Control of the Complex Economy through Fiscal Variables

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Abstract: The aim of this work is that of exemplifying some applications of the modern theory of the complexity to the economic sector; we will highlight some of the possibilities of control of chaotic systems and some of that possibilities which are opened by the study of such systems. Remembering how a simple traditional macroeconomic model can give place to deterministic chaotic phenomena we will highlight: a) how it is possible to control such a system using opportune values of the fiscal variables; b) how it is possible to foresee the trend of the objective variable through a neural network, and, therefore, subsequently to control it on the basis of the value instruments chosen by the neural network. This will be done either in the presence of casual noises or in the case of a completely deterministic model; c) finally a different and more recent method of controlling chaotic systems will be indicated.
Keywords: Public Finance, Complexity, Economic Control,

1. Introduction.

The idea that control theory is not applicable to economic systems is agreed upon in the economic literature for two main reasons. These are: a) economic forecasts which are the basis of a possible implementation of controls, based on normal econometric instruments, are not reliable; b) economic agents understand the controls in action, and adjust to them. On this basis economists have sanctioned the bankruptcy of the application of such a methodology to the economic sector. It is, however, well-known that: a) economic systems are characterised by complex dynamics and, moreover, that there exists a strong disagreement on which models to apply; b) when the system is complex it is not always describable in an analytical form, or, at the least, such a description proves of little use. In this case, it is obvious that econometric forecasts are also of little use. It would seem, therefore, preferable to use an instrument able to describe and to forecast the behaviour of complex systems. Neural networks are an example of such an instrument. If the forecasts are of a neural type, then the control will also have to be decided on this basis. We will use a chaotic model strongly favoured in the literature (Gabish-Samuelson), emphasising how this proves equivalent to Baumol's model. We will then demonstrate that a neural network can learn the chaotic behaviour of such a system and how - the level of the objective variable being established - the network quickly learns what the level of the control instrument has to be. Finally, exploiting the sensitivity to the initial conditions, we have optimised the entity of the control variable. In such a way it is shown how a neurocontrol can be both implemented in the economy and the way in which we can exploit one of the main characteristics of chaos. In the last few years, the physical sciences have concentrated on the possibility of exploiting some of the characteristics of complexity; in particular the sensitivity to initial conditions of chaotic systems, in order to control in an "economic" way - namely with limited resources of control - complex systems. It is our intentional so to hint at the possible applicability of such a method to economic systems. With respect to the effects of the public sector on the economic cycle, the literature has often followed the approach that considers modifications to the public sector as being one of the exogenous shock elements from which economic cycles either arise or, in this way, are modified. The present tendency to give evidence for the endogenous causes of economic cycles means that this type of layout must be more usefully replaced by a layout of the structural type. That is, a layout which tries to understand the way in which the presence of the public sector modifies the structure of the economic system and if this causes a different cyclical behaviour possibility. In such attempts they have often been limited to considering - as an element of fiscal policy - public expenditure only[1] or taxes in a fixed sum. Although in the literature there are many recent works which confront the question of the endogenous causes of economic cycles, there are few which analyse the consequences of the public activity from this visual angle[2]. It seems, therefore, useful to go deeper into what might be the effects of fiscal variables on the economic cycle. After a brief introduction to the more recent theories of the endogenous economic cycle, in particular to the chaotic models, the accelerator - multiplier model of Gabisch will be recalled. Then, following the classic layout of Musgrave[3], the public sector will be introduced in such a model. Thus, we are looking to emphasise public expenditure and fixed sum tax effects, both in the case of a balanced budget and in the case of surplus and deficit.

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[1] See Goodwin (1967).

[2] See, amongst the few works which deal with this specific aspect, those of Grandmont (1985, 1986, 1989) and Goodwin (1967, 1984).

[3] Musgrave R., (1959), pp. 480-483.

2. The simple mathematics of chaotic dynamics.

According to the theorem of Li and Yorke (1975), chaotic dynamics is manifested in a system after a point has been reached where a cycle of period 3 appears; there can then appear cycles of every and all periodicities: another way of characterising chaos. A crucial aspect of such chaotic dynamics is local instability: namely the sensitive dependence upon the initial conditions. This is highlighted by examining the Lyapunov's exponent of the system. In fact, the Lyapunov's exponent characterises the behaviour of a dynamic process by measuring its degrees sensitive dependence upon such initial conditions. Some have thought that this is the defining element of chaotic dynamics rather than that of the dimension of the attractor (Eckmann and Ruelle [1985]).

If the maximum real value is positive, then the system is locally unstable and exhibits sensitive dependence upon the initial conditions. As a consequence it is chaotic. If an economic model can be explicitly expressed as a difference equation system, the numerical estimation of the larger Lyapunov's exponent is possible[4].

3. Chaos in discrete dynamic systems

Since many typical phenomena of the discrete continuos dynamic systems of greater dimensionality can be illustrated with mono-dimensional maps[5] and since, moreover, many economic examples are already verifiable in mono - dimensional difference equations it would seem opportune to concentrate on such a type of system[6] because of their simplicity.

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[4] For the case in which the dynamic specification is unknown, it has been demonstrated that the broadest Lyapunov exponent can be evaluated on the basis of historical data; see von Stahlecker P. and K. Schmidt, (1991).

[5] For a good introduction see the works of Collet and Eckmann (1980), Grandmont (1988), Preston (1983), Singer (1978) and Whitley (1983). General surveys of chaos models in the economy can be found in: Boldrin and Woodford (1988), Baumol and Behnabib (1989), Brock and Malliaris (1989) and Rosser (1990).

[6] In fact, although this complex behaviour can occur in discrete continuous systems of varying dimensionalities, recent mathematical studies show that very simple systems also, linear, discrete and mono-dimensional, can behave in a very complex dynamic way. It should be remembered,

In general these can be represented by a non-linear difference equation $x_{t+1}=f(x_t)$. This will converge, diverge, oscillate or will be chaotic depending upon the slope at the point of meeting with the line at 45 degrees which represents the stability positions $x_{t+1}= x_t$.

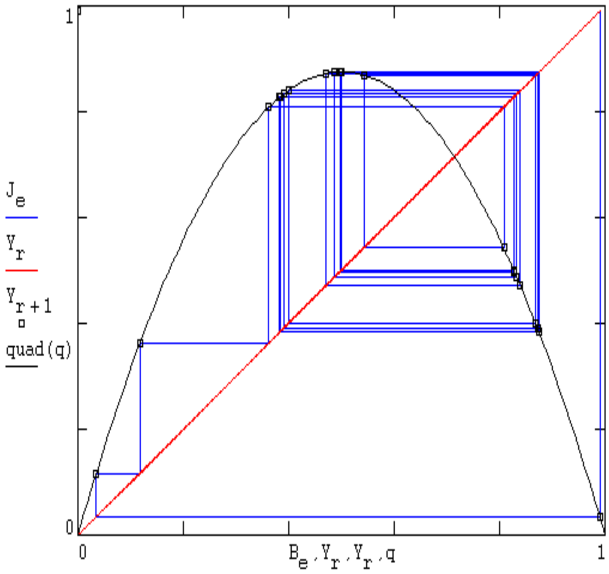

Figure 1: *Mapping di* $x_{t+1}=x_t$

4. The equivalence between the Gabish-Samuelson and Baumol models.

In economics, as already with other disciplines, there is a central controversy between those who consider the world to be governed by a mechanism of a strongly deterministic type and for whom irregular fluctuations are the exception (regular and constant trends, steady or equilibrium states, are the rule) and those who hold an opposite point of view. This means there are two strongly differentiated approaches

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however, that Stutzer's (1980) work tended to emphasise the differences between discrete and continuous models.

to the theory of the economic cycle[7] - the theories of exogenous shock and those of the endogenous cycle[8]. Obviously, the first approach - hypothesising environmental factors or exogenous shocks that are unpredictable and completely unconsidered (i.e. casual external noise) -means that the economic agent is reduced to the role of shock processor function. We will concentrate on the second approach, within whose ambit the economic agent has a greater function. The approach often followed- with respect to the effects of the public sector on the economic cycle -has been to consider modifications of the public sector as one of the exogenous shock elements from which the economic cycles originates, or from a more cautious point of view, is just modified. The present tendency to give evidence for the endogenous causes of economic cycles means that this type of layout must be more usefully replaced by a layout of the structural type which looks to understand how the presence of the public sector modifies the structure of the economic system itself, causing a greater or lesser possibility of cyclical behaviour[9]. Lately, along with a growing consciousness that the economic cycle is, in some cases, characterised by a behaviour that can be defined as erratic, several authors have shown that, for an opportune class of discrete processes - also in the apparent erratic dynamics of economic systems - there is an order[10]. The dynamics of these processes are dominated by a periodic behaviour that is, moreover, persistent in the small and smooth perturbation hypothesis[11]

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[7] A good recent survey of the theories of the economic cycle is given by Gabisch and Lorenz (1989). Also, we can usefully consider the works of Chang and Smyth (1971) and Schinasi (1982) on Kaldor and Tower's (1977) model on a complete Keynesian system. A discussion of the endogenous theories of the cycle close to the layout which will follow here is supplied by Average (1979).

[8] On the theories of the endogenous and exogenous cycle see the works of Blatt (1983). He presents a strong case against the theories of exogenous shocks; Mullinex (1984) and Zarniwitz (1985) affirm that there are not many empirical confirmations that casual shocks of every kind play a great role in the economic cycle as has been assigned to them in the recent literature. Also the weight of the exogenous factors of economic politics seems, more often than not, over-considered.

[9] Goodwin (1967); Grandmont (1985, 1986, 1989) and Goodwin (1967, 1984).

[10] Rosser J. B. Jr., (1990), pp. 265-291.

[11] See Nusse H. E. and C. H. Hommes, (1990).

In what follows we will use one of the simpler non-linear discrete growth models[12]: an accelerator - multiplier model of Samuelsonian type, which has been modified by Gabisch (1984). In it we have introduced fiscal variables following the classic layout of Musgrave[13].

5. The modification of fiscal variables.

With the aim of concentrating our attention on a concrete model, from the various models which have appeared in the literature, one of the simpler will be utilised: an accelerator - multiplier model of the Samuelsonian kind, as modified by Gabisch (1984). Such a model has the advantage of being easy to use and can help with understanding the type of elements in action.

A meaningful Keynesian answer to the Neo and New Classic hypotheses[14] is the development of macroeconomic models that present economic cycles that are based either on: a) rational expectations and competitive equilibria; or b) chaotic endogenous economic cycles (potentially removable from knowledge and from a trust in systematic stabilisation politics).

6. The theory of the economic cycle - The historical antecedents

6.1 The dependence of the economic cycle on exogenous shock: The necessity for a non-linear theory of economic dynamics.

In recent years, there have been many criticisms about the concept of equilibrium that appears in the standard neo classical scheme. The traditional method of dynamic

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[12] We had the possibility of choosing between the many non-linear discrete growth models with complex dynamic behaviour recently introduced into the literature: a discreet version of Haavelmo's model on the part of Stutzer (1980), a model regarding the rational choices of Benhabib and Day (1981), a discreet version of Goodwin's growth model on the part of Pohjola (1981), a (neo)-classical growth model on the part of Day (1982, 1983), a Cobweb modified model on the part of Cugno and Montrucchio (1984), a modified version of Samuelson's model on the part of Gabisch (1984),and competitive economic cycles on the part of Nusse E. and Hommes C. H., (1990), pp. 1-19).

[13] Musgrave R., (1959), cit.

[14] Hypotheses that regard the inevitable ineffectiveness and inefficiency of such a stabilisation.

analysis, which has recently attracted the criticisms of Blatt (1983) and Kregel (1980), foresees the use of the comparative statics in conjunction with the theory of linear stability. As is well known, the basic idea of this type of analysis[15] is that of expressing the relations of the model of the economic system under analysis in terms of an entity of algebraic equations whose solution represents the equilibrium of the system. The object of the comparative statics analysis is that of determining how the equilibrium points are influenced by the changes of the parameters implicated by the model. The theory of linear dynamic systems, and, therefore, of economic dynamics, intervenes when it is asked if the economic system will finally approach the new equilibrium, introducing the concept of stability and instability points. In such an analysis, the theoretical economist is interested only in the points of stable equilibrium[16] and the only relevant dynamic behaviour is governed by the local linearisation around such equilibrium.

Amongst the various criticisms one of particular interest to our ends is that of Kaldor (1972) who attacks the axiomatic nature of the theory of general equilibrium emphasising that a correct theory of economic dynamics would have to take into account endogenous modifications.

6.2 Endogenous cycles: the onset of chaos in the economic cycle models

Many of the oscillations that can be noted in economic variables following the ordinary techniques of linear statistics appear to be casual, but in fact they are not. Rather, some regularity in oscillations could be localised (see Lorenz [1963]) using graphical representations in which the level of the parameter is indicated horizontally and the equilibrium solutions vertically, inserting for each parameter one or more points which represent the final result once the system has reached equilibrium (cyclical or steady-state).

Chaos seems to carry a surprising message: simple deterministic models can produce that which seemed casual behaviour. In fact, such behaviour has a fine sand with a very precise structure, even though every one of its parts seems indistinguishable from noise. This, moreover, as is emphasised by the authors previously quoted, involves the endogeneity of the cycle. From a methodological point of view, the

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15 Chiarella C., (1990), pp. 1-9.

16 The unstable equilibria are discarded invoking Samuelson's correspondence principle.

theory of chaos involves an attempt to understand the behaviour of the system not locally but globally analysing the critical limit between a stationary condition and cyclicity[17]. This involves an increasing interest in theories of the endogenous cycle that manifests itself in a period in which the qualitative theory of the dynamic systems (see Chiaretta [1990]; Cap. 2) gives to the theoretical economist a group of instruments and concepts more useful for developing models of the economic cycle, as in the work of Bothwell (1952). Chaotic dynamics can arise from the models of the New Classic and the Orthodox Keynesian, as well as in those of the New Keynesian school. In fact, it can be shown that there exists in the traditional basic models non-invertible maps[18] also without any successive follow up. Besides, the majority of the models that are usual in the theory of descriptive growth can be reformulated in such a way that their dynamical equations are similar to the unimodal maps. A key in all cases is the sufficient non-linearity of the dynamic processes.

Various types of chaos have been recognised in economics: a) Neo Classical Chaos; b) Malthusian Classic Chaos; c) Orthodox Keynesian Chaos; d) The evolutionary Chaos of Haavelmo; e) The pulsating Chaos of Goodwin; f) Modified Samuelson Chaos. We will concentrate on the modified Samuelson chaos.

7. The modified model of Samuelson

The original accelerator - multiplier model of Samuelson (1939a) can be modified to produce a model that generates endogenous chaotic cycles. The key, which appeared in a work by the same Samuelson (1939b), consists in assuming a non-linear function of consumption[19]. As Blatt (1983) noted, for some opportune level of the values of the parameters, this modified model of the multiplier accelerator of

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[17] In fact, an observer of the real world would see only the vertical section corresponding to one parameter at a time; he would observe, therefore, an apparent causality, or stationary condition or a cycle of *n* years, losing touch with the unitary nature of the problem.

[18] An example of this type of model has been supplied by Stutzer (1980).

[19] An hypothesis that is based, at least in part, on the results of empirical analysis, especially of high income levels.

Samuelson can generate chaotic cycles[20]. An analogous model, with non-linear investment functions, which has received greater attention in the literature, is that of Hicks (1950), which as Brock (1988b) has demonstrated, can also generate chaotic dynamics.

8. A simple chaotic model of the economic cycle

Gabisch (1984)[21] is responsible for the specification of a chaotic behaviour. This is the simplest chaotic economic model and is based on the interrelationship between accelerator and multiplier. He modifies the original accelerator - multiplier model of Samuelson (1939a) giving rise to a model which generates endogenous chaotic cycles, on the basis of the ideas of Samuelson (1939b) himself. This assumes a non-linear consumption function; a hypothesis born, at least in part, for high income levels, from empirical analysis[22]. The model, based fundamentally on a non-linear difference equation $Y_{t+1}=F(Y_t)$, may be summarised as follows:

$$Y_t = C_t + I_t \qquad k > 1 \qquad (1)$$

$$I_t = k[Y_t - Y_{t-1}] \qquad k > 1 \qquad (2)$$

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20 Obviously, such a model is subject to the usual NeoClassical criticisms. The transition to chaos can be observed by considering a very simple model of non-linear difference equations due to Baumol and Quandt (1983). This takes the form: $y_{t+1} = y_t\, w\, (1 - y_t)$.
It has a simple equilibrium in: $y^* = (w - 1) / w$. For $2 < w < 3$, the system is stable.

If w increases beyond the value 3, the equilibrium becomes twofold and an oscillation of period 2 appears. Beyond w 3.45 the system forks into an oscillation of period four. At the following fork, it passes to an oscillation of period eight and so on. This is “Feigenbaum’s cascade” (Feigenbaum [1978]), and is well-known as a universality because of the fact that it is common to many chaos models. At the level w = 3.9, the system has entered into the chaotic zone and the oscillations have begun to be irregular. In the figure, the solid lines indicate stable equilibria and the dotted unstable equilibria. It can thus be seen that the forking process consists essentially in the destabilisation of a previously stable equilibrium path.

21 In reality, because of its conformation the model used seems close to the model used by Hicks (1950). In fact, while Samuelson uses the investment function $I_t=k\,(C_t - C_{t-1})$, Gabisch’s model uses, as Hicks does, a dependence on the level of product income $I_t = k\,(Y_t - Y_{t-1})$.

22 Blatt (1983) was amongst the first to notice that this model, for a correct grouping of the values of the parameters, can generate chaotic cycles.

$$C_t = c[Y_{t-1}]^a \qquad a \Rightarrow 1,\ 0 < c < 1 \qquad (3)$$

where Y is the national product, C consumption, I investment and c, k have the obvious usual meanings[23].
It should be noted that the formulation of the investment function is the same as that used by Musgrave (1959), less than the parameter *a*.
Thus, by substitution, we obtain:

$$Y_{t+1} = \frac{Y_t\{k - c[Y_t]^{a-1}\}}{(k-1)}$$

or:

$$Y_{t+1} = Y_t\{c[Y_t]^a - k[Y_t]\}(1-k) \qquad (4)$$

And, therefore, we can consider the family map:

$$F(Y;a,k,c) = Y(k - cY^{a-1})/(k-1) \qquad (5)$$

Gabisch[24] has established that, for a certain spectrum of the value of the parameters, there is chaos in the sense of Li and Yorke (1975). In fact if:

$$k = \left[1 + a^{\frac{a}{(1-a)}} - a^{\frac{1}{(1-a)}}\right] \qquad (6)$$

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23 The justification for this can be found in the possible differences between the effective realisation of a behavioural accession standards lightly divergent from a theoretical ideal. In this case, it is important to consider a restricted interval of the parameter a=1.

24 The derivative of F is given by (k-a) / (k-1). We will call F “the map of Gabisch”. It should be noted that the point 0 is an unstable fixed point of the map F. Clearly the point Y* = [1 / c] 1 / (a-1) is a fixed point of F and the derivative of F in this point is -1 if and only if, k = (a+1) / 2.

then there exists a periodic orbit with period three. We are, therefore, in the presence of chaos in the sense of Li and Yorke. Moreover, in this chaotic spectrum of the values of the parameters the solutions are highly sensitive to variations of the parameters or the initial values. In such a model, it is easy to introduce the fiscal variable. If both the budget expenses G and the taxes T are fixed we easily obtain:

$$Y_t = C_t + T_t + G \qquad (7)$$

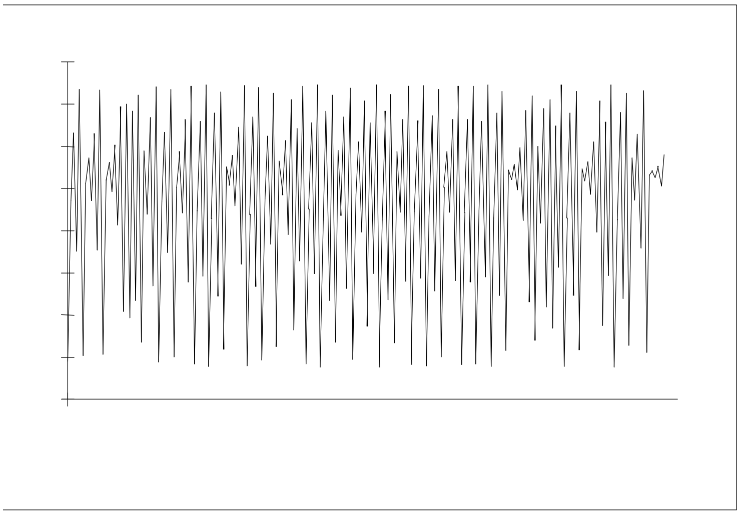
$$I_t = k[Y_t - Y_{t-1}] \qquad k > 1 \qquad (8)$$

$$C_t = c[Y_{t-1} - T]^a \qquad a \Rightarrow 1, 0 < c < 1 \qquad (9)$$

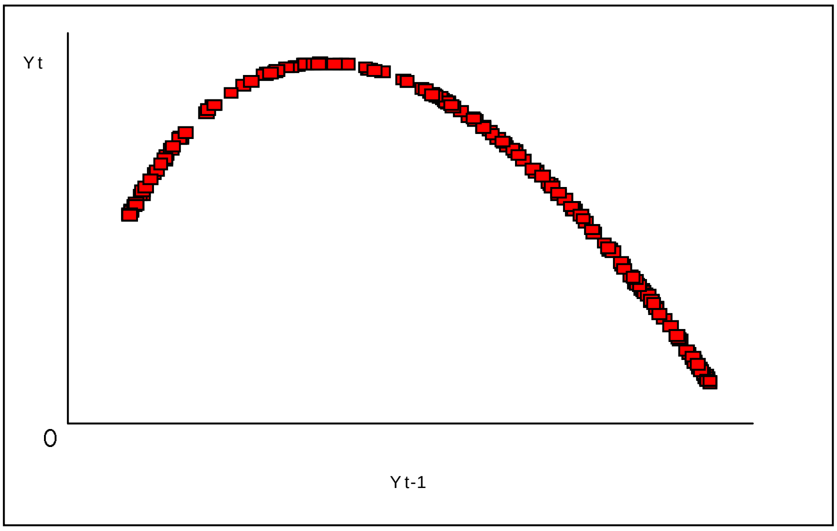
The equilibrium solution becomes:

$$Y_{t+1} = ([k * Y_t - c(Y_t - T)^a] - G)/(k-1) \qquad (10)$$

In the following figure, the temporal trend of the model is shown; from this it is easy to identify the chaotic trend.



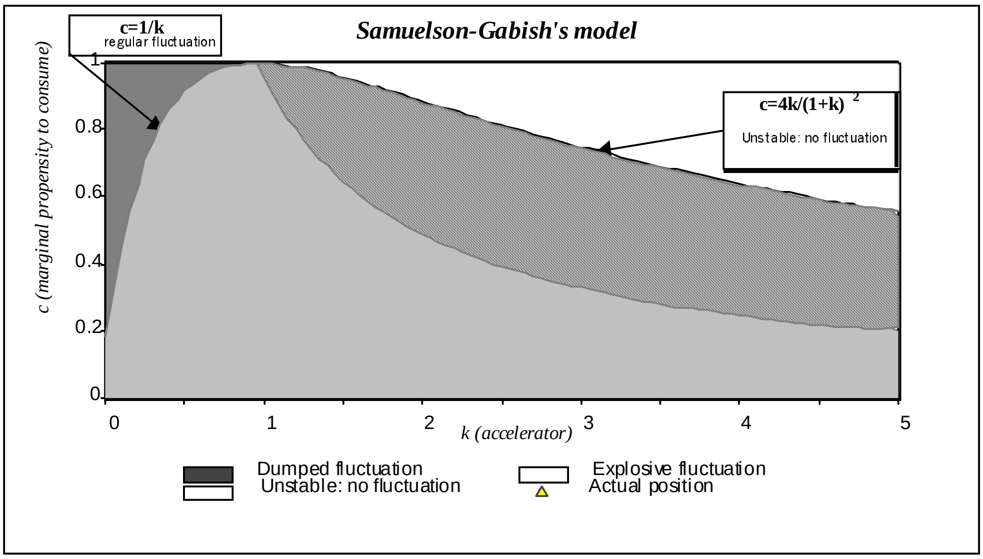
It is easy to see the similarities between this model and that analysed by Baumol and Behnabib (1989) or by Yorke. It is, in fact, sufficient to represent the variable Y with respect to the value it had in the preceding temporal instant (see the following Figure).



8.1 Highlighting of similarities and differences

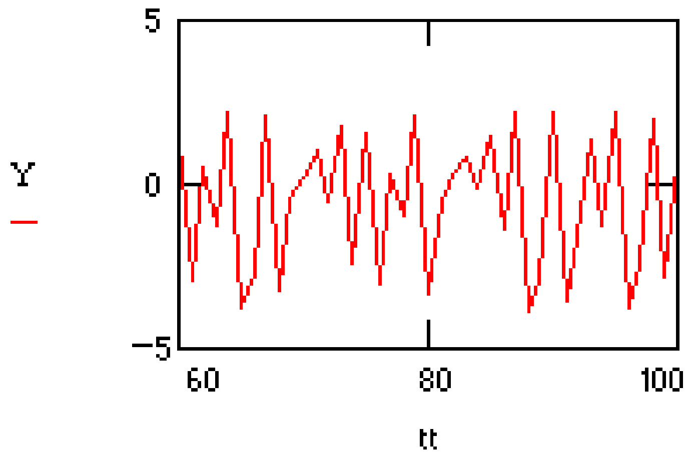
This model is already well known since Samuelson has shown that the multiplier-accelerator interrelationship can give place to stability and to various types of cyclical behaviour. The values of the parameters that give rise to such effects, for a slightly different[25] model, are represented in the following figure.

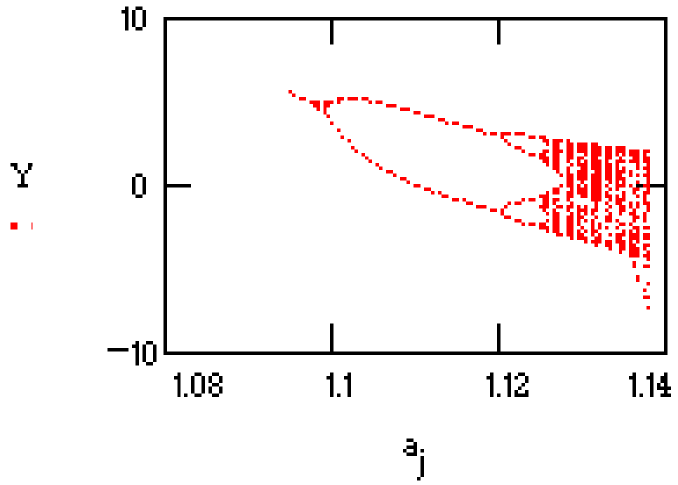
[25] In fact, this model consider $I_t=k*(C_{t-1}-C_{t-2})$.



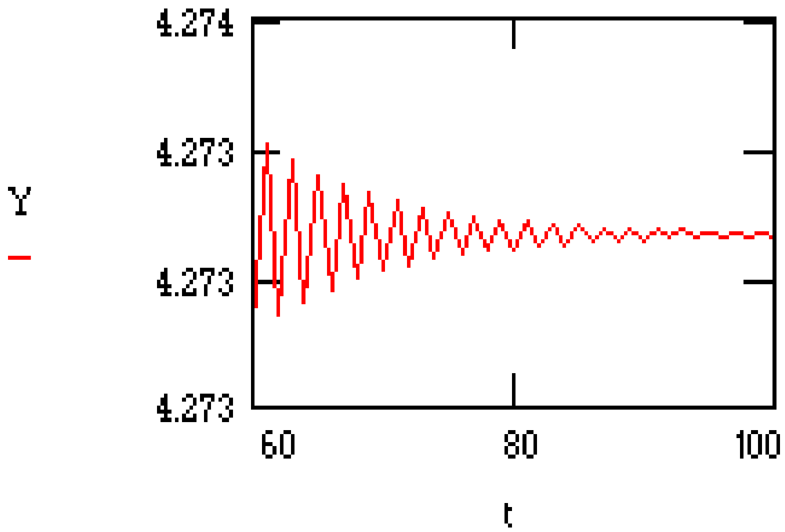
The explicit introduction of fiscal variables into the model does not change the structure of the results. These are given in the following figures (Y indicates different equilibrium values for aj (different exponents) and tt (different times). In the first couple of figures, the values of the parameters are the following: T=.27; rg=0.5;G=T/rg; G=0.54 (where rg is the pre-fixed relation T/G), and:

$$Y_{t+1} = \frac{k * Y_t - c * (Y_t - T)^a - G}{(k-1)}$$

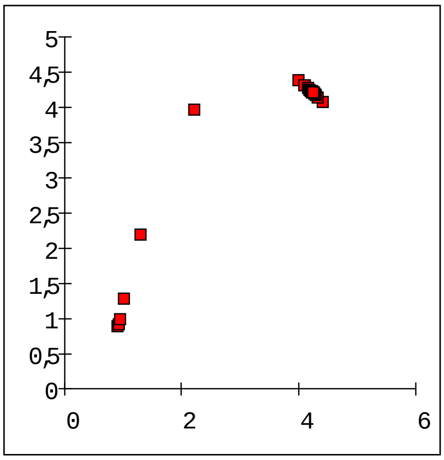


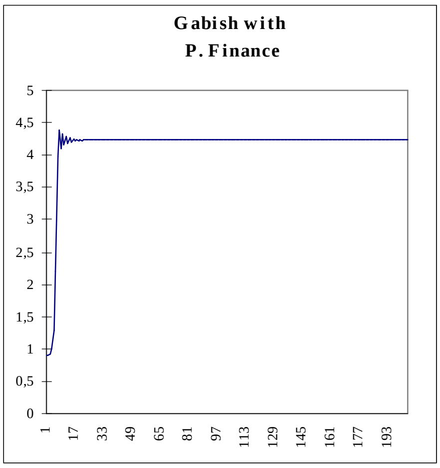


Modifying the relationship (rg) between G and T (T constant) we easily obtain a modification of the behaviour of the system. So with: T=0.27; rg=1.52; G= T*rg; G=0.41 we obtain the convergent trend represented in the following figure:

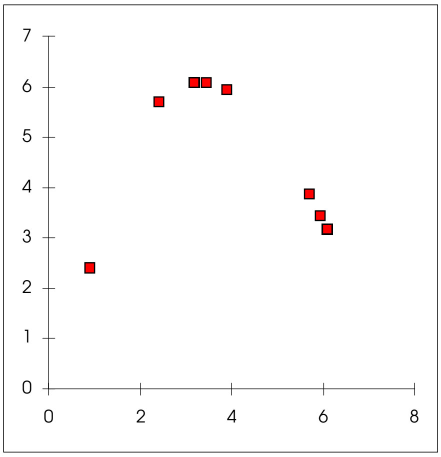


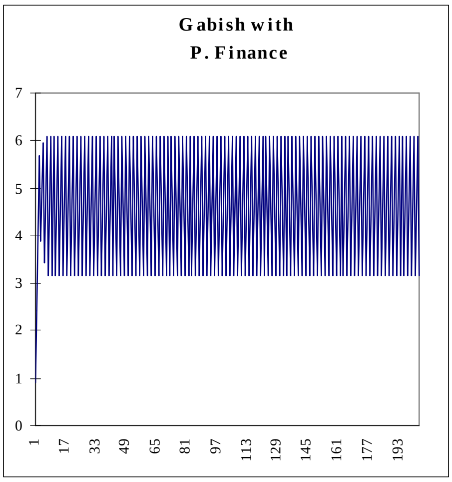
While with 1.33 < g / T < 1.577 we obtain a behaviour of the following type:





Finally, with G/T close to the value 1.3 we obtain fork trends of the type outlined in the following figures:

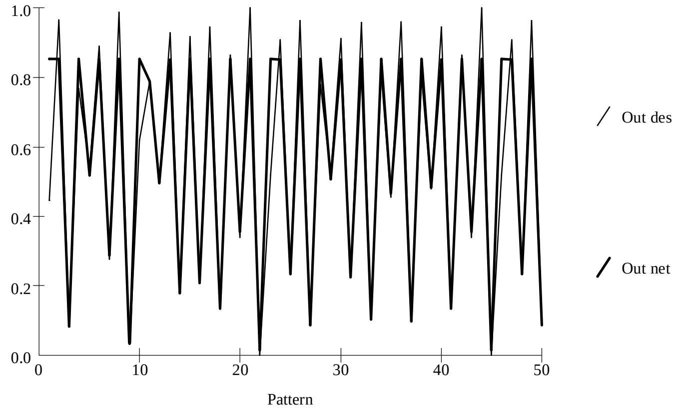




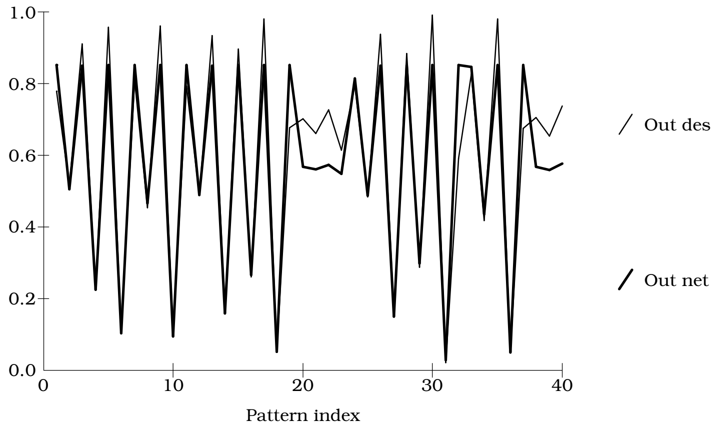
G / T=1.3

9. The learning and forecasts of a neural network

On the data of Gabish's model, for economically significant values of the variables, we have used a neural network of the back-propagation type. Given the type of input and output data, we have used a network with one input, one output and one hidden layer. The learning, as is shown in the following figure, has been very efficient and rapid (Algorithm of Marquant with program NNDT120), with almost negligible error.



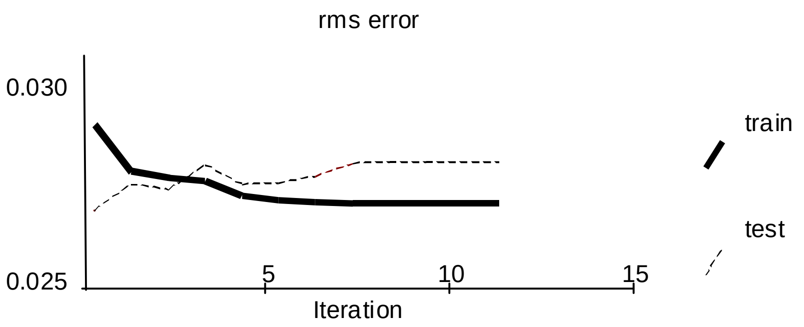
Training data: Description



Test date: Forecasting

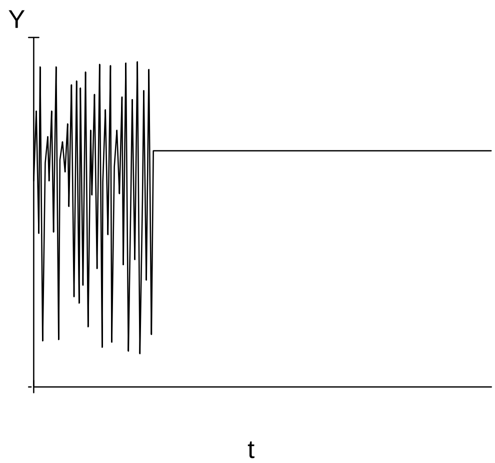
Training and Test date error

Starting from these premises, some attempts at empirical application are now being made, utilising the data of the Italian economy. Here, the main problem is the existence of disturbances. For this reason we have also run the neural network on data with noises. In this case too we have obtained excellent results (see next figure).

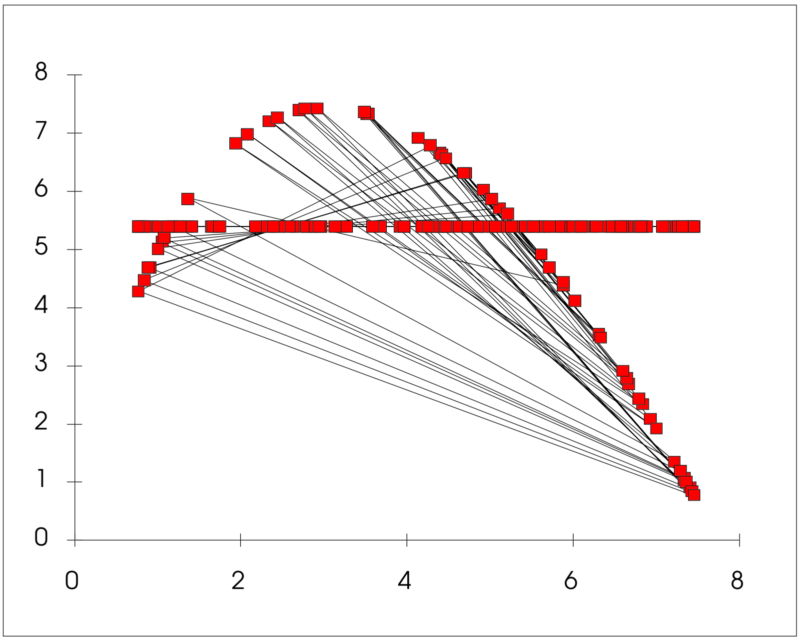


10. Sensitivity to initial conditions

We have already seen that local instability – namely the sensitive dependence upon initial conditions – is a crucial aspect of chaotic dynamics.As is known, this is indicated by examining the Lyapunov exponent of the system. The Lyapunov exponents characterise the behaviour of a dynamic process by measuring its degree of sensitive dependence upon initial conditions. If the maximum real value is positive, then the system is locally unstable and exhibits sensitive dependence upon initial conditions. Therefore it is said to be chaotic. Since an economic model can be explicitly expressed as a difference equation system, the numerical calculation of the greater Lyapunov exponent is possible. Some writers have maintained that this is the defining element of chaotic dynamics, rather than the dimension of the attractor (Eckmann and Ruelle [1985]). This implies that a very small variation of a parameter can cause large movements of the objective. Profiting from this characteristic of sensitivity to initial conditions and blocking the system when it reaches an orbit next to the desired condition, Pecora [1994] has succeeded in controlling a chaotic system by using weak perturbations. Following Pecora's approach we have obtained a trend of the type indicated in the figure below:



For reasons of clarity, the behaviour on Poincarré's map is given. From this it can be seen how, after the initial chaotic trend, the system stabilises itself in an optimal way as soon as it reaches a value next to that desired.



11. The public sector and chaotic models: Some conclusions

With respect to the effects of taxes in this model some conclusions seem possible. Obviously, such conclusions are only partially extendible to the grouping of models of the economic cycle in general. The main conclusion regards the ease of controlling the cyclicity of a chaotic economic system through variations of the public sector. Obviously, the most realistic hypothesis of variable coefficients will modify the results that we obtained. A second conclusion concerns the fact that the presence of chaos in the economic system can be positively exploited with the aim of achieving the stability of the same system. This could involve the advantage of introducing structural variations into the parameters of public finance such that the system is definitely of a chaotic type. By exploiting the strong sensitivity to initial conditions in this way, the system could be controlled through weak interventions. This would eliminate one of the principal problems of stabilisation policy, which is constituted by the scarcity of resources.

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