Precommitment and State and Local Policy Coordination

William H. Hoyt
and
Richard A. Jensen

Department of Economics
College of Business and Economics
University of Kentucky
Lexington, KY 40506-0034

September 1994

Abstract

Local government tax and service policies, for reasons including tax competition, spillovers of service benefits, and a limited choice of tax instruments, may not be globally efficient. Policies used by state governments to remedy these inefficiencies include matching grants, mandates, and tax subsidies. We consider an additional strategy the state government may employ to influence local policies -- choosing or "precommitting" to its policies before the local governments choose their policies. We argue that the conditions necessary for the state government to credibly precommit to its policies with respect to local government policies do exist. The unlikelihood of the state government being able to alter its policies after local governments have chosen their policies reduces problems associated with time-inconsistency.

We first derive the general conditions necessary for precommitment to improve social welfare. Essentially we find that precommitment can increase social welfare if the state government does not have "sufficient" policy instruments to eliminate any change in welfare in one locality from a change in another locality's government policies and if the state government's policies influence the local government policies that create the uncompensated externality. We then illustrate the conditions when precommitment can improve social welfare using two examples from the `tax competition' literature.

Send all correspondence to:

William Hoyt
Department of Economics
College of Business and Economics
University of Kentucky
Lexington, KY 40506-0034
Telephone: 606-257-2518
Fax: 606-323-1920
E-Mail ECO183@UKCC.UKY.EDU

JEL Classifications: H7, R51

We wish to thank the comments and suggestions of seminar participants at Georgetown University, the Conference of the Southern Economic Association, and the Public Choice Society. In addition we wish to thank Carol Rogers, Matt Canzoneri, and David Wildasin for helpful discussions.

1. Introduction

The possibility that governments, whether at the local, state, or even federal level, may not set efficient policies, because they ignore the benefits or costs of their policies received by residents of other government jurisdictions, arises in a variety of contexts in public economics. For local governments, examples include the provision of public services with "spillover" benefits such as recreational facilities and cultural events; environmental policies such as water treatment and zoning; and even `fiscal' externalities such as the movement of capital among jurisdictions as a result of their tax policies, the basis of the "tax competition" literature. Examples at the state level include externalities created by tax and welfare policies. Recently, attention has been brought to the international environmental externalities, "global warming" and "acid rain," for example.

These externalities suggest grounds for intervention by another level of government. Actions that state governments, for example, could consider to influence the level of local government services include grants-in-aid [Wildasin (*1984; 1989*)]; mandates; the consolidation of local governments [Hoyt (*1991*)]; or tax subsidies. Both state and federal governments frequently employ these policies to influence policies at other levels of government.

In this paper rather than considering additional or alternative fiscal instruments that state governments can use to influence local policies, we consider the potential effects of the timing of the budget decisions made by state and local governments. In particular, we compare the policies found when state governments `precommit' to their policies with respect to local government policies, that is when state governments choose their policies first, to the traditional simultaneously determined policies of the Nash equilibrium. We show that, under very general conditions, the levels of state and local public services are different with precommitment than without precommitment. Moreover, under these same conditions precommitment improves social welfare.

While the original motivation for the institutional structure of state and local budget determination may not have been for the purposes of enabling state governments to precommit to their policies, current legislative practices and constitutional restrictions allow state governments to credibly precommit to their policies. One difficulty often encountered in leadership games is ensuring that a determinant leader emerges. This problem

does not arise if state governments choose to act as leaders. The United States Constitution specifically mentions state governments and provides a role for them, but does not do the same for local governments. The powers of local governments are strictly derivative, generally defined by state constitutions and statutes.[1] Hence, state governments can dictate not only the acceptable policies of local governments, but also timing of their budget decisions. The calendars for state budget processes suggests that state governments usually do, in fact, set their policies before local governments do. Most state constitutions require that the state budget be passed at least two or three months prior to the date when it becomes effective.[2] Municipal budgets must then be passed before the state budget becomes effective.[3] Further the heavy reliance of local governments on intergovernmental aid -- in *1989, 37.9* percent of all local revenue was from intergovernmental aid--means that in practice localities either delay making their budget plans until they have learned the magnitude of state aid, or alter budget plans when state aid is determined.[4]

Another difficulty in sustaining a setting with precommitment by a single player is the time-consistency of its policies. In this case, if the state government chooses its policies first and then localities choose their policies, does the state have an incentive to change its policies? If the answer is yes, then the state policies are not credible and will not influence local policies in the way the state had intended because local governments realize it is not in the interest of the state government to remain committed to its policies. While the time consistency of government policies has been the focus of a number of studies, we believe that it is not an issue in this context because it is very difficult and costly for a state government to alter its budget once it has been set.[5] Most state constitutions not only legally restrict the length of time their legislatures can meet, but also make reconvening of their legislatures very difficult. In 24 states the legislature cannot call a special session. In most of the remaining states two-thirds or more of the legislature must approve having a special session.[6]

Traditionally, policy determination has been examined using the Nash equilibrium concept in which all governmental units choose their policies simultaneously. A number of studies have demonstrated how one level of government (the state, for example) can, in the Nash equilibrium, eliminate any jurisdictional externalities and

increase social welfare by influencing the policies of lower level governments (local), through matching grants [e.g. Wildasin (*1989*)]. More precisely, the state government can obtain the same level of social welfare as if it controlled all the policy instruments of the local governments. While there are conditions when the state or federal government can "correct" inefficiencies arising from externalities at the local level, there may be instances in which limitations on the policy instruments they have makes it impossible to obtain the same level of social welfare as if they controlled local policies. When state governments lack the policy instruments to obtain an efficient outcome, we demonstrate that under certain conditions the state may increase social welfare by precommitting to some or all of their policies. Our results therefore complement those of Cubitt (1992), who provides conditions that are sufficient, though not necessary, to ensure that precommitment cannot increase social welfare. Instead, we provide conditions that are necessary and sufficient for precommitment to increase social welfare.

We believe that the conditions when precommitment can improve social welfare are not unusual or uncommon. After outlining these conditions, we present two examples of when precommitment improves social welfare based on the tax competition literature. The possibility that local government goods and services are underprovided as the result of tax competition among localities has been examined in a number of studies including Wilson *(1986)*, Wildasin *(1988)*, Zodrow and Mieszkowski *(1986)*, and Hoyt *(1991)*. Essentially the local governments ignore the external benefit of an increase in their tax rates, the flow of capital to other localities, and, therefore, they set inefficiently low tax rates. We discuss an example in which state governments can improve social welfare by precommitment when state governments are unable to use matching grants and another example when they can use a matching grant but are unable to tax the local governments' tax base. While we focus on examples based on tax competition, the advantages of precommitment are not limited to these fiscal externalities alone.

In addition to increasing social welfare, precommitment results in different prescriptions for policy intervention by state governments than those derived from a model of Nash competition. In our first example

in which the state government does not have the ability to use a matching grant, precommitment may result in either a higher or lower level of the state public service than in the Nash equilibrium depending on the relationship between the state and local public services. If the state and local public services are strong complements, precommitment by the state government to a higher level of the state public service than in the Nash equilibrium can increase the benefits of increasing local public services. However, if the state public service level has little or a negative effect on the demand for local public services, the state government can improve social welfare by reducing its public service level relative to the Nash equilibrium to increase the resources available to localities. The implications of precommitment are very different if the state government can use matching grants but has a different tax base than localities. In this case, if the state public service has no effect on the demand for the local public service, if the state social welfare increases if the state government precommits to both a higher tax rate and matching grant than in the Nash equilibrium. As both a predictor of state and local government policies and a setting for prescribing appropriate policies, precommitment may be a more useful concept than Nash equilibrium.

In the next section we derive general conditions under which precommitment is welfare improving. Section *3* presents a simple example of tax competition in which precommitment improves social welfare when the state government does not have the ability to use matching grants. In section *4* we present an example in which the state government has the use of matching grants but is unable to tax the local tax base and Section *5* concludes.

2. Precommitment and Policy Externalities

We wish to contrast the policy outcomes when states and local governments simultaneously choose policies, the Nash equilibrium, with the outcome obtained when the state is able to choose its policies prior to the local governments or "precommit". Our purpose is to derive conditions under when the state government can improve social welfare by precommitting to its policies. These conditions, we suggest, are not unusual as the examples in the following sections illustrate.

We begin by presenting formal definitions and comparing the social welfare maximizing outcome and the two equilibrium concepts, Nash equilibrium and precommitment. This comparison is necessary to determine when precommitment is likely to occur.

2.1 Social Welfare Maximization and the Equilibrium Concepts

Assume that a single state government has a set of $S = (S_1,...,S_T)$ of T independent policy instruments that include taxes, public service provision, regulations, and possibly matching grants and other aid to localities. The notion of independent policy instruments refers to the fact that each government budget constraint will determine the value of one policy given the values of the others. For example, a single tax and a single public service give one independent instrument.

Each of the n localities has K independent policy instruments that include local taxes, provision of local public services, and regulations. Let $L_i = (L_{i1},...,L_{iK})$ represent the set of independent instruments for locality i and $L = (L_1,...,L_n)$ be the vector consisting of all the localities' instruments. Given these state and local policies, the single resident of each locality i is assumed to choose his private consumption to maximize utility. As residents maximize utility subject to the state and local policies they face, we can express their utility as a function of the state and local policies with the utility level for resident i given by $V_i(L,S)$.

Each locality i is assumed to choose its instruments, L_i, to maximize its resident's indirect utility function $V_i(L,S)$. The state government chooses its policies to maximize the social welfare function $W(V_1(L,S),...,V_n(L,S))$ with social welfare increasing in each locality's utility, $\partial W/\partial V_i > 0$, $i = 1,...,n$ but at a decreasing rate, $\partial^2 W/\partial V_i^2 < 0$, $i = 1,...,n$.

As our benchmark, we consider the outcome if the state government could maximize social welfare by choosing both its own instruments, S, and the local instruments, L. Thus, for example, if the state government does not have the power to `tax' discriminate, setting different tax rates across localities, then being able to choose local tax rates gives them this ability. Formally, the social welfare maximizing policies, (L^W,S^W) solve

$$\underset{L,S}{\text{Maximize}} \quad W(V_1(L,S),\ldots,V_n(L,S)) \tag{2.1}$$

where $W(L^W,S^W) = W(V_1(L^W,S^W),\ldots,V_n(L^W,S^W))$ is the associated social welfare level. Following Cubitt (1992), we assume that *(2.1)* has a unique solution and no other allocation satisfies the first order conditions.

The traditional framework for analyzing state and local policy determination has used the Nash equilibrium concept where policies are set simultaneously. Formally in the Nash equilibrium (L^N,S^N) the state policies, S^N, solve

$$\begin{aligned} &\underset{S}{\text{Maximize}} \quad W(V_1(L_1^N,S),\ldots,V_n(L_n^N,S)), \\ &\text{and for each locality } i,\ L_i^N \text{ solves} \\ &\underset{L_i}{\text{Maximize}} \quad V_i(L_i,L_{-i}^N,S^N), \end{aligned} \tag{2.2}$$

where $L_{-i}^N = (L_1^N,\ldots,L_{i-1}^N,L_{i+1}^N,\ldots,L_n^N)$ and $W(L^N,S^N) = W(V_1(L^N,S^N),\ldots,V_n(L^N,S^N))$, the associated social welfare level. Again following Cubitt, we assume the Nash equilibrium of this game exists and is unique.

We wish to contrast the Nash equilibrium with the equilibrium that exists when the state government precommits to its policies. Conceptually we can imagine the state government picking a policy instrument, *S*, and then, given *S*, all local governments simultaneously choose their policies. For each mix of state policies, *S*, there is an associated set of local policies, $L^*(S)$, determined in the Nash equilibrium. The state government, knowing how local policies depend on its choice of policies, chooses *S* based on both its direct effect on welfare and how it influences the local policies.

We solve this leadership game by backward induction. First we consider the subgame between localities for any given set of state policies, *S*. We assume the Nash equilibrium of this subgame exists and is unique and locally stable.[7] The equilibrium policies of this subgame simultaneously satisfy the first order conditions for all localities, $\partial V_i/\partial L_{ik} = 0$, $i = 1,\ldots,n$ and $k = 1,\ldots,K$. We then solve the state government's problem, which is to choose its policies to maximize social welfare subject to the constraint that $L_i = L_i^*(S)$ for all i. That is, the state

government, when precommitting to a policy, explicitly considers the effect of its policy on the policies of the localities. As is well-known, solving the game using this approach gives the subgame perfect equilibrium. Formally, when the state precommits to its policies, (L^P,S^P), the state solves the problem:

$$\underset{S}{\text{Maximize}} \; W(V_1(L^*(S),S),\ldots,V_n(L^*(S),S)). \tag{2.3}$$

Again, we assume this optimization problem has a unique solution. To determine the subgame equilibrium, observe that the first order conditions for any given locality *i* implicitly define that locality's best policy reply. We denote locality *i*'s best reply by the *K*-dimensional vector-valued function, $\phi_i(L_{-i},S)$. That is, given any state policies and any set of policies for the other localities, $\phi_i(L_{-i},S)$ is the vector of policies for locality *i* that maximizes the utility of its resident. Then the subgame perfect equilibrium policies must satisfy $L_i^*(S) = \phi_i(L_{-i}^*(S),S)$ for all *i*. To examine the reply by locality *i* to changes in state policy instrument S_t we totally differentiate the first order condition for locality *i*. For a single local policy, L_{i1} (a tax on capital, for example), the first order condition is simply,

$$\frac{\partial V_i(L,S)}{\partial L_{i_1}} = 0. \tag{2.4}$$

Then differentiating (2.4) with respect to L_{i1} and S_t gives

$$\frac{\partial \phi_{i_1}}{\partial S_t} = -\frac{\partial^2 V_i/\partial L_{i_1}\partial S_t}{\partial^2 V_i/\partial L_{i_1}^2} \gtrless 0 \text{ as } \frac{\partial^2 V_i}{\partial L_{i_1}\partial S_t} \gtrless 0. \tag{2.5}$$

where the sign of (2.5) depends on the sign of the numerator because the denominator is negative by the second order sufficient condition. Thus if the marginal utility of an increase in local policy L_{i1} increases with an increase in state policy S_t, then in response to an increase in S_t locality *i*'s best response is to increase L_{i1}. For simplicity, we consider examples with only a single independent local policy.

While we express the equilibrium policies in terms of the best reply functions, in general, we can not determine the change in the equilibrium local policies from a change in a state policy even when we know the best reply for a locality. This is because changes in equilibrium local policies depend on how other locality's policies

change with a change in the state policies. In the case of identical localities, however, the sign of the change in the equilibrium local policies, $\partial L_i^*/\partial S_t$ depends only on the sign of $\partial \phi_i/\partial S_t$.[8]

2.2 Instruments

We wish to characterize the policy instruments of the state government according to its ability to use these instruments to influence private consumption and public service patterns to achieve social welfare goals. Let $X(L,S) = (X_1(L,S),G_1(L,S);...;X_n(L,S),G_n(L,S),G_s(L,S))$ represent a vector of private consumption (X_i) and public services (G_i) chosen in all of the n localities and the state public services (G_s) given state policies S. The state government is said to have *sufficient policy instruments* if:

$$X(L^N, S^N) = X(L^W, S^W)$$
$$\text{and consequently} \qquad (2.6)$$
$$W(L^N, S^N) = W(L^W, S^W)$$

However, the state is said to have *insufficient policy instruments* if:

$$X(L^N, S^N) \neq X(L^W, S^W)$$
$$\text{and consequently} \qquad (2.7)$$
$$W(L^N, S^N) < W(L^W, S^W)$$

To better understand these terms, consider the example we discuss in the next section. Each of the n localities provides its resident a single public service and the state provides a single public service to the residents of all localities. Local governments tax capital in their locality and the state taxes capital throughout the economy. While capital is inelastically supplied to the economy, each local government faces an elastic supply. Since local governments ignore the positive externality of a tax increase in their locality, the increase in capital in other localities, taxes and local public services will be inefficiently low.

In this example, if the state government does not value all localities equally in its social welfare function, it has *sufficient policy instruments* only if it can set different tax rates on capital and different matching grants for each locality. In this way, the state, as long as the local government has some positive level of taxation, entirely determines the consumption patterns of the localities.

If all localities are identical (as we assume in later analysis) and the state values all localities equally in its social welfare function, then the state has no need to tax localities differentially or set different matching grants. The state government will have sufficient policy instruments if it can use a uniform matching grant because social welfare maximization only requires a uniform level of local public service provision that exceeds the Nash equilibrium level [e.g. Wildasin (*1988*)]. If the state government does not have a matching grant as an instrument, then $W(L^N,S^N) < W(L^W,S^W)$ and it is said to have *insufficient policy instruments*.

2.3 Conditions for Precommitment

Our focus is on conditions when state governments can increase social welfare above the level achieved in the Nash equilibrium by precommitting to policy instruments. We present two propositions and a proof followed by discussion and interpretation.

Proposition 1. Necessary conditions for the state government to be able to improve welfare by precommitting to some policy instrument(s) are:

1) *The state has insufficient policy instruments or, equivalently,*

$$W(L^W, S^W) > W(L^N, S^N)$$

and

$$2) \quad \sum_{j \neq i} \frac{\partial W(L^N, \underline{S}^N)}{\partial V_j} \frac{\partial V_j(L^N, \underline{S}^N)}{\partial L_{i_k}} \neq 0, \tag{2.8}$$

for some policy instrument k and locality i.[9]

Proposition 1 is proved with *Proposition 2*. Part *1)* merely states that there is no gain to precommitting to a policy or set of policies if the Nash equilibrium maximizes social welfare. There are two distinct reasons why the Nash and social welfare maximizing outcomes may differ. One reason is that the utility levels for the different localities in the Nash equilibrium do not maximize social welfare. Thus, in our example, if the state government desires a higher utility level in locality *1* than the other localities but can only offer a uniform matching grant and not differently tax capital, the Nash equilibrium is not social welfare maximizing. The second reason that the

Nash equilibrium is not social welfare maximizing is because externalities from one locality's policies on other localities have not been eliminated by the state's instruments -- part *2)* summarizes this condition. The first problem, the relative weighing of utility, can not be alleviated by precommitment; the second problem, externalities, may be.

Proposition 2. A sufficient condition for precommitment to some policy or set of policies to improve social welfare is if there exists some state policy S_t such that:

$$\sum_{k=1}^{K}\sum_{i=1}^{n}\sum_{j\neq i}\frac{\partial W}{\partial V_j}\cdot\frac{\partial V_j}{\partial L_{i_k}}\cdot\frac{\partial L_{i_k}}{\partial S_t} \neq 0 \qquad (2.9)$$

when evaluated at the Nash equilibrium policies, (S^N,L^N).

Proof: If the state government precommits to policy S_t the effect on social welfare is:

$$\frac{dW}{dS_t} = \underbrace{\frac{\partial W}{\partial S_t}}_{(a)} + \underbrace{\sum_{k=1}^{K}\sum_{i=1}^{n}\frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial L_{i_k}}\cdot\frac{\partial L_{i_k}}{\partial S_t}}_{(b)} + \underbrace{\sum_{k=1}^{K}\sum_{i=1}^{n}\sum_{j\neq i}\frac{\partial W}{\partial V_j}\cdot\frac{\partial V_j}{\partial L_{i_k}}\cdot\frac{\partial}{}}_{(c)} \qquad (2.10)$$

At the Nash equilibrium policies, (S^N,L^N), the terms (a) and (b) of *(2.9)* are equal to zero by the first order conditions in Nash equilibrium. Then, if term (c) is nonzero, dW/dS_t does not equal zero at the Nash equilibrium and social welfare can be improved by precommitting and deviating from the Nash strategy.

This proposition complements, rather than contradicts, Cubitt's result that precommitment cannot improve social welfare for a class of restricted models. To replicate Cubitt's result in our model, assume that all agents (localities) are identical, and that there exists a function $\epsilon(L,S)$ such that utility of the resident of locality *i* can be written as $V_i = U_i(L_i, \epsilon(L,S))$ where the state always has some effect on the subutility function ϵ at the margin, $\partial\epsilon/\partial S_t \neq 0$. Then condition *(2.9)* becomes $(\partial V_j/\partial L_{ik})(\partial L_{ik}/\partial S_t) = 0$ (for some k, t, and i = j). Next note that, in the policy game without precommitment, the Nash equilibrium condition for the choice of a state policy S_t is $\partial W/\partial S_t = [\Sigma_{j=1}(\partial W/\partial U_j)((\partial U_j/\partial\epsilon)](\partial\epsilon/\partial S_t) = 0$. Because localities are identical and as Cubitt assumes that $\partial\epsilon/\partial S_t \neq 0$, this condition implies $\partial U_j/\partial\epsilon = 0$ for all *j* and as a consequence, $\partial V_i/\partial L_j = 0$, $j \neq i$, so *(2.9)* cannot hold and precommitment cannot improve social welfare.

Cubitt's restricted models have the property that the state's policy choices will eliminate any effect one

locality's policies have on the utility of resident of other localities. This happens because changes in policy by locality *i* and the state affect locality *j*'s utility in essentially the same way, through the function ϵ. Our analysis shows that precommitment can improve social welfare not only when localities are heterogeneous (as is clear from Cubitt's work), but also when they are identical if utility does not have the form $U_i(L_i, \epsilon(L,S))$ where the state always has some marginal effect on ϵ. Indeed, our result shows that when utility does not display this type of separability, then precommitment will improve social welfare if: a change in some policy *k* in some locality *i* at the Nash equilibrium will change the level of utility in some other locality *j* ($\partial V_j/\partial L_{ik} \neq 0$); the local policy *k* depends on some state policy S_t ($\partial L_{ik}/\partial S_t \neq 0$); and the total change in social welfare at Nash equilibrium from a change in S_t is nonzero. Essentially, precommitment may improve social welfare if the policy instruments of the state do not eliminate any externalities associated with a change in one locality's policies on utility in another locality.

3. Tax Competition and Precommitment

To demonstrate situations when a state government can increase social welfare by precommitting to its policies, we develop several examples based on the "tax competition" literature, [see Wilson *(1986)*]. First we present an example in which the state has *insufficient policy instruments* because it does not have the use of a matching grant to correct the policy externality associated with the local taxation of capital. In the next section we consider examples in which the state can offer a uniform matching grant.

3.1 The Model

We employ the same basic model used in Wildasin *(1988)* to examine tax competition. The primary difference in our model is the inclusion of a public service provided by the state government and consumed by residents of all localities. There are *n* localities, each inhabited by a single resident. To simplify the analysis and obtain definitive results we assume that all localities are identical, that is, all residents have identical tastes and localities have the same production functions. Each locality uses a variable input, capital, along with some fixed input, land or labor, to produce a single homogenous private good. Let the amount of capital used in locality *i*

be given by K_i. Then production in locality *i* is given by $F(K_i)$ with $F' > 0 > F''$. This private good can be consumed or used to produce either the local or state public service, where g_i the local public service level in locality *i* and g_s is the state public service level. The local and state public service are publicly produced private goods with one unit of the private good providing one unit per resident of either public service. Each locality *i* sets a tax on its capital at a rate τ_i to finance a public service g_i. The state government also sets a tax on capital in all localities at a rate τ_s to finance a public service g_s.

As capital is freely mobile, equilibrium requires that the rate of return on capital must be equal in all localities and that the demand for capital across localities equals the capital stock, or

$$F_i'(K_i) - \tau_i - \tau_s = \rho, \; i = 1,\ldots,n, \qquad \textbf{(3.1a)}$$

and

$$\sum_{i=1}^{n} K_i = \overline{K} \qquad (3.1b)$$

where ρ is the net rate of return on capital and K is the total stock of capital in the economy. Let $K = K/n$, the amount of capital in each locality in the symmetric equilibrium.

Equations *(3.1a)* and *(3.1b)* provides a system of $n + 1$ equations that determine the equilibrium allocation of capital and the net return on capital as functions of the tax rates in the localities and the state, $\underline{\tau} = (\tau_1,...,\tau_n,\tau_s)$. As usual, an increase in locality *i*'s tax rate decreases the amount of capital it uses ($\partial K_i/\partial \tau_i < 0$), increases the amount of capital used in all other localities ($\partial K_j/\partial \tau_i > 0$), and decreases the net return on capital ($\partial \rho/\partial \tau i < 0$). A change in the state's tax rate will not change the allocation of capital among localities ($\partial K_i/\partial \tau_s = 0$), although it will affect the net return on capital ($\partial \rho/\partial \tau_s = -1$).[10]

The income of the resident of locality *i* is composed of the local rents from the fixed factor of production there, $F(K_i) - (\rho+\tau_i+\tau_s)K_i$, and the return on his share of capital, $\rho K/n$. Private consumption is equal to income and is a function of tax rates through their effects on the return on and stock of capital in the jurisdiction.

3.2 Policy Choices without Precommitment

Each locality chooses its tax rate to maximize the utility of its resident, with the utility of the resident of locality *i* given by $V_i(\underline{\tau}) = U(x_i(\underline{\tau}),g_i(\underline{\tau}),g_s(\tau_s))$. Since we are considering the case in which all localities are identical, we assume the state government sets its tax rate to maximize the utility of a representative resident. We also assume governments compete in a Nash equilibrium in tax rates when they are unable to precommit.[11]

Each locality's government chooses its tax rate (and, implicitly, its public service level) to maximize its resident's utility given the tax rates set by the rest of the localities and the state government. For locality *i* the utility maximizing tax rate must satisfy

$$\frac{\partial V_i}{\partial \tau_i} = \frac{\partial U}{\partial x} \cdot \frac{\partial x}{\partial \tau_i} + \frac{\partial U}{\partial g} \cdot \frac{\partial g_i}{\partial \tau_i} = 0. \qquad (3.2)$$

Using the facts that $\partial x/\partial \tau_i = -K_i$ and $\partial g_i/\partial \tau_i = K_i + \tau_i(\partial K_i/\partial \tau_i)$ in *(3.2)* we find that the equilibrium local tax rate, τ_i^*,

satisfies

$$\frac{\partial V_i}{\partial \tau_i} = -K + MRS_i K\left[1 + \frac{\tau_1^*}{K}\frac{\partial K_i}{\partial \tau_i}\right] = 0 \quad or \quad MRS_i = \frac{1}{\left[1 + \frac{\tau_1^*}{K}\frac{\hat{c}}{\hat{c}}\right]} \qquad \textbf{(3.3)}$$

where $MRS_i = (\partial U/\partial g_i)/(\partial U/\partial x)$. Equation *(3.3)* is analogous to equation *(8.1)* in Wildasin (1988), the first order condition in a Nash equilibrium in the tax rate.

We assume, for now, that the state takes the localities' tax rates as given. The state tax rate that maximizes its social welfare function must satisfy:

$$\frac{\partial W(\tau)}{\partial \tau_s} = \frac{\partial U}{\partial x}\frac{\partial x}{\partial \tau_s} + \frac{\partial U}{\partial g_s}\frac{\partial g_s}{\partial \tau_s} = 0. \qquad \textbf{(3.4)}$$

Using the facts that $\partial x_i/\partial \tau_s = -K/n = -K$ and $\partial g_s/\partial \tau_s = K$ to simplify *(3.4)* gives $MRS_s = 1$, the first-best rule, where $MRS_s = (\partial U/\partial g_s)/(\partial U/\partial x)$.

Local governments underprovide their public services relative to the first-best outcome, $MRS_i > 1$, while the state government provides the efficient level of its public service. Local governments underprovide their public services because each locality's tax base is elastic, decreasing with increases in its tax rate. The state government's tax base is inelastic and the state tax, therefore, is lump-sum. In the first-best outcome, the public service levels are set so that $MRS_i = MRS_s = 1$.

3.3 Precommitment and Tax Leadership

In the Nash equilibrium, an increase in the tax rate in one locality will affect the utility in another with:

$$\frac{\partial V_j}{\partial \tau_i} = \frac{\partial V_j}{\partial g_j}\frac{\partial g_j}{\partial \tau_i} = \frac{\partial V_j}{\partial g_j}\tau_1^*\frac{\partial K_j}{\partial \tau_i} > 0, \; j \neq i. \qquad \textbf{(3.5)}$$

Because locality *i* ignores the fact that an increase in its tax rate increases the tax bases and, consequently, the public service levels in other localities, tax competition leads to an underprovision of local public services.

Equation *(3.5)* indicates that the necessary conditions for precommitment to be welfare improving exist. To determine if *Proposition 2*, the sufficient condition, is satisfied, we derive the best reply function for the local tax rate with respect to a change in the state tax rate, $\partial\phi_i/\partial\tau_s$. Recall from *(2.5)* that the sign $[\partial\phi_i/\partial\tau_s] = \text{sign}\,[\partial V_i^2/\partial\tau_i\partial\tau_s]$. Then differentiating the first order condition for locality *i*, *(3.3)*, gives

$$\frac{\partial^2 V_i}{\partial\tau_i\partial\tau_s} = \left[-\frac{\partial MRS_i}{\partial x_i} + \frac{\partial MRS_i}{\partial g_s}\right] K \frac{\partial U_i}{\partial x_i}\frac{\partial g_i}{\partial\tau_i}. \qquad \textbf{(3.6)}$$

The sign of $\partial\phi_i/\partial\tau_s$ depends on the sign of the bracketed term of *(3.6)*. This term reflects both the effect of the decrease in private consumption from an increase in τ_s on the marginal rate of substitution between the local public and the private good, $-(\partial MRS_i/\partial x)$, and the effect of an increase in the state public service on MRS_i, $(\partial MRS_i/\partial g_s)$. If an increase in g_s decreases MRS_i, the state and local public services are substitutes and $\partial\phi_i/\partial\tau_s$ is negative. If an increase in g_s increases MRS_i, the two goods are complements and $\partial\phi_i/\partial\tau_s$ could be positive.

In general we can not determine the change in the equilibrium tax rate from a change in the state tax rate even when we know the best reply for a locality. This is because changes in equilibrium local tax rates depend on how other locality's tax rates change with a change in the state tax rate. In the case of identical localities, however, the sign of the change in the equilibrium local tax rate, $\partial\tau_i^*/\partial\tau_s$, depends only on the sign of $\partial\phi_i/\partial\tau_s$.

The state, when it can precommit, will again choose it tax rate to maximize the utility of a representative resident. With precommitment, τ_s^* must satisfy

$$\frac{dV_i}{d\tau_s} = \underbrace{\left[\frac{\partial U}{\partial x}\frac{\partial x}{\partial\tau_s} + \frac{\partial U}{\partial g_s}\frac{\partial g_s}{\partial\tau_s}\right]}_{(a)} + \underbrace{\sum_{j=1}^{n}\left[\frac{\partial U}{\partial x}\frac{\partial x_i}{\partial\tau_j} + \frac{\partial U}{\partial g_i}\frac{\partial g_i}{\partial\tau_j}\right]\frac{\partial\tau_j^*}{\partial\tau_s}}_{(b)} = \qquad \textbf{(3.7)}$$

Term (a) is the direct effect of the increase in the state's tax rate on the utility of the resident, the

increase in the level of the state public service and the decrease in private consumption from the increased state taxes. Term (b) is the effect due to changes in the localities' tax rates. As the bracketed term in (b) is positive, the sign of (b) depends upon the sign of $\partial\tau^*_j/\partial\tau_s$. The direct effect was the only way the state government could affect utility in the Nash game; when it precommits it affects utility by altering local tax rates. Simplifying *(3.7)* gives[12]

$$MRS_s = 1 + \left(1 - \frac{1}{n}\right) MRS_l \tau^*_l \left(\frac{\partial K_i}{\partial \tau_j}\right)\left(\frac{\partial \tau^*_l}{\partial \tau_s}\right) \quad \begin{array}{l} < 1 \text{ if } \dfrac{\partial \tau^*_l}{\partial \tau_s} > 0 \\ > 1 \text{ if } \dfrac{\partial \tau^*_l}{\partial \tau_s} < 0 \end{array} \qquad \textbf{(3.8)}$$

If an increase in the state public service increases the provision of the local public service and if the state is able to precommit, then it will set a higher tax rate and provide a higher service level than if it is unable to precommit. If increases in the state public service level decrease the provision of the local public service, then the state government will set a lower tax rate and service level when it precommits.

Regardless of whether increases in the state tax rate decrease or increase local taxes, when the state can precommit it will choose a tax policy that increases local public services above the level when it does not precommit. This is consistent with the finding that in the Nash equilibrium the local public service is underprovided. By increasing the underprovided local public service, the state government can increase utility when it precommits. Utility is increased by this precommitment regardless of whether precommitment reduces or increases state government expenditures.

Table 1 presents the results of a numerical example that compares the Nash equilibrium policies with those found when the state precommits to its tax rate. A Cobb-Douglas utility function is used with the parameters chosen to reflect the 1990 U.S. average for local expenditures of $2123 and state expenditures of $2111 per capita.[13] The example is also chosen to approximate a 33% tax on the annual returns to property that Wildasin (1989) used. The elasticity of demand for capital is set at -1.00.[14]

4. Precommitment and Matching Grants

In the simple model discussed in the preceding section, if the state can set matching grants, a first-best allocation can be achieved. The matching rate is set equal to the external benefits received by other localities when a locality raises its tax rate. In this way the externality, the flow of capital, is internalized to the locality. With identical localities and a single tax base, a matching grant gives the state *sufficient policy instruments*. The state, then, has no incentive to attempt to precommit, since the matching grant gives the state the ability to determine the local service level as well as the state public service in a simultaneous move game. With a single tax base, matching grants and identical localities, the state can effectively set the tax rate and the levels of both goods, thereby determining all relevant policy variables. In this example the failure of precommitment to improve social welfare is obtained with a utility function that does not impose the separability assumptions used by Cubitt.[15]

For precommitment to be advantageous to the state, it must face limits on the policies it can use. One reasonable example of a policy limitation that gives the state the incentive to precommit is the inability to use the same tax base as the localities. In many states, the state government applies no taxes to property, allowing local governments exclusive rights to use property values as a source of revenue. States also often prohibit or severely restricting the use of sales or income taxes by local governments.

In this section we present a simple example in which the state has the ability to determine both public services levels by using a matching grant. The state, however, is restricted to taxing only labor and gives localities the exclusive right to tax capital (property). While the state can ensure that tax revenue is allocated efficiently between the two public services, it can not ensure that taxes are collected efficiently, that is, the taxation of labor relative to capital is optimal.

4.1 The Model

We now assume there are two inputs in production, capital (K) and labor (L), with production of the private commodity in locality i given by $x_i = F(K_i,L_i)$ where K_i and L_i are the amounts of capital and labor employed in locality i and it is assumed that $F_{iKL} = \partial^2 F_i/\partial K_i \partial L_i = 0$[16]. While capital is inelastically supplied to

the economy, the supply of labor is not inelastic as it is determined by the utility maximizing choices of residents. Each resident *i* has an endowment of human capital $\theta_i\Lambda$ to be used as either labor, λ_i, or leisure, l_i, and making the total labor supply in the economy, $L = \Sigma\lambda_i$. Labor, like capital, is mobile across localities. As in the preceding section the private commodity can be consumed or used to produce either the local public service (g_l) or the state public service (g_s) with one unit of x_i producing one unit per resident of either g_i or g_s. Utility in this case is given by the separable utility function $V_i(\underline{\tau}) = x_i(\tau) + U^l(l_i(\tau)) + U^{gl}(g_i(\underline{\tau})) + U^{gs}(g_s(\underline{\tau}))$.[17] Consumer utility maximization gives the labor supply function, $\lambda_i(w)$, and the total labor supply in the economy equal to $L(w) = \Sigma\lambda_i(w)$, where w is the wage rate. As there is no income effect on labor supply, $\partial L/\partial w > 0$. Private consumption for *i*, x_i, equals the sum of rent in locality *i*, $(F(K_i,L_i) - (1+\tau_i)K_i - (1+\tau_s)L_i)$, income from i's endowment of capital, $\rho\mu_i K$, and labor earnings, $w\lambda_i$, where τ_i and τ_s are taxes by locality *i* on capital and the state on labor supply.

Local governments tax capital and receive a matching grant from the state government *(m)* making the budget constraint for locality *i* equal to $(1+m)\tau_i K_i = g_i$. The state government budget constraint is $m\Sigma\tau_i K_i + ng_s = \tau_s L(w)$.

In addition to equilibrium conditions for the capital market found in section 3 the market for labor must also clear,

$$\sum_{i=1}^{n} L_i(w) = L(w), \qquad \textbf{(4.1a)}$$

and

$$F_L^i - \tau_s = w, \; i = 1,\ldots,n \qquad \textbf{(4.1b)}$$

Expression *(4.1a)* simply states that the demand for capital in the economy must equal the total supply and *(4.1b)* states that marginal product of labor equals the gross-of-tax wage.

The effects of changes in the local tax rate on the capital market are identical to those found in section *3*, with the exception of $\partial g_i/\partial\tau_i$, which increases by a factor of $(1+m)$.[18] Because the state's tax base is elastic, the state tax will affect its base $(\partial L/\partial\tau_s < 0)$ and the wage rate $(-1 < \partial w/\partial\tau_s < 0)$.[19]

4.2 Nash Equilibrium

To simplify the analysis we assume that localities are small enough to ignore any effects their policies may have on the wage rate, the return on capital, or the provision of the state's public service. Since each locality chooses its tax rate to maximize the utility of its resident the first order condition for the locality is

$$\frac{\partial V_i}{\partial \tau_i} = \frac{\partial U_i}{\partial x_i}\left[-K_i + MRS_i \frac{\partial g_i}{\partial \tau_i}\right] = 0, \qquad \textbf{(4.2)}$$

where MRS_i denotes $(\partial U_i/\partial g_i)/(\partial U_i/\partial x_i)$. The tax increase in locality i, by reducing rent by $-K_i$, reduces x_i by K_i.

The state government has three policy variables, its tax rate, the public service level, and the matching grant. We assume that it chooses its tax rate, τ_s, and matching grant, m, and lets the public service level be determined by the budget constraint. The first order condition for the tax rate is

$$\frac{\partial V_i}{\partial \tau_s} = \frac{\partial V_i}{\partial x_i}\left[-L_i + MRS_s \frac{\partial g_s}{\partial \tau_s}\right] = 0 \qquad \textbf{(4.3a)}$$

where MRS_s denotes $(\partial U_i/\partial g_s)/(\partial U_i/x_i)$.[20] The first order condition for the matching grant is

$$\frac{\partial V_i}{\partial m} = \frac{\partial V_i}{\partial x_i}\left[MRS_i \frac{\partial g_i}{\partial m} + MRS_s \frac{\partial g_s}{\partial m}\right] = \frac{\partial V_i}{\partial x_i}\left[MRS_i - MRS_s\right]\tau_i K_i = 0 \qquad \textbf{(4.3b)}$$

The state sets it matching grant so that the marginal rates of substitution are equated for the public services. While a first best allocation is obtained for the distribution of revenue between services, a first best allocation would also have all revenue collected on the inelastic supply of capital, but the state government does not have the ability to tax this base.

4.3 Precommitment

As before, we wish to determine the changes in the best reply functions, $\partial\phi/\partial\tau_s$ and $\partial\phi/\partial m$, for the localities; that is, how they will adjust their tax rates when the state government changes its tax rate and now also its match. Recall that sign $[\partial\phi/\partial\tau_s]$ = sign $[\partial^2{}_i V/\partial\tau_i\partial\tau_s]$ and in the symmetric equilibrium, sign $[\partial\tau_i^*/\partial\tau_s]$ = sign $[\partial\phi/\partial\tau_s]$. Analogously, for the matching grant sign $[\partial\phi/\partial m]$ = sign $[\partial^2 V_i/\partial\tau_i\partial m]$ and in the symmetric

equilibrium, sign $[\partial \tau_i^*/\partial m]$ = sign $[\partial \phi/\partial m]$.

Differentiating the locality's first order condition, (*4.2*), with respect to τ_s gives

$$\frac{\partial^2 V_i}{\partial \tau_i \partial \tau_s} = \frac{\partial V_i}{\partial x_i}\left[\frac{\partial MRS_i}{\partial x_i}\frac{\partial x_i}{\partial \tau_s}\right]\frac{\partial g_i}{\partial \tau_i} = 0. \qquad \textbf{(4.4a)}$$

Given that $\partial U_i/\partial x_i$ is a constant, even with a change in x_i there is no change in MRS_i as g_i is unaffected by τ_s.

Differentiating *(4.2)* with respect to *m*, gives the response to an increase in the matching grant,

$$\frac{\partial^2 V_i}{\partial \tau_i \partial m} = \frac{\partial V_i}{\partial x_i}\left[MRS_i\left[K_i + \tau_i \frac{\partial K_i}{\partial \tau_i}\right] + \frac{\partial MRS_i}{\partial g_i}\frac{\partial g_i}{\partial m}\frac{\partial g_i}{\partial \tau_i}\right]. \qquad \textbf{(4.4b)}$$

The first term in (*4.4b*) is the direct effect of the increase in the match and is positive and the second term is negative, making the sign of *(4.4b)* indeterminant. While indeterminant, it seems reasonable to expect that the direct effect may dominate and the local tax rate will increase with matching grant.

To determine if the state should precommit we evaluate the first order conditions for the state when it precommits at the Nash equilibrium policies. As *(4.4a)* suggests there will be no response to precommitting with the tax rate, we evaluate the first order condition for the matching grant,

$$\frac{dV_i}{dm} = \frac{\partial V_i}{\partial m} + \frac{\partial V_i}{\partial \tau_i}\frac{\partial \tau_1^*}{\partial m} + \sum_{j \neq i}\frac{\partial V_i}{\partial \tau_j}\frac{\partial \tau_1^*}{\partial m} \qquad \textbf{(4.5)}$$

The first two terms of the right side of (*4.5*) are equal to zero in the Nash equilibrium and in the symmetric equilibrium the third term is equal to[21]

$$(n-1)\frac{\partial V_i}{\partial \tau_j} = (n-1)\frac{\partial U_i}{\partial x_i}\left[MRS_i\frac{\partial g_i}{\partial \tau_j} + MRS_s\frac{\partial g_s}{\partial \tau_j}\right]\frac{\partial \tau_j}{\partial \tau_s} = (n-1)(1+m)\frac{\partial U_i}{\partial x_i}MR \qquad \textbf{(4.6)}$$

The first term of (*4.6*) is a positive externality, the increase in the tax base in locality *i* from an increase in the other localities' tax rates. This externality should induce the state to precommit in a way to increase local taxes. The second term is an additional externality imposed by local taxes, the reduction in state public services due to the use of a matching grant. This is a negative externality and should induce the state to attempt to lower local tax rates. The sign of $\partial V_i/\partial \tau_j$, then, is indeterminant and will depend on the elasticity of the local tax base ($\partial K_i/\partial \tau_j$) and the size of the matching grant. A similar result is discuss in Hoyt *(1992)*.

In general, *(4.6)* will not equal zero so the state has an incentive to precommit to its matching grant and deviate from the Nash equilibrium.

As in the preceding section, we again solve a numerical example. As *Table 2* shows, the differences in policies can be dramatic particularly with only a few localities. In fact with only a few localities the matching grants become negative and the state income (labor) tax is approximately equal to zero with precommitment. For two localities the welfare gain from precommitment is $325, almost ten percent of local expenditures. As the number of localities increase, the changes in local policy and the benefits from precommitment decrease, but sizeable differences in state policies remain.

5. Conclusion

We have argued that institutional characteristics of state and local budget determination suggest that state governments do generally determine their budgets before local government budgets are determined. Legal restrictions and costs also make it difficult for state governments to change their budgets after local budgets are determined. These institutional features of the budget process provide state governments with the opportunity to precommit to its policies with respect to the local governments.

In this paper we demonstrate that in addition to having the ability to precommit to its policies, state governments have compelling reasons to do so. We have derived the conditions necessary for precommitment by state governments to their policies to improve social welfare relative to the level achieved in the Nash equilibrium. Essentially the conditions are simply that the state government does not have enough policies to eliminate all externalities generated by the policies of localities and the state government must have some policy that will influence the local policies that generate the externalities.

Our examples from the tax competition literature suggest that how precommitment changes the government polices from its Nash policies depends on the relationships between the state and local government services and the policy instruments available. When local public services are underprovided due to tax competition, the state government may precommit to a higher or lower tax rate than the Nash rate in

attempt to increase local services depending on whether the state and public goods are complements and the tax bases of the two levels of government. Our examples also suggest that how the state government's policies differ when it precommits depends critically on the policy instruments available to the state and local governments, making general conclusions about the effects of precommitment difficult.

While our examples are from the literature on tax competition, the gains to precommitment apply to any locally-generated policy generated externalities or externalities generated by individuals. Precommitment can also apply to interactions between the federal and state governments as well.

References

Advisory Council on Intergovernmental Relations, 1991, *Significant Features of Fiscal Federalism, 1991*, volumes 1 and 2, Washington, DC.

_____. 1992, *Significant Features of Fiscal Federalism, 1992*, volume 2, Washington, DC.

Barro, R.J. and D.B. Gordon, 1983, "A Positive Theory of Monetary Policy with Incomplete Information," *Journal of Political Economy*, 91, 589-610.

Blackburn, K. and M. Christensen, 1989, "Monetary Policy and Policy Credibility: Theories and Evidence," *Journal of Economic Literature*, 27, 1-45.

Bucovetsky, S., 1989, "Asymmetric Tax Competition," *Journal of Urban Economics*, forthcoming.

_____ and J. D. Wilson. 1991, "Tax Competition with Two Tax Instruments, *Regional Science and Urban Economics*, 21, 333-350.

Chari, V.V., P. J. Kehoe, and E. C. Prescott, 1989, "Time Inconsistency and Policy," in: R.J. Barro (ed.), *Modern Business Cycle Theory* (Harvard, Cambridge) 265-305.

Council of State Governments, 1988, *Book of States, 1988-1989*, Lexington, KY.

Cubitt, R. P., 1992, "Economic Policy Precommitment and Social Welfare," *Journal of Public Economics*, 49, 191-201.

Fisher, R., 1988, *State and Local Public Finance* (Scott, Foresman, and Co., New York).

Hoyt, W. H., 1991, "Property Taxation, Nash Equilibrium, and Market Power," *Journal of Urban Economics*, 30, 123 - 131.

Jensen, R. A. and E. F. Toma, 1991, "A Model of Debt Competition," *Regional Science and Urban Economics*, 21, 371-392.

Martin, D. L., 1982, *Running City Hall: Municipal Administration in America* (University of Alabama Press, University, AL).

Rogers, C. A., 1987, "Expenditure Taxes, Income Taxes, and Time-Inconsistency, *Journal of Public Economics*, 32, 215-230.

Tesfatsion, L., 1986, "Time Inconsistency of Benevolent Government Economies," *Journal of Public Economics*, 31, 25-52.

Wildasin, D. E., 1984, "The Welfare Effects of Intergovernmental Grants in an Economy with Distortionary Local Taxes," *Journal of Public Economics*, 25, 103-125.

_____., 1988, "Nash Equilibria in Models of Fiscal Competition," *Journal of Public Economics*, 35, 229-240.

_____. 1989, "Interjurisdictional Capital Mobility: Fiscal Externality and a Corrective Subsidy, *Journal of Urban Economics*, 25, 193-212.

Wilson, J. D., 1986, "A Theory of Inter-regional Tax Competition," *Journal of Urban Economics*, 19, 296-315.

Zodrow, G. R. and P. Mieszkowski, 1986, "Pigou, Tiebout, Property Taxation, and the Underprovision of Local Public Goods," *Journal of Urban Economics*, 19, 356-370.

Notes

1. State government supremacy over local governments was established in Dillon's Rule, the 1868 Iowa State Supreme Court opinion of John M. Dillon who wrote that municipalities were "the mere tenants at the will of the legislature (John F. Dillon, *Commentaries on the Law of Municipal Corporations*, 1911, Boston: J:Crockfott, 5th ed., vol. 1, p. 448). See David L. Martin *Running City Hall: Municipal Administration in America*, 1982, University of Alabama Press, University, AL, for further discussion of Dillon's Rule and its implications for state-municipal government relations.

2. From *Significant Features of Fiscal Federalism*, 1992, volume 1, Table 2, pages 4-5.

3. Failure to comply with this deadline can result in legal recourse by the state. For example, recently the commissioners of Breathitt County in Kentucky were jailed by a state judge for failing to provide a balanced budget in a timely fashion, and not released until they were willing to do so (*Lexington Herald-Leader*, page 1, August 1, 1992).

4. From *Significant Features of Fiscal Federalism, 1991*, volume 2, *Tables 73* and *74*, pages 136 - 137. See Ronald Fisher (1988) Chapter 20 on the discussion of the budget process.

5. See as examples V.V. Chari et. al. (*1989*); Tesfatsion (*1986*); and Rogers (*1987*) as examples. Blackburn and Christensen (*1989*) for a review of the literature on policy credibility and monetary policy.

6. From *The Book of States, 1988-1989*, Table 3.2, pages 85-87. While governors face fewer restrictions on calling special sessions, a number of state constitutions explicitly state that the governor may only call special sessions in the event of grave crises or for the purpose of addressing specific issues. A typical example of a state constitution is the Constitution of the State of Arkansas (1874), Article VI, Section 19:

> Sec. 19. Extraordinary sessions of general assembly--Calling--Purposes.--The Governor may, by proclamation, on extraordinary occasions convene the General Assembly at the seat of government... and he shall specify in his proclamation the purpose for which they are convened, and no other business than that set forth therein shall be transacted until the same shall have been disposed of...

7. We mean that this equilibrium is unique in the standard sense that, if there is a small perturbation from the equilibrium, and if localities use their best replies in sequence, then the policies return to their equilibrium

values.

8. For example, when all localities are identical, and the state and local governments each set a single tax on capital and provide a single public service, the change in the equilibrium tax rate in locality *i* from a change in the state tax rate can be expressed as

$$\frac{\partial \tau_i^*}{\partial \tau_s} = \left[1 + (n-1)\frac{\partial \Phi_i}{\partial \tau_j}\right]\frac{\partial \Phi_j}{\partial \tau_s}$$

where there are n -1 other identical localities represented by locality *j*. For a stable equilibrium to exist the bracketed term must positive, that is, the effects of changes in other localities taxes must not dominate the changes in its own tax rate.

9. These conditions alone are not the necessary conditions. It must also be the case that the Nash equilibrium allocation does not satisfy the first order conditions for the welfare maximizing policy, that is, it cannot be a local maximum.

10. With identical policies in all localities, we have

$$\frac{\partial \rho}{\partial \tau_i} = -\frac{1}{n}, \quad \frac{\partial K_i}{\partial \tau_i} = \left(1-\frac{1}{n}\right)\frac{1}{F''} < 0, \text{ and } \frac{\partial K_j}{\partial \tau_i} = -\frac{1}{n}\frac{1}{F''} > 0, \; j \neq i.$$

11. Alternatively we could consider a Nash equilibrium in the government service. However, since our interest is in policy coordination between the state and local government rather than the type of strategy chosen by localities, we focus on the simpler case of a Nash equilibrium in the taxes. Qualitatively our results are identical regardless of the strategy chosen by the localities. Only for the strategy of the local governments does it matter if the strategy are in taxes or the public service. Since the state government's policies have no effect on the aggregate tax base or the tax base of the localities, they will set the same level of public service regardless of their strategy.

12. By the envelope theorem we have

$$\frac{\partial U_i}{\partial x_i}\frac{\partial x_i}{\partial \tau_i} + \frac{\partial U_i}{\partial g_i}\frac{\partial g_i}{\partial \tau_i} = 0.$$

This leaves the effects of the remaining n - 1 localities to consider or (1-1/n) of the market.

13. See *Significant Features of Fiscal Federalism, 1991*, volume 2, Table 42, page 82.

14. Alternative elasticities of demand of capital of -0.50 and -2.00 were also tried with little difference in results.

15. More precisely, in Cubitt's model utility (welfare) is given by $g^i(s_i,\epsilon(s_g,\theta))$ where s_i is agent i's strategy, s_g is the government's policies, and $\theta = \Sigma_{j=1} s_j$. Assume that the number of agents is large enough so that agent i treats $\partial\epsilon/\partial s_i = 0$. Then if $g^i(s_i^*,\epsilon(s_g^*,\Sigma_{j=1}s_j^*)) = g^i(s_i^*,\epsilon(s_g',\Sigma_{j=1}s_j'))$ it follows that

$$\frac{\partial g^i}{\partial s_i}(s_i^*, s_g^*, s_{-i}^*) = \frac{\partial g^i}{\partial s_i}(s_i^*, s_g', s_{-i}').$$

In the model described in section 3 with a matching grant we have

$$\frac{\partial g^i}{\partial s_i} = \left[-\frac{\partial U_i}{\partial x_i}k_i + \frac{\partial U_i}{\partial g_i}(1+m)\left(k_i + \frac{\tau_i}{F''}\right)\right],$$

where *m* is the matching rate. The capital stock in locality *i* (k_i), the level of private consumption (x_i), the local public service level (g_i), the state public service (g_s) and the matching grant (m) all are likely to differ between (s_i^*,s_g^*,s_{-i}^*) and (s_i^*,s_g',s_{-i}'). Then as $\partial U_i/\partial x_i$ and $\partial U_i/\partial g_i$ depend on the mix of (x_i, g_i, g_s, m) then $\partial g^i/\partial s_i = \partial U_i/\partial \tau_i$ is not likely to be the same for the two combinations.

16. The assumption that $F_{LK} = 0$ is done to simplify the determination of the responses of capital and labor to changes in tax rates and does not affect the qualitative nature of our results.

17. Separability of the utility function means that here, unlike in the preceding section, the policy chosen by the state when it precommits does not depend on whether the state and local public services are complements or substitutes. The constant marginal utility of *x* means that labor supply is independent of income.

18. Of course since F(K,L) has two arguments F' and F" should now be interpreted as $F_K = \partial F/\partial K$ and $F_{KK} = \partial^2 F/\partial K^2$.

19. Totally differentiating *(4.1)* gives

$$\frac{\partial w}{\partial \tau_s} = \frac{-L'}{(1-\frac{1}{n}F_{«}L')} < 0, \quad \frac{\partial L}{\partial \tau_s} = L'\frac{\partial w}{\partial \tau_s} < 0, \text{ and } \partial\frac{L_i}{\partial \tau_s} = \frac{1}{n}L'\frac{\partial w}{\partial \tau_s} < 0.$$

Note that the assumption that $F_{LK} = 0$ and the fact that λ_i depends only on w imply that $\partial K_i/\partial \tau_s = 0$ and $\partial L_i/\partial \tau_i = 0$.

20. Totally differentiating the utility function with respect to τ_s gives

$$\frac{dU_i}{d\tau_s} = \frac{\partial U_i}{\partial x_i}\left[F_L^i \frac{\partial L^i}{\partial \tau_s} - (w+\tau_s)\frac{\partial L^i}{\partial \tau_s} - L^i\left(\frac{\partial w}{\partial \tau_s}+1\right) + \lambda_i\left(\frac{\partial w}{\partial \tau_s}\right) + w\frac{\partial \lambda}{\partial \tau_s}\right] + \frac{\partial U_i}{\partial l}\frac{\partial l_i}{\partial \tau_s} + \frac{\partial U_i}{\partial g_s}\frac{dg_s}{d\tau_s}$$

Using the facts that $F_L = (w + \tau_s)$ by *(4.1b)*; that $L_i = \lambda_i$ in a symmetric equilibrium; that $d\lambda_i = -dl_i$; and that by the consumer's first order condition $\partial U_i/\partial l_i = w(\partial U_i/\partial x_i)$ gives *(4.3a)*.

21. We obtain *(4.6)* by using the fact that $\partial g_i/\partial \tau_j = \tau_i(\partial K_i/\partial \tau_j) = -(1+m)(1/n)(1/F_{KK})$ in a symmetric equilibrium; $\partial g_s/\partial \tau_j = -(1/n)(1+m)K_j$; and in the Nash equilibrium $MRS_i = MRS_s$.