EXTERNALITIES, PROPERTY RIGHTS AND PROFITABILITY:

A Data Envelopment Analysis Investigation

by

Rolf Färe*†, Shawna Grosskopf*† and Wen-Fu Lee**††

September, 1995

*Financial support from USA EPA is acknowledged

**Financial support from ROC NSC is acknowledged

†Department of Economics
Southern Illinois University at Carbondale
Carbondale, Illinois 62901

††Department of Economics
National Chengchi University
Taipei 11623
Taiwan

Abstract

In this paper we introduce DEA models that can be used to compute the way firms' profitability changes when the assignment of property rights changes.

I. Introduction

Externalities in production are frequently observed. Generally, such externalities result in market failure, i.e., even a perfectly competitive market does not necessarily lead to a Pareto efficient outcome. To correct such failures, one remedy is the creation and enforcement of property rights, often referred to as a Coase (1960) solution. Although such a solution should be efficient, it will affect the profitability of the agents involved, which will depend on which party receives the property right. The purpose of the paper is to investigate the relationship between property rights and profitability using DEA (Data Envelopment Analysis)[1] and the network theory of production.

Externalities may be positive (economies) or negative (diseconomies). The paper is confined to the case of diseconomies, in which an upstream agent produces good and bad outputs, and the bad outputs adversely affect the downstream agent's production opportunity. We leave the case of external economies to the reader.

Specifically, section II sets up the production technology based on a DEA model and network theory. Section III compares how property rights affect agents' profits under three regimes: (1) the upstream agent has the property right, (2) the downstream agent has the property right, and (3) the externality is internalized to an efficient network. Comparison of individual profits under the three regimes provides an estimate of the redistribution associated with the assignment of property rights.

---

[1]This expression was coined by Charnes, Cooper and Rhodes (1978).

I. The Production Network

In this paper we take a network approach (see Färe, 1991) to modeling the interaction between the polluter and the receptor. Our model is initially restricted to one polluter and one receptor, a restriction that may be relaxed. To illustrate the interaction, we assume there are two technologies, represented by their output sets P^1 and P^2. We illustrate these sets and their interactions in Figure 1.

y^1 $\qquad\qquad\qquad\qquad$ y^2

P^1 $\qquad$ ${}^2_1y$ $\qquad$ P^2

x^1 $\qquad\qquad\qquad\qquad$ x^2

Figure 1: The Network Model

The polluter technology, P^1 uses input vector $x^1 \in \mathbb{R}^N_+$ to produce two sets of outputs. The first set $y^1 \in \mathbb{R}^M_+$ denotes final products that are traded on the market, and the second set, ${}^2_1y \in \mathbb{R}^I_+$ are those outputs that are used as inputs in the second technology, and that are potentially detrimental to this technology. These two sets of outputs are produced jointly. Here we use the terminology of joint in the sense of Shephard and Färe (1980) and say that y^1 is nulljoint with ${}^2_1y$ if $(y^1, {}^2_1y) \in P(x^1)$ and ${}^2_1y\ 0$ imply that $y^1=0$. Nulljointness thus implies that positive final outputs $y^1 \geq 0,\ y^1 \neq 0$ can only be produced if some bad output ${}^2_1y$ is also produced.

The outputs ${}^2_1y$ are here assumed to be pollutants, which have a negative impact on the second technology $P^2(x^2, {}^2_1y)$. The receptor technology cannot avoid using the ${}^2_1y$ as input, and we assume that all outputs ${}_{1}^{2}y$ from the polluter are inputs to the receptor. This assumption may be relaxed.

To model the unfavorable impact of ${}_{1}^{2}y$ on the receptor, we assume that this technology exhibits weak disposability of inputs, i.e.,

(2.1) $$P^2(\lambda x^2, \lambda {}_{1}^{2}y) \supseteq P^2(x^2, {}_{1}^{2}y),\ \lambda \geq 1$$

and that ${}_{1}^{2}y$ are congesting in the sense that

(2.2) $$P^2(x^2, {}_{1}^{2}y) \supseteq P^2(x^2, {}_{1}^{2}y') \text{ if } {}_{1}^{2}y' \geq {}_{1}^{2}y.$$

In words, (2.1) says that if all inputs are increased proportionally, then output does not decrease, while (2.2) expresses the idea that if bad inputs ${}_{1}^{2}y$ are increased output y^2 does not increase, it may even decrease.

The network model illustrated in Figure 1 may be formalized as two interacting activity analysis or DEA (Data Envelopment Analysis) models. To see this we assume that there are k = 1,...,K observations of the two technologies, and we start by modelling the polluter technology.

(2.3) $$P^1(x^1) \quad \Big\{(y^1, {}_{1}^{2}y):\ y_m^1 \leq \sum_{k\ 1}^{K} z_k^1 y_{km}^1,\ m\ 1,...,M,$$
$$\qquad {}_{1}^{2}y_i \quad \sum_{k\ 1}^{K} z_k^1\, {}_{1}^{2}y_{ki},\ i\ \ 1,...,I,$$
$$\qquad x_n^1 \geq \sum_{k\ 1}^{K} z_k^1 x_{kn}^1,\ n\ \ 1,...,N,$$
$$\qquad z_k^1 \geq 0,\ k\ 1,...,K,\ \sum_{k\ 1}^{K} z_k^1 \leq 1\Big\}$$

Here the observations are $y_{km}^1,\ {}_{1}^{2}y_{ki}$ and $x_{kn}^1;$ they denote the kth observation of final output m, intermediate or "bad" output i, (which is an input into the second technology) and the input n, respectively.

The polluter technology (2.3) satisfies nonincreasing returns to scale due to the restriction on the intensity variables $z_k^1,\ k\ 1,...,K,$ which imply nonnegative profit. The outputs

$(y^1, {}^2_1y)$ are weakly disposable in the sense that

(2.4)

$$\text{if } (y^1, {}^2_1y) \in P^1(x^1) \text{ and } 0 \le \theta \le 1 \text{ then } (\theta y^1, \theta\, {}^2_1y) \in P^1(x^1).$$

In addition the final outputs y^1 are freely disposable. These two conditions follow from the first M inequalities and the I equalities. Finally inputs x^1 are freely disposable, i.e.,

$$(2.5) \qquad x^1 \ge \hat{x}^1 \text{ implies that } P^1(x^1) \supseteq P^1(\hat{x}^1).$$

To model the receptor technology P^2, again we assume that there are k = 1,...,K observations of inputs x^2_{kn}, ${}^2_1y_{ki}$ and outputs y^2_{km}. The DEA model of the receptor may be written as

$$(2.6) \qquad P^2(x^2, {}^2_1y) \quad \Big\{ y^2:\ y^2_m \le \sum_{k=1}^{K} z^2_k y^2_{km},\ m = 1,...,M,$$

$${}^2_1y_i = \sum_{k=1}^{K} z^2_k\, {}^2_1y_{ki},\ i = 1,...,I,$$

$$x^2_n \ge \sum_{k=1}^{K} z^2_k x^2_{kn},\ n = 1,...,N,$$

$$z^2_k \ge 0,\ k = 1,...,K,\ \sum_{k=1}^{K} z^2_k \le 1 \Big\}$$

The outputs y^2_m and inputs x^2_n are freely disposable as seen by the corresponding inequalities. One may prove that the receptor technology satisfies (2.1), weak disposability of inputs.

So far we have modelled the polluter and receptor as independent technologies. Next we integrate them into a network. The network output is equal to the sum of the two internalized technologies' outputs. At this point it should be noted that both technologies' final output vectors belong to $\mathbb{R}^M_+$. However, this does not imply that they need to produce the same outputs, since some components may be zero.

The network exogenous input is x which is at least as large as $x^1 + x^2$. The vector ${}^2_1y$ now becomes an intermediate product; it is produced and used within the network.

Under these conditions our technology is

(2.7) $$P(x) \quad \{y: \ y \ y^1 \ y^2 \\ (y^1, {}^{2}_{1}y) \in P^1(x^1) \\ y^2 \in P^2(x^2, {}^{2}_{1}y) \\ x^1 \ x^2 \leq x\}.$$

One can prove that in this model, inputs (x,...x_n) and outputs (y_1,...,y_M) are freely disposable and that it satisfies nonincreasing returns to scale in the sense that

(2.8) $$P(\lambda x) \supseteq \lambda P(x), \ \lambda \geq 1.$$

Disposability follows from the inequalities in (2.7) plus the corresponding inequalities in (2.3) and (2.6). Nonincreasing returns to scale is a consequence of the restrictions of the intensity variables z_k^1 and z_k^2, k 1,...,K.

3. Property Rights and Profit

In this section we develop profit maximization models for several cases. We first grant the polluter the property right, and then the receptor. We also develop a profit maximization model for the case where the bad outputs or externalities are internalized.

Initially we assume that the polluter has the property right. This situation is modelled by allowing the polluter to maximize profit without any constraints on the production of bad outputs ${}^{2}_{1}y$. The solution yields profit and the optimal ${}^{2}_{1}y^*$. This output vector is then taken as input for the receptor, which in the second problem maximizes its profit given ${}^{2}_{1}y^*$. Formally, given output prices $p \in \mathbb{R}_+^M$ and input prices $w \in \mathbb{R}_+^N$, the polluter's profit maximization problem is

(3.1) $$\Pi^1(p,w) \quad \max_{(y^1, {}^{2}_{1}y, \ x^1)} \sum_{m \ 1}^{M} p_m y_m^1 \quad \sum_{n \ 1}^{N} w_n x_n^1$$

$$\text{s.t.} \quad y_m^1 \leq \sum_{k \ 1}^{K} z_k^1 y_{km}^1, \ m \ 1,...,M,$$

$$ {}_1^2y_i \quad \sum_{k\;1}^{K} z_k^1 \, {}_1^2y_{ki}, \; i \; 1,\ldots,I, $$

$$ x_n^1 \geq \sum_{k\;1}^{K} z_k^1 x_{kn}^1, n \; 1,\ldots,N, $$

$$ z_k^1 \geq 0, \; k \; 1,\ldots,K, \; \sum_{k\;1}^{K} z_k^1 \leq 1. $$

The solution to (3.1) shows the maximal profit when the polluter has the property right. The solution also yields a vector of optimal outputs ${}_1^2y_i^*, \;\; i \; 1,\ldots,I.$ Taking this as given for the receptor, we can compute the receptor's profit, which is then

$$ (3.2) \qquad \Pi^2(p,w,{}_1^2y^*) \quad \max_{(y^2,x^2)} \sum_{m\;1}^{M} p_m y_m^2 \quad \sum_{n\;1}^{N} w_n x_n^2 $$

$$ \text{s.t.} \;\; y_m^2 \leq \sum_{k\;1}^{K} z_k^2 y_{km}^2, \; m \; 1,\ldots,M, $$

$$ {}_1^2y_i^* \quad \sum_{k\;1}^{K} z_k^2 \, {}_1^2y_{ki}, \; i \; 1,\ldots,I, $$

$$ x_n^2 \geq \sum_{k\;1}^{K} z_k^2 x_{kn}^2, n \; 1,\ldots,N, $$

$$ z_k^2 \geq 0, \; k \; 1,\ldots,K, \; \sum_{k\;1}^{K} z_k^2 \leq 1. $$

Before turning to our second case, a few remarks are in order. If there is no regulation and no property right is granted, we might expect (3.1) and (3.2) to be a likely outcome, i.e., it could serve as a model of the unregulated case. The fact that the polluter is assumed to be given the property right (i.e., the right to pollute), does not, in this case, necessarily lead to the Coase efficient outcome, since there are no side payments included in the model. However, we could use these two problems to compute the loss in profit to the polluter and gain in profit to the receptor of reducing emissions below ${}_1^2y^*$.[2] Turning to our second case, if we give the property

---

[2]In principle, by reducing ${}_1^2y^*$ to zero, we would have the case in which the receptor

right to the receptor, we have a second pair of profit maximization problems. Now the receptor's problem is solved first, i.e.,

$$(3.3) \qquad \Pi^2(p,w) = \max_{(y^2,\,{}^{2}_{1}y,\,x^2)} \sum_{m=1}^{M} p_m y_m^2 - \sum_{n=1}^{N} w_n x_n^2$$

$$\text{s.t.} \quad y_m^2 \leq \sum_{k=1}^{K} z_k^2 y_{km}^2,\ m = 1,...,M,$$

$${}^{2}_{1}y_i \quad \sum_{k=1}^{K} z_k^2\, {}^{2}_{1}y_{ki}^2,\ i = 1,...,I,$$

$$x_n^2 \geq \sum_{k=1}^{K} z_k^2 x_{kn}^2, n = 1,...,N,$$

$$z_k^2 \geq 0,\ k = 1,...,K,\ \sum_{k=1}^{K} z_k^2 \leq 1.$$

In addition to obtaining the maximal profit for the receptor when the receptor has the property right, expression (3.3) yields a vector of optimal inputs ${}^{2}_{1}y_i^{**},\ i = 1,...,I.$ Note that one would expect ${}^{2}_{1}y_i^{**},\ i = 1,...,I,$ to be zero, since they are undesirable to the receptor. However, without data, one could not exclude the possibility of positive ${}^{2}_{1}y$ from consideration. These inputs are outputs from the polluter and they become parameters in the polluter's problem when the receptor has the property right. Thus we have for the polluter,

$$(3.4) \qquad \Pi^1(p,w,{}^{2}_{1}y^{**}) = \max_{(y^1,x^1)} \sum_{m=1}^{M} p_m y_m^1 - \sum_{n=1}^{N} w_n x_n^1$$

$$\text{s.t.} \quad y_m^1 \leq \sum_{k=1}^{K} z_k^1 y_{km}^1,\ m = 1,...,M,$$

$${}^{2}_{1}y_i^{**} \quad \sum_{k=1}^{K} z_k^1\, {}^{2}_{1}y_{ki},\ i = 1,...,I,$$

---

would in effect have the property right.

$$x_n^1 \geq \sum_{k\ 1}^{K} z_k^1 x_{kn}^1, n\ 1,\ldots,N,$$

$$z_k^1 \geq 0,\ k\ 1,\ldots,K,\ \sum_{k\ 1}^{K} z_k^1 \leq 1.$$

Again, this solution does not account for potential bargaining á la Coase. Comparisons of (3.1) and (3.4) for the polluter, and (3.2) and (3.3) for the receptor would, however, give an estimate of the maximum potential redistribution of "income" that changes in the assignment of property rights could entail. Clearly we expect (3.1) > (3.4) and (3.3) > (3.2). In turn, this would provide an estimate of potential "rents" to be gained.

Our final profit maximization problem deals with the situation when the "externalities" ${}^{2}_{1}y_i,\ i\ 1,\ldots,I,$ are internalized. This corresponds to maximization of profit with the network technology as the constraints. In this case we have

(3.5) $$\Pi(p,w) \quad \max_{(y^1,y^2,x,{}^{2}_{1}y)} \sum_{m\ 1}^{M} p_m y_m \quad \sum_{n\ 1}^{N} w_n x_n$$

$$\text{s.t. } y_m \quad y_m^1\ y_m^2,\ m\ 1,\ldots,M$$

$$y_m^1 \leq \sum_{k\ 1}^{K} z_n^1 y_{km}^1, m\ 1,\ldots,M$$

$${}^{2}_{1}y_i \quad \sum_{k\ 1}^{K} z_k^1\, {}^{2}_{1}y_{ki},\ i\ 1,\ldots,I,$$

$$x_n^1 \geq \sum_{k\ 1}^{K} z_k^1 x_{kn}^1,\ n\ 1,\ldots,N,$$

$$z_k^1 \geq 0, k\ 1,\ldots K,\ \sum_{k\ 1}^{K} z_k^1 \leq 0$$

$$y_m^2 \leq \sum_{k\ 1}^{K} z_k^2 y_{km}^2,\ m\ 1,\ldots,M$$

$${}^{2}_{1}y_1 \quad \sum_{k\ 1}^{K} z_k^2\, {}^{2}_{1}y_{ki}^2,\ i\ 1,\ldots,I$$

$$x_n^2 \geq \sum_{k\ 1}^{K} z_k^2 x_{kn}^2,\ n\ 1,\ldots,N,$$

$$z_k^2 \geq 0,\ k\ 1,\ldots,K,\ \sum_{k\ 1}^{K} z_k^2 \leq 1,$$

$$x_n^1\ x_n^2 \leq x_n,\ n\ 1,\ldots,N.$$

In principle, these problems are very simple to compute if the relevant data are available. Despite their simplicity, they explicitly model the key aspects involved in production externalities: 1) the joint production of good and bad outputs, 2) the role of bads as an "intermediate" input which adversely affects production of other firms, and 3) the potential lack of free disposability of bads. Formulation of these problems in a profit maximizing framework allows us to simulate the redistribution of "income" which would result from a change in ownership of property rights as well as allowing us to solve for the efficient outcome under merger or internalization of the externality.

To summarize, in this paper we illustrate how property rights affect agents' profitability and how that can be analyzed in a DEA model. It should be noted that the network output vector is not necessarily greater than the sum of the two independent technologies' output vectors, but the network profits are at least equal to the sum of the two independent technologies' profits. The potential profit gain from internalization of the externality can then be measured, which in turn could be used as a compensation benchmark for the agents involved when considering merging or buying out. Alternatively, this could be used to derive optimal quantity constraints for the effluents, or to verify whether existing restrictions are "optimal."[3]

---

[3]Brännlund, Färe and Grosskopf (1995) use a similar activity analysis model to compute the loss in profits due to regulatory constraints restricting emissions in the paper and pulp industry in Sweden. Brännlund, Chung, Färe and Grosskopf (1995) modify that model to simulate emissions trading in the same industry.

References

Brännlund, R., R. Färe and S. Grosskopf (1995), Environmental Regulation and Profitability: An Application to Swedish Paper and Pulp Mills, *Environmental and Resource Economics* 5: 1-14.

Brännlund, R., Y. Chung, R. Färe and S. Grosskopf (1995), Emissions Trading and Profitability: The Swedish Paper and Pulp Industry, SIUC Working Paper 95:14.

Charnes, A., W. W. Cooper and E. Rhodes (1978), "Measuring the Efficiency of Decision Making Units," *European Journal of Operational Research* 2:6, 429-444.

Coase, Ronald W. (1960), The Problem of Social Cost, *Journal of Law and Economics* 3, 31-44.

Färe, R. (1991), Measuring Farrell Efficiency for a Firm with Intermediate Inputs, *Academia Economic Papers*, 19:2, 329-340.

Shephard, R. W. and R. Färe (1980), *Dynamic Production Correspondences*, Verlag Anton Hain.