Efficiency in Economies with Jurisdictions and Public Projects [1]

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Abstract

Traditionally in the literature on local public goods it is assumed that each local public good is a selection from a convex space. In this paper existence is shown for a class of finite models where local public goods are selections from abstract, possibly non-convex, commodity spaces. These equilibria are shown to contain the core.

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1. Introduction

Tiebout (1956) theorized that, with a sufficiently large number of jurisdictions, migration would lead to near efficient provision of local public goods. In modelling local public good economies, Tiebout imposed no restrictions on the nature of the local public goods provided except that consumption by any one consumer does not diminish the quantity of any local public good available for any other consumer and local public goods are not excludable within the region. Any consumer who resides in the region in which a local public good is provided may consume the public good. Since Tiebout, a number of papers have clarified when Tiebout equilibria exist and when Tiebout equilibria are Pareto optimal. However, it is traditional in the literature that each local public good may easily be quantified by picking a number on the real line and that each local public good is infinitely divisible. This is true of Bewley (1981), Ellickson (1973) and Wooders (1978, 1980, 1989). It is clear that not all public goods may be infinitely divisible or characterized by picking a number on the real line (for example, building a bridge over a river or the exploration of outer space). For instance, Edwards (1992) shows how changes in income change consumer desires for collective consumption goods that are indivisible.

In Mas-Colell (1980), Mas-Colell introduced the concept of a valuation function to allow Pareto optimal allocations of pure public goods to be implemented where the pure public goods are elements of an abstract (possibly non-convex) commodity space. An element of such a commodity space is called a public project. A valuation function assigns, for each element of the commodity space, an individual value for each consumer. This valuation is interpreted as the amount each consumer must pay to consume that public project. In this paper it is shown that first best allocations of local public projects may be supported with a valuation system. The form of valuation system required depends on how each consumers preferences change as any consumer migrates. Further, in this paper it is shown that if the concept of valuation equilibrium is restricted in such a way that the valuation of each public project must be non-negative then the set of "cost share" valuation equilibrium is equivalent to the core.

In Section 2 I describe the class of models used to demonstrate the Propositions to follow. An allocation consists of an assignment of a local public project to each region, money to each consumer and the assignment of consumers to regions. Each assignment of local public projects to regions is an element of an abstract (possibly non-convex) commodity space. An allocation is first best if it is Pareto optimal. Three candidate dual spaces are considered. Each dual space is an analogue of a dual space to be found in Manning (1993) and Wooders (1989) in models with infinitely divisible local public goods. Production in this paper and Mas-Colell (1980) may be viewed as performed by a Social Planner. Unlike Diamantaras and Gilles (1993), all consumers may affect production opportunities. Production of the local public project may therefore be interpreted as involving the services of all consumers in that jurisdiction.

The class of models with local public projects I introduce allows for any form of congestion in consumption and production.

In Section 3 a First Welfare Theorem and three Existence Theorems are proved for classes of models with complete personalized, personalized and non-personalized valuation systems. A complete personalized valuation system is a valuation system such that any consumer's valuation may change as he (she) or any other consumer migrates. In the first Existence Theorem, non-anonymous crowding in consumption and production may occur. It is shown that any Pareto optimal allocation may be supported as a complete personalized valuation system. In the second Existence Theorem, the crowding in consumption is restricted. Consumer preferences over money and public projects do not change as any consumer migrates. As a consequence, it is shown that the dual space may be constrained to the set of personalized valuation systems. A personalized valuation system is a valuation system such that any consumer's valuation of a local public project may differ from that of any other consumer but will not change as he (she) or any other consumer migrates. In the third Existence Theorem consumer preferences are further restricted to be globally the same. Therefore, the dual space may be constrained to the set of non-personalized valuation systems. A non-personalized valuation system is a valuation system such that every consumer's valuation of each local public project is the same.

Unlike models with infinitely divisible public goods and linear Lindahl prices, residence taxes are never needed. Valuations may be chosen such that residence taxes are implicit.

All proofs appear in an Appendix.

2. Model

In this section I describe the class of models I will use to investigate the existence of valuation equilibrium.

2.1 Regions, Public and Private Goods

Each model consists of a Social Planner and a finite number of distinct regions, $J \equiv$

$\{1, \ldots, J\}$. Each region is indexed by $j \in J$.

Associated with each region j is a nonempty space K_j of projects. Let there be a status quo project, $0 \in K_j$, in each region j. Generically, $y_j \in K_j$, where y_j is a public project in region j. In addition there is a unique private good, which can be thought of as "money," denoted by $m \in I\!R$.

2.2 Consumers

Each consumer is identified by an index, $i \in I \equiv \{1, \ldots, I\}$. Each consumer is said to be of different *type* if any one of the following two characteristics differ: his (her) preferences or endowments.

Consumers may choose to reside or not to reside in a region. Consumer residence choice is indicated by a partition of I, $\mathcal{S}$, where $\#\mathcal{S} = J$. The partition may be denoted by $\mathcal{S} \equiv \{\{R_1\}, \ldots, \{R_J\}\}$. The set of all such partitions is denoted $\mathbf{S}$. Constrain attention to the subset $C \subset I$ of consumers. The constrained partition is denoted by $\mathcal{S}_C \equiv \{\{R_1 \cap C\}, \ldots, \{R_J \cap C\}\}$. The set of all such constrained partitions is denoted by $\mathbf{S}_C$. The Social Planner is assumed to be able to identify each consumer's index, type and region of residence.

Define $\underline{\mathrm{y}}_j \equiv (0, \ldots, y_j, \ldots, 0) \in \prod_{j \in J} K_j$ and $\underline{\mathrm{R}}_j \equiv (\emptyset, \ldots, R_j, \ldots, \emptyset) \subset \prod_{j \in \{1, \ldots, j-1\}} \emptyset \times I \times \prod_{j \in \{j+1, \ldots, J\}} \emptyset$. The consumption vector of consumer i when residing in region j is

$$x^i \equiv (\underline{\mathrm{y}}_j,\ m^i,\ \underline{\mathrm{R}}_j)$$

where m^i is the money consumed by i.

The aggregate allocation is $x \equiv (y,\ m,\ \mathcal{S})$, where $y = \{y_j\}_{j \in J}$ and $m = \sum_{i \in I} m^i$.

Each consumer endowed with a non-negative amount of money w^i.

Each consumer's utility is a function of his (her) consumption of public projects, money and the composition of the population of the region in which he (she) resides. If consumer i resides in region j then the presence of consumer i provides a service (disservice) to all residents in region j. Further, the presence of each resident of region j provides a service (disservice) to consumer i. Therefore, the commodity space of each consumer is taken to include the residence choice of all consumers. Preferences for each consumer i are represented by the complete preordering $\succeq^i$ over $\prod_{j\in J} K_j \times I\!R_+ \times \mathbf{S}$.

2.3 Production

Production is not decentralized. Production of public projects is performed by the Social Planner. Let $K \equiv \prod_{j\in J} K_j$. The cost of local public projects, over a population C, is represented by a function $c : K \times \mathbf{S}_C \mapsto (-\infty, \infty]$. c is assumed to be proper. This ensures that the set of feasible allocations is compact. A proposal $(y, m, \mathcal{S}_C)$ is *feasible* if

$$(y, m, \mathcal{S}_C) \in Y_C \equiv \{(y, m, \mathcal{S}_C) \in K \times I\!R \times \mathbf{S}_C |\ c(y, \mathcal{S}_C) \leq \sum_{i\in C} w^i - \sum_{i\in C} m^i\}.$$

It is assumed that $Y_C \subseteq Y_I$, for all $C \subset I$. That is, there are no decreasing returns to coalition size. This ensures that the maximum profit over the "grand coalition" must be no less than the maximum profit over some sub-coalition.

2.4 Valuation Systems

Define $\underline{K}_j \equiv 0 \times \ldots \times K_j \times \ldots \times 0 \in \prod_{j\in\{1,\ldots,j-1\}} 0 \times K_j \times \prod_{j\in\{j+1,\ldots,J\}} 0$. A *complete personalized* valuation system is a vector v, where

$$v \equiv \{v^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j \cap C)\}_{i \in R_j, j \in J}$$

is a set of upper semicontinuous functions $v^i: \underline{\mathrm{K}}_j \times \mathbf{S}_{R_j \cap C} \mapsto [-\infty, \infty),\ i = 1, \ldots, n$.

A *personalized* valuation system is a vector v, where

$$v \equiv \{v^i(\underline{\mathrm{y}}_j)\}_{i \in R_j, j \in J}$$

is a set of upper semicontinuous functions $v^i: \underline{\mathrm{K}}_j \mapsto [-\infty, \infty),\ i = 1, \ldots, n$.

A *non-personalized* valuation system is a vector v, where

$$v \equiv \{v(\underline{\mathrm{y}}_j)\}_{i \in R_j, j \in J}$$

is an upper semicontinuous function $v: \underline{\mathrm{K}}_j \mapsto [-\infty, \infty),\ i = 1, \ldots, n$.

2.5 Valuation Equilibrium

Let the budget set of any consumer $i \in R_j$ be

$$B^i(w^i) \equiv \{(\underline{\mathrm{y}}_j, m^i, \underline{\mathrm{R}}_j) \in \underline{\mathrm{K}}_j \times \mathbb{R}_+ \times \mathbf{S}_{R_j} \,|\, v^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j) + m^i \leq w^i\}.$$

Because of the freedom in the form of pricing of the public projects it is possible to "incorporate" any profit distribution and/or residence taxes in the valuations of the public projects.

An allocation $\bar{x} \equiv (\bar{y}, \bar{m}, \bar{\mathcal{S}})$ is an *equilibrium at a complete personalized valuation system* v if

(a) $(\bar{y}, \bar{m}, \bar{\mathcal{S}})$ maximizes $\sum_{j \in J} \sum_{i \in R_j} v^i(\underline{\bar{\mathrm{y}}}_j, \underline{\bar{\mathrm{R}}}_j) - c(\bar{y}, \bar{\mathcal{S}})$ on Y_I,

(b) for all $i,$ if $x^{i\star} \succ^i \bar{x}^i$ then $x^{i\star} \notin B^i(w^i)$.

Condition (a) says that profits are maximized for the grand coalition. Condition (b) says that allocations preferred to the equilibrium allocation cost more than the equilibrium allocation.

Equilibrium at a personalized valuation system and *equilibrium at a non-personalized valuation system* are defined analogously.

2.6 Pareto Optimality and the Core

Definition 2.1 A state x is *Pareto optimal* if it is feasible and there is no feasible state x' such that $x^{i\prime} \succ^i x^i$ for all consumers i.

Definition 2.2 A state x belongs to the *Core* if it is feasible and there is no $C \subset I$ such that $C \neq \emptyset$ and for some state $x' = (\bar{y}, \bar{m}, \bar{R}) \in Y_C$ and $x^{i\prime} \succ^i x^i$ for all consumers $i \in C$.

Definition 2.3 Let there be a status quo $0 \in \prod_{j \in J} K_j$. Let consumers be "endowed" with the partition $\hat{\mathcal{S}}$. An allocation is *maximally Pareto improving* if it is Pareto optimal and $(\underline{\mathrm{y}}_j, m^i, \underline{\mathrm{R}}_j) \succ^i (0, w^i, \underline{\mathrm{R}}_j)$ for all consumers i.

3. Results

In this section a First Welfare Theorem and Existence results are given for the class of models described in Section 2. In addition, conditions are given that are sufficient to characterize valuation equilibrium as maximally Pareto improving and further, as closely related to the core. The results in this section show similarity to those of Mas-Colell ((1980), Proposition 1, p. 628).

Assumption 1 For every consumer i, $\succeq^i$ continuous, complete, reflexive, transitive preorder on $\prod_{j \in J} K_j \times I\!R_+ \times \mathbf{S}$.

Assumption 2 For every consumer i, each bundle $(\underline{y}_j, m^i, \underline{R}_j)$ and for each region k, residence in k, R_k, and public project in region k, y_k, there exists $m^{i\prime}$ such that $(\underline{y}_k, m^{i\prime}, \underline{R}_k) \succ^i (\underline{y}_j, m^i, \underline{R}_j)$.

Assumption 3 For every consumer i, for all $(\underline{y}_j, m^i, \underline{R}_j)$ such that $y_j \in K_j$, $m^i > 0$, $(\underline{y}_j, m^i, \underline{R}_j) \succ^i (\underline{y}'_k, 0, \underline{R}'_k)$ for any $\underline{y}'_k$ and $\underline{R}'_k$.

Assumption 4 For every consumer i, for each j and for each $(y_j, R_j) \in K_j \times \mathbf{S}_{R_j}$ if $m^{i\prime} > m^i$ then $(y_j, m^{i\prime}, R_j) \succ^i (y_j, m^i, R_j)$.

Assumption 2 is closely related to the Essentiality condition in Diamantaras and Gilles ((1992), p. 6). However, the statement of 2 is somewhat more complicated. Assumption 2 says that for any allocation and any other partition and associated public project, there exists an amount of money that ensures the second allocation is strictly preferred to the first allocation.

The Pareto optimality of equilibrium at a complete personalized, personalized and non-personalized valuation system is demonstrated in Theorem 1.

Theorem 1 *(First Welfare Theorem) Under 2, if the state $\bar{x}$ is a valuation equilibrium at a complete personalized, personalized or non-personalized valuation system then $\bar{x}$ is Pareto optimal.*

That every consumer is better than at his endowment point is demonstrated in Corollary 2.

Corollary 2 *Let there be a status quo project* $(0, w, \mathcal{S})$*. If the state* $\bar{x}$ *is a valuation equilibrium at a complete personalized, personalized or non-personalized valuation system, such that* $v^i(\underset{-}{0}, \hat{R}_j) \leq 0$ *for all* i*, then* $\bar{x}$ *is maximally Pareto improving.*

That there exist no opportunities for defection from the grand coalition at a valuation equilibrium is demonstrated by Corollary 3 when there are constant returns to scale in coalition size.

Corollary 3 *Let* $c(y, \mathcal{S}) = c(y, \mathcal{S}')$ *and* $c(y, \mathcal{S}) \geq 0$ *for all* $y \in \prod_{j \in J} K_j$ *and* $\mathcal{S}, \mathcal{S}' \in \mathbf{S}$*. Under 2, if the state* $\bar{x}$ *is a valuation equilibrium at a complete personalized, personalized or non-personalized valuation system then* $\bar{x}$ *is in the core.*

That any Pareto optimal allocation may be implemented as an equilibrium at a complete personalized valuation system is demonstrated in Theorem 4.

Theorem 4 *(Existence) Under 1, 2, 3 and 4, any Pareto optimal allocation* $\bar{x}$*, such that* $\bar{m}^i > 0$ *for all* i*, may be supported as a valuation equilibrium at a complete personalized valuation system.*

The Proposition given here differs from Mas-Colell (1980) in one important respect: The addition of assumption 3 that it is always better to have some money and some public project than no money and any public project. The valuation system is developed by constructing a compensation function g^i. Assumption 3 ensures that g^i exists for every public project. Because Mas-Colell did not have the assumptions necessary to ensure the existence of g^i everywhere Mas-Colell imposed the valuation $-\infty$ to those public projects for which g^i has no value. However, this ensures that, while the Social Planner would not want to supply such an allocation because it would be associated with an infinite loss, any consumer may wish to consume such an allocation since it would ensure infinite wealth and the potential to consume an infinite amount of the private good. As a consequence, consumer and producer plans may be inconsistent and an equilibrium need not exist.

That all allocations that leave all consumers better off than their endowments may be implemented as an equilibrium at a complete personalized valuation system is demonstrated in Corollary 5. Corollary 5 is the converse of Corollary 2.

Corollary 5 *Let there be a status quo project* $(0, w, \hat{\mathcal{S}})$. *Under 1, 2, 3 and 4, if the state* $\bar{x}$, *such that* $\bar{m}^i > 0$ *for all consumers* i, *is maximally Pareto improving then* $\bar{x}$ *is a valuation equilibrium at a complete personalized valuation system such that* $v^i(\underline{0}, \hat{\underline{R}}_j) \leq 0$ *for all* i.

If, further, the allocation is in the core, then the allocation may be implemented as an equilibrium at a complete personalized valuation system, where each valuation is non-negative, in Corollary 6. Corollary 6 is the converse of Corollary 3.

Corollary 6 *Let* $c(y, \mathcal{S}) \geq 0$ *for all* $y \in \prod_{j \in J} K_j$ *and* $\mathcal{S} \in \mathbf{S}$. *Under 1, 2, 3 and 4, if* $\bar{x}$ *is in the core then* $\bar{x}$ *is a valuation equilibrium at a complete personalized non-negative valuation system.*

Define

$$I_k^i(\bar{\underline{y}}_j, \bar{m}^i, \bar{\underline{R}}_j) \equiv \{(y_k, m^i) \in \underline{K}_k \times I\!R | \; (\underline{y}_k, m^i, \underline{R}_k) \sim^i (\bar{\underline{y}}_j, \bar{m}^i, \bar{\underline{R}}_j)\}.$$

If each consumer's preferences over money and public projects do not change as any consumer migrates then the dual space may be constrained to one of a personalized valuation system. Assumption 5 is weaker than this in that it recognizes that public projects possible in one region may not be possible in another.

Assumption 5 For all consumers, i and regions, j and j', $I^i_j = I^i_{j'}$ on $(\underline{\mathrm{K}}_j \times I\!\!R) \cap (\underline{\mathrm{K}}_{j'} \times I\!\!R)$.

Theorem 7 *(Existence) Under 1, 2, 3, 4 and 5, any Pareto optimal allocation $\bar{x}$, such that $\bar{m}^i > 0$ for all i, may be supported as a valuation equilibrium at a personalized valuation system.*

In Manning (1993), assumption 5 is relaxed to the requirement that consumers value allocations of money and public projects associated with partitions of consumers other than the equilibrium partition no more highly than they do the same allocation under the equilibrium partition.

Given assumption 5 the following two Corollaries are immediate.

Corollary 8 is the converse of Corollary 2.

Corollary 8 *Let there be a status quo project $(0, w, \hat{\mathcal{S}})$. Under 1, 2, 3, 4 and 5, if the state is maximally Pareto improving then $\bar{x}$ is a valuation equilibrium at a personalized valuation system such that $v^i(\underline{0}, \hat{\underline{R}}_j) \leq 0$ for all i.*

Corollary 9 is the converse of Corollary 3.

Corollary 9 *Let $c(y, \mathcal{S}) \geq 0$ for all $y \in \prod_{j \in J} K_j$ and $\mathcal{S} \in \mathbf{S}$. Under 1, 2, 3, 4 and 5, if $\bar{x}$ is in the core then $\bar{x}$ is a valuation equilibrium relative to a non-negative personalized valuation system.*

Assumption 6 For all consumers, i and i' and regions, j, $I_j^i = I_j^{i'}$.

If, in addition to assumption 5, each consumer's preferences over money and public projects are identical, assumption 6, then the dual space may be constrained to one of a non-personalized valuation system.

Theorem 10 *(Existence) Under 1, 2, 3, 4, 5 and 6, any Pareto optimal allocation $\bar{x}$ may be supported as a valuation equilibrium at a non-personalized valuation system.*

Given assumptions 5 and 6 the following two Corollaries are immediate.

Corollary 11 is the converse of Corollary 2.

Corollary 11 *Let there be a status quo project $(0, w, \hat{\mathcal{S}})$. Under 1, 2, 3, 4, 5 and 6 if the state $\bar{x}$, such that $\bar{m}^i > 0$ for all consumers i, is maximally Pareto improving then $\bar{x}$ is a valuation equilibrium at a non-personalized valuation system such that $v^i(\underline{0}, \hat{\underline{R}}_j) \leq 0$ for all i.*

Corollary 12 is the converse of Corollary 3.

Corollary 12 *Let $c(y, \mathcal{S}) \geq 0$ for all $y \in \prod_{j \in J} K_j$ and $\mathcal{S} \in \mathbf{S}$. Under 1, 2, 3, 4, 5 and 6, if $\bar{x}$ is in the core then $\bar{x}$ is a valuation equilibrium at a non-negative non-personalized valuation system.*

Conclusions

Some effort has already been made in the literature to show that Lindahl pricing schemes may be implemented as lump sum taxes rather than linear prices. For instance, Wooders (1992) has shown that any Lindahl equilibrium, in a sufficiently replicated economy, may be implemented using lump sum taxes. Barro and Romer (1987) call the equivalence between the linear pricing mechanism and the lump sum pricing mechanism the *package deal effect.* By a specialisation of the class of models presented here, this paper shows that the package deal effect holds for finite economies with local public goods. For instance, it is immediate that the package deal effect holds for the class of models in Manning (1993).

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Appendix

The proofs in this Appendix are a direct adaptation of those in Mas-Colell (1980).

Proof of Theorem 1: Suppose that for some $C \subset I$, $C \neq \emptyset$, there is $x' \in Y_C$ such that $x' \succ^i \bar{x}$ for every i in C (by (2)). By (a) and (b) of the definition of an equilibrium at a complete personalized valuation system,

$$\sum_{j \in J} \sum_{i \in R_j} v^i(\bar{\underline{\mathrm{y}}}_j, \bar{\underline{\mathrm{R}}}_j) - c(\bar{y}, \bar{\mathcal{S}}) \geq \sum_{j \in J} \sum_{i \in R_j} v^i(\underline{\mathrm{y}}'_j, \underline{\mathrm{R}}'_j) - c(y', \mathcal{S}'),$$

$$\sum_{j \in J} \sum_{i \in C \cap R_j} \left(v^i(\underline{\mathrm{y}}'_j, \underline{\mathrm{R}}'_j) + m^{i\prime} \right) > \sum_{i \in C} w^i,$$

and

$$\sum_{i \in C} (w^i - m^{i\prime}) - c(y', \mathcal{S}_C) \geq 0.$$

So, if $C = I$, we have

$$0 = \sum_{j \in J} \sum_{i \in R_j} v^i(\bar{\underline{y}}_j, \bar{\underline{R}}_j) - c(\bar{y}, \bar{\mathcal{S}}) > \sum_{i \in I} (w^i - m^{i\prime}) - c(y', \mathcal{S}') \geq 0,$$

a contradiction. *Q.E.D.*

Proof of Corollary 2: Immediate by the utility maximization hypothesis. *Q.E.D.*

Proof of Corollary 3: Consider $\bar{x}$ and x' as in the proof of Theorem 1.

$$0 = \sum_{j \in J} \sum_{i \in R_j} v^i(\bar{\underline{y}}_j, \bar{\underline{R}}_j) - c(\bar{y}, \bar{\mathcal{S}}) \geq \sum_{j \in J} \sum_{i \in R_j} v^i(\underline{y}'_j, \underline{R}'_j) - c(y', \mathcal{S}'),$$

by profit maximization and

$$\sum_{j \in J} \sum_{i \in R_j \cap C} v^i(\underline{y}'_j, \underline{R}'_j) - c(y', \mathcal{S}'_C) > \sum_{j \in J} \sum_{i \in R_j \cap C} (w^i - m^{i\prime}) - c(y', \mathcal{S}'_C) \geq 0,$$

by the proof of Theorem 1. When $c(y', \mathcal{S}') = c(y', \mathcal{S}'_C)$, by the definition of an equilibrium at a cost share valuation system

$$\sum_{j \in J} \sum_{i \in R_j} v^i(\underline{y}'_j, \underline{R}'_j) - c(y', \mathcal{S}') \geq \sum_{j \in J} \sum_{i \in R_j \cap C} v^i(\underline{y}'_j, \underline{R}'_j) - c(y', \mathcal{S}'_C)$$

and a contradiction is obtained. *Q.E.D.*

Proof of Theorem 4: Let $\bar{x}$ be feasible. For every i and $\bar{x}$, let

$$g^i(\underline{y}_k, \underline{R}_k) = \{w^i - z^i \in I\!R \mid (\underline{y}_k, z^i, \underline{R}_k) \sim^i (\bar{\underline{y}}_j, \bar{m}^i, \bar{\underline{R}}_j)\}.$$

g^i is defined for any (y_k, R_k). To see this consider

$$L_{(y_k,R_k)} \equiv \{m^i \in I\!R_+ |\ (\bar{\text{y}}_j, \bar{m}^i, \bar{\text{R}}_j) \succ^i (\text{y}_k, m^i, \text{R}_k)\}$$

$$\text{and} \quad B_{(y_k,R_k)} \equiv \{m^i \in I\!R_+ |\ (\text{y}_k, m^i, \text{R}_k) \succ^i (\bar{\text{y}}_j, \bar{m}^i, \bar{\text{R}}_j)\}.$$

By (3), $L_{(y_k,R_k)} \neq \emptyset$ and by (2), $B_{(y_k,R_k)} \neq \emptyset$. If $g^i(\text{y}_k, \text{R}_k)$ is not defined for every (y_k, R_k), for all k, then $L_{(y_k,R_k)} \cup B_{(y_k,R_k)} = I\!R_+$ for some (y_k, R_k). However, by continuity (4.1) $L_{(y_k,R_k)}$ and $B_{(y_k,R_k)}$ are open. Further, $I\!R_+$ is connected. Therefore, it cannot be the case that $L_{(y_k,R_k)} \cup B_{(y_k,R_k)} = I\!R_+$ and therefore

$$g^i(\text{y}_k, \text{R}_k) = I\!R_+/(L_{(y_k,R_k)} \cup B_{(y_k,R_k)}) \neq \emptyset, \quad \text{for every } (y_k, R_k).$$

We have $g^i(\bar{\text{y}}_j, \bar{\text{R}}_j) = w^i - \bar{m}^i$ and therefore

$$\sum_{j \in J} \sum_{i \in R_j} g^i(\bar{\text{y}}_j, \bar{\text{R}}_j) \geq c(\bar{y}, \bar{\mathcal{S}}).$$

Suppose now that $\bar{x}$ is Pareto optimal. Assume for some $x \in Y_I$ (not necessarily different from $\bar{x}$),

$$a = \sum_{j \in J} \sum_{i \in R_j} g^i(\text{y}_j, \text{R}_j) - c(y, \mathcal{S}) > 0;$$

then $g^i(\text{y}_j, \text{R}_j) > -\infty$ for all i. Consider the state x defined by $m^{i\prime} = w^i - g^i(\text{y}_j, \text{R}_j) + a/I$. If for all i, $(\bar{\text{y}}_j, \bar{m}^i, \bar{\text{R}}_j) \sim^i (\text{y}_j, w^i - g^i(\text{y}_j, \text{R}_j), \text{R}_j)$ we have $(\text{y}_j, m^{i\prime}, \text{R}_j) \succ^i (\bar{\text{y}}_j, \bar{m}^i, \bar{\text{R}}_j)$ for all i, which is impossible because $(\bar{\text{y}}_j, \bar{m}^i, \bar{\text{R}}_j)$ is Pareto optimal and $(\text{y}_j, m^{i\prime}, \text{R}_j)$ is feasible. We therefore conclude that

$$\sum_{j\in J}\sum_{i\in R_j} g^i(\underline{y}_j, \underline{R}_j) \leq c(y, \mathcal{S})$$

for all $(\underline{y}_j, m^{i\prime}, \underline{R}_j)$. Therefore, if we put $v^i = g^i$ the profit maximization condition (a) follows.

Define $\{(\underline{y}_k, m^i, \underline{R}_k)\}_{i\in I}$ to be such that $(\underline{y}_k, m^i, \underline{R}_k) \succ^i (\bar{\underline{y}}_j, \bar{m}^k, \bar{\underline{R}}_j)$. We claim that

$$g^i(\underline{y}_k, \underline{R}_k) + m^i > g^i(\bar{\underline{y}}_k, \bar{\underline{R}}_k) + \bar{m}^i = w^i.$$

By definition of g^i there exists a $\{z^i\}_{i\in I}$ such that

$$g^i(\underline{y}_k, \underline{R}_k) + z^i = g^i(\bar{\underline{y}}_k, \bar{\underline{R}}_k) + \bar{m}^i.$$

By (4.4) $m^i > z^i$ and so the claim follows.

Therefore, if we put $v^i = g^i$ the utility maximization condition (b) follows. *Q.E.D.*

Proof of Corollary 5: Let there be a status quo project $(0, w^i, \hat{R}_j)$ and let $\bar{x}$ be maximally Pareto improving. Since $\bar{x} \succeq^i (\underline{0}, w^i, \hat{\underline{R}}_j)$ for all $i \in I$, $g^i(\underline{0}, \hat{\underline{R}}_j) \leq 0$ for all $i \in I$. Hence, if, as in the Proof of Theorem 4, we have $v^i = g^i$, we are finished. *Q.E.D.*

Proof of Corollary 6: Now let $\bar{x} = (\bar{y}, \bar{m}, \bar{\mathcal{S}})$ belong to the core. By definition of the core and the functions g^i, we must have

$$\sum_{j\in J}\sum_{i\in R_j\cap C} g^i(\bar{\underline{y}}_j, \bar{\underline{R}}_j) \leq c(\bar{y}, \bar{\mathcal{S}} \cap C),$$

for all $C \subset I, C \neq \emptyset$ and $\underline{\bar{x}} \in \prod_{j\in J} K_j \times I\!R_+ \times \mathbf{S}$. Then define $v^i \colon K_j \times \mathbf{S}_{R_j} \mapsto I\!R_+$ by $v^i(\underline{y}_j, \underline{R}_j) = \max\{0, g^i(\underline{y}_j, \underline{R}_j)\}$ and note that v^i is then non-negative. For all $x \in$

$\prod_{j\in J} K_j \times I\!R_+ \times \mathbf{S}$,

$$\sum_{j\in J}\sum_{i\in R_j} v^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j) \leq c(y, \mathcal{S}),$$

since, letting $C = \{i \in I | g^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j) \geq 0\}$,

$$\sum_{j\in J}\sum_{i\in R_j} v^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j) = \sum_{j\in J}\sum_{i\in R_j} g^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j) \quad \text{if } C \neq \emptyset$$

and

$$\sum_{j\in J}\sum_{i\in R_j} v^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j) = 0 \leq c(y, \mathcal{S}) \quad \text{if } C = \emptyset.$$

On the other hand,

$$\sum_{j\in J}\sum_{i\in R_j} v^i(\bar{\underline{\mathrm{y}}}, \bar{\underline{\mathrm{R}}}_j) \geq \sum_{j\in J}\sum_{i\in R_j} g^i(\bar{\underline{\mathrm{y}}}, \bar{\underline{\mathrm{R}}}_j) \geq c(\bar{y}, \bar{\mathcal{S}}),$$

So,

$$\sum_{j\in J}\sum_{i\in R_j} v^i(\bar{\underline{\mathrm{y}}}, \bar{\underline{\mathrm{R}}}_j) = c(\bar{y}, \bar{\mathcal{S}}),$$

and $\bar{x}$ maximizes profits. Also, for all $i \in I$, if $v^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j) + m^i \leq w^i$, then $\bar{x} \succeq^i x$ because $v^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j) \geq g^i(\bar{\underline{\mathrm{y}}}_j, \bar{\underline{\mathrm{R}}}_j)$ and so the utility maximization hypothesis is satisfied. *Q.E.D.*

Proof of Theorem 7: Replace $g^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j)$ by $g^i(\underline{\mathrm{y}}_j)$ in the proof of Theorem 4. *Q.E.D.*

Proof of Corollary 8: Replace $g^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j)$ by $g^i(\underline{\mathrm{y}}_j)$ in the proof of Corollary 5. *Q.E.D.*

Proof of Corollary 9: Replace $g^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j)$ and $v^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j)$ by $g^i(\underline{\mathrm{y}}_j)$ and $v^i(\underline{\mathrm{y}}_j)$, respectively, in the proof of Corollary 6. *Q.E.D.*

Proof of Theorem 10: Replace $g^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j)$ by $g(\underline{\mathrm{y}}_j)$ in the proof of Theorem 4. *Q.E.D.*

Proof of Corollary 11: Replace $g^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j)$ by $g(\underline{\mathrm{y}}_j)$ in the proof of Corollary 5. *Q.E.D.*

Proof of Corollary 12: Replace $g^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j)$ and $v^i(\underline{\mathrm{y}}_j, \underline{\mathrm{R}}_j)$ by $g(\underline{\mathrm{y}}_j)$ and $v(\underline{\mathrm{y}}_j)$, respectively, in the proof of Corollary 6. *Q.E.D.*

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