A VERY SIMPLE MODEL OF CONFLICT WITH ASYMMETRIC EVALUATIONS AND INSTITUTIONAL CONSTRAINT

by

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This paper considers a partial equilibrium model of conflict where two asymmetric, rational and risk-neutral opponents conflict in order to appropriate a positive fraction of a stake. An institutional constraint is modelled through an exogenously fixed element as a feature of a modified Contest Success Function.
However, this very simple model shows that even if conflicting agents are willing to commit themselves to ease the conflict joining an institutional setting they do not 'disarm'. The findings of the model show that: (a) both agents prefer to settle under an institutional constraint if and only if an exogenous institutional fee is fixed under a critical value; (b) the critical value of the institutional fee is directly related to the evaluation of the stake each agent does retain; (c) the agents with a higher evaluation of the stake has a higher willingness to settle.

Jel Code: D50, D74, H8

Keywords: Contest Success Function, Institution, Conflict, Settlement.

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1. INTRODUCTION

This paper considers a partial equilibrium model of conflict where two rational and risk-neutral opponents conflict in order to appropriate a positive fraction of a stake. The pioneering work on modelling conflict in recent economic literature is by Jack Hirshleifer, whose foundations are in Hirshleifer (1987, 1988, 1989). The economic theory of conflict[1] rests to a large extent upon the assumption that agents involved in conflict interactions have to choose an optimal level of efforts or resources devoted to the unproductive activity of conflicts.

Economic literature on conflict is also akin to both with rent-seeking and contest models. The main difference between the conflict models and the rent-seeking and contest models is that conflict models are generally general equilibrium models whilst the latter are partial equilibrium models. This means that conflict models involve a trade-off between productive and unproductive activities and the contested prize (or the rent) is endogenous. That is, the stake of the conflict is interpreted as a joint production which depends upon the productive efforts of agents. At the same time the cost function is represented by the foregone production. In such a construction the larger the number of the agents the larger will be the 'pie' to be shared. Instead, in rent-seeking and contest models the prize (or the rent) is given exogenously. In such a case, even if the number of contestants becomes larger the rent does not change.[2]

However, conflicts, contests and rent-seeking can be considered directly unproductive activities (DUP) in the spirit of the definition provided by Bhagwati (1982) that proposes a general taxonomy for a broader range of economic activities which represent ways of making profit despite being directly unproductive. This is rationale behind the labelling *directly unproductive profit-seeking activities* (DUP). Their output is clearly zero in terms of the flow of goods and services entering a conventional utility function.

This paper is intended to extend the literature on this subject dealing with two main points: (1) the incentives to settle peacefully

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[1] In more recent years several studies extended Hirshleifer's basic model. See among others: Grossman (1991), Skaperdas (1992), Garfinkel (1994), Grossman and Kim (1995), Neary (1997), Anderton and Carter (1999), Garfinkel and Skaperdas (2000).

[2] For a deep comparison of conflict and rent-seeking models see Hauken (2005).

under an institutional constraint; (2) the impact of asymmetry in evaluations on such willingness to settle.

The following framework the stake of the conflict is exogenously fixed as in rent-seeking and contest models. The stake of the conflict can be interpreted in different ways. It might be for example a contested natural resource, a territory or a homogenous input. It is assumed that the evaluation of the stake is different. In particular, agent 1 has a higher evaluation than agent 2. Being a partial equilibrium model with an exogenous prize, it would fall into the category of rent-seeking and contest models. However, what this paper draws from literature on conflict is the destructiveness of conflict itself. That is, even if agents contest over a positive stake they are able to get only a fraction of the contestable stake. Then, differently from rent-seeking and contest models throughout this paper I will assume that conflict is destructive. That is, a positive fraction of the contested stake will be destroyed in the conflict and an exogenous parameter will be used in order to capture this impact. The expected destruction of conflict captures exactly the idea of opportunity cost of conflict. Moreover, differently from rent-seeking and contest models there is no winner-take-all mechanism. In particular, under the assumption of risk-neutrality, agents interpret the outcome of the non-cooperative interaction as deterministic, giving each party control over a positive fraction of the contested stake in order to maximise its own payoff.

Although this partial equilibrium model will not consider the traditional trade-off between 'guns' and 'butter' for sake of simplicity throughout the paper I will call 'guns' the efforts invested in appropriative activity. In a similar fashion I will call 'pie' the positive stake of the conflict.

The remainder of the paper is organised as follows: In the first section the basic hypothesis, definitions and model are presented. In the second section, the basic model is extended following Garfinekl and Skaperdas (2000) in order to show the incentives to settle considering also the existence of an institutional constraint. In a third section, by means of comparative statics analysis the incentives to settle will be investigated. Eventually, the concluding remarks summarise the results and present suggestions for future research agenda.

2. THE CONFLICT INTERACTION

There are two risk-neutral agents indexed by $i = 1,2$. They conflict over a positive stake denoted by $V_i \in (0, \infty), i = 1,2$. The agents have different evaluations of the stake in the conflict. Then, in this two-agents model it is possible to write that $V_i \neq V_j; \forall i \neq j; i,j = 1,2$. In order to appropriate the maximum positive fraction of the contested stake, say the 'pie', both agents expend efforts, say 'guns'. Let $z_i, i = 1,2$ denote the amount of violent efforts. The 'guns' are measured in the same units as the 'pie'. The conflict is destructive, namely a destruction parameter denoted $\beta \in (0,1)$ captures the foregone fraction of the positive stake due to the violent activity. In other words, as β increases, the conflict becomes less and less destructive. The destruction parameter can be interpreted as an *ex-ante* perceived evaluation of conflict losses. Both agents share the same perception of expected destruction. Then, in their general form the payoff functions can be written as $\pi_i = \pi_i(z_i, V_i, \beta), i = 1,2$.

The outcome of the conflict is determined through a Contest Success function (hereafter CSF for brevity). It summarizes the relevant aspects of what Hirshleifer defines the *technology* of conflict. In particular, even if the CSF can take different forms, I apply the *ratio* form of the CSF.[3]

$$p_i(z_i, z_j) = \frac{z_i}{z_i + z_j} \qquad \text{for } i = 1,2 \text{ and } j \neq i \tag{1}$$

where, under the assumption of risk-neutrality p_i denotes the proportion of appropriation going to agent i for $i = 1,2$ and follows the conditions below:

$$\begin{cases} p(0,0) \equiv 1/2 \\ p_1 + p_2 = 1 \\ p(.,.) \text{ is twice differentiable} \\ \partial p_i / \partial z_i > 0 \qquad \partial p_i / \partial p_j < 0 \\ \partial^2 p_i / \partial z_i^2 \leq 0 \qquad \partial^2 p_i / \partial z_j^2 \geq 0 \end{cases} \tag{1.1}$$

Eventually, each agent's payoff function is given by:

$$\pi_1 = \frac{\beta z_1}{z_1 + z_2} V_1 - z_1 \tag{2.1}$$

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3 Hirshleifer (1989) analyses the different impact of two different function form for CSF: the *ratio form* and the *logistic form*. In the first case, the contest outcome depends upon the ratio of the efforts applied, whilst in the second case it depends upon the difference between the resources committed.

$$\pi_2 = \frac{\beta z_2}{z_1 + z_2} V_2 - z_2 \tag{2.2}$$

I assume $V_1 > V_2 > 0$. That is, player 1 has a higher evaluation than player 2. Expending efforts in violence is costly according to a simple cost function $c(z_i) = z_i, i = 1,2$. The first order conditions for an interior Nash equilibrium in the maximization mechanism are given by:

$$\frac{\partial \pi_1}{\partial z_1} = \frac{\beta z_2 V_1}{(z_1 + z_2)^2} - 1 = 0 \tag{3.1}$$

$$\frac{\partial \pi_2}{\partial z_2} = \frac{\beta z_1 V_2}{(z_1 + z_2)^2} - 1 = 0 \tag{3.2}$$

Solving the first order conditions it is possible to get the equilibrium level of 'guns' for both agents:

$$z_1^* = V_1 \frac{V_1 V_2}{(V_1 + V_2)^2} \beta \tag{4.1}$$

$$z_2^* = V_2 \frac{V_1 V_2}{(V_1 + V_2)^2} \beta \tag{4.2}$$

Since $V_1 > V_2 > 0$ it follows that $z_1^* > z_2^*$. That is, the agent with a higher evaluation of the stake will expend more violent efforts than the opponent. The payoffs for both agents are given by:

$$\pi_1^* = \frac{V_1^3 \beta}{(V_1 + V_2)^2} \tag{5.1}$$

$$\pi_2^* = \frac{V_2^3 \beta}{(V_1 + V_2)^2} \tag{5.2}$$

The agent with a higher evaluation of the stake will have a higher payoff than the opponent, namely $\pi_1^* > \pi_2^*$.

2. Conflict and Settlement with an Institutional Constraint

Garfinkel and Skaperdas (2000) propose a model where the contestable stake can be disposed in one of two ways: through conflict or through a peaceful and predefined division of the 'pie'. The exploitation of violence plays a role in both cases: in case of breakout of a violent conflict, 'guns' would determine directly the positive fraction and then the attainable payoffs for each party; in case of a settlement they constitute a credible threat and they influence each party's negotiating position and therefore again the share of the 'pie' ad the attainable payoffs of parties. Hence each agent's share of the 'pie' will be a weighted combination of two possible rules: (i) the CSF denoting the technology of conflict and (ii) a symmetric split-of-surplus rule of division which is commonly indicated in the literature on bargaining as the appropriate axiomatic outcome. The relative weights are determined by the destruction parameter denoted by $\beta \in (0,1)$. According to this construction, when a settlement occurs, it does arise only *in the shadow of conflict* borrowing an expression from Anderton and Carter.

Caruso (2006) extends the Garfinkel-Skaperdas model in order to consider the existence of an institution committed to conflict management and resolution. The agents are assumed to join an institutional set of rules by giving up a certain amount of resources that would be spent for violent efforts. This behaviour is intended to modify the payoff functions of both agents. Throughout this paper I will call h the *institutional fee*. For analytical simplicity, it is assumed to be exogenous and equal for both contestants. Being exogenous, the *institutional fee* does fit more with a scenario where an institution already does exist and third parties are allowed to join it. At the same time it could also be interpreted as an exogenous reciprocal concession fixed by a mediator. Anyway, paying this *institutional fee*, agents also signal the intention to comply with obligations emerging under an institutional regime. The intuition behind the nature of such *institutional fee* relates to Boulding's idea of 'grant economy' in integrative systems.[4] The reduction of resources devoted to unproductive 'guns' is also a pillar of contractarian approach as expounded in Skogh and Stuart (1982).

Let $h \in (0,\infty)$ denote the positive *institutional fee*. Therefore, the payoff functions now can be described in their general form as $\pi_i^{ins} = \pi_i^{ins}(z_i, V_i, \beta, h)$. The payoff function has as cornerstone a modified version of the CSF:

$$p_i(z_i, z_j, h) = \frac{z_i - h}{z_i + z_j - 2h} \qquad \text{for } i = 1,2 \text{ and } j \neq i \qquad (6)$$

[4] Boulding expounded in a comprehensive manner the theory of 'grants economics' in Boulding et al. (1972) and in Boulding (1973).

which follows the condition presented in "(1.1)" and also:

$$\begin{cases} \dfrac{\partial p_i}{\partial h} > 0 \quad \Leftrightarrow \quad z_i > z_j \\ \dfrac{\partial^2 p_i}{\partial^2 h} > 0 \quad \Leftrightarrow \quad z_i > z_j \\ \lim\limits_{h \to \infty} \left(\dfrac{z_i - h}{z_i + z_j - 2h} \right) = \dfrac{1}{2} \end{cases} \tag{6.1}$$

Then, the payoffs function in their functional specification become:

$$\pi_1^{ins}(z_1, V_1, \beta, h) = \left[\frac{\beta(z_1 - h)}{z_1 + z_2 - 2h} + \frac{(1-\beta)}{2} \right] V_1 - z_1 - h \tag{7.1}$$

$$\pi_2^{ins}(z_2, V_2, \beta, h) = \left[\frac{\beta(z_2 - h)}{z_1 + z_2 - 2h} + \frac{(1-\beta)}{2} \right] V_2 - z_2 - h \tag{7.2}$$

where the superscripts $'ins'$ denote the institutional scenario. In the CSF, h can be considered a constant vector that affects the ordinary outcome of the contests.[5] In the modified CSF the *institutional fee* decreases the impact of expenditures devoted to 'guns'. The first order conditions for the maximization problem in this case are:

$$\frac{\partial \pi_1^{ins}}{\partial z_1} = \frac{\beta(z_2 - h)V_1}{(z_1 + z_2 - 2h)^2} - 1 = 0 \tag{8.1}$$

$$\frac{\partial \pi_2^{ins}}{\partial z_2} = \frac{\beta(z_1 - h)V_2}{(z_1 + z_2 - 2h)^2} - 1 = 0 \tag{8.2}$$

The second order conditions for a maximum are:

$$\frac{\partial^2 \pi_1^{ins}}{\partial^2 z_1} = -2\beta(z_2 - h)\frac{V_1}{(z_1 + z_2 - 2h)^3} < 0 \tag{9.1}$$

$$\frac{\partial^2 \pi_2^{ins}}{\partial^2 z_2} = -2\beta(z_1 - h)\frac{V_2}{(z_1 + z_2 - 2h)^3} < 0$$

(9.2)

5 As pointed out by Skaperdas (1996) this is not true for 'logit' form of CSF.

Solving the first order conditions the interior Nash equilibrium level of violent efforts for both agents are given by:

$$z_1^* = V_1 \frac{V_1 V_2}{(V_1 + V_2)^2} \beta + h \tag{10.1}$$

$$z_2^* = V_2 \frac{V_1 V_2}{(V_1 + V_2)^2} \beta + h \tag{10.2}$$

As above, it is simple to verify that $z_1^* > z_2^*$. Moreover, since $h > 0$ by assumption the equilibrium level of guns after agents joined an institution is higher than in other scenario. Thus, the model suggests the following result:

PROPOSITION 1: *If agents are asymmetric in evaluations of the stake and are both committed to join an institution through the disbursement of a positive equal and exogenous institutional fee, then (i) the agents with a higher evaluation of the stake will spend more in violent efforts than the opponent; (ii) both agents will expend more violent efforts than in pure conflict scenario.*

The Nash-equilibrium payoffs (denoted by stars superscripts on the left) are given by:

$$^*\pi_1^{ins} = \frac{V_2^2 \beta (3V_1 + 2V_2)}{(V_1 + V_2)^2} + \frac{V_1(\beta + 1)}{2} - 2(V_2 \beta + h) \tag{11.1}$$

$$^*\pi_2^{ins} = \frac{V_2^3 \beta}{(V_1 + V_2)^2} - \frac{V_2(\beta - 1) + 4h}{2} \tag{11.2}$$

It would be simple to verify that also in this case $^*\pi_1^{ins} > {}^*\pi_2^{ins}$. That is, the agent with a higher evaluation of the stake will arm more than the rival.

3. COMPARATIVE STATICS FOR A SETTLEMENT CONDITION

Given the assumption of rationality agents will choose the strategy in order to maximize their payoffs. Hence, it is possible to say that each agent would prefer to settle with an institutional constraint if and only if the attainable payoffs are greater than payoffs achievable under pure conflict, namely if and only if:

$$ {}^{*}\pi_i^{ins} > \pi_i^{*}, i = 1,2 \qquad (12) $$

Inequality "(12)" does identify a *settlement condition*. That is, whenever it holds the agents will choose to settle with an institutional constraint although they still arm. Therefore, comparing equations "(5)" and "(11)", re-arranging and manipulating the settlement condition becomes:

$$ h_i * < V_i \frac{(1-\beta)}{4}, i = 1,2 \qquad (13) $$

Inequality (12) expresses the settlement condition in relation to the *institutional fee*. More precisely, the settlement condition holds if and only if the level of *institutional fee* is below a critical value $h*$. This can be interpreted in a broader sense as the willingness to settle of each agent. The critical value of each agent depends upon the valuations of the 'pie' it retains. Therefore, agents with different evaluations of contestable stake retain different willingness to settle. Then, since $V_1 > V_2$ it would simple to say that $h_1^* > h_2^*$. Therefore, a settlement between parties under an institution will take place if only if the exogenous membership fee is fixed below the critical value for the party with a lower fraction of the contestable output.

PROPOSITION 2: *under an institution (a) both agents prefer to settle under an institutional constraint if and only if the exogenous institutional fee is fixed under a critical value; (b) the critical value of the institutional fee is directly related to the evaluation of the stake each agent does retain; (c) the agents with a higher evaluation of the stake has a higher willingness to settle.*

4. CONCLUDING REMARKS

This very simple model showed that even if conflicting agents are willing to commit themselves to solve the conflict through joining an institutional setting of rules they do not 'disarm'. The reasonable explanation is in the traditional explanation for non-cooperative Nash equilibria. No agent is going to disarm because if one agent is going to lower its expenditure in 'guns' the other would prefer to increase it in order to appropriate a higher fraction of the contested 'pie'.

However, at the same time both agents interpret as rational the strategy of settling under an institutional set of rules committed to solve the conflict. Such a strategy is feasible if and only if an exogenously-fixed *institutional fee* lies below a critical value. The critical value captures in some sense the willingness to settle of agents. Under the assumption of different evaluations of the contestable 'pie' agents show different willingness to settle. In particular, the agent with a higher willingness to settle is also the agent with a higher level of 'guns'. For sake of simplicity, call this equilibrium 'armed settlement'.

Finally, this short paper is of course little more than a 'spare part' of a larger study to be developed. There is a large agenda for future research, mainly concerning how to relax some simplifying assumptions I employed throughout the work: (i) only two party interaction; (ii) full information; (iii) a not-decisive technology of conflict; (iv) no distinction between offensive and defensive technologies; (v) agents as unitary actors; (vi) risk-neutrality of both actors; (vii) no definition of market structure and prices. The formalization presented here is intended to be used towards the construction of broader models. The aim of the larger study is to enrich the theory of exchange between asymmetric economic agents taking into account the existence of appropriative activities from the start. Brilliant analyses have been presented by some scholars as Anderton and Carter (1999) and Anderson and Marcouiller (1997), but much remains to be done. This line of theoretical analysis, which considers conflict from the beginning, can have remarkable implications for the designing of economic policies in societies where conflict is a characteristic element. Consider for instance, the case of post-conflict societies, some LDC countries or mafia-infiltrated states.

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