enough' redistributions performed between agents which are in interior solutions to their individual optimization problems (1). Warr-type redistributions lead to Warr-type neutrality results when they do not affect the total provision of the public good and the private consumption vector at equilibrium —see Warr (1982) and BBV. In this model, unless $f_k'(\cdot) = f_l'(\cdot) =$ constant in the redistribution range, then Warr-type redistributions involving agents k and l are non-neutral. In general, then, wealth redistributions among agents will lead to different equilibria and to a different provision of the public good. Redistributions towards the most efficient agents in the production of the public good not only will enhance productive efficiency but they also may *sometimes* lead to Pareto-superior allocations. However, in the particular case where the technologies are linear in a neighbourhood around the NE then there always exist Pareto-improving transfers.

Proposition 1. *(Existence of Welfare-Enhancing Redistributions with Linear Technologies) Suppose that $f_i(y_i) = \theta_i y_i$ in a neighbourhood of y_i^e when $y_i^e > 0$ and to the right of zero when $y_i^e = 0$. Provided that $y_k^e, y_l^e > 0$, if $\theta_k < \theta_l$, a small redistribution of wealth from k to l will increase the provision of G, and the welfare of all agents.*

Proof. We only need to show that the level of G in the equilibrium after the redistribution must increase (since that implies larger consumption of the private goods for agents in interior solutions and no smaller for agents in corner solutions, and, hence, increased welfare). Suppose $\Delta G \leq 0$. Then, since x_i and G are normal goods, we must have $\Delta x_i \leq 0$ for all agents. By the individual budget constraints, it follows that $\Delta y_i \geq 0$ for all agents except, possibly, country k which gives the transfer. Differentiating the sum of k and l's resource constraints we get $\Delta x_k + \Delta x_l + \Delta y_k + \Delta y_l = 0$ which (using the inequalities discussed above) implies $\Delta y_l = -(\Delta x_k + \Delta x_l + \Delta y_k) \geq -\Delta y_k$. On the other hand, by assumption, $\Delta G = \sum_i \theta_i \Delta y_i \leq 0$, which implies

$$\Delta y_l \leq -\frac{\theta_k}{\theta_l}\Delta y_k - \sum_{i \neq k,l} \frac{\theta_i}{\theta_l}\Delta y_i \leq -\frac{\theta_k}{\theta_l}\Delta y_k < -\Delta y_k,$$

which establishes the contradiction. Therefore, we must have $\Delta G > 0$. ∎

This proposition —which is a straightforward n-agent generalization of Proposition 3 in Konrad and Lommerud (1995)— has important implications. If all the agents increase their welfare by virtue of these transfers, not only will they be tolerated but they will continue until they are not possible anymore. As a result, the marginal costs of producing the public good will be equalized across all agents in *interior* solutions when they all have linear technologies.

The game agents play changes now as we allow for wealth transfers. In the first stage, agents choose how much to transfer to other agents. In the second stage agents solve the same problem (1) they did before with their wealth, w_i, properly adjusted for the transfers. In a subgame-perfect equilibrium of this game, the marginal costs producing the public good must be the same for all agents which engage in the production of the public good. Note, however, that not all agents need to be producing the public good; the less productive agents will be in corner solutions. If all agents have linear technologies (so that proposition 1 applies everywhere) then the marginal rates of transformation of all agents engaged in the production of the public good must be equalized when agents are allowed to do voluntary transfers of wealth. (In a CO_2-emission context, the marginal costs of carbon abatement would be equalized across countries.)

Unfortunately proposition 1 does not hold for general production functions —*i.e.*, concave technologies. In the next section, we shall provide a counter-example with a strictly concave production function. The reason why proposition 1 does not hold when we replace linear technologies with strictly concave technologies is that the marginal productivity differential can easily be reversed when the agents readjust their production levels. The change in marginal products induces substitution effects which did not exist before —in the Rotten Spouse case we only have parallel displacements of the budget lines. We discuss the role of this substitution effect in the final section.

2.2. A Counter-Example with a Concave Technology

Consider two agents with identical Cobb-Douglas utility functions, $u(x, G) = xG$. Total wealth is 1 and $w_1 \in [0, 1]$ represents agent one's share. In the contribution game, agent one takes g_2 as given and solves, $\max_{x_1} x_1(f_1(w_1 - x_1) + g_2)$; while agent two solves a similar problem, $\max_{x_2} x_2(g_1 + f_2((1 - w_1) - x_2))$, taking, in turn, g_1 as given.

Let $g_1 = \sqrt{y_1}$ and $g_2 = 2y_2$. Agent one is more productive than agent two at the margin when $y_1 < 1/16$. The NE, for all possible distributions of the total wealth, are given by:

$$x_1^* = \begin{cases} \{w_1 - (1 - w_1)[(1 - w_1) - \sqrt{1 + w_1^2}\,]\}/2 & \text{if } w_1 < 3/4 \\ \frac{2}{3}w_1 & \text{otherwise} \end{cases}$$

and

$$x_2^* = \begin{cases} (1 - w_1)/2 + \frac{\sqrt{2}}{8}\{w_1 + (1 - w_1)[(1 - w_1) - \sqrt{1 + w_1^2}\,]\}^{\frac{1}{2}} & \text{if } w_1 < 3/4 \\ (1 - w_1) & \text{otherwise} \end{cases}$$

The question is whether, at wealth allocations that lead to interior NE where the agents produce the public good with different marginal costs, both agents will improve their welfare by transferring resources from the marginally less productive agent to the more productive one before the one-shot game is played. The answer is "not always".

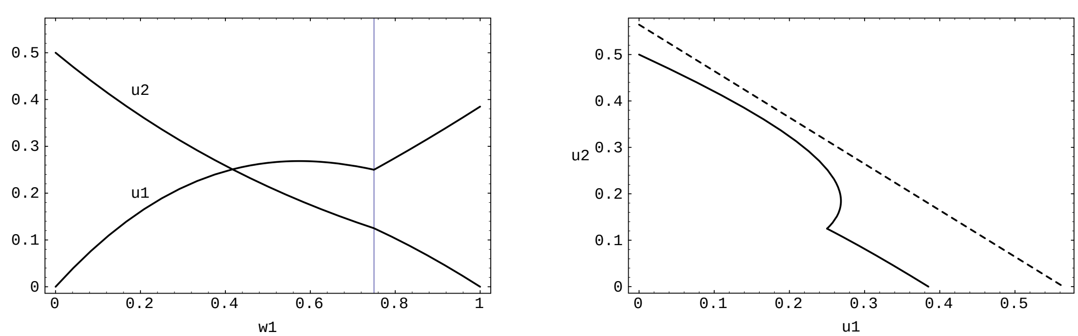

Fig. 1. General Technologies: Rotten Spouse Theorem does not hold.
Left: Utilities at the Nash equilibria associated with different wealth distributions.
Right: Utility Possibilities Frontier (dashed) and Nash-Equilibria utility pairs (solid).

The left panel in figure 1 shows the utility levels at all NE associated with every possible distribution of wealth. There is a region to the left of $\frac{3}{4}$ where both graphs decrease simultaneously which means that wealth redistributions towards agent two will increase both agents' welfare.

However, as an example, to the right of the origin, both agents are in interior solutions and agent one is more efficient at the margin so we should see both graphs increasing if a generalized version of proposition 1 held true.[2] However, the utility of agent two goes down as wealth gets transferred to agent one. The reason is that now there is also a substitution effect playing a role when redistributions of wealth take place. The Pareto-Efficient allocations are given by $G^* = \frac{17}{16}$ and $x_1^* + x_2^* = \frac{17}{32}$. The right panel in figure 1 shows the utility-possibility frontier (dashed line) and the utility pairs at all possible NE (solid graph). Pareto-improving wealth redistributions are possible along the positive-sloped portion of the graph.

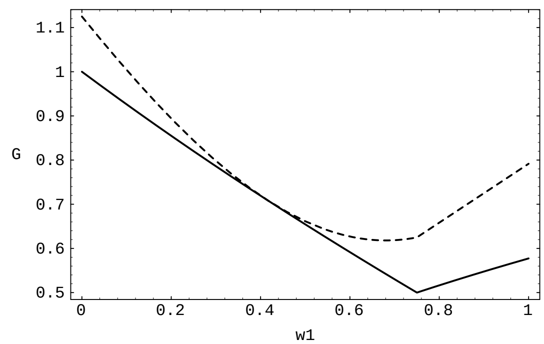

Fig. 2. G^e (solid) and cost-efficient G (dashed) given (x_1^e, x_2^e).

Fig. 2 shows the level of public good provided at different NE (solid) and the amount that could be provided if production took place in an efficient way. Thus, the dashed line is obtained by efficiently transforming $y = 1 - x_1^e - x_2^e$ into G. Note that the level of production inefficiency can be substantial when provision and production are linked together.[3] (The point of tangency corresponds to the NE where $y_1^e = 1/16$ and therefore $f_1'(y_1^e) = f_2'(y_2^e)$.)

2.3. Piece-Wise Linear Technologies

What about piece-wise linear technologies? The theorem holds when everybody stays away from the kinks. In that case it is always possible to find redistributions which are small enough that lead to NE where the agents stay positioned inside the same line segments. Once any agent moves to another segment with a different slope, there will be a substitution effect playing a role and anything is possible.

To understand the intuition behind the theorem and why it does not work in the previous example, let us consider first the linear case of Konrad and Lommerud (1995)'s paper; refer to fig. 3. Agent one is able to transform 1 unit of the private good in θ_1 units of the public good, while agent two obtains θ_2 units; with $\theta_1 > \theta_2$. At the initial NE, when agent two contributes g_2^*, agent one chooses e_1^*, contributing g_1^* towards the public good which amounts to G^*.

[2] The reader might want to check out the case when $g_1 = y_1$ and the *Rotten Spouse Theorem* holds. In that case, both agents would prefer to redistribute wealth from agent one to agent two whenever $w_1 \in (\frac{1}{2}, \frac{4}{5})$, which is the range of wealth distributions that lead to interior NE.

[3] In the CO_2 context, some countries have realized that provision and production need not go hand in hand. Often, an industrial country interested in providing G contracts the production in a less developed country where the production is cheaper. The concept of joint implementation implies joint ventures involving both industrial countries and developing countries: Norway and Mexico (replacement of small electric appliances in Mexico); The Netherlands and Poland and India (substitution of coal by natural gas). Private firms and foundations are also active participants in joint implementation projects: a Netherlands foundation, Forest Absorbing Carbon Emission (FACE) is paying for sequestration in selected Latin American countries; the New England Electric System has supported reduced impact logging in Malaysia, and Applied Energy Systems has funded agroforestry in Guatemala.

Suppose that prior to playing the contribution game, a transfer of Δw is made from agent two to agent one so that one's initial wealth moves from w_1 to w_1'. In order to characterize the new NE, it is useful to do the following thought experiment. Assume that, after the transfer, agent two simply reduces his contribution by $\theta_2 \Delta w$. Then agent one's new budget line moves from abw_1 to cdw_1'. If agent one increased his contribution by exactly $\theta_2 \Delta w$, two's budget line would be kmw_2' and his choice e_2^*. However, facing cdw_1', one's choice would be to the north-east of e_1^* by the strict normality of both goods. This means that one would increase his contribution by more than two's decrease —*i.e.*, one would be choosing an amount of public good greater than G^*. The new NE will look like (e_1^{**}, e_2^{**}). Note that one's budget line will move further out since agent two will react to one's larger contribution by increasing his own and $g_2^{**} > g_2^* - \theta_2 \Delta w$.[4]

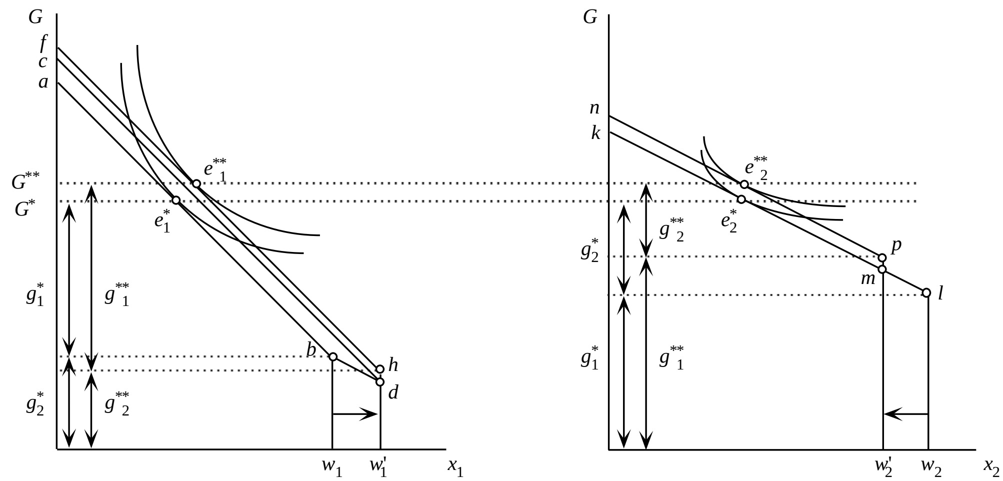

Fig. 3. Rotten Spouse Theorem at work.

Let us now assume that agent one's technology is piece-wise linear and that e_1^* is located at a kink: $g_1 = \vartheta_1 y_1$ if $y_1 > y_1^* = g_1^*/\theta_1$; with $\vartheta_1 < \theta_1$. Assume again, for a moment, that after the transfer, agent two reduces his contribution by $\theta_2 \Delta w$. If $\vartheta_1 < \theta_2$, it is possible to prove that the new equilibrium will necessarily have a lower level of G, and while agent one could end up better or worse off, agent two will always be worse off.

However, if $\vartheta_1 \geq \theta_2$, agent one's new budget set expands outwards along all its borders and Pareto-improving transfers are possible. The crucial difference relative to the Rotten Spouse case is that now, if agent one wanted to provide $G \geq G^*$, his marginal product has decreased from θ_1 to ϑ_1. This happens because he needs to use his technology more intensively in order to offset the decrease in agent two's contribution and therefore arrives to his less productive region earlier. This change in 'relative prices' that agent one would now be facing introduces a substitution effect that was not present in the Rotten Spouse case. This substitution effect goes against the income effect due to the transfer. Tastes and technology parameters will determine whether the transfer is Pareto improving —or, similarly, whether the final effect on the provision of G is positive or negative.

[4] If $\theta_1 = \theta_2$ then d will be aligned with ab, and (e_1^*, e_2^*) will also be the NE after the redistribution. Both agents would be consuming the same amounts of the two goods before and after the transfer. This is Warr's neutrality result.

3. Concluding Remarks

We have examined the extent and limitations of Konrad and Lommerud (1995)'s *Rotten Spouse Theorem* in an n-player non-cooperative public-good provision game. While the theorem generalizes to n agents, it fails to hold for general technologies for producing the public good. When the theorem holds, the production of the public good is achieved in the most efficient way.

In the case of smooth concave technologies, where the theorem does not hold with generality, there might still be room for Pareto-improving transfers in each particular instance. The possibility of these transfers depends on the shape of the technologies and the indifference maps of the agents involved.

In a CO_2-emissions context, there are large differences in marginal productivities —*i.e.*, in the cost of CO_2-emission reductions— among countries —*e.g.*, India and Germany. However, there are also big differences in (per capita) GDP. Therefore, even if the tastes of different countries were similar it is likely that the marginal rates of substitution at the NE will be substantially different —*i.e.*, India is probably at a corner solution maximizing its GDP without much environmental concerns about CO_2. In such case, large transfers would be needed before India moved away from its corner solution. Technology transfers —which would allow India to generate the same output with a lower level of associated emissions— would then probably be a more effective way of reducing CO_2 emissions.

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