Designing Optimal Taxes With a Microeconometric Model of Household Labour Supply

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Abstract
This paper is concerned with the empirical analyses of optimal taxation, adopting Equality of Outcome (EO) as well as Equality of Opportunity (EOp) as evaluation criteria. The EOp- and EO-criteria provide alternative methods for summarizing the efficiency-equality trade-off in the distribution of individual welfare. We also compare the results depending on whether we use income or money-metric utility as a measure of individual welfare. We estimate micro-econometric models of household labour supply and corresponding individual welfare measures based on 1995 Norwegian data for both married couples and singles. We then use these models to simulate behavioural responses and welfare gains and losses of various constant-revenue four-parameter tax rules, i.e. the tax rules defined by a lump-sum transfer (positive or negative), two marginal tax rates and a “kink point” that produces the same revenue collected with the observed 1995 rules. Using the various EOp- and EO-critera as a basis for evaluating and comparing these tax rules, EOp- and EO-optimal tax rules are identified.

Keywords:

JEL classification

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1. Introduction

This paper presents an empirical analysis of optimal taxation. The exercise differs in many important ways from most other attempts to empirically compute optimal taxes. Typically, those computational exercises start with some version of the optimal taxation framework originally set up in the seminal paper by Mirlees (1971) and go on by trying to feed with empirical measures the formulas produced by the theory (e.g. Tuomala 1990). There are two main problems with this procedure: 1) the theoretical results become amenable to an operational interpretation only by adopting some very special assumptions concerning the preferences, the composition of the population and the structure of the tax rule; 2) the empirical measures used as counterparts of the theoretical concepts are typically derived from previous estimates obtained under assumptions that are usually very different from those used in the theoretical model. As a consequence the consistency between the theoretical model and the empirical measures is dubious. At the end, it remains therefore unclear the significance of the numerical results. Recently, this literature has received an important contribution by Saez (2001), who shows the existence of a very neat link between Mirlees's formulas and labour supply elasticities, which in principle might ease the empirical implementation of the theoretical results. Still, even the approach developed by Saez has been so far implemented using very restrictive assumptions on preferences and smooth tax rules. As further limitations of this literature, simultaneous household decisions and quantity constraints to labour supply choices have been ignored and do not seem to be easily accounted for anyway.

The approach adopted here is completely different. We do not start with theoretical results. Instead we start with a microeconometric model of labour supply that uses a rather flexible representation of preferences and accounts for simultaneous decisions of household components and for quantity constraints on the on the labour supply choices. We then identify optimal tax rules – within a class of 6-parameter piece-wise linear rules - by iteratively running the model and maximizing a social welfare function (which takes the household utility levels as arguments) under a constant tax revenue constraint. The closest previous example adopting a similar approach is probably represented by Fortin B, Truchon M, Beausejour L (1993). However, the model they use is still rather restrictive in terms of behavioural responses (a Stone-Geary utility function) and is not estimated but rather calibrated. The class of tax rules within which we search for the optimum is also more general with respect to Fortin et al. (1993).

We use and compare two alternative social evaluation criteria: Equality of Opportunity (EOp) and the more traditional Equality of Outcome (EO). The latter consists in maximizing a weighted sum of individual welfare levels (the "outcomes" of households' choices). The former is a computable concept of equality of opportunity developed by Roemer (1998). The idea motivating the development of this new criterion is that "outcomes" are the joint result of "opportunities" and "effort", and that the social planner might wish to account for the inequality due to unequal "opportunities" but not for the inequality due to unequal "effort". This concept is interesting from the policy point-of-view, since the majority of citizens in most industrialised countries, although not unfavourable to redistribution, seem sensitive to the way that a certain outcome has been attained. Redistribution is more likely to receive support if it is designed to correct circumstances that are beyond people's control (i.e. opportunities). On the other hand, if a bad outcome is associated with a lack of effort, redistribution would be much less acceptable.

In a previous contribution that originated from an international research project, this concept has been applied to evaluate the EOp performance of income tax rules in various countries, using a relatively simple common model of labour supply behaviour with calibrated parameters[1]. This paper extends the previous study in several respects. First, to allow for alternative weighting profiles in the treatment of income differentials that arise from factors beyond the individuals' control, a generalised version of Roemer's (1998) EOp-criterion is introduced. Secondly, we employ a relatively sophisticated model of labour supply that provides a simultaneous treatment of partners' decisions and

[1] See Roemer et al. (2001).

accounts for quantity constraints on the distribution of hours. Finally, while the previous study only concerned male heads of household's 25-40 years old this study deals with approximately the entire labour force. To our knowledge, this is the first tax evaluation based on models for both married couples and single individuals. Most tax evaluations are either based on representative agent models or microeconometric models for single individuals or married females conditional on husbands' income.

In Section 2 we illustrate the min features of the microeconometric model of household labour supply, estimated on 1994 Nowegian data. A more detailed description of the model is given in the Appendix.

In Section 3 we explain how we compute the individual welfare levels so that they can be compared and aggregated into social welfare functions.

In section 4 we present the EO and Eop criteria and the associated social welfare functions used to evaluate the effects of the tax rules.

In Section 5 the model is used to identify the optimal tax rules according to the alternative welfare evaluation criteria.

2. The microeconometric labour supply model

The labour supply model used in this study is detailed described in the Appendix. Here we give a bird-eye presentation. The model can be considered as an extension of the standard multinomial logit model, and differs from the traditional models of labour supply in several respects[2]. First, it accounts for observed as well as unobserved heterogeneity in tastes and choice constraints, which means that it is able to take into account the presence of quantity constraints in the market. Second, it includes both single person households and married or cohabiting couples making joint labour supply decisions. A proper model of the interaction between spouses in their labour supply decisions is important as most of the individuals are married or cohabiting. Third, by taking all details in the tax system into account the budget sets become complex and non-convex in certain intervals. For expository simplicity we consider in what follows only the behaviour of a single person household.

In the model, agents choose among jobs characterized by the wage rate *w*, hours of work *h* and other characteristics. The problem solved by the agent looks like the following:

(2.1) $$\max_{(w,h,j)\in B} U(c,h,j,e)$$

subject to the budget constraint $c = f(wh,m)$, where *h* denotes hours of work, *w* is the pre-tax wage rate, j and e indicates other respectively observed and unobserved job and/or household characteristics, *m* is the pre-tax non-labour income (exogenous), *c* is disposable income, *f(.,.)* represents the tax rule that transforms pre-tax incomes (*wh,m*) into net income c, *B* denotes the set of all opportunities available to the household (including non-market opportunities, i.e. a "job" with $\mathrm{w}=0$ and $\mathrm{h}=0$).

Agents can differ not only in their preferences and in their wage (as in the traditional model) but also in the number of available jobs of different type. Note that for the same agent, wage rates (unlike in the traditional model) can differ from job to job. As analysts we observe the chosen *h* and *w*, but we do not know exactly what opportunities are contained in *B*. Therefore we use a probability density function to represent *B*. Let $p(h,w,j)$ denote the density of jobs of type (h,w,j). By specifying a probability density function on *B* we can for example allow for the fact that jobs with hours of work in a certain range are more or less likely to be found, possibly depending on agents' characteristics; or for the fact that for different agents the relative number of market opportunities may differ. We assume that the utility function can be factorised as

(2.2) $$U\left(f(wh,m),h,j,e\right) = V\left(f(wh,m),h,j\right)e ,$$

where *V* and *e* are the systematic and the stochastic component, respectively. Moreover, we assume that e is i.i.d. according to:

(2.3) $$\Pr(e \le u) = \exp\left(-u^{-1}\right)$$

[2] Examples of previous applications of this approach are found in Aaberge, Dagsvik and Strøm (1995), and Aaberge, Colombino and Strøm (1999, 2000). The modeling approach used in these studies differs from the standard labour supply models by characterizing behaviour in terms of a comparison between utility levels rather than between marginal variations of utility. These models are close to other recent contributions adopting a discrete choice approach such as Dickens and Lundberg (1993), van Soest (1995) and Euwals and van Soest (1999).

The term *e* is a random taste-shifter that accounts for the effect on utility of all the characteristics of the household-job match observed by the household but not by us. It can be shown that under the assumptions (2.1), (2.2) and (2.3) we can write the probability density function of a choice (h,w,j) as[3]:

$$(2.4) \qquad j\,(h, w, j) = \frac{V\big(f(wh,m),h,j\big)p\,(h,w,j)}{\sum_{(x,y,z)\in B} V\big(f(xy,m),y,z\big)p\,(y,x,z)}.$$

The intuition behind expression (2.4) is that the probability of a choice (*h*,,*w*,*j*) can be expressed as the relative attractiveness – weighted by a measure of "availability" *p(h,w,j)* – of jobs of type (*h*,*w*,*j*).

The tax rule, however complex, enters the expression as it is, and there is no need to simplify it in order to make it differentiable or manageable as in the traditional approach. While the traditional approach derives the functions representing household behaviour on the basis of a comparison of marginal variations of utility, our approach is based on comparison of discrete levels of utility.

As explained in the Appendix, the model contains 78 parameters that capture the heterogeneity in preferences and opportunities among households and individuals. This version of the model is used to simulate the choices given a particular tax rule. Those choices are therefore generated by preferences and opportunities that vary across the decision units. For the purpose of welfare evaluation, however, we also estimate a model with a common utility function (comparable individual welfare function). It is this common utily function that is used to compute the individual welfare levels that will form the basis of the social welfare evaluation of tax reforms. More details, together with the estimates of the common utility are given in Secton 3.

[3] See Dagsvik (1994) and Aaberge et al. (1999), who provide two alternative methods for deriving (2.4).

3. Specification and estimation of individual welfare functions

As is universally recognized one needs to compare gains in welfare of some to losses in welfare of others when concern is turned to the distributional impact of a tax reform. It is non-controversial to assume that each individual's welfare increases with increasing income and leisure as is also captured by the household-specific utility functions. However, since some individuals live as singles whereas others form families and live together the estimated utility functions cannot be considered as comparable individual welfare functions. To solve the comparability problem we treat all individuals as singles and introduce an individual welfare function that is allowed to vary with age and number of children (at various ages), and where we adjust for scale economics in consumption by dividing couples' income by the square root of 2. The resulting income (y) is assumed to be enjoyed by each of the two adult partners. The formal definition of the individual welfare function is given by

(3.1)
$$\ln v(c,h,s) = g_2\left(\frac{c^{g_1}-1}{g_1}\right) + \left(g_4 + g_5 \log A + g_6 (\log A)^2 + g_7 s + g_8 C_1 + g_9 C_2 + g_{10} C_3 + g_{11} s C_1 + g_{12} s C_2 + g_{13} s C_3\right)\left(\frac{L^{g_3}-1}{g_3}\right)$$

where L is leisure, defined as $L = 1 - (h/8736)$, $s = 1$ if he/she works in the public sector (= 0 otherwise), A is age, C_1, C_2, and C_3 are number of children below 3, between 3 and 6 and between 7 and 14 years old, respectively, and y is the individual's income after tax defined by

(3.2)
$$c = \begin{cases} f(wh, m) & \text{for singles} \\ \frac{1}{\sqrt{2}} f(w_F h_F, w_M h_M, m) & \text{for couples.} \end{cases}$$

Since the possibility for realizing the various combinations of leisure and disposable income depend on the market opportunities, the impact of constraints in market opportunities has to be accounted for by the method used for estimating the parameters of the individual welfare functions. Thus, the density (2.4) where the systematic part of the utility function is replaced by the individual welfare function (3.1) may form the basis of the likelihood function. Note, however, that the estimated distributions of offered hours and wages will be inserted for p in (2.4). The estimated parameters for the individual welfare functions are reported in Table 3.1.

Table 3.1. Estimates of the parameters of the welfare functions for individuals 20 – 62 years old, Norway 1994

Variable	Parameter	Esimate	Stand.dev.
Consumption			
	g_1	-0.694	0.086
	g_2	3.155	0.144
Leisure			
	g_3	-11.862	0.590
	g_4	4.552	1.236
Log age	g_5	-2.425	0.666
Log age squared	g_6	0.326	0.090
# children, 0 – 2 years old	g_7	-0.015	0.007
# children, 3 – 6 years old	g_8	-0.010	0.006
# children, 7 – 14 years old	g_9	-0.003	0.004
Employed in public sector	g_{10}	-0.032	0.011
Empl. in pub. sec. * # child., 0 – 2 years old	g_{11}	0.045	0.030
Empl. in pub. sec. * # child., 3 – 6 years old	g_{12}	0.079	0.033
Empl. in pub. sec. * # child., 7 – 14 years old	g_{13}	0.039	0.016

The results in Table 3.1 demonstrate that the curvature parameters of the income and leisure terms are statistically significant and make these terms increasing concave. Moreover, the impact of leisure on individual welfare is found to depend on age and on the number of children at the age of 0-2 years. Moreover, leisure appears to be more important for people working in the public sector except for those with children at the age of 3-14 years. The latter effect may be due to the flexibility in hours of work arrangements in the public sector[4].

[4] Statistics Norway has, for example, more than 90 different hours of work arrangement. On top of that, many employees are allowed to spend up to three days of work in their home office.

4. The EO and EOp criteria

This informational structure of the individual welfare functions defined by (3.1) allows welfare gains and losses of different individuals due to a policy change to be compared. When evaluating the welfare effects of a tax system and/or a tax reform it may be useful to summarize the gains and losses by a social welfare function. The simplest welfare function is the one that adds up the comparable welfare gains (Vs) over individuals. The objection to the linear additive welfare function is that the households are given equal welfare weights, independent of whetherof they are poor or rich. Concern for distributive justice requires, however, that poor households are assigned larger welfare weights than rich households. This structure is captured by the following family of welfare functions that have their origin from Mehran (1976) and Yaari (the 1988)[5],

(4.1) $$W_k = \int_0^1 p_k(t)F^{-1}(t)dt, \quad k=1,2,...,$$

where F^{-1} is the left inverse of the cumulative distribution function of the individual welfare levels V with mean μ, and $p_k(t)$ is a weight function defined by

(4.2) $$p_k(t) = \begin{cases} -\log t, & k=1 \\ \dfrac{k}{k-1}\left(1-t^{k-1}\right), & k=2,3,.... \end{cases}$$

Note that the inequality aversion exhibited by W_k decreases with increasing k. As $k \to \infty$, W_k approaches inequality neutrality and coincides with the linear additive welfare function defined by

(4.3) $$W_\infty = \int_0^1 F^{-1}(t)dt = \mu.$$

It follows by straightforward calculations that $W_k \le \mu$ for all j and that W_k is equal to the mean μ for finite k if and only if F is the egalitarian distribution. Thus, W_k can be interpreted as the equally distributed individual welfare level. As recognised by Yaari (1988) this property suggests that I_k, defined by

(4.4) $$I_k = 1 - \frac{W_k}{\mu}, \quad k=1,2,...$$

can be used as a summary measure of inequality and moreover is a member of the "illfare-ranked single-series Ginis" class introduced by Donaldson and Weymark (1980). As noted by Aaberge (2000), I_1 is actually equivalent to a measure of inequality that was proposed by Bonferroni (1930), whilst I_2 is the Gini coefficient.[6] In this paper we will measure individual welfare level with a common utility function (see Section 3), although we will also present a comparative exercise where individual welfare is measure by monetary income.

For a given total welfare (i.e. the sum of individual welfare levels) the welfare functions W_1, W_2, and W_3 take their maximum value when everyone receives the same income and may thus be

[5] Several other authors have discussed rationales for this approach, see e.g. Sen (1974), Hey and Lambert (1980), Donaldson and Weymark (1980, 1983), Weymark (1981), Ben Porath and Gilboa (1992) and Aaberge (2001).

[6] For further discussion of the family $\{I_k: k=1, 2, ...\}$ of inequality measures we refer to Mehran (1976), Donaldson and Weymark (1980, 1983), Bossert (1990) and Aaberge (2000, 2001).

interpreted as Equality-of-Outcome criteria (EO) when employed as a measure for evaluating tax systems.

However, as indicated by Roemer (1998) the EO criterion is controversial and suffers from the drawback of receiving little support among citizens in a nation.[7] This is due to the fact that differences in outcomes resulting from differences in efforts are, by many, considered ethically acceptable and thus should not be the target of a redistribution policy. An egalitarian redistribution policy should instead seek to equalise those differentials in individual welfare arising from factors beyond the control of the individual. Thus, not only the outcome, but its origin and how it was obtained, matters. This is the essential idea behind Roemer's (1998) theory of equality of opportunity, where people are supposed to differ with respect to *circumstances*, which are attributes of the environment of the individual that influence her earning potential, and which are "beyond her control".

This study defines circumstances by family background, and classifies the individuals into three types according to father's years of education:

- less than 5 years (Type 1),
- 5-8 years (Type 2), and
- more than 8 years (Type 3).

Assume that $F_j^{-1}(t)$ is the welfare level level of the individual located at the t[th] quantile of the income distribution (F_j) of type j. The differences in welfare levels within each type are assumed to be due to different degrees of effort for which the individual is to be held responsible, whereas welfare differences that may be traced back to family background are considered to be beyond the control of the individual. As indicated by Roemer (1998) this suggests that we may measure a person's effort by the quantile of the welfare distribution where he is located. Next, Roemer declares that two individuals in different types have expended the same degree of effort if they have identical positions (rank) in the welfare distribution of their type. Thus, an EOp (Equality of Opportunity) tax policy should aim at designing a tax system such that $\min F_j^{-1}(t)$ is maximised for each quantile t. However, since this criterion is rather demanding and in most cases will not produce a complete ordering of the tax systems under consideration a weaker ranking criterion is required. To this end Roemer (1998) proposes to employ as the social objective the average of the lowest welfare levels at each quantile,

(4.5) $$\tilde{W}_\infty = \int_0^1 \min_j F_j^{-1}(t)dt$$

Thus, $\tilde{W}_\infty$ ignores income differences *within* types and is solely concerned about differences that arise from differential circumstances. By contrast, the EO criteria defined by (2.1) does not distinguish between the different sources that contribute to welfare inequality. As an alternative to (2.1) and (2.5) we introduce the following extended family of EOp welfare functions,

(4.6) $$\tilde{W}_k = \int_0^1 p_k(t)\min_j F_j^{-1}(t)dt,\ \ k = 1,2,...,$$

where $p_k(t)$ is defined by (2.2).

The essential difference between $\tilde{W}_k$ and $\tilde{W}_\infty$ is that $\tilde{W}_k$ gives increasing weight to the welfare of lower quantiles in the type-distributions. Thus, in this respect $\tilde{W}_k$ captures also an aspect of inequality within types. As explained above, the concern for within type inequality is greatest for the most disadvantaged type, i.e. for the type that forms the largest segment(s) of $\left\{\min_j F_j^{-1}(t) : t \in [0,1]\right\}$.

Note that $\min_i F_i^{-1}(t)$ defines the inverse of the following cumulative distribution function $\left(\tilde{F}\right)$

[7] See also Dworkin (1981a, 1981b), Arneson (1989, 1990), Cohen (1989) and Roemer (1993).

(4.7) $$\tilde{F}(x) = \Pr\left(\tilde{F}^{-1}(T) \leq x\right) = \Pr\left(\min_i F_i^{-1}(T) \leq x\right) = 1 - \prod_i \left(1 - F_i(x)\right),$$

where T is a random variable with uniform distribution function (defined on [0,1]). Thus, we may decompose the EOp welfare functions $\tilde{W}_k$ as we did the EOp welfare functions W_k. Accordingly, we have that

(4.8) $$\tilde{W}_k = \tilde{W}_\infty \left(1 - \tilde{I}_k\right), \quad k = 1,2,...$$

where $\tilde{I}_k$, defined by

(4.9) $$\tilde{I}_k = 1 - \frac{\tilde{W}_k}{\tilde{W}_\infty}, \quad k = 1,2,...$$

is a summary measure of inequality for the mixture distribution $\tilde{F}$.

Expression (4.8) shows that the EOp welfare functions $\tilde{W}_k$ for $k < \infty$ take into account value judgements about the trade-off between the mean income and the inequality in the distribution of welfare for the most EOp disadvantaged people. Thus, $\tilde{W}_k$ may be considered as an inequality within type adjusted version of the pure EOp welfare function that was introduced by Roemer (1998). As explained above, the concern for within type inequality is greatest for the most disadvantaged type, i.e. for the type that forms the largest segment(s) of the mixture distribution $\tilde{F}$. Alternatively, $\tilde{W}_k$ for $k < \infty$ may be interpreted as an EOp welfare function that, in contrast to $\tilde{W}_\infty$, gives increasing weight to individuals who occupy low effort quantiles.

Note that the EOp criterion was originally interpreted as more acceptable—from the point of view of individualistic-conservative societies. Our extended EOp welfare functions can be considered as a mixture of the EO welfare functions and the pure EOp welfare function; they are concerned about inequality between types as well as inequality within the worst-off distribution defined by (4.7). EOp looks at what happens to the distribution formed by the most disadvantaged segments of the intersecting type-specific distributions (defined by (4.7)). Moreover, the pure version of the criterion only looks at the mean of the worst-off distribution. By contrast, EO takes into account the whole income distribution. For a given sum of incomes, EO will consider equality of welfare (everyone attains the same level of welfare) as the most desirable welfare distribution. The pure EOp will instead consider equality in mean welfare across types as the ultimate goal. Since the extended EOp combines these two criteria, transfers that reduce the differences in the mean welfare between types as well as the welfare differentials between the individuals within the worst-off distribution are considered equalising by the extended EOp. Thus, in the case of a fixed total welfare also the extended EOp will consider equality of income as the most desirable distribution. However, by transferring money from the most advantaged type to the most disadvantaged type, EOp inequality may be reduced although transfers may be conflicting with the Pigou-Dalton transfer principle. Whether it is more "efficient" to reduce inequality between or within types depends on the specific situation. When labour supply responses to taxation are taken into account the composition of types in the worst-off distribution will change and depend on the chosen welfare function $\left(\tilde{W}_k\right)$ as well as on the considered tax rule. Thus, the large heterogeneity in labour supply responses to tax changes that is captured by our model(s) makes it impossible to state anything on EOp- or EO-optimality before the simulation exercises have been completed.

5. Optimal tax-transfer rules

The purpose of this section is to present an exercise where we locate the optimal tax rules given a fixed total net tax revenue, from the point of view of EO and EOp criteria. To this end we employ the labour supply model and simulation framework explained in section 2 and in the Appendix to simulate the labour supply behaviour of single females, single males, and couples that are between 18 and 54 years old. To capture the heterogeneity in preferences we have estimated three separate models of labour supply: one for single females, one for single males and one for couples.

The search for the optimal tax rule is limited to the class of 3-brackets, piecewise-linear rules, with

$$y = \begin{cases} Z \text{ if } Z \leq E \\ Z - t_1(Z-E) \text{ if } E < Z \leq \bar{Z}_1 \\ Z - t_1(\bar{Z}_1 - E) - t_2(Z - \bar{Z}_1) \text{ if } \bar{Z}_1 < Z \leq \bar{Z}_2 \\ Z - t_1(\bar{Z}_1 - E) - t_2(\bar{Z}_1 - \bar{Z}_2) - t_3(Z - \bar{Z}_2) \text{ if } \bar{Z}_2 < Z \end{cases} \quad (5.1)$$

where y is net available income, Z is gross income, E is the exemption level, (t_1, t_2, t_3) are the marginal tax rates applied to the three brackets of income above the exemption level, $\bar{Z}_1$ is the upper limit of the first bracket and $\bar{Z}_2$ is the upper limit of the second bracket. Thus, each particular tax rule is characterized by the six parameters: E, t_1, t_2, t_3, $\bar{Z}_1$ and $\bar{Z}_2$.

The tax rule specified by (5.1) replaces the current rule as of 1995, which is described in Table 5.1. All transfers implemented by welfare policies (social assistance, income support related to disability etc.) are kept unchanged under the alternative tax rules.

Table 5.1. Current tax rule in Norway as of 1995

Tax function for singles without children and couples without children and with two wage earners. NOK 1994.

Gross income	Tax
[0 – 17000)	0
[17000 – 24709)	0.25Y - 4250
[24709 – 28250)	0.078Y
[28250 – 140500)	0.302Y - 6328
[140500 – 208000)	0.358Y - 14196
[208000 – 234500)	0.453Y - 33956
[234500 –)	0.495Y - 43804

Tax function for couples without children and with one wage earner. NOK 1994

Gross income	Tax
[0 – 17000)	0
[17000 – 24709)	0.25Y - 4250
[24709 – 56500)	0.078Y
[56500 – 140500)	0.302Y - 12656
[140500 – 252000)	0.358Y - 20524
[252000 – 263000)	0.453Y - 44464
[263000 –)	0.495Y - 55510

The identification of optimal tax rules consists of six main steps:

1. The tax rule is applied to individual earners' gross incomes in order to obtain disposable incomes. New labour supply responses in view of a new tax rule are taken into account by the household labour supply model. Note that the utility functions (and choice sets) of the underlying microeconometric model(s) are stochastic. Thus, we use stochastic simulation to find, for each individual/couple, the optimal choice given a tax-transfer rule. The simulations are made under the conditions of constant total tax revenue and non-negative disposable household incomes.
2. To each decision maker (wife or husband) between 18 and 54 years old, an *equivalent income* is imputed, computed as total disposable household income divided by the square root of the number of household members.
3. As a result of the previous steps, we now have for each individual a simulated quadruple (c, h, j, e). We then compute the individual welfare levels by applying to the chosen (c, h, j, e) the common utility function (see section 3).
4. When adopting the Eop criterion we build the individual welfare distributions F_1, F_2 and F_3 for the types defined according to parental (actually father's) education: less than 5 years (type 1), 5-8 years (type 2) and more than 8 years (type 3).
5. Finally, we compute W_k and $\tilde{W}_k$ for k =1, 2, 3 and ∞.
6. Optimization is performed by iterating the above steps, in order to find the tax rule that produces the highest value of W_k or $\tilde{W}_k$ for each value of k under the constraint of unchanged tax revenue.

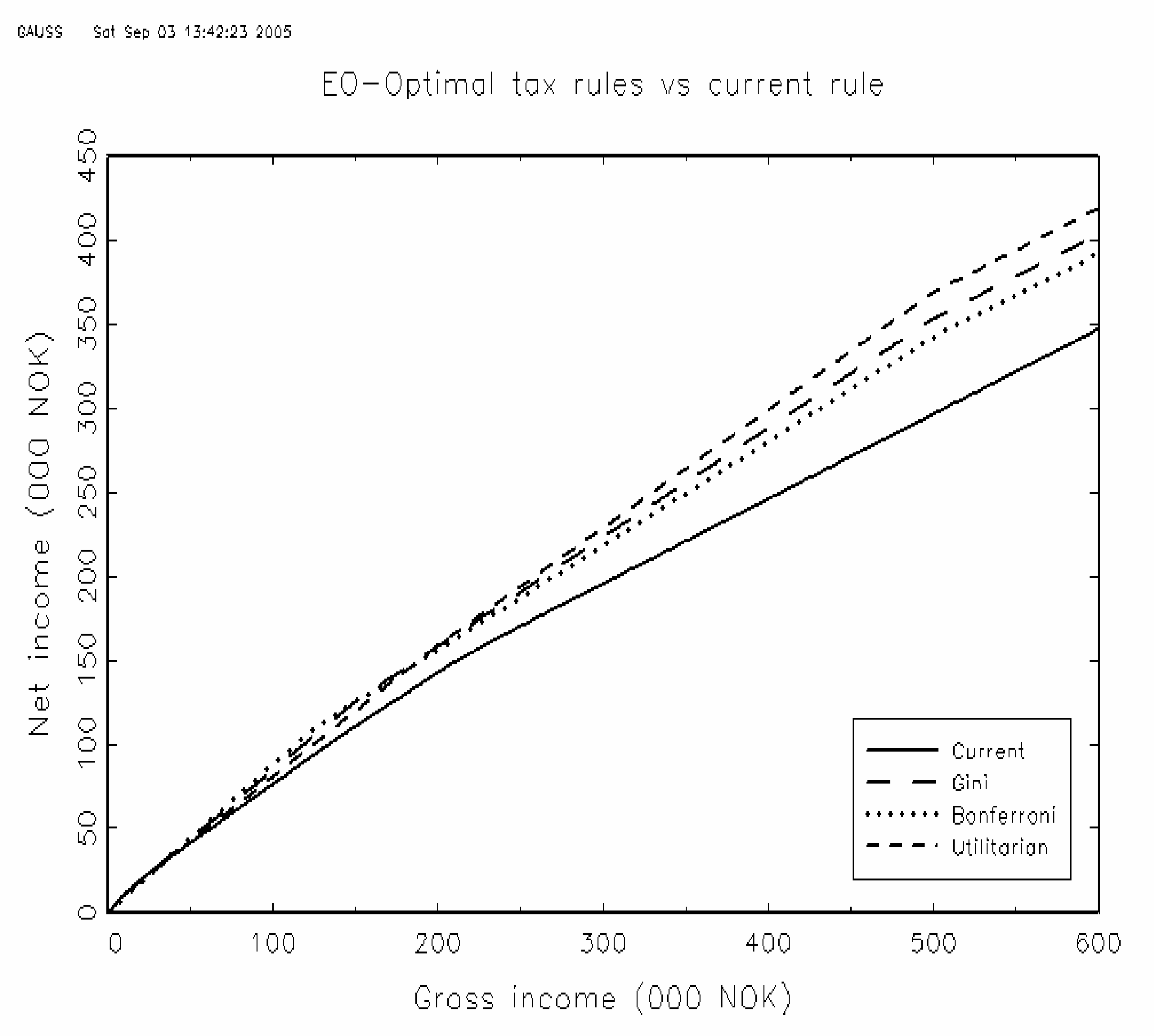
The results are reported in Table 5.2 and in Graphs 5.1 – 5.4. The first two graphs cover the whole range of income values, while graphs 5.3 and 5.4 zoom on low and average incomes levels.

- The Table and the Graphs show that the more egalitarian the criterion is, the more progressive is the optimal tax rule. For example the optimal rule according to Bonferroni is more progressive than the optimal rule according to Gini, which in turn is more progressive than the optimal utilitarian rule.
- The differences implied by using the EO or the EOp criterion seem negligible. This is interesting since EOp is usually interpreted as a less interventionist criterion than EO: still, when empirically implemented they both seem to require very similar tax rules, even slightly more progressive the one implied by EOp .
- Overall, the structure of the optimal rules is not dramatically different from the current rule: all the rules envisage a smooth sequence of increasing marginal tax rates.
- There are however also two important differences between the current and the optimal rules. First, all the optimal rules imply a higher net income for most levels of gross income. In other words, the optimal rules are able to extract the same total tax revenue from a larger total gross income (i.e. applying a lower average tax rate). The result is due to a sufficiently high labour supply response estimated and accounted for by the model. Second, the marginal tax rates applied to very high income are essentially identical in the current and in the optimal tax rule (around 50%). It is on low and average income brackets that the optimal rules apply markedly lower marginal rates as compared to the current rule.
- The last comment provides a controversial perspective in view of the tax reforms implemented in many developed countries during the last decades. In most cases those reforms embodied the idea of improving efficiency and labour supply incentives through a lower average tax rate and lower marginal tax rates on higher incomes. Our optimal tax computations give support to the first part (lowering the average tax rate), much less to the second: on the contrary our results suggest that a lower average tax rate should be obtained by lowering the marginal tax rates particularly on low and average income brackets.

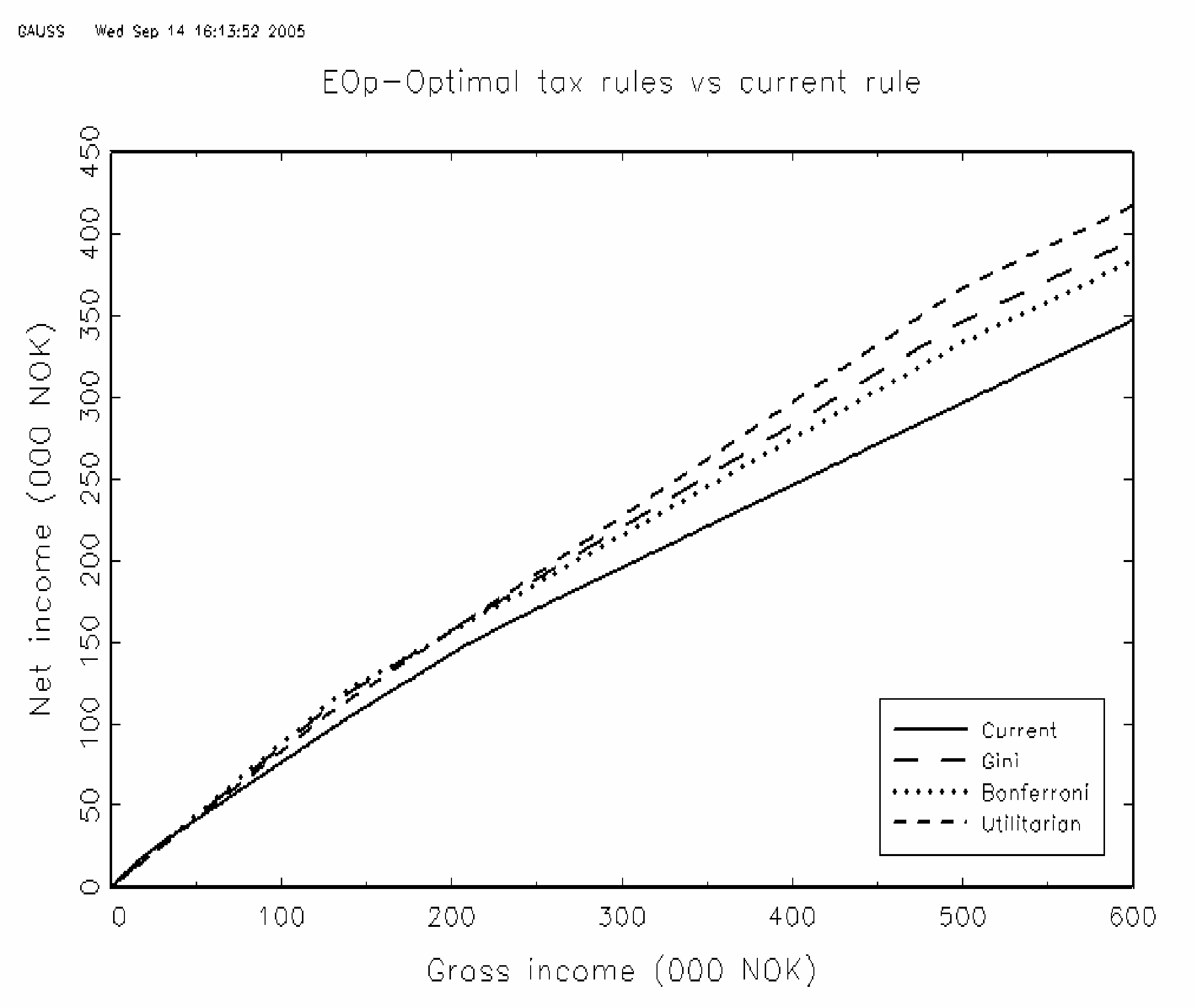
Table 5.2 Optimal tax rules according to alternative social welfare criteria

	EO-social welfare				**EOp-social welfare**			
	W_1 (Bonferroni)	W_2 (Gini)	W_3	W_∞ (Utilitarian)	$\tilde{W}_1$ (Bonferroni)	$\tilde{W}_2$ (Gini)	$\tilde{W}_3$	$\tilde{W}_\infty$ (Utilitarian)
t_1	.12	.18	.23	.22	.12	.14	.15	.17
t_2	.38	.35	.35	.30	.41	.37	.35	.30
t_3	.50	.50	.50	.50	.50	.50	.50	.50
E	0	11224	24436	11980	.0	.0	.0	.0
$\bar{Z}_1$	150000	150000	200000	200000	131880	134278	135178	128397
$\bar{Z}_2$	500000	500000	500000	500000	500000	500000	500000	500000

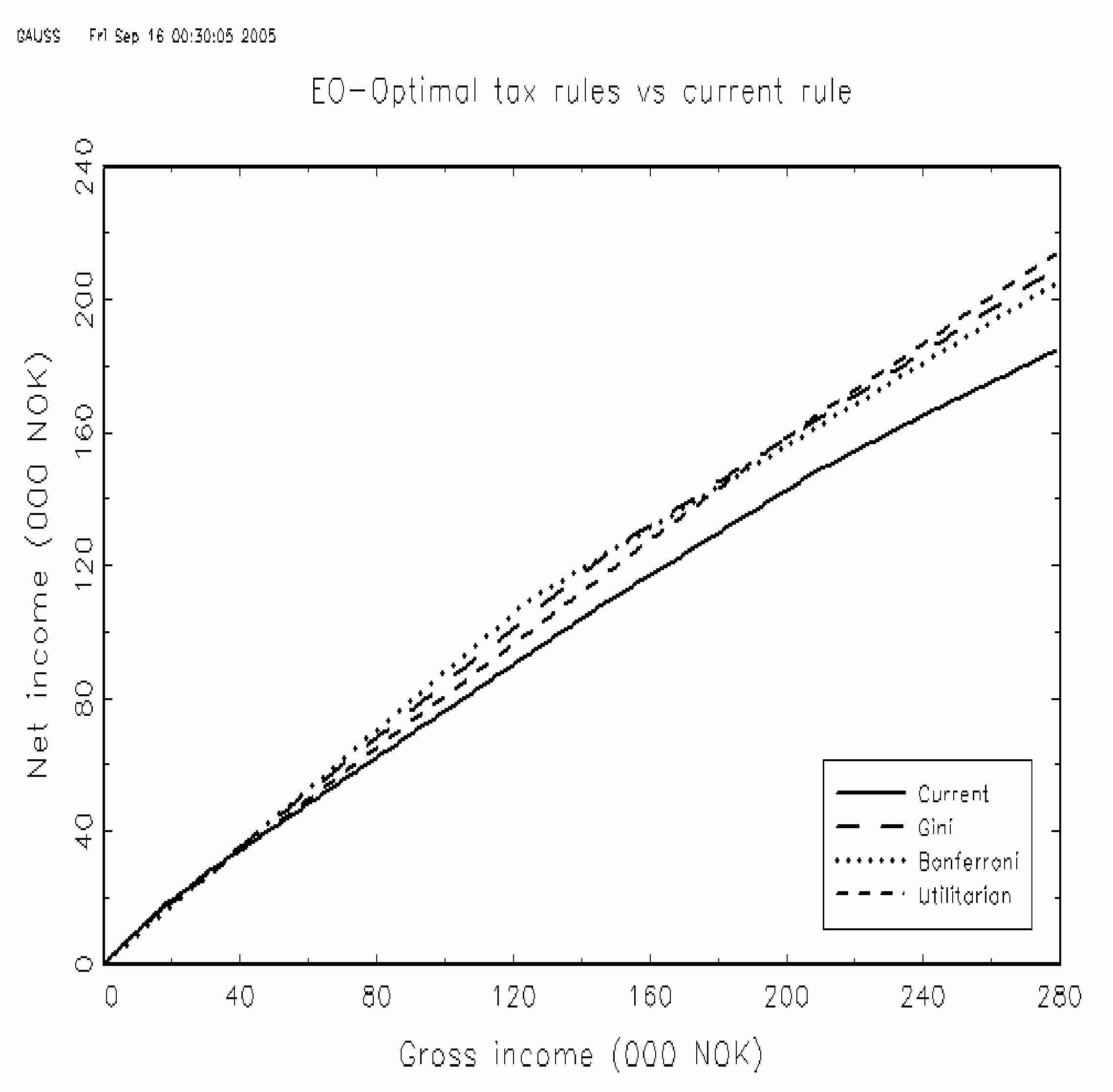
Graph 5.1


GAUSS Sat Sep 03 13:42:23 2005
EO-Optimal tax rules vs current rule
Net income (000 NOK)
0
50
100
150
200
250
300
350
400
450
0
100
200
300
400
500
600
Gross income (000 NOK)
Current
Gini
Bonferroni
Utilitarian

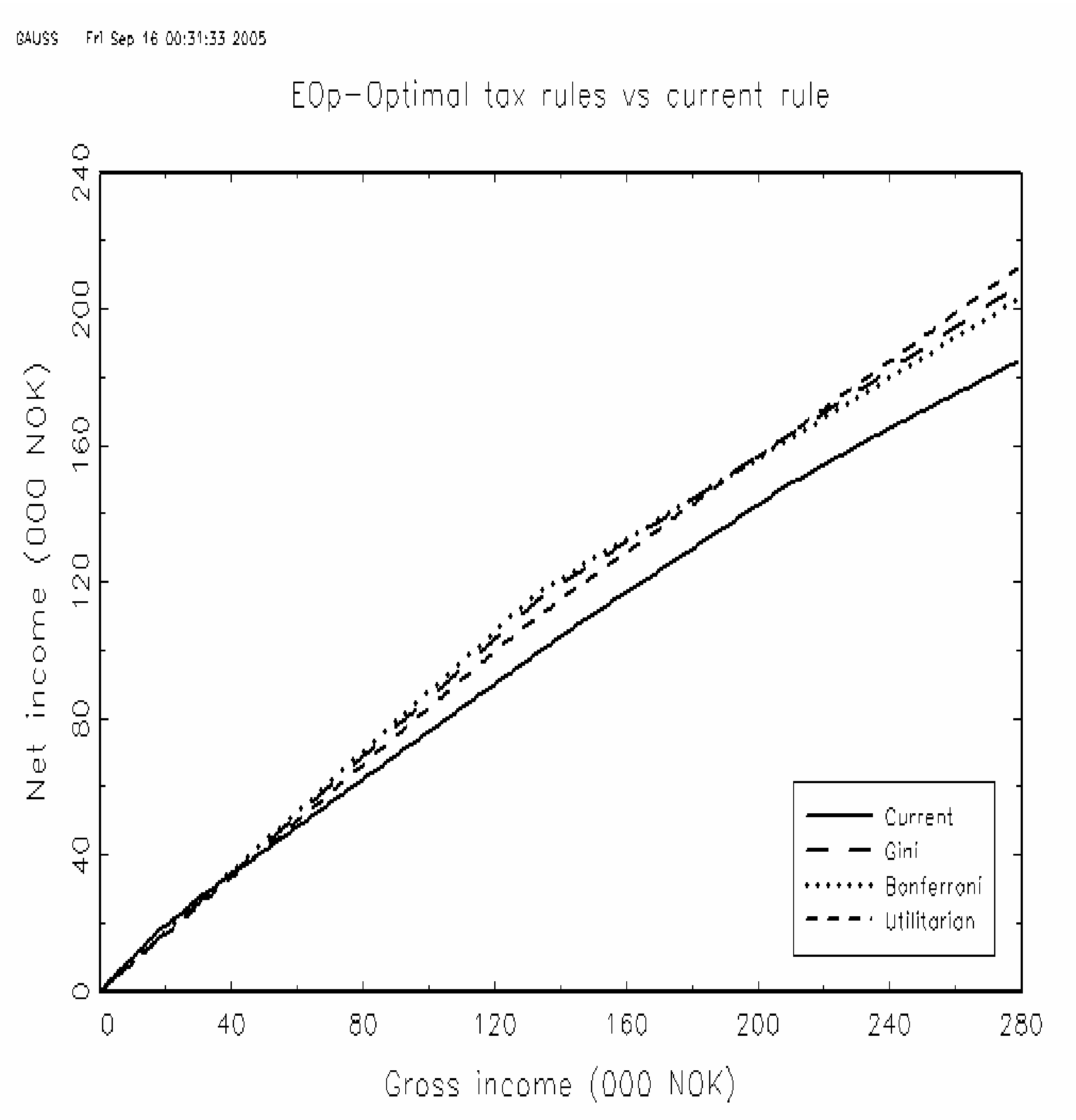
Graph 5.2


GAUSS Wed Sep 14 16:13:52 2005
EOp−Optimal tax rules vs current rule
Net income (000 NOK)
0
50
100
150
200
250
300
350
400
450
0
100
200
300
400
500
600
Gross income (000 NOK)
Current
Gini
Bonferroni
Utilitarian

Graph 5.3


GAUSS Fri Sep 16 00:30:05 2005
EO-Optimal tax rules vs current rule
Net income (000 NOK)
0
40
80
120
160
200
240
0
40
80
120
160
200
240
280
Gross income (000 NOK)
Current
Gini
Bonferroni
Utilitarian

Graph 5.4


GAUSS Fri Sep 16 00:31:33 2005
EOp-Optimal tax rules vs current rule
Net income (000 NOK)
Gross income (000 NOK)
0
40
80
120
160
200
240
280
Current
Gini
Bonferroni
Utilitarian

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Appendix

The micro-model - Empirical specification and estimation results

The modelling approach of this paper differs from the traditional textbook model by treating the utility function as a random variable and analysing labor supply as a random utility maximization problem. This framework can be considered as an extension of the standard multinomial logit model; see Dagsvik (1992) and Aaberge et al. (1999) for further details. For the sake of completeness we give a brief outline of this modelling framework.

To account for the fact that single individuals and married couples may face different choice sets and exhibit different preferences over income and leisure we estimate separate models for single females and males and married couples.

A.1. Single females and males

The utility functions for single females and males is assumed to be of the following form

$$U\left(f(wh,I),h,s\right)=v(h,w,s)e \qquad \text{A1.}$$

where

w = wage rate
h = hours of work
I = exogenous income
s = 1 if the job belongs to the Public Sector (= 0 otherwise),

$f\left(wh,I\right)$ is disposable income (income after tax) measured in 100 000 NOK

and e follows a Type III estreme value distribution.

The systematic part is specified as follows

$$\ln(v(h,w,s))=a_2\left(\frac{f(wh,I)^{a_1}-1}{a_1}\right)$$
$$+\left(a_4+a_5\log A+a_6\left(\log A\right)^2+a_7 s+a_8 C_1+a_9 C_2+a_{10} C_3+a_{11} sC_1+\right. \qquad \text{A2.}$$
$$\left.a_{12} sC_2+a_{13} sC_3\right)\left(\frac{L^{a_3}-1}{a_3}\right)$$

where

L is leisure, defined as $L = 1 - (h/8736)$,

A is age,

C_1, C_2, and C_3 are number of children below 3, between 3 and 6 and between 7 and 14 years old, respectively.

The parameters α are gender-specific.

The children terms are dropped in the utility function for single males since we observe very few children living with single males.

The stochastic components e are assumed to be independently drawn from a Type IIII extreme value distribution.

The individuals maximise their utility by choosing among opportunities defined by hours of work, hourly wage and sector of employment. Opportunities with $h = 0$ (and $w = 0$) are non-market opportunities (i.e. alternative allocations of "leisure").

We write the density of opportunities in sector s requiring h hours of work and paying hourly wage w

$$p(h,w,s) = \begin{cases} p_0 g_{1s}(h) g_{2s}(w) g_3(s) & \text{if} \quad h > 0 \\ 1 - p_0 & \text{if} \quad h = 0 \end{cases} \qquad \text{A3.}$$

where

p_0 is the proportion of market opportunities in the opportunity set;
$g_{1s,}$ g_{2s} and g_3 are respectively the densities of hours, wages, and opportunities in sector S, conditional upon the opportunity being a market job.

Given the above assumption upon the stochastic component and upon the density of opportunities, it turns out that the probability (density) that an opportunity (h,w,s) is chosen is

$$j(h,w,s) = \frac{v(h,w,s)\,p(h,w,s)}{\sum_{s=0,1} \int\int v(x,y,s)\,p(x,y,s)\,dxdy} \quad . \qquad \text{A4.}$$

In view of the empirical specification it is convenient to divide both numerator and denominator by $1 - p_0$ and define $g_0 = \frac{p_0}{1 - p_0}$. We can then rewrite the choice density as follows:

$$j(h,w,s) = \frac{v(h,w,s)\,g_0 g_{1s}(h) g_{2s}(w) g_3(s)}{v(0,0,0) + \sum_{s=0,1} \int_{x>0} \int_{y>0} v(x,y,s)\,g_0 g_{1S}(h) g_{2S}(w) g_3(s)\,dxdy} \qquad \text{A5.}$$

for $\{h, w\} > 0$ and

$$j\,(0,0,0) = \frac{v(0,0,0)}{v(0,0,0) + \sum_{s=0,1} \int_{x>0} \int_{y>0} v(x, y, s)\, g_0 g_{1s}(h) g_{2s}(w) g_3(s) dxdy} \qquad \text{A6.}$$

for $\{h, w\} = 0$

Except for possible peaks corresponding to part time (*pt*, 18-20 weekly hours) and to full time (*ft*, 37-40 weekly hours) we assume that the distribution of offered hours is uniformly distributed. Thus, g_1 is given by

$$g_{1s}(h) = \begin{cases} g_s & \text{if} \quad h \in [1,17] \\ g_s \exp(p_1 + p_2 s) & \text{if} \quad h \in [18,20] \\ g_s & \text{if} \quad h \in [21,36] \\ g_s \exp(p_3 + p_4 s) & \text{if} \quad h \in [37,40] \\ g_s & \text{if} \quad h \in [41, w] \end{cases} \qquad \text{A7.}$$

where w is the maximum observed value of h.

Since the density values must add up to 1, we can also compute g_s according to:

$$g_s \left((17-1) + (20-18) \right) \exp(p_1 + p_2 s) + (36-21) + (40-37) \exp(p_3 + p_4 s) + (w - 41) \big) = 1 .$$

We also specify:

$$g_0 g_3(s) = \exp\left(m_1 s + m_2 (1-s)\right) \qquad \text{A8.}$$

The above parameters p and m vary by gender. In the Tables we refer to p and m as the parameters of the *job opportunity density*.

The density of offered wages is assumed to be lognormal with mean that depends on length of schooling (*Ed*) and on past potential working experience (*Exp*), where experience is defined to be equal to age minus length of schooling minus five, i.e.

$$\log w = b_0 + b_1 Exp + b_2 Exp^2 + b_3 Ed + s\,h \qquad \text{A9.}$$

where h is standard normally distributed. The parameters b vary by gender and sector of employment.

The estimation of the models for single individuals and married couples is based on data from the 1995 Survey of Level of Living. For a more detailed description of the data and definition of variables we refer to the appendix. We have restricted the ages of the individuals to be between 18 and 54 in order to minimize the inclusion in the sample of individuals who in principle are eligible for retirement, since analysis of retirement decisions is beyond the scope of this study.

The parameters appearing in expressions (A1)-(A5) are estimated separately for single females and males by the method of maximum likelihood. The likelihood functions are equal to the products of the individual-specific labor supply densities for single females and males, respectively. The estimates of the preference and opportunity density parameters are reported in Table A1 and A3.

Table A1. Estimates of the parameters of the utility functions for single females and males. Norway 1994

Variable	Parameter	Single females		Single males	
		Estimate	Std. Dev.	Estimate	Std. Dev.
Consumption					
	α_1	-0.59	(0.28)	0.24	(0.33)
	α_2	4.37	(0.52)	2.27	(0.44)
Leisure					
	α_3	0.65	(0.92)	0.76	(0.99)
	α_4	498.50	(145.18)	337.40	(128.84)
Log age	α_5	-265.77	(79.22)	-180.89	(70.63)
Log age squared	α_6	36.36	(10.89)	24.81	(9.75)
# children, 0 – 2 years old	α_7	3.62	(2.43)		
# children, 3 – 6 years old	α_8	-0.36	(7.87)		
# children, 7 – 14 years old	α_9	-2.24	(1.42)		
Employed in public sector	α_{10}	-2.97	(0.87)	-2.20	(0.90)
Empl. in pub. sec. * # child., 0 – 2 years old	α_{11}	-7.29	(7.46)		
Empl. in pub. sec. * # child., 3 – 6 years old	α_{12}	-1.02	(2.10)		
Empl. in pub. sec. * # child., 7 – 14 years old	α_{13}	1.15	(1.10)		

A2. Married couples

The labor supply model for married couples accounts for both spouses' decisions through the following specification of the structural part of the utility function for couples

$$\log v\left(h_M, h_F, w_M, w_F, s_M, s_F\right) = a_2\left(\frac{f(w_F h_F, w_M h_M, I)^{a_1} - 1}{a_1}\right) +$$
$$\left(a_4 + a_5 \log A_F + a_6\left(\log A_F\right)^2 + a_7 s_F + a_8 C_1 + a_9 C_2 + a_{10} C_3 + a_{11} s_F C_1 + a_{12} s_F C_2 + a_{13} s_F C_3\right)\left(\frac{L_F^{a_{14}} - 1}{a_{14}}\right) +$$
$$\left(a_{15} + a_{16} \log A_M + a_{17}\left(\log A_M\right)^2 + a_{18} s_M + a_{19} C_1 + a_{20} C_2 + a_{21} C_3 + a_{22} s_M C_1 + a_{23} s_M C_2 + a_{24} s_M C_3\right)\left(\frac{L_M^{a_3} - 1}{a_3}\right)$$
$$+ a_{25}\left(\frac{L_M^{a_3} - 1}{a_3}\right)\left(\frac{L_F^{a_{14}} - 1}{a_{14}}\right).$$

A10.

where the leisure L_i is defined as $L_i = 1 - \left(h_i / 8736\right)$, i = F, M. We allow for sector- and gender-specific job opportunities in accordance with the functional forms ((A2)-(A6)) that were used for single females and males.

In this case the households choose among opportunities defined by a vector $\left(h_M, h_F, w_M, w_F, s_M, s_F\right)$. Here $S_k = 1$ if the partner of gender k is employed in the public sector, with k = M, F. Analogously to what we have done with singles, we specify the corresponding density function as

$$p\left(h_M, h_F, w_M, w_F, s_M, s_F\right) = \begin{cases} p_{0M}\, g_{1s_M}(h_M) g_{2s_M}(w_M) g_3(s_M)\, p_{0F}\, g_{1s_F}(h_F) g_{2s_F}(w_F) g_3(s_F) \text{ if } h_M > 0, h_F > 0 \\ p_{0M}\, g_{1s_M}(h_M) g_{2s_M}(w_M) g_3(s_M)\left(1 - p_{0F}\right) \text{ if } h_M > 0, h_F = 0 \\ \left(1 - p_{0M}\right) p_{0F}\, g_{1s_F}(h_F) g_{2s_F}(w_F) g_3(s_F) \text{ if } h_M = 0, h_F > 0 \\ \left(1 - p_{0M}\right)\left(1 - p_{0F}\right) \text{ if } h_M = 0, h_F > 0 \end{cases}$$

A11.

The choice density of an opportunity $\left(h_M, h_F, w_M, w_F, s_M, s_F\right)$ is:

$$j\left(h_M, h_F, w_M, w_F, s_M, s_F\right) = \frac{v\left(h_M, h_F, w_M, w_F, s_M, s_F\right) p\left(h_M, h_F, w_M, w_F, s_M, s_F\right)}{\sum_{s_M = 0,10} \sum_{s_F = 0,1} \iiiint v\left(x_M, x_F, y_M, y_F, s_M, s_F\right) p\left(x_M, x_F, y_M, y_F, s_M, s_F\right) dx_M\, dy_F\, dx_M \ldots}$$

A12.

For the purpose of empirical specification and estimation it is convenient to divide the density $p(\ )$ by $\left(1 - p_{0M}\right)\left(1 - p_{0F}\right)$ and define

$$g_{0M} = \frac{p_{0M}}{(1 - p_{0M})}$$

$$g_{0F} = \frac{p_{0F}}{(1 - p_{0F})} \qquad \text{A13.}$$

$$g_{0MF} = \frac{p_{0M}\, p_{0F}}{(1 - p_{0M})(1 - p_{0F})}$$

Now the choice density can be written as follows:

$$j\,(h_M, h_F, w_M, w_F, s_M, s_F) = \frac{v(h_M, h_F, w_M, w_F, s_M, s_F)\, g_{0MF}\, g_{1s_M}(h_M) g_{2s_M}(w_M) g_3(s_M) g_{1s_F}(h_F) g_{2s_F}(w_F) g_3(s_F)}{D}$$

A14.

if both work;

$$j\,(h_M, 0, w_M, 0, s_M, 0) = \frac{v(h_M, 0, w_M, 0, s_M, 0)\, g_{0M}\, g_{1s_M}(h_M) g_{2s_M}(w_M) g_3(s_M)}{D} \qquad \text{A15.}$$

if only the husband works;

$$j\,(0, h_F, 0, w_F, 0, s_F) = \frac{v(0, h_F, 0, w_F, 0, s_F)\, g_{0F}\, g_{1s_F}(h_F) g_{2s_F}(w_F) g_3(s_F)}{D} \qquad \text{A16.}$$

if only the wife works;

$$j\,(0,0,0,0,0,0) = \frac{v(0,0,0,0,0,0)}{D}$$

A17.

if none of them work, where we have defined

$$D = v(0,0,0,0,0,0) +$$

$$\sum_{s_M=0,1} \iint_{\substack{x>0\\y>0}} v(x_M, 0, y_M, 0, s_M, 0)\, g_{0M} g_{1s_M}(x_M) g_{2s_M}(y_M) g_3(s_M)\, dx_M\, dy_M +$$

$$\sum_{s_M=0,1} \iint_{\substack{x>0\\y>0}} v(0, x_F, 0, y_F, 0, s_F)\, g_{0F} g_{1s_F}(x_F) g_{2s_F}(y_F) g_3(s_F)\, dx_F\, dy_F +$$

$$\sum_{s_M=0,1} \sum_{s_F=0,1} \iiint\int_{\substack{x>0\\y>0}} v(x_M, x_F, y_M, y_F, s_M, s_F) g_{0MF}\, g_{1s_M}(x_M) g_{2s_M}(y_M) g_3(s_M) p_{0F} g_{1s_F}(x_F) g_{2s_F}(y_F) g_3(s_F)\, dx_M\, dy_F\, dx_M\, dy_M$$

A18.

The hours densities and the wage densities are the same as specified for singles. The same applies to $g_{0M}\, g_3(s_M)$ and $g_{0F}\, g_3(s_F)$. Moreover:

$$g_{0MF}\, g_3(s_M) g_3(s_F) = \exp(m_0 + m_{1M}\,(s_M) + m_{2M}\,(1 - s_M) + m_{1F}\,(s_F) + m_{2F}\,(1 - s_F)) \qquad \text{A19.}$$

The estimates of the parameters for couples are reported in Table A2 and A3.

Table A2. Estimates of the parameters of the utility function for married/cohabitating couples. Norway 1994

Variable	Parameter	Estimate	Std. Dev.
Consumption			
	α_1	0.14	(0.09)
	α_2	6.49	(0.43)
Wife's leisure			
	α_3	-3.81	(0.43)
	α_4	194.89	(28.53)
Log age	α_5	-107.09	(15.88)
Log age squared	α_6	15.14	(2.23)
# children, 0 – 2 years old	α_7	0.34	(0.31)
# children, 3 – 6 years old	α_8	1.31	(0.31)
# children, 7 – 14 years old	α_9	1.70	(0.26)
Employed in public sector	α_{10}	-0.95	(0.30)
Empl. in pub. sec. * # child., 0 – 2 years old	α_{11}	0.40	(0.33)
Empl. in pub. sec. * # child., 3 – 6 years old	α_{12}	0.39	(0.32)
Empl. in pub. sec. * # child., 7 – 14 years old	α_{13}	-0.97	(0.24)
Husband's leisure			
	α_{14}	-1.01	(039)
	α_{15}	222.99	(41.03)
Log age	α_{16}	-116.55	(22.34)
Log age squared	α_{17}	15.85	(3.06)
# children, 0 – 2 years old	α_{18}	-0.08	(0.40)
# children, 3 – 6 years old	α_{19}	-0.30	(0.35)
# children, 7 – 14 years old	α_{20}	-0.15	(0.25)
Employed in public sector	α_{21}	-0.60	(0.51)
Empl. in pub. sec. * # child., 0 – 2 years old	α_{22}	-0.16	(0.39)
Empl. in pub. sec. * # child., 3 – 6 years old	α_{23}	-0.93	(0.31)
Empl. in pub. sec. * # child., 7 – 14 years old	α_{24}	-0.16	(0.25)
Leisure interaction between spouses	α_{25}	4.84	(1.12)

*) Standard deviations in parentheses.

Table A3. Job, Hours and Wage densities, Norway 1994

		Females		Males	
	Parameter	Estimate	Std. Dev.	Estimate	Std. Dev.
Job opportunity					
	μ_1	-2.10	(0.18)	-3.17	(0.23)
	μ_2	-1.51	(0.18)	-2.68	(0.20)
	μ_3	1.39	(0.17)	1.39	(0.17)
Hours					
	p_1	0.49	(0.13)	-0.50	(0.22)
	p_2	-0.23	(0.23)	0.09	(0.51)
	p_3	1.47	(0.09)	1.81	(0.07)
	p_4	0.03	(0.14)	0.06	(0.13)
Wage – Private sector					
	b_0	3.62	(0.07)	3.50	(0.06)
	b_1	3.93	(0.50)	5.38	(0.41)
	b_2	2.60	(0.30)	2.83	(0.31)
	b_3	-4.04	(0.64)	-4.41	(0.64)
	s	0.24	(0.00)	0.28	(0.01)
Wage - Public sector					
	b_0	3.71	(0.08)	3.62	(0.09)
	b_1	3.59	(0.46)	4.95	(0.47)
	b_2	2.14	(0.33)	2.46	(0.44)
	b_3	-3.37	(0.71)	-3.82	(0.91)
	s	0.18	(0.01)	0.22	0.01