Benefit-Cost Analysis and Distortionary Taxes: A Public Choice Approach

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The fundamental rule of benefit-cost analysis is that if taxes are non-distortionary, then a necessary condition for efficient supply of any public good is that the sum of individual marginal rates of substitution between public and private goods equals the marginal cost of the public good. In a world where taxes distort and where most public decisions are not between Pareto-ranked alternatives, decision makers have a right to ask economic theorists for more detailed and useful advice.

Arthur Cecil Pigou (1947) maintained that benefit-cost analysis of public projects should discount benefits relative to costs because the taxes that finance public projects typically cost the private economy more than the amount of money raised.

> ‘‘*Where there is indirect damage, it ought to be added to the direct loss of satisfaction involved in the withdrawal of the marginal unit of resources by taxation, before this is balanced against the satisfaction yielded by the marginal expenditure. It follows that, in general, expenditure ought not be carried so far as to make the real yield of the last unit of resources expended by the government equal to the real yield of the last unit left in the hands of the representative citizen.*’’[1]

Modern public finance theorists have used the theory of optimal commodity taxation to analyze this problem. They hypothesize a *social welfare function* which is to be maximized subject to appropriate microeconomic incentive constraints for individual taxpayers. Important contributions along these lines were made by Peter Diamond and James Mirrlees (1971), Joseph Stiglitz and Partha Dasgupta (1971), Anthony Atkinson and Nicholas Stern (1974) and Atkinson and Stiglitz (1980). Stiglitz and Dasgupta dispute Pigou’s conclusion and argue that with distortionary taxes, the fundamental rule may either overstate or understate the incremental benefits from public expenditure. Atkinson and Stern clarify the issue with a more careful delineation of the conditions under which public goods paid for by a distortionary excise tax would be over or underprovided by a government using the fundamental rule of benefit cost.

To many economists, the use of social welfare functions in benefit-cost analysis is problematic.

[1] This passage is quoted in Atkinson and Stern (1974)

The modern formulation of social welfare functions dates from the work of Bergson (1938) and Samuelson (1947). Bergson-Samuelson social welfare functions offer a convenient articulation of the preferences of a single individual over possible allocations of resources in a society. Maximizing a social welfare function would certainly be an appropriate method of public choice in an economy governed by a benevolent monarch or of a rational deity of transparent disposition. But in a pluralistic society, where the preferences of many people ‘‘count’’, one can not comfortably assume the existence of a complete ranking of social states that is be agreed to by all or most reasonable people--under any reasonable definition of ‘‘reasonable’’. [2] If application of benefit cost analysis must await consensus on a social welfare function, it will provide little or no guidance to policy.

Samuelson (1947) suggested that in the absence of a monarch or a universal consensus, social welfare functions remain useful tools for exploring the policy implications of alternative hypothetical social preference orderings .

> *‘‘it is a legitimate exercise of economic analysis examine the consequences of various value judgments...just as the study of comparative ethics is itself a science like any other branch of anthropology.’’*

Samuelson’s proposed exploration in comparative ethics has been admirably conducted in the existing literature on optimal taxation. Moreover, some of the discoveries from this exploration are sufficiently general to apply regardless of the specific choice of social welfare function. It is the purpose of this paper to show that benefit-cost analysis under distortionary taxes yields clean, decisive results that are better understood without the artifice of social welfare functions.

In his survey of the theory of optimal taxation, Stern (1987) suggests that

> *‘‘this is not to claim that optimal taxation is the only way to think about tax policy...And one may want to focus on taxes as the outcome of pressures and influences on government by different groups, self-interested or otherwise, rather than choices taken by a government interested in the welfare of its citizens.’’*

[2] Perhaps the use of social welfare functions for benefit cost analysis is an intellectual vestige from an earlier ‘‘age of absolutism.’’ This appears to be the view of Knut Wicksell (1896) who remarked: ‘‘If I may make bold to risk a general observation against men who in many respects are above even my praise, I would venture to suggest that with some few exceptions, the whole theory now rests on the outdated political philosophy of absolutism...Even the most recent manuals on the science of public finance frequently leave the impression, at least on me, of some sort of philosophy of enlightened and benevolent despotism...Imagine how an enlightened and benevolent ruler--one imbued, say, with the sense of equity of our modern educated classes--would organize the expenditures and taxes of his country...He would spend this income on the satisfaction of public wants ranked in the order of their importance so that the people would be assured of receiving the highest attainable total utility...On the other hand, our ruler would probably not worry overmuch about the thorny question whether the activities of the State adequately compensate his subjects for their sacrifice.’’

This paper pursues the alternative focus that Stern suggests. It seeks useful conditions relating individual preferences to willingness to support specific governmental tax-expenditure proposals. This discussion follows Wicksell in considering only tax-expenditure proposals that specifically tie expenditures to taxes that finance them. [3] The target of this effort is an empirically meaningful formula that quantifies Pigou's advice on discounting the public benefits paid for by distortionary taxes.

1. An Economy with Distortionary Taxation on Labor Supply

The ideas of this paper are illustrated by a relatively simple economy in which the distortionary tax is a tax on labor income.

Consumption, production, and taxation

Consider an economy with n consumers. Each consumer i has a utility function, $U^i(G, C_i, \ell)$, where G is the total amount of a pure public good made available per year, C_i is consumer i's annual consumption of a private consumption good and ℓ_i is the number of leisure hours per year that consumer i enjoys. Both the private good and the public good can be produced at constant unit cost, using one unit of labor per unit of output. Then the competitive prices of labor, public goods and private goods will all be $1 per unit.

Consumers have no source of income other than sale of their labor. Consumer i is able to supply w_i units of labor per hour that he works. He is paid $1 per unit of labor supplied, or equivalently $\$w_i$ per hour. Let H_i be the total number of hours that i has to divide between labor and leisure. If he takes ℓ_i units of leisure, he will be able to supply $Z_i = w_i(H_i - \ell_i)$ units of labor and his before-tax wage income will be Z_i. The public good is paid for entirely by a proportional tax on labor income at the rate t, so his after-tax labor income is $(1-t)Z_i$. The budget equation of consumer i is therefore $C_i = (1-t)Z_i$. Total government revenue is $t\sum_i^n Z_i$. Government budget balance requires that $G = t\sum_i^n Z_i$.

[3] Wicksell proposed that ''no public expenditure ever be voted upon without simultaneous determination of the means of covering their cost.'' (*op. cit.* page 91)

Nash equilibrium in work effort for fixed tax rates.

Because of the interplay between public goods supply and the many individual decisions about work effort, the effects of taxes and expenditures are best understood if the economy is modelled explicitly as a game. The effects of changes in the government's rules for taxes expenditures can then be studied by analyzing their effects on Nash equilibrium outcomes. Once we are able to predict the consequences of a given policy change, we can consider the question of which consumers will be in favor and which will oppose the change.

We model public choice with a three-stage game. In the first stage, the ‘‘government’’ sets a tax rate t. This rate may be set as the result of a voting game or some other kind of political process. In the second stage, given the government's choice of tax rate, individuals simultaneously choose their consumption quantities, C_i, and work efforts, Z_i. In the third stage, the government collects taxes tZ_i from each consumer and spends its revenue, $t\sum_i Z_i$ on the public good. This paper concentrates on the the second stage of this game, analyzing the effects of changes in the tax rate on labor supply, tax revenue, public goods supply, and the utility of various kinds of consumers.

The game in which consumers simultaneously determine their labor supplies is nontrivial because the best labor supply for consumer i in general will depend on the amount of public goods provided. The amount of public goods provided depends in turn on tax revenue which depends on labor supplies. For fixed t, we seek a Nash equilibrium vector, $Z_1(t), \ldots, Z_n(t)$ of labor supplies.

Uniqueness of Nash Equilibrium

The path of comparative statics will be much smoother if it is known that for any specified tax rate, there is only one Nash equilibrium. But without further assumptions, uniqueness can not be taken for granted. In general, there can be more than one equilibrium level of tax revenue and public expenditure corresponding to a given tax rate if public goods are sufficiently complementary with work effort. This can happen if, for example, starting from one equilibrium, there is an increase in public goods supply that would be sufficient to induce enough additional labor supply to pay for itself in taxes.

This possibility seems not entirely fanciful for some economies. For example, it might be that in a primitive economy without roads, police services, and other social overhead there is little incentive for working and hence little tax revenue to pay for social overhead. For the same economy with the same tax rate, there might be another ‘‘high-level’’ Nash equilibrium. In the high-level equilibrium, everyone works harder than in the low-level equilibrium. Greater work effort yields more income and more tax revenue for the government. The extra tax revenue is used to support more social overhead, which in turn motivates the high-level equilibrium level of labor supply.

Let $Z_{\sim i}$ denote the total labor suppy by persons other than i, then $G = tZ_{\sim i} + tZ_i$. Consumer i's money budget implies that $C_i = (1-t)Z_i$, and his time budget implies that $\ell_i = H_i - Z_i/w_i$. When the tax rate is t and others supply $Z_{\sim i}$ units of labor, consumer i can therefore achieve the utility level

$$V_i(t) = \max_{Z_i} U^i(tZ_{\sim i} + tZ_i, (1-t)Z_i, H_i - Z_i/w_i). \tag{1}$$

by choosing a best response Z_i.[4]

In general, labor supply will depend both on after-tax wages and on the amount of the public good available. Let $L^i(G, w)$ be Consumer i's labor supply function[5] and $L(G, w) = \sum_i L^i(G, w)$ be the aggregate labor supply function. Then $Z_1(t), \ldots, Z_n(t)$ is a Nash equilibrium with tax rates t if for all i, $Z_i(t) = L^i(t \sum Z_i(t), (1-t))$.

Equilibrium will be unique if an extra dollar of *per capita* spending on public goods induces less than an extra dollar's worth of taxes throught additional labor supply from each individual. In particular, the following assumption guarantees existence and uniqueness of Nash equilibrium.

Assumption 1. *Each consumer has a continuous labor supply function, $L^i(G, t)$. If $G' > G$, then $tL(G', t) - tL(G, t) < G' - G$.*

Theorem 1. *If Assumption 1 holds, then for any t such that $0 \le t < 1$, there exists a unique Nash equilibrium, $Z_1(t), \ldots Z_n(t)$.*

Proof:

[4] When he decides how much labor to supply, a consumer who maximizes (1) takes into the account the effect of his taxes on the amount of public good. For a large economy, where individual consumers' marginal valuations of the public good are small, this effect can safely be neglected. On the other hand, for small groups, or in situations where relatively little of the public good is supplied by the government, these effects may be important.

[5] This will be a well-defined function if individual preferences are continuous and strictly convex.

Existence of at least one Nash equilibrium follows from the usual Brouwer's fixed point argument. The set of possible strategies is a closed bounded convex set. The ''best response'' function which maps a labor supply vector $Z_1, \ldots, Z_n$, into the vector $L_1(t\sum_i Z_i, 1-t), \ldots, L_n(t\sum_i Z_i, 1-t)$ is a continuous function from this set into itself and therefore has a fixed point. This fixed point is a Nash equilibrium.

Suppose that there are two distinct Nash equilibria, $Z_1, \ldots, Z_n$, and $Z'_1, \ldots, Z'_n$. Let $G = t\sum_i Z_i$ and $G' = t\sum_i Z'_i$. If $G = G'$, then for all i, $Z_i = L^i(G, 1-t) = L^i(G', 1-t) = Z'_i$. Since the two equilibria are assumed to be distinct, it must be that $G' \neq G$. Without loss of generality, let $G' > G$. But then, according to Assumption 1, $tL(G', 1-t) - tL(G, 1-t) < G' - G$. But this is impossible since it must be that $G' = tL(G', 1-t)$ and $G = tL(G, 1-t)$. ∎

2. Benefit-Cost Analysis for Selfish Individuals

Since we do not assume that consumers share a social welfare function, we do not seek a ''social optimum''. Instead, we establish principles that govern individual ''benefit-cost analysis'' of tax-expenditures proposals for selfish consumers, who want to know ''what's in it for me.'' When Nash equilibrium is unique for given tax rates, this exercise amounts to calculating the changes in Nash equilibrium and evaluating these changes according to the utility function of Consumer i. To do this, we simply differentiate the function $V_i(t)$ defined by Equation 1. Consumer i will prefer an increase (decrease) in the tax rate if $V'_i(t)$ is positive (negative). Calculating this derivative:

$$V'_i(t) = U^i_G \frac{d}{dt} tZ_{\sim i}(t) + Z_i(t)(U^i_G - U^i_C) \tag{2}$$

Expression (2) has a simple and interesting interpretation. For Consumer i, an increase in the tax rate t has both a ''good effect and a bad effect''. The good effect is that an increase in the tax rate will typically[6] increase the amount of tax revenue collected from *other* consumers and this revenue will pay for more public goods. This effect increases i's utility by $U^i_G \frac{d}{dt} tZ_{\sim i}(t)$. The bad effect is that i, himself, has to pay more taxes. If i did not change his labor supply, the extra tax would reduce his utility by the amount of extra tax times the difference between his marginal

[6] But not always, as we will see.

utilities for private and public goods consumption. This effect is measured by $Z_i(t)(U^i_G - U^i_C)$. [7]

With a bit of algebra, Equation (2) can be restated in terms of marginal rates of substitution, elasticities of labor supply, and tax shares. The resulting Equation (3)[8] uses the following definitions:

$m_i(t) = u^i_G/u^i_C$--Person i's marginal rate of substitution between public and private goods in the equilibrium that corresponds to tax rate t.

$\theta_i(t) = Z_i(t)/\sum_j Z_j(t)$--The share of the economy's taxes paid by person i in the equilibrium that corresponds to tax rate t.

$\xi_i(t) = tdZ_i(t)/Z_i(t)dt \;\; \xi(t) = tdZ(t)/Z(t)dt$--The elasticities respectively of individual i's work effort and total work effort with respect to the tax rate.

$$V'_i(t)/Z(t)U^i_C = m_i(t)(1 + \xi(t) - \theta_i(t)\xi_i(t)) - \theta_i(t). \tag{3}$$

Therefore we can conclude that:

Theorem 2. *Given Assumption 1, consumer i will favor an increase in taxes and public expenditure if $m_i(t)(1+\xi(t)-\theta_i(t)\xi_i(t))) > \theta_i(t)$ and a decrease if the inequality is reversed.*

By adding the inequalities in Theorem 2, we can find a necessary condition for a tax increase or decrease, accompanied by a corresponding change in expenditures, to be Pareto improving.

Corollary 1. *A necessary condition for a tax increase (decrease) to be Pareto improving is that $\sum_i m_i(t)(1 + \xi(t) - \theta_i(t)\xi_i(t))) - 1$ is positive (negative).*

The expressions in Theorem 1 and Corollary 1 have simple interpretations. Recall that $\xi(t)$ is the elasticity of work effort with respect to an increase in taxes and the corresponding increase in the supply of public goods. If the net effect of taxes and the public goods is to reduce work effort, then $\xi(t)$ is negative. The expression $1 + \xi(t)$ is the elasticity of total tax revenue with

[7] The observant reader will notice that the expression for $V'_i(t)$ contains no accounting for the fact that the tax will make i change his own work effort. This effect disappears because in the initial equilibrium, the partial derivative of utility with respect to Z_i was zero. This is an application of the well-known principle of the ‘‘envelope theorem.’’

[8] The steps that lead from Equation (2) to Equation (3) are in the Appendix.

respect to tax rate, and $1+\xi(t)-\theta_i(t)\xi_i(t)$ is the elasticity of tax revenue collected from persons other than i. If taxes were non-distortionary, the elasticities $\xi(t)$ and $\xi_i(t)$ would be zero and the necessary condition in Corollary 1 would reduce to the fundamental condition: $\sum m_i(t) = 1$. In a large economy, where consumer i's influence is small, the elasticity $1+\xi(t)-\theta_i(t)\xi_i(t)$ of revenue from persons other than i is very close to the elasticity of revenue $1+\xi(t)$ from the entire population. Therefore we have

Corollary 2. *Consumer i will favor a tax increase if $m_i(t)(1+\xi(t)) > \theta_i(t)$ and if i is sufficiently small relative to the entire economy. If this inequality is reversed, and i is sufficiently small relative to the entire economy, then i will favor a tax decrease.*

Corollary 3. *If each consumer is small relative to the economy, then a necessary condition for a tax increase to be a Pareto improvement is that $\sum m_i(t)(1+\xi(t)) \geq 1$ and a necessary condition for a tax decrease to be a Pareto improvement is that $\sum m_i(t)(1+\xi(t)) \leq 1$.*

In the special case where consumers have identical tastes and incomes, Pigou's remark about the ‘‘representative’’ citizen takes simple concrete form. Increased or reduced public expenditure will increase or decrease the well-being of the representative citizen as follows:

Corollary 3. *If there are n consumers with identical tastes and incomes, then a small increase in the tax rate will benefit all consumers if $(1+\xi(t)-\xi(t)/n))\sum_i m_i(t) > 1$ and a small decrease in the tax rate will benefit all consumers if $(1+\xi(t)-\xi(t)/n))\sum_i m_i(t) < 1$.*

3. Conclusion

This paper is intended to extend and clarify earlier studies of optimal provision of public goods under distortionary taxation. Although some of the results found here are closely parallel to propositions found by Atkinson and Stern and by Atkinson and Stiglitz, it is hoped that removing the extraneous scaffold of social welfare functions reveals a simpler and more attractive theory beneath.

Appendix

Steps from Equation 2 to Equation 3.

We start with

$$V_i'(t) = U_G^i \frac{d}{dt} t Z_{\sim i}(t) + Z_i(t)(U_G^i - U_C^i) \tag{2}$$

On dividing both sides of this equation by U_C^i, one has

$$\begin{aligned} V_i'(t)/U_C^i &= m_i(t)\frac{d}{dt} t Z_{\sim i}(t) + Z_i(t)(m_i(t) - 1) \\ &= m_i(t)(Z_{\sim i}(t) + t dZ_{\sim i}/dt) + Z_i(t) m_i(t) - Z_i(t) \\ &= m_i(t)(Z(t) + t dZ(t)/dt - t dZ_i(t))/dt - Z_i(t) \\ &= Z(t) m_i(t)(1 + \xi(t) - \xi_i(t)\theta_i(t)) - \theta_i(t) \end{aligned}$$

Equation (3) is then immediate.

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