Agenda Networks and Farsightedly Stable Agenda Formation*

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Abstract

We model the agenda formation process as a network. In an *agenda network* nodes represent agendas while arcs represent coalition preferences over agendas and coalitional moves from one agenda to another. We show that all agenda networks have agenda nodes which are farsightedly consistent. These nodes represent agendas which are likely to emerge and persist if agents behave farsightedly in forming agendas. We demonstrate the usefulness of our approach by computing the farsightedly consistent agendas for three examples of agenda networks.

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1 Introduction

The potential existence of cycles under majority rule has arguably been *the* central preoccupation in the political economy literature (Condorcet, 1785, Arrow, 1963, and McKelvey, 1976). "Chaos" and manipulability are the twin interpretations of the existence of cycles: anything may happen or, alternatively, an election's ultimate outcome may be determined by a skilled agenda setter.

Yet, as Tullock (1981) pointed out, even in the absence of a manipulative agenda setter, observed political outcomes have been surprisingly stable. A potential explanation for such divergence between theory and facts may be the endogeneity of the proposed or amended bills or agendas that legislators choose to vote on.

Indeed, in a typical democratic legislative process, legislators determine successful agendas (that is, bills that are likely to pass) through the formation of implicit or explicit coalitions. This first stage of the political process is far less understood than the actual voting stage, yet it is this key stage, one hopes, that might ultimately allow for a more narrow prediction of the aggregation process from individual preference profiles to political outcomes.

Unfortunately, agenda formation has resisted formal game-theoretic treatments because such treatments would have to be protocol-specific with the decision-making sequence preset. Put bluntly, the complexity and fuzziness of the process have defied the ability to construct formal and tractable game-theoretic models of agenda formation. Our main contribution is to simplify the complexity and clear away the fuzziness by modeling the agenda formation process as a directed network. In an *agenda network* each node represents a particular agenda, and depending on coalition preferences and the rules of agenda formation, each pair of agenda nodes can be connected by two types of arcs: move arcs and preference arcs. In an agenda network, a move arc belonging to coalition c (denoted by m_c) running *from* agenda a_0 *to* agenda a_1 (denoted, $a_0 \xrightarrow{m_c} a_1$) represents the fact that under the rules of agenda formation, coalition c can change agenda a_0 to agenda a_1. A preference arc belonging to coalition c (denoted by p_c) running *from* agenda a_0 *to* agenda a_1 (denoted, $a_0 \xrightarrow{p_c} a_1$) represents the fact that all members of coalition c prefer agenda a_1 to agenda a_0.

A network representation of the agenda formation process has several advantages, but three deserve specific mention: (1) By modeling the agenda formation process as a network, we are able to introduce equilibrium notions which are independent of the specification of the move sequence. This reduces some of the complexity associated with purely game-theoretic models of agenda formation. (2) Using networks, we are able to bring to bear on the problem equilibrium notions which reflect both the noncooperative and cooperative aspects of agenda formation, as well as allow for the possibility that agents behave farsightedly in forming agendas. Here, we shall focus on the notion of farsighted consistency introduced by Chwe (1994) in an abstract game setting and extended to network formation models by Page, Wooders, and Kamat (2005).[1] As shown by Chwe (1994), the notion of farsighted consistency represents a refinement of Rubinstein's stability notion (see Rubinstein (1980), Le Breton and Salles (1990), Li (1993), and Martin and Merlin (2002)). (3) Using networks we are able to move directly and more easily to the computation of equilibrium agendas. In fact we compute the farsightedly consistent sets corresponding to three examples of agenda formation using an algorithm developed by Page and Kamat (2001).

In a recent paper, Dutta, Jackson, and Le Breton (2003) introduce a definition of equilibrium for endogenous agenda formation that also does not depend on a specific game form or move-sequence protocol. They manage to dispense with any reference to a specific protocol by merely defining an agenda as equilibrium if no agent prefers any continuation equilibrium. Their reliance on non-myopic agents is also inspired by Chwe's (1994) concept of farsighted consistency.

2 The Model

We begin with a formal definition of agenda networks. Suppose that there are finitely many issues indexed by $i = 1, 2, \ldots, M$ and that for each issue i, there is a finite set of positions I_i that can be taken. For example, I_i could be $\{\text{yes}, \text{no}, \text{don't care}\}$. Let typical element p_i denote a particular position in I_i. We define the set of agendas, A, as $A = I_1 \times \cdots \times I_M$, with typical element $a = (p_1, \ldots, p_M)$.

[1]Other equilibrium notions could be applied (e.g., the farsighted core - see Page and Wooders (2004)). For a given agenda network, an agenda (that is, a node in the network) is said to be *farsightedly consistent* if no agent (or coalition of agents) is willing to alter the agenda for fear that such an alteration might induce further agenda alterations by other agents or coalitions that, in the end, would be detrimental to the initially deviating agent or coalition. It follows from Page, Wooders, and Kamat (2005) that all agenda networks possess a unique nonempty farsightedly consistent set of agendas.

Let D be the set of agents (the committee), with typical element d, and thus let 2^D denote the set of coalitions. Denote $C \subseteq 2^D$ the set of *decisive* coalitions with typical element c. For example, if a two-thirds majority is required, then $C = \left\{ c \in 2^D \mid \frac{|c|}{|D|} \geq \frac{2}{3} \right\}$.

Each agent's preferences over agendas in A are specified via an agenda payoff function,

$$v_d : A \rightarrow R$$

Agent $d \in D$ then prefers agenda a' to agenda a, if

$$v_d(a') > v_d(a).$$

Moreover, coalition $c \in C$ prefers agenda a' to agenda a if and only if

$$v_d(a') > v_d(a) \, \forall d \in c.$$

Let $M = \{m_c \mid c \in C\}$ be the set of *move arcs,* let $P = \{p_c \mid c \in C\}$ be the set of *preference arcs,* and define *arc set* $E = M \cup P$. Thus $(m_c, (a, a'))$, also written $a \xrightarrow{m_c} a'$, means that coalition c can change agenda a to agenda a', whereas $(p_c, (a, a')) \in G$, also written $a \xrightarrow{p_c} a'$, means that coalition c *prefers* agenda a' to agenda a.

Definition 1 (Agenda Networks)
Given agenda set A, agent payoff functions $\{v_d(\cdot) \mid d \in D\}$, and arc set $E \equiv M \cup P$, an agenda network*, G, is a subset of $E \times (A \times A)$, such that for all agendas a and a' in A and for all coalitions $c' \in 2^D$,*

$$(m_{c'}, (a, a')) \in G$$

if and only if coalition c' can change agenda a to agenda a', $a' \neq a$, and

$$(p_{c'}, (a, a')) \in G$$

if and only if $v_d(a') > v_d(a)$, for all $d \in c'$.

Thus an agenda network G specifies how the agendas in A are connected via coalition moves and coalitional preferences and thus provides a *network representation* of both agent preferences and the rules governing agenda formation.

Definition 2 *(Farsighted Dominance)*

Given agenda network $G \subset E \times (A \times A)$, we say that agenda $a' \in A$ farsightedly dominates agenda $a \in A$ if there is a finite sequence of agendas $a_0, a_1, \cdots, a_h$, with $a = a_0$, $a' = a_h$, and $a_k \in A$ for $k = 0, 1, \cdots, h$, and a corresponding sequence of coalitions, $c_1, c_2, \cdots, c_h$, such that for $k = 1, \cdots, h$,

$$(m_{c_k}, (a_{k-1}, a_k)) \in G, \text{ and}$$

$$(p_{c_k}, (a_{k-1}, a_h)) \in G.$$

We shall denote by $a' \triangleright\triangleright a$ the fact that network $a' \in A$ farsightedly dominates network $a \in A$.

Definition 3 *(Farsightedly Stable Agendas)*
Let $G \subset E \times (A \times A)$ be an agenda network. A subset F_A of agendas in A is said to be farsightedly consistent if

$$\text{for all } a_0 \in F_A \text{ and } (m_{c_1}, (a_0, a_1)) \in G$$

there exists $a_2 \in F_A$ with $a_2 = a_1$ or $a_2 \triangleright\triangleright a_1$ such that

$$(p_{c_1}, (a_0, a_2)) \notin G.$$

Thus, a subset of agendas F_A is farsightedly consistent if given any agenda $a_0 \in F_A$ and any m_{c_1}-deviation to agenda $a_1 \in A$ by coalition c_1, there exists further deviations leading to some agenda $a_2 \in F_A$ where the initially deviating coalition c_1 is not better off -and possibly worse off. There can be many farsightedly stable sets. We shall denote by F_A^* the largest farsightedly stable set. Thus, if F_A is a farsightedly stable set, then $F_A \subset F_A^*$.

Extending Chwe's (1994) existence and nonemptiness results to the agenda network framework, we are able to conclude that any agenda network has a nonempty largest farsightedly consistent set of agendas. The proof of this fact is identical to the proof of Theorem 1 in Page, Wooders, and Kamat (2005).

Proposition 1 $F_A^* \neq \emptyset$
Given any agenda network $G \subset E \times (A \times A)$, there exists a unique, nonempty, largest farsightedly consistent set F_A^*. Moreover, F_A^* is externally stable with respect to farsighted dominance, that is, if agenda a is contained in $A \backslash F_A^*$, then there exists an agenda a' contained in F_A^* that farsightedly dominates a.

3 Examples

3.1 Example 1: Condorcet

Let $D = \{x, y, z\}$ be the set of agents.[2] Under simple majority rule, there are four decisive coalitions:

$$C = \{c^1, c^2, c^3, c^4\} = \{\{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}.$$

Furthermore, let there be three agendas:

$$A = \{a_1, a_2, a_3\}.$$

Preferences: Let individual preferences over the agendas be as follows:

$$v_x(a_3) > v_x(a_2) > v_x(a_1)$$
$$v_y(a_1) > v_y(a_3) > v_y(a_2)$$
$$v_z(a_2) > v_z(a_1) > v_z(a_3)$$

Without loss of generality, let the utility levels take the values 0, 1, 2. Then the preferences can be summarized in the payoff matrix of Table 1.

	x	y	z
a_1	0	2	1
a_2	1	0	2
a_3	2	1	0

Table 1: Individual preferences

The alternative presentation of preferences on the left hand side of Table 2 helps visualize the preference arcs. For instance, both x and y prefer a_2 over a_3, whereas only y prefers a_2 over a_1. Thus there exists a coalition (c^1) that prefers a_2 over a_1, but there is no decisive coalition that prefers a_2 over a_1. A simplified representation, on the right, only shows the preferences of the *decisive* coalitions. The corresponding coalitional preference arcs are drawn in Figure 1.

Moves: Any decisive coalition has the "legislative ability" to pass any

↙	a_1	a_2	a_3
a_1		y	y, z
a_2	x, z		z
a_3	x	x, y	

↙	a_1	a_2	a_3
a_1			c^3
a_2	c^2		
a_3		c^1	

Table 2: Individual and coalitional preferences

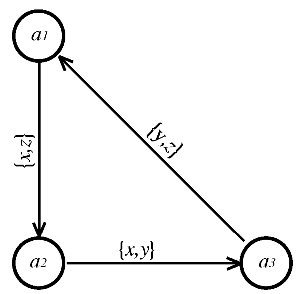

Figure 1: Preferences of Decisive Coalitions in Network Form

agenda over any other, as captured in Table 3.

Computation Result:[3] The farsightedly stable outcomes are agendas a_1, a_2, and a_3, i.e., every agenda is farsightedly stable. In this example, thus, our solution concept does not allow for a more narrow prediction than heretofore. Suppose, for instance, that we are at agenda a_1. Agent z will think twice before forming a coalition with x to move to a_2, since she would expect x to then form a coalition with y for a move to agenda a_3, which z does not prefer to a_1.

Certainly, one could remark that since x would hold a similar thought once she is at a_2, she would be dissuaded to instigate a move (with y) to a_3, knowing that it opens the door to a subsequent move to a_1. Satisfied by this reasoning and thus expecting a_2 to be stable, agent z would be tempted to instigate a move from a_1 to a_2. Clearly, however, if we can reason that z and x would form a coalition to move away from a_1, then we can just as well form the same expectations for x and y at a_2, and y and z at a_3. This cannot be

[2] For instance, let there be one issue admitting three positions, p_1, p_2, or p_3. Thus $A = \{a_1, a_2, a_3\} = \{p_1, p_2, p_3\}$.

[3] We use the *Mathematica* algorithm described in Kamat and Page (2001).

$\swarrow$	a_1	a_2	a_3
a_1		c^1, c^2, c^3, c^4	c^1, c^2, c^3, c^4
a_2	c^1, c^2, c^3, c^4		c^1, c^2, c^3, c^4
a_3	c^1, c^2, c^3, c^4	c^1, c^2, c^3, c^4	

Table 3: Move arcs

an equilibrium.

3.2 Example 2

Again, let $D = \{x, y, z\}$ be the set of agents, but let's add a fourth agenda. [4]

Preferences: Let individual preferences over the agendas be as follows:

$$v_x(a_3) > v_x(a_2) > v_x(a_4) > v_x(a_1)$$
$$v_y(a_1) > v_y(a_3) > v_x(a_4) > v_y(a_2)$$
$$v_z(a_2) > v_z(a_1) > v_x(a_4) > v_z(a_3)$$

Without loss of generality, let the utility levels take the values 0, 1, 2, 3. Then the preferences can be summarized in the payoff matrix of Table 4.

	x	y	z
a_1	0	3	2
a_2	2	0	3
a_3	3	2	0
a_4	1	1	1

Table 4: Individual preferences

The alternative presentation of preferences on the left hand side of Table 5 helps visualize the preference arcs. A simplified representation, on the right, shows the decisive coalitions only.
The corresponding coalitional preference arcs are drawn in Figure 1.

[4]For instance, let there be two issues A and B, each one admitting two positions, 0 or 1, for a total of four agendas.) Thus $A = \{a_1, a_2, a_3, a_4\} = \{(0,0), (0,1), (1,0), (1,1)\}$.

$\swarrow$	a_1	a_2	a_3	a_4
a_1		y	y,z	y,z
a_2	x,z		z	x,z
a_3	x	x,y		x,y
a_4	x	y	z	

$\swarrow$	a_1	a_2	a_3	a_4
a_1			c^3	c^3
a_2	c^2			c^2
a_3		c^1		c^1
a_4				

Table 5: Preferences of Decisive Coalitions in Tabular Form

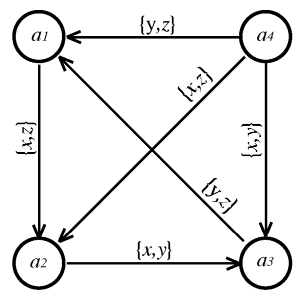

Figure 2: Preferences of Decisive Coalitions in Network Form

Moves: See Table 6.

$\swarrow$	a_1	a_2	a_3	a_4
a_1		c^1, c^2, c^3, c^4	c^1, c^2, c^3, c^4	c^1, c^2, c^3, c^4
a_2	c^1, c^2, c^3, c^4		c^1, c^2, c^3, c^4	c^1, c^2, c^3, c^4
a_3	c^1, c^2, c^3, c^4	c^1, c^2, c^3, c^4		c^1, c^2, c^3, c^4
a_4	c^1, c^2, c^3, c^4	c^1, c^2, c^3, c^4	c^1, c^2, c^3, c^4	

Table 6: Move arcs

Computational Result: The farsightedly stable set is the singleton $\{a_4\}$. This result may surprise since every coalition strictly prefers a different agenda. Agent x, however, will think twice before forming a coalition with, say, y to force agenda a_3 over agenda a_2. Indeed, agent x correctly anticipates that once the proposed agenda is a_3, she will be abandoned by agent y, who will form a coalition with z to move to a_1.

Certainly, once a_1 has been proposed, there would be a move to a_2, which agent x would actually prefer over the original agenda a_4. Now, if a_2 were indeed the final resting point, then agent x would have been willing to form the original coalition with y to depart from a_4. However, since a_2 is not a stopping point and the cycle is expected to continue, leading back to a_1 via a_3, agent x cannot exclude a_1 from the set of possible outcomes, which is why she will not deviate from a_4 in the first place.

It is thus fair to say that the concept of farsighted stability implies an extreme form of risk aversion: the mere possibility of realizing a_1 is enough of a deterrent for agent x, this despite the fact that both a_3 and a_4 are strictly preferred.

It is also worth pointing out that the result is independent of the *status quo* or starting point. Agents who start out at, say, agenda a_1 will be far-sighted enough to move to agenda a_4: indeed agent y will prefer the sure (but relatively smaller) loss of utility associated with a move to a_4 to the merely possible (but larger) loss associated with the possibility of ending up at a_2. Similarly, agent z will prefer a_4 for sure over a gamble that includes the possibility of realizing a_3. In short, y and z, who strictly prefer a_1 over a_4, will form a coalition actually pushing for a move from a_1 to a_4.

3.3 Example 3:

We now add a fourth agent, w, to the previous example; thus $D = \{w, x, y, z\}$ is the new set of agents. Under strict majority rule, there are five admissible coalitions:

$$C = \{c^1, c^2, c^3, c^4, c^5\} = \{\{w, x, y\}, \{w, x, z\}, \{w, y, z\}, \{x, y, z\}, \{w, x, y, z\}\}.$$

Preferences: Let the preferences of agents x, y, and z be the same as in the previous example, while agent w's preferences over the agendas are:

$$v_w(a_4) > v_w(a_3) > v_w(a_2) > v_w(a_1)$$

Then the coalitional preference and move arcs can be summarized in Table 7:

The corresponding coalitional preference arcs are drawn in Figure 3.

Moves: See Table 8.
Computation Result:
The farsightedly stable set is $\{a_1, a_3, a_4\}$. a_1 is stable because agent z will not want to form a coalition with x and y to move to a_2 in anticipation of

	w	x	y	z
a_1	0	0	3	2
a_2	1	2	0	3
a_3	2	3	2	0
a_4	3	1	1	1

$\swarrow$	a_1	a_2	a_3	a_4
a_1		y	y, z	y, z
a_2	w, x, z		z	x, z
a_3	w, x	w, x, y		x, y
a_4	w, x	w, y	w, z	

$\swarrow$	a_1	a_2	a_3	a_4
a_1				
a_2	c^2			
a_3		c^1		
a_4				

Table 7: Individual and coalitional preferences

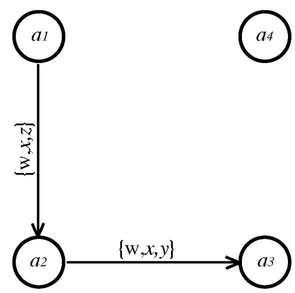

Figure 3: Preferences of Decisive Coalitions in Network Form

x and y subsequently turning their back on her and forming a coalition with w to move to a_3. a_3 and a_4 are stable since no decisive coalition prefers an alternative. a_2 is the only non-stable agenda since w, x, and y can move to a_3 without having to fear further moves.

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$\swarrow$	a_1	a_2	a_3	a_4
a_1		c^1, c^2, c^3, c^4, c^5	c^1, c^2, c^3, c^4, c^5	c^1, c^2, c^3, c^4, c^5
a_2	c^1, c^2, c^3, c^4, c^5		c^1, c^2, c^3, c^4, c^5	c^1, c^2, c^3, c^4, c^5
a_3	c^1, c^2, c^3, c^4, c^5	c^1, c^2, c^3, c^4, c^5		c^1, c^2, c^3, c^4, c^5
a_4	c^1, c^2, c^3, c^4, c^5	c^1, c^2, c^3, c^4, c^5	c^1, c^2, c^3, c^4, c^5	

Table 8: Move arcs

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