An optimal auction perspective on lobbying

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Proposed Running Head: An optimal auction perspective on lobbying

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Abstract

The lobbying process has been described as an auction (see, for instance, Bernheim and Whinston [5]). The auction rules picked are supposed to be descriptive, however they vary from author to author.

An optimal auction for a government official leads to the same policy as in [5], although contributions are different. A necessary condition for an auction to be optimal is that it allows contributions from the government official to the lobby. The proof of these results depends on an extension of the work by Bernheim and Whinston [6] on implementation in environments with complete information. In particular all choice functions are Coalition-Proof Nash equilibrium implementable when individuals preferences can be represented by quasi-linear utility functions bounded with respect to all variables – except for money.

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An optimal auction perspective on lobbying

The outcome of the lobbying process is a policy picked by a government official and a vector of contributions from the lobbies to the government official. Bernheim and Whinston [5] assume that the government does not know the preferences of the lobbies over the policy, but lobbies know each other's preferences. Bernheim and Whinston show that if the lobbying process can be described by a menu auction (defined in the next section), then the policy picked is the one that maximizes the sum of the utilities of the lobbies and the government official.

Recently this model of lobbying has received quite a bit of attention. Grossman and Helpman [11] use this model of lobbying to determine the tariffs selected. Dixit [8] extends [11] by allowing the government to choose separately producer and consumer taxes. In [12], Grossman and Helpman use this model of lobbying to study the feasibility of free-trade agreements.

As discussed in the next section, a menu auction is not the only way to describe the lobbying process and different ways of modeling the lobbying process lead to different predictions over which policy is picked. A natural mechanism to look at is the one that maximizes the utility of the government official. Such a mechanism leads to the same policy as the one predicted in [5] but to higher contributions.

An optimal mechanism for the government official has the property that negative contributions (i.e., contributions from the government official to lobbies) are possible (Corollary 1 and Proposition 2). The intuition for this result is that in order to extract all the surplus the government official needs to learn the preferences of all lobbies. This can only be done by bribing lobbies to reveal each other's preferences.[1] A mechanism can be constructed in such a way that in equilibrium the government official does not need to make payments to any of the lobbies. The intuition for this result is that since all lobbies have exactly the same information, the outcome of Bertrand competition is for the government official to obtain the information for free.

The proof of these results depends on an extension of the work by Bernheim and Whinston [6] on implementation in environments with complete information. In particular, Lemma 4 states that all choice functions are

[1] Kerry Back suggested this explanation.

Coalition-Proof Nash equilibrium implementable when individuals preferences can be represented by quasi-linear utility functions bounded with respect to all variables – except for money. Further, the mechanism that implements all choice functions is particularly simple; in particular it does not use any lotteries, integer games, or modulo games (see Lemma 3).

Section 1 discusses several existing models for the lobbying process and gives examples that illustrate how the different models lead to different predictions. Section 2 is self-contained and describes the model and the results. Section 3 relates the results to the literature on implementation in environments with complete information. Section 4 ties the results to Becker's model of lobbying. The appendix gives some anecdotal evidence that government officials try to construct mechanisms that maximize contributions.

1 Problem formulation

Bernheim and Whinston [5] model the lobbying process as a **menu auction**, more precisely as a first-price menu auction. In the first stage, each lobby announces a menu; for instance lobby i announces a menu s_i. A menu states the following: for any possible policy x, if the government official selects x, then lobby i pays the government official $s_i(x)$. In the second period, the government official selects the policy and gets the promised contributions by all lobbies.

Each lobby and the government official has a quasi-linear utility function (over the policy selected and the transfers). Specifically, the utility for lobby i of policy x and contribution t_i is $u_i(m, x, t) = m_i(x) - t_i$. The utility over policies by the government official can be interpreted as the effect that the choice of policy has on the likelihood of reelection.

Baye et al [2] model the lobbying process as an **all-pay auction**. In such an auction all lobbies announce a most preferred alternative and a bid. The most preferred alternative of the highest bidder is implemented and all lobbies have to pay their bid.

Groseclose and Snyder [10] consider the case with two lobbies, $\{1, 2\}$ and two possible decisions, $\{x, y\}$. They assume that 1 always prefers x and 2 always prefers y. The lobbying process works as follows. In the first period, lobby 1 selects a vector of bribes s_1. In the second period, lobby 2 selects a vector of bribes s_2. In the third period the legislature selects an alternative by majority rule. The utility for legislator i who votes for x is $v_i(x) + s_1(i)$. The utility for legislator i who votes for y is $v_i(y) + s_2(i)$.

This **sequential auction** can easily be extended to be comparable to a menu auction. Specifically, there is only one legislator, at stage i lobby i announces a bid and selects an alternative, and finally the alternative picked by the highest bidder is implemented. Only the highest bidder pays his bid.

Finally, in a **first-price auction** each lobby announces a pair (x, t_i), the alternative with the highest bid is implemented.[2]

The rest of the section discusses examples that illustrate how the contribution and the policy selected vary according to the modeling of the lobbying process. The solution concept used throughout, CPNE, is defined in the next section.[3] Throughout this section the government official has flat preferences and hence only cares about the total amount of contributions, or revenue.

Example 1 Let $m(x) = (3, 0, 0)$, $m(y) = (0, 2, 2)$, $m(z) = (-1, -1, -1)$. So, for instance, the utility of lobby 1 for policy z is -1. In a menu auction, the alternative picked is y and the revenue raised is 3.[4] (One possible CPNE is $((3, 0, 0), (0, 1.5, 0), (0, 1.5, 0))$.) In the first-price auction, the alternative picked is x and the revenue raised is 2. (The CPNE is $((2, x), (2, y), (2, y))$.) In the sequential auction, the outcome is y and the revenue raised is 3. (One equilibrium path is as follows: lobby 1 select $(3, x)$, lobby 2 selects $(1.5, y)$, lobby 3 selects $(1.5, y)$.) One can construct a mechanism such that the outcome selected is y and the revenue raised is 7.[5] □

Example 2 For simplicity this example has only two lobbies. However one could easily include more lobbies by giving them flat preferences. Let

[2]Note, in case of ties, the tie-breaking procedure must be picked in such a way to guarantee existence of equilibrium. Instead of using a device such as ϵ-equilibrium or a finite set of bids, include the government official as a player in the game. At this stage the government official is assumed to be perfectly informed of everyone's preferences and the only move consists of deciding on whose favor to break a tie.

[3]More precisely, for the sequential auction, the solution concept is subgame perfect equilibrium. The generalization of CPNE to extensive form games is the perfectly CPNE. For normal form games, perfectly CPNE corresponds to CPNE. For finite games of perfect information where preferences form linear orders, perfectly CPNE corresponds to subgame perfect Nash equilibria. See Peleg [14].

[4]If X is finite, then in a menu auction the policy selected is $x(m) \in \text{argmax}_x \sum_i m_i(x)$ and the revenue raised is $\sup_{I,x,y}[\sum_{i\in I} m_i(x) + \sum_{i\notin I} m_i(y)] - \sum_{i\in N} m_i(x(m))$.

[5]See Example 5 on page 14.

$m(x) = (0, 0)$, $m(y) = (3, -100)$, $m(z) = (-100, 3)$. Then in a menu auction, alternative x is picked and the revenue raised is 6. (The CPNE is $((3, 6, 0), (3, 0, 6))$.) However with a first-price auction either y or z are picked and the revenue raised is 103. (The CPNE is $((103, y), (103, z))$.) In the sequential auction, alternative x is picked and the revenue raised is 3. (One equilibrium path is: 1 moves first and offers 3 for x, 2 offers 0 for z.) One can construct a mechanism such that x is selected and the revenue raised in 200. □

Example 3 Baye et al. [2] consider the following model. For all i, $v_i > v_{i+1}$, $X = \{x_1, \ldots, x_n\}$, $m_i(x_i) = v_i$ and if $i \neq j$, $m_i(x_j) = 0$. They show that in an all-pay auction the revenue raised is $(1 + v_2/v_1)(v_2/2)$, which is less than v_2.[6] In a first-price auction, a menu auction, and a sequential auction, the revenue raised is v_2. □

2 Results

The previous section describes several auctions discussed in the literature on the lobbying process. Examples showed that none of these auctions are optimal from the perspective of the government official. The auctions discussed suggest that the lobbying process should be analyzed as a deterministic outcome of CPNE of a normal form game, as a deterministic subgame perfect Nash equilibrium of an extensive game of perfect information, or a mixed strategy Nash equilibrium of a normal form game. Only the first approach is pursued in this paper.

This section has three parts. The first part lays out the model of lobbying. The second part defines the property WM and proves that it is equivalent to CPNE implementability. The third part applies the CPNE implementation result to the design of optimal lobbying rules.

2.1 Model and assumptions

The lobbying process is defined by the game $\Gamma(m) = (N = \{1, \ldots, n\}, (S_i)_{i \in N}, (u_i)_{i \in N})$ which is defined below. The number of lobbies is at least three;

[6] The result in [2] is for Nash equilibrium, but it is also true for CPNE.

$n \geq 3$. The set X describes the set of all possible public decisions, or policies. The set $T \subset \mathbf{R}^n$ describes the set of all possible contributions, or transfers.[7] The outcome function $g: S \rightarrow X \times T$ maps the moves by all the players into a decision and a vector of transfers. The set of all payoffs is parametrized by the set of preference over policies M; specifically, the utility of lobby i for policy x and contribution t is $u_i(m, x, t) = m_i(x) - t_i$. Lobbies know m but the government official does not.[8] The utility of the government official for policy x and transfer t is $m_0(x) + \sum_{i=1}^n t_i$. To simplify the notation m_0 does not have m as an argument; however, the function m_0 can depend on the realized m.

For any δ, let $B(\delta)$ be the set of preferences over public decisions that are bounded (in absolute value) by δ and satisfy the following two properties: each lobby has a worst possible public decision and there is a public decision that maximizes the sum of the utility of all individuals (the lobbies and the government official). Formally,

$$\mathcal{B}(\delta) = \{m: N \times X \rightarrow (-\delta, \delta): \quad \begin{array}{l} \exists z(m,i) \text{ s.t. } \forall y \neq z(m,i), m_i(z(m,i)) \leq m_i(y) \\ \exists x(m) \text{ s.t. } \forall y \neq x(m), \sum_{i=0}^n m_i(x(m)) > \sum_{i=0}^n m_i(y) \end{array} \quad \}.$$

Note that if the set of policies X is finite (as assumed in [5]), the first condition is trivially satisfied.

By defining the map $\mathcal{B}$, the assumption regarding M, the set of preferences over policies, takes a very simple form.

Assumption 1 *The set M is such that $\mathcal{B}(C) \subset M \subset \mathcal{B}(B)$, where $B > C > 0$.*

Assumption 1 insures that the the government official is uninformed of the preferences of the lobbies and there are punishments that can deter any type

[7]These transfers should not be construed as campaign contributions made to affect the probability with which a candidate is going to be elected, but as a means of influencing a current office holder. One estimate puts the campaign contribution-to-lobbying expenditure ratio at around 1:10 ([1]). As explained by Richard Armstrong, president of the Public Affairs Council: "If you're trying to buy access, giving after the election is like buying tickets on a horse race once the race is over." (See, Sabato [15], page 92.)

[8]Of course the government official knows the set M or at least the constant C defined below.

of lobby.[9] Note that Assumption 1 does not restrict m_0, the preference of the government official. This is because m_0 is not a component of $m \in M$, although m_0 can be a function of m.

Assumption 2 *The function m_0 is such that for all $x \in X$, $|m_0(x)| < C/4$.*

Assumption 2 insures that there are some circumstances in which the government official picks a policy that he does not like as much in order to get higher contributions.

2.2 CPNE implementability

For a strategy choice for the players in $I \subset N$, $s_I \in \times_{i \in I} S_i$, let $\Gamma(m, s_I)$ be the game involving players $N \setminus I$ such that if the players select the strategy $s_{N \setminus I} \in \times_{i \in N \setminus I} S_i$, the outcome is $g(s)$.

In a single player game, s is a **Coalition-Proof Nash equilibrium** (CPNE), as defined by Bernheim, Peleg and Whinston [4], if it maximizes the payoff of the individual. Suppose that CPNE has been defined for all games with fewer than n players. Then, (i) s is self-enforcing if for all $\emptyset \neq I \subset N$, the strategy $s_{N \setminus I}$ is a CPNE for the game $\Gamma(m, s_I)$; (ii) s is a CPNE if it is self-enforcing and the outcome is not Pareto dominated by an outcome of any other self-enforcing strategy.

A **choice function** $h: M \to T \times X$ is such that for all m and i, if $(t, x) = h(m)$, then $m_i(x) - t_i \geq \min_y m_i(y)$.[10] A choice function is CPNE-implementable if there is a game $\Gamma = (g, S)$ such that for all m, if s is a CPNE of the game $\Gamma(m)$, then $g(s) = h(m)$.

[9]The assumption $\exists z(m,i)$ s.t. $\forall y \neq z(m,i), m_i(z(m,i)) \leq m_i(y)$ simplifies the participation constraint. The assumption $\exists x(m)$ s.t. $\forall y \neq x(m), \sum_{i=0}^{n} m_i(x(m)) > \sum_i m_i(y)$ simplifies the notation; otherwise the choice function f would have to be a correspondence, see page 13.

[10]The participation constraints, or individual rationality constraints, are not obvious in this problem. The conditions given in this paper are clearly necessary. If transfers can be negative, the game can be written so that the conditions in the paper are sufficient to guarantee participation. If a lobby decides not to participate in the lobbying process, then all the lobbies who do participate receive a very high transfer. For the case where negative transfers are not allowed, the participation constraints compound the problems described in Proposition 2.

For all a and m, let $UL'(m,a)$ be the set of all m' such that

$$\begin{aligned} u_i(m,a) \geq u_i(m,b) &\Rightarrow u_i(m',a) \geq u_i(m',b), \\ u_i(m,a) > u_i(m,b) &\Rightarrow u_i(m',a) > u_i(m',b), \\ u_i(m',c) \geq u_i(m',a), c, d \neq a &\Rightarrow \\ \big(u_i(m,c) > u_i(m,d) &\Leftrightarrow u_i(m',c) > u_i(m',d), \\ u_i(m,c) \geq u_i(m,d) &\Leftrightarrow u_i(m',c) \geq u_i(m',d)\big). \end{aligned}$$

A choice function $h\colon M \to X \times T$ satisfies weak monotonicity (WM) when: if $a = h(m)$, $m' \in UL'(m,a)$, then $h(m') = a$.[11]

Proposition 1 *The allocation $h\colon M \to X \times T$ is CPNE implementable if and only if it satisfies WM.*

The proposition follows from the next three lemmata. The next result and proof are very similar to Lemma 1 in [6] and its proof.

Lemma 1 *If h is CPNE implementable, then h satisfies WM.*

Proof: Suppose s_i is a CPNE for the one person game $\Gamma(m, s_{-i})$, $a = g(s)$, and $m' \in UL'(m,a)$. Let $s'_i \neq s_i$, $s' = (s'_i, s_{-i})$, $a = g(s)$, $b = g(s')$. Since s is a CPNE for $\Gamma(m, s_{-i})$, $u_i(m,a) \geq u_i(m,b)$. Since $m' \in UL'(m,a)$, $u_i(m',a) \geq u_i(m',b)$. Hence, s_i is a CPNE for $\Gamma(m', s_{-i})$.

Suppose that for all I where $|I| > k$, if $s_{N\setminus I}$ is a CPNE of $\Gamma(m, s_I)$, $a = g(s)$, $m' \in UL'(m,a)$, then $s_{N\setminus I}$ is a CPNE of $\Gamma(m', s_I)$. Let I be such that $|I| = k$, $s_{N\setminus I}$ is a CPNE of $\Gamma(m, s_I)$, $a = g(s)$, $m' \in UL'(m,a)$. By the induction hypothesis $s_{N\setminus I}$ is self-enforcing for the game $\Gamma(m', s_I)$. Let $s'_{N\setminus I} \neq s_{N\setminus I}$ be self-enforcing for the game $\Gamma(m', s_I)$, let $b = g(s'_{N\setminus I}, s_I)$, and suppose for all $i \in N \setminus I$, $u_i(m',b) > u_i(m',a)$. Since $m' \in UL'(m,a)$, then for all $i \in N \setminus I$, $u_i(m,b) > u_i(m,a)$.

Since $s_{N\setminus I}$ is a CPNE of $\Gamma(m, s_I)$, $s'_{N\setminus I}$ cannot be self-enforcing in the game $\Gamma(m, s_I)$. Hence, there must be some $J \supset I$, such that $s'_{N\setminus J}$ is not

[11] Some readers will notice the close resemblance of this condition to LSPA. The connection between the two conditions is discussed in the next section. In the social choice literature, weak monotonicity has a different meaning; a more precise name for the property is LSPA'.

a CPNE of the game $\Gamma(m, s_I, s'_{J\setminus I})$. Consequently, there is a self-enforcing strategy profile of $\Gamma(m, s_I, s'_{J\setminus I})$, $s''_{N\setminus J}$, such that $c = g(s_I, s'_{J\setminus I}, s''_{N\setminus J})$ and for all $i \in N \setminus J$, $u_i(m, c) > u_i(m, b)$. Since $m' \in UL'(m, a)$, for all $i \in N \setminus J$, $u_i(m', c) > u_i(m', b)$. Further, by the induction hypothesis, $s''_{N\setminus J}$ is a self-enforcing strategy for the game $\Gamma(m', s_I, s'_{J\setminus I})$. Hence, $s'_{N\setminus I}$ is not self-enforcing for the game $\Gamma(m', s_I)$. Contradiction.

Let (S, g) be a game that implements h. Let s be a CPNE of $\Gamma(m)$, so that $g(s) = h(m) = a$ and let $m' \in UL'(m, a)$. Then, s is a CPNE of the game $\Gamma(m')$ and hence $a = g(s) = h(m')$. ∎

The condition WM can be simplified by using the fact that lobbies' preferences are quasi-linear. Let $a = h(m)$ and define the following set:[12]

$$I(m, m') = \{i \in N \colon \exists b, c \text{ s.t. } \quad u_i(m', a) < u_i(m', b), u_i(m, c) > u_i(m, b), \; u_i(m', c) < u_i(m', b) \}.$$

If $i \in I(m, m')$, then there are two outcomes b and c such that individual i prefers c over b when preferences over policies are m, and b over c when preferences are m'.

Lemma 2 *Let $h \colon M \to X \times T$ satisfy WM. If $h(m) \neq h(m')$, then $I(m, m') \neq \emptyset$.*

Proof: Let $a = (x, t)$. Suppose there is some $b = (y, t')$ and i for which $u_i(m, a) \geq u_i(m, b)$ and $u_i(m', a) < u_i(m', b)$. Let $d = (x, t_{-i}, t''_i)$ where $t''_i \in (-m'_i(y) + t'_i + m'_i(x), t_i)$. Then, $u_i(m', d) > u_i(m', a)$, $u_i(m, d) > u_i(m, b)$, and $u_i(m', d) < u_i(m', b)$.

Let $c = (z, s)$, $u_i(m', b) \geq u_i(m', a)$, $u_i(m, b) > u_i(m, c)$, and $u_i(m', b) = u_i(m', c)$. Let $d = (y, t'_{-i}, t''_i)$, $e = (z, s_{-i}, s'_i)$, $\delta > 0$, $t''_i = t'_i + \delta$, $s'_i = s_i - [m_i(y) - t'_i - m_i(z) + s_i]/2 - \delta$. Then, $u_i(m', d) > u_i(m', a)$, $u_i(m, d) > u_i(m, e)$, and $u_i(m', d) < u_i(m', e)$.

Let $u_i(m', b) \geq u_i(m', a)$, $u_i(m, b) = u_i(m, c)$, and $u_i(m', b) > u_i(m', c)$. Let $d = (y, t'_{-i}, t''_i)$, $e = (z, s_{-i}, s'_i)$, $\delta > 0$, $t''_i = t'_i + \delta$, $s'_i = s_i - [m'_i(y) - t'_i - m'_i(z) + s_i]/2 - \delta$. Then, $u_i(m', d) > u_i(m', a)$, $u_i(m, d) < u_i(m, e)$, and $u_i(m', d) > u_i(m', e)$.

[12]The set $I(m, m')$ should have h as an argument, however the omission should not cause any confusion.

Let $u_i(m', b) \geq u_i(m', a)$, $u_i(m, b) = u_i(m, c)$, and $u_i(m', b) < u_i(m', c)$. Let $d = (y, t'_{-i}, t''_i)$ where $t''_i \in (-m'_i(z) + s_i + m'_i(y), t'_i)$. Then, $u_i(m', d) > u_i(m', a)$, $u_i(m, d) > u_i(m, c)$, and $u_i(m', b) < u_i(m', c)$.

Let $u_i(m', b) \geq u_i(m', a)$, $u_i(m, b) < u_i(m, c)$, and $u_i(m', b) = u_i(m', c)$. Let $d = (y, t'_{-i}, t''_i)$ where $t''_i \in (-m_i(z) + s_i + m_i(y), t'_i)$. Then, $u_i(m', d) > u_i(m', a)$, $u_i(m, d) < u_i(m, c)$, and $u_i(m', d) > u_i(m', c)$.

Hence the Lemma follows from Lemma 1. ∎

Lemma 3 *If $h: M \to X \times T$ be such that $I(m, m') \neq \emptyset$ when $h(m) \neq h(m')$. Then h is CPNE implementable.*

Proof: The proof consists of constructing a mechanism that implements h. The mechanism works as follows. Each lobby reports the preferences of every lobby. If every lobby reports the same vector of preferences, m, the government official implements the corresponding choice, $h(m)$. If one lobby deviates, this lobby is severely punished. Hence, it is a Nash equilibrium for all lobbies to make the same announcement (whether or not it is the truth). If two individuals deviate (not any two individuals, but the ones defined in the mechanism), the government official rewards the deviators and punishes everyone else.

The key to this construction is that under CPNE any deviation must be free of counter-deviations.[13] In particular, if the deviators are deviating from the truth, one of the deviators has an incentive to counter-deviate; i.e., announce the truth. If the deviators are not deviating from the truth, then neither deviator has an incentive to counter-deviate. Consequently, truth-telling is the only CPNE. Another way to gain intuition is to look at Example 4 on page 14.

The mechanism just described does not work when there are three or four lobbies. For instance, if two lobbies announce a vector of preferences m and two lobbies announce a vector of preferences m', the government official does not know which lobbies are deviating. For this reason, the mechanism is modified so that in addition to announcing preferences, lobbies announce whether they are deviating, D, or not, ND.

Define $P: M \times M \to (X \times T)^2$ to be such that if $h(m) \neq h(m')$, then $P(m, m') = (b, c)$ where: $i = \min I(m, m')$, $l = i + 1 \pmod{n}$, $a = (x, t) =$

[13]Of course, since the game is of normal form, none of these deviations and future deviations occur anywhere other than the players introspective reasonings.

$f(m)$, $b = (y, t')$, $c = (z, s)$, $m_l(y) - t'_l > m_l(x) - t_l$, $\forall j \neq i, l, t'_j = s_j = 2B$, $s_j = 2B$, $u_i(m', b) > u_i(m', a)$, $u_i(m', b) > u_i(m', c)$, $u_i(m, b) < u_i(m, c)$. By Lemma 2, the function P is well defined.

Let $\Gamma = (S, g)$ where $S_i \equiv M \times \{D, ND\}$.[14] The function $g: S \to X \times T$ is such that if $s = (s_i)_{i \in N}$, $h(m) = a = (x, t)$, $h(m) \neq h(m')$, $(b, c) = P(m, m')$, $i = \min I(m, m')$, $l = i + 1 \pmod n$, then:

1. if for all j, $s_j = (m, ND)$, then $g(s) = h(m^i)$
2. if $s_l = (m', D)$ and $\forall j \neq l$, $s_j = (m, ND)$, then $g(s) = c$;
3. if $\forall j \neq i, l$, $s_j = (m, ND) \neq s_i = s_l = (m', D)$, then $g(s) = b$;
4. in all other cases, $g(s) = (w, \underline{t})$ where for all i, $\underline{t}'_i = 2B$ and w is arbitrary.

For all j, let $s_j = (m, ND)$. Note that s_j is the unique CPNE of the game $\Gamma(m, s_{-j})$. Suppose that for all I where $|I| > k$, $s_{N \setminus I}$ is the only CPNE of the game $\Gamma(m, s_I)$. Let I be such that $|I| = k$. Then, $s_{N \setminus I}$ is self-enforcing for $\Gamma(m, s_I)$. Let $s_{N \setminus I}{}'$ be self-enforcing for $\Gamma(m, s_I)$ and suppose $g(s_{N \setminus I}{}', s_I)$ Pareto dominates $g(s)$. By the construction of g, $I = \emptyset$ and $s'_j \equiv (m', ND)$. Again, by the construction of g, $s'_j \equiv (m', ND)$ is self-enforcing if and only if $m' = m$. Hence $s_j \equiv (m, ND)$ is the unique CPNE of $\Gamma(m)$ and the choice is $g(s) = h(m)$. ∎

Lemma 4 *Let $T = \mathbf{R}^n$. Then any choice function h is CPNE implementable.*

Proof: Let $m \neq m'$, $f(m) = (x, t) = a$, and without loss of generality suppose $m_i(y) - m_i(z) > \epsilon > m'_i(y) - m'_i(z)$, where $y, z \neq x$. Let $b = (z, s)$, $c = (y, t')$, where $s_i = L$, $t'_i = \epsilon + L$, and L is picked in such a way that $u_i(m', b) \geq u_i(m', a)$. Then $u_i(m, b) - u_i(m, c) < 0$, $u_i(m', b) - u_i(m', c) > 0$, so that $i \in I(m, m')$ and $I(m, m') \neq \emptyset$. Hence the result follows from Lemma 3. ∎

2.3 The design of optimal lobbying rules

The **optimal choice** for the government official is such that when preferences are m, the choice is $f(m) = (x, t)$ where $x = \text{argmax}_y \sum_{i=0}^n m_i(y)$ and $t_i =$

[14] If $n > 4$, it is sufficient to have $S_i = M$.

$m_i(x) - \min_y m_i(y)$. This is the immediate consequence of maximizing the utility of the government official conditional on the participation constraints of the lobbies. An immediate corollary to Lemma 4 is the following.

Corollary 1 *Let $T = \mathbf{R}^n$. Then, the government official's optimal choice is CPNE implementable.*

Example 4 The following example illustrates the mechanisms constructed in Lemma 3 with one of the preference profiles that make the implementation of f the most problematic. Let $L > \epsilon \geq 0$, and for all $i \in N$, let $m_i(x) = L$, $m'_i(x) = \epsilon$, and for all $y \neq x$, $m_i(y) = m'_i(y) = 0$.

Suppose preferences are m, then it should not be an equilibrium for each lobby to announce m'. Let $a = f(m') = (x, \epsilon)$, $b = (y, t')$, and $c = (x, t'')$ where t' is such that the two lobbies who deviated receive L while everyone else pays B, and t'' is such that the person who deviates pays B while everyone else receives ϵ.

If two lobbies deviate and announce m, the outcome is c which makes the deviating lobbies better off.[15] Notice that any further deviation leads to an outcome b which makes the deviating individual worse off.

Suppose the preferences are m' and everyone announces m'. Then, two lobbies by announcing m can make themselves better off. However, this deviation is not immune to a further deviation. □

Example 5 As another illustration of the mechanism constructed in Lemma 3, consider the situation described in Example 1, namely, $m(x) = m'(x) = (3, 0, 0)$, $m(y) = m'(y) = (0, 2, 2)$, $m(z) = (-1, -1, -1)$, $m'(z) = (0, 0, 0)$. Let $a = f(m) = (y, -1, 3, 3)$, $a' = f(m') = (y, 0, 2, 2)$, $b = (z, -2, -2, 2B)$, $c = (y, -3/2, 2B, 2B)$. Notice that if preferences are m, reporting m' – which leads to an outcome a' – is not a CPNE. This is because player 1 and player 2 can profitably deviate by reporting m and getting an outcome b. If the preferences are m' such a deviation is not self enforcing because player 1 has an incentive to deviate back to m'. □

Proposition 2 *Let $T = \mathbf{R}^n_+$. Then the government official optimal's choice is not CPNE implementable.*

[15]Notice that in order for this to work, negative transfers must be possible.

Proof: Let m and m' be as follows: $m_1(x) = m'_1(x) = C$, $m_2(x) = 0$, $m'_2(x) = C$, and if $y \neq x$ or $i \neq 1, 2$, $m_i(y) = m'_i(y) = 0$. Then, $f(m) = (x, C, 0, 0, \ldots, 0) = a$, $f(m') = (x, C, C, 0, \ldots, 0)$, $m' \in UL'(m, a)$ in violation of WM.[16] ∎

Notice that a corollary of this result is that none of the normal form mechanisms considered in Section 1 result in an optimal choice for the government official.

3 Relationship of results with implementation theory

This section answers the following questions: is the government official's optimal choice Nash implementable, dominant strategy implementable? How about iteratively weakly dominated strategy implementation? Does the revelation principle hold for CPNE? Is the condition WM a trivial strengthening of LSPA?

3.1 Nash implementation

For all a and m, let $U(m, a)$ be the set of all m' such that

$$u_i(m, a) \geq u_i(m, b) \Rightarrow u_i(m', a) \geq u_i(m', b), u_i(m, a) > u_i(m, b) \Rightarrow u_i(m', b) > u_i(m', b).$$

A choice function $h: M \rightarrow X \times T$ satisfies Maskin monotonicity (MM) when: if $a = h(m)$, $m' \in U(m, a)$, then $h(m') = a$.

Lemma 5 *The optimal choice does not satisfy MM.*

Proof: For all i, $m_i(x) = C/2$, $m'_i(x) = C$, for all $y \neq x$, $m_i(y) = m'_i(y) = 0$. Then, $f(m) = (x, C/2, \ldots, C/2) = a$, $m' \in U(m, a)$, but $f(m') = (x, C, \ldots, C) \neq a$. ∎

[16] If $T = \mathbf{R}^n$, then $m' \notin UL'(m, a)$. For instance, let $b = (y, -\mathbf{C})$, $c = (x, -\mathbf{C}/2)$, where $\mathbf{C} = (C, C, \ldots, C)$. Then, $u_2(m', c) = 3C/2 > u_2(m', b) = C \geq u_2(m', a)$, but $u_2(m, c) = C/2 < u_2(m, b) = C$.

Hence, by Maskin's theorem [13], the optimal choice is not Nash implementable; i.e., there does not exist a game $\Gamma = (S, g)$ such that if s is a Nash equilibrium of $\Gamma(m)$, then $g(s) = f(m)$. A further consequence of Lemma 5 is that f does not satisfy the sufficient conditions for CPNE implementation given in [5].

3.2 Dominant strategy implementation

A choice function h is independently person by person monotonic (IPM) when: if $h(m) = a$, $m' = (m'_1, m_{-i})$, $u_i(m, a) \geq u_i(m, b)$ implies $u_i(m', a) > u_i(m', b)$, then $f(m') \neq b$. Dasgupta, Hammond and Maskin [7] (Theorem 4.1.1, Theorem 4.3.1) show that IPM is a necessary condition for dominant strategy implementation.[17]

Lemma 6 *The optimal choice does not satisfy IPM.*

Proof: Let m and m' be as follows: $m_1(x) = m'_1(x) = C$, $m_2(x) = 0$, $m'_2(x) = C$, and if $y \neq x$ or $i \neq 1, 2$, $m_i(y) = m'_i(y) = 0$. Let $a = f(m) = (x, C, 0, 0, \ldots, 0) = a$, and let $b = (x, C, C, 0, \ldots, 0)$. Then, $u_i(m, a) \geq u_i(m, b)$, $u_i(m', a) > u_i(m', b)$, but $f(m') = b$. ∎

Hence, the optimal choice is not dominant strategy implementable; i.e., there does not exist a game $\Gamma = (S, g)$ such that if s is a dominant strategy equilibrium of $\Gamma(m)$, then $g(s) = f(m)$.

3.3 Iteratively weakly dominated strategy implementation

An alternative approach to the one in this paper is to implement the optimal choice of the government official under the assumption that lobbies do not select actions that are iteratively weakly dominated. To compare CPNE implementation and iteratively weakly dominated strategy implementation, consider the game in Figure 1. In this game there are two pure strategy Nash equilibria: (T, l) and (M, r): the former is the outcome of iterative elimination of dominated strategies, while the latter is the CPNE.[18]

[17] Note, IPM does not imply MM (see Example 7.2.1 in [7]).

[18] John Nachbar suggested this example.

Player 2

Player 1		l	r
	T	1,1	0,0
	M	0,100	100,100
	D	0,0	99,99

Figure 1: Comparison of CPNE and iterative elimination of weakly dominated strategies

Player 2

Player 1		S1	S2	S3
	S1	1,1	0,0	0.5,0
	S2	0,0	1.5,1.5	0,2
	S3	0,0.5	2,0	0,0

Figure 2: Game when preferences are m

Player 2

Player 1		S1	S2	S3
	S1	1.5,1.5	0,0	0,2
	S2	0,0	1,1	0.5,0
	S3	2,0	0,0.5	0,0

Figure 3: Game when preferences are m'

3.4 The revelation principle

Does the revelation principle imply that, without loss of generality, the set of games can be restricted to the case where $S_i = M_i$? First, in the revelation game each player must announce his type which in this context is the preference profile of *all* players. Second, the revelation principle does not hold for CPNE.

Example 6 Consider the game where $n = 2$, $S_i = \{S1, S2, S3\}$. The CPNE when preferences are m (see Figure 2) is $(S1, S1)$ which leads to a payoff of $(1, 1)$. The CPNE when preferences are m' (see Figure 3) is $(S2, S2)$ which leads to a payoff of $(1, 1)$.

Consider the corresponding revelation games, see Figure 4 and Figure 5. The CPNE when preferences are m is (m', m') which leads to a payoff of $(1.5, 1.5)$. The CPNE when preferences are m' is (m, m) which leads to a payoff of $(1.5, 1.5)$. Hence, the CPNE is different in the revelation game than in the original game, and truth-telling is not a CPNE. □

		Player 2	
		m	m'
Player 1	m	1,1	0,0
	m'	0,0	1.5,1.5

Figure 4: Revelation game when preferences are m

		Player 2	
		m	m'
Player 1	m	1.5,1.5	0,0
	m'	0,0	1,1

Figure 5: Revelation game when preferences are m'

3.5 Lower stronger positive association

Bernheim and Whinston [5] prove that LSPA is a necessary condition for CPNE implementation. This section discusses the relationship between LSPA and the new condition WM.

For all a and m, let $UL(m, a)$ be the set of all m' such that

$$\begin{aligned} u_i(m, a) \geq u_i(m, b) &\Rightarrow u_i(m', a) \geq u_i(m', b), \\ u_i(m, a) > u_i(m, b) &\Rightarrow u_i(m', b) > u_i(m', b), \\ u_i(m, c) \geq u_i(m, a), c, d \neq a &\Rightarrow \\ \Big(u_i(m, c) > u_i(m, d) &\Leftrightarrow u_i(m, c) > u_i(m, d), \\ u_i(m, c) \geq u_i(m, d) &\Leftrightarrow u_i(m', c) \geq u_i(m', d)\Big). \end{aligned}$$

A choice function $h\colon M \rightarrow X \times T$ satisfies lower stronger positive association (LSPA) when: if $a = h(m)$, $m' \in UL(m, a)$, then $h(m') = a$. It is very hard to see that the conditions WM and LSPA are not identical. (Hint: the second line in the definition of $UL(m, a)$ is the same as the second line in the definition of $UL'(m, a)$ except for m replacing m'.) It is even harder to see how the new condition WM represents a contribution to the literature on implementation. The next result shows that.

Proposition 3 *Let $T = \mathbf{R}^n_+$. The optimal choice satisfies LSPA.*[19]

Proof: Let $f(m) = (x, t)$ and let $m' \in UL(m, a)$.

If $m_i(x) \geq m_i(y)$, then $m_i(x) \geq m_i(y)$ implies that $m'_i(x) \geq m'_i(y)$. If $m_i(x) \leq m_i(y)$, then $m_i(x) \geq m_i(y) - [m_i(y) - m_i(x)]$ implies that $m'_i(x) \geq$

[19] Of course the proposition holds also for the case when $T = \mathbf{R}^n$ since WM is stronger than LSPA.

$m_i'(y) - [m_i(y) - m_i(x)]$ or $m_i'(x) - m_i'(y) \geq m_i(x) - m_i(y)$. Hence, $x \in \text{argmax}_y \sum_i m_i(y)$ implies $x \in \text{argmax}_y \sum_i m_i'(y)$.

Let $x' \in \text{argmin}_y m_i(y)$ and $y' \in \text{argmin}_y m_i'(y)$. Let K be such that $m_i(x) - m_i(x') = m_i'(x) - m_i'(y') + K$. (i) Suppose $K < 0$. Notice that $m_i(y') \geq m_i(x) - [m_i(x) - m_i(x')]$ and $m(y') > m_i(x) - [m_i'(x) - m_i'(y')]$. Then, $m_i'(y') > m_i'(x) - [m_i'(x) - m_i'(y')] = m_i'(y')$. Contradiction. (ii) Suppose $K > 0$. Note that $m_i(y') \geq m_i(x) - [m_i(x) - m_i(x')]$ and $m_i(y') > m_i(x) - [m_i(x) - m_i(x') - K/2]$. Then,

$$\begin{aligned} m_i'(y') &> m_i'(x) - [m_i(x) - m_i(x') - K/2] \\ &= m_i'(x) - [m_i'(x) - m_i'(y') + K - K/2] \\ &= m_i'(y') + K/2. \end{aligned}$$

Hence, $f(m') = (x, t)$. ■

Consequently, identifying the condition WM is crucial in proving Proposition 1 and Proposition 2.

4 Concluding remarks

An optimal mechanism for the government official leads to the same policy predicted in [5] but to higher contributions. In a more general model of lobbying, the level of contributions should affect which firms and individuals decide whether or not to form a lobby. This in turn should affect which policy is selected by the government official.

The results in this paper can be used to justify other models of lobbying. Becker [3] adopts an aggregate production function approach to lobbying.[20] Suppose the set of policies is $X = [0, 1]$. Lobby 1 prefers policies that are closer to 0, lobby 2 prefers policies that are closer to 1, while the other lobbies are indifferent. Then, the policy picked is $x = F(t_1, t_2)$, where t_i is the contribution of lobby i, and the function F is assumed to be decreasing and strictly convex in the first argument while increasing, strictly concave in the second argument.[21] If the government official is not too uncertain about

[20] This terminology is borrowed from Austen-Smith [1].

[21] Becker considers both an influence function and a pressure function. However, as pointed out in [1], in Becker's model, the two can be combined without loss of generality.

the preferences of the lobbies, then the mechanism defined by the function F can be optimal. Specifically, let $M = \{(a\sqrt{1-x}, b\sqrt{x}) \colon a, b \in [0, K]\}$. Then, it turns out that the government official's optimal choice is CPNE implementable and $x = t_2/(t_1 + t_2)$.

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Appendix

The appendix attempts to give some anecdotal evidence that government officials try to construct mechanisms to maximize contributions. The first example shows how at the end of the nineteenth century and the beginning of the twentieth century "combines" in St. Louis extracted bribes, or boodle.[22] Steffens [16] (page 22) gives the following quote from a grand jury investigating corruption in St. Louis:

> Combines in both branches of the Municipal Assembly are formed by members sufficient in number to control legislation. To one member of this combine is delegated the authority to act for the combine, and to receive and to distribute to each member the money agreed upon as the price of his vote in support of, or opposition to, a pending measure.

Steffens (page 35) reports the following vote agreement:

> You will vote on roll call after Mr. —. I will place $ 45,000 in the hands of your son, which amount will become yours, if you have to vote for the measure because of Mr. —'s not keeping his promise. But if he stands out for it you can vote against it, and the money shall revert to me.

On page 83, Steffens writes:

> ...though boodling was a business by itself, it was a good business, and so easy that anybody could learn it by study. ...a fellowship had grown up among boodling aldermen of the leading cities in the United States. Committees from Chicago would come to St. Louis to find out what "new games" the St. Louis boodlers had, and they gave the St. Louisans hints as to how they "did business" in Chicago.

Eventually many of the St. Louis boodlers were put in jail, so one may wonder if such practices still exist. Sabato [15], page 110, quotes Peter Lauer of the American Medical Political Action Committee (AMPAC):

[22] Gary Miller suggested this example.

> Some of those [anti-PAC] officeholders call me a whore so I'll call them hypocrites. The best example is [Sen.] Bob Dole [of Kansas] who says, "When the PAC's give, they want something in return besides good government." Then I get a letter from him asking for money for his PAC. I also get a telegram from him two weeks out of the election saying, "I see you have not supported [a Dole-backed candidate in Kansas]. Deeply disappointed, Signed, Bob Dole, Chairman, Senate Finance Committee." What kind of sledgehammer is that? I find those things deeply disturbing.

Fritsch [9] claims that the following is the legal behavior of "a smart politician working the system like so many of his colleagues."

> A lobbyist for Ace Framus Company sends a $ 1,000 check to Senator Wily's campaign fund. Senator Wily dips into the fund to lease a new car and introduces a bill to deregulate the framus industry. ...

As Representative Andrew Jacobs of Indiana states (the quote is on page 127 of [15]):

> The only reason it isn't considered bribery is that Congress gets to define bribery.

More specifically, Fritsch [9] writes:

> Technically, nothing in the law bars a prosecutor from charging an elected official with bribery and for taking an official action on behalf of a campaign contributor. But prosecutors may hesitate to pursue such cases because of the difficulty in proving a quid pro quo, said Philip B. Heynman, a Harvard law professor and former Deputy Attorney General in the Clinton Administration.
> ...
>
> Stanley M. Brand, a Washington lawyer who has represented a number of politicians in trouble, said it is hard for a prosecutor to make a compaign contribution look like a bribe unless there is "the most explicit evidence" of quid pro quo. "In 1996 people seem to have smartened up to where they don't have those conversations and writings," Mr. Brand said.