KEYNESIAN AND NEOCLASSICAL FISCAL SUSTAINABILITY INDICATORS, WITH APPLICATIONS TO EMU MEMBER COUNTRIES

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Abstract – The purposes of this paper are twofold: first, it aims at critically evaluating the solvency criterion, pioneered by Hamilton and Flavin (1986), which is nowadays almost hegemonic in the analysis of public debt sustainability, and at illustrating alternative measures of sustainability grounded on the dynamic stability approach originated by Domar (1944); secondly, it looks at sustainability in EMU member countries, with particular attention given to the relations between sustainability and the design of fiscal rules. The results show that the 3% rule imposed by the Maastricht treaty can be justified as a sustainability requirement for an "average" EMU member country. At the same time, the dispersion around this average is quite substantial: this questions the viability of uniform deficit caps across EMU member countries.

Keywords: public debt sustainability, dynamic analysis, solvency constraint, EMU, fiscal rules.

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0. Introduction

Public debt sustainability is an important issue in the current policy debate. This applies in particular to the countries belonging to the European Monetary Union (EMU), where the need to ensure fiscal sustainability was often invoked as a rationale for the fiscal rules set out in the Maastricht treaty and in the Stability and Growth Pact (SGP); see for instance Buiter *et al.*, (1993), Canzoneri and Diba, (1999), Buti and Giudice, (2002). But what do we know about public debt sustainability in EMU countries?

The evidence available so far for these countries is scarce and does not reach unambiguous conclusions; see Corsetti and Roubini (1991), Payne (1997), Artis and Marcellino (1998), Bravo and Silvestre (1999). Moreover, these results are exclusively grounded on empirical testing of the intertemporal budget constraint, also called solvency constraint, or NPG (No Ponzi Game[1]) condition.

This paper provides fresh evidence on public debt sustainability in the EMU member countries using a different empirical approach, which goes back to Domar (1944), and identifies sustainability with the dynamic stability of the public debt/GDP ratio around a constant steady state. In fact, a critical analysis of the solvency criterion shows that it is questionable both on operational and theoretical grounds, as it leads to useless and inconsistent definitions of sustainability, and is of little or no help in the design of fiscal rules. This motivates our proposition of alternative sustainability indicators grounded on dynamic analysis. To illustrate this alternative approach we adopt two indicators, which take both the form of a sustainability "threshold", i.e., a value of the public debt/GDP ratio above which the economic system is dynamically unstable. The first indicator was obtained by Zee (1988) using a neoclassical overlapping-generation model; the second one was derived by Bagnai (1995) from the stability conditions of the dynamic Keynesian model of Tobin and Buiter (1976). These indicators depend in a meaningful way on a restricted number of key macroeconomic parameters. Using estimates of these

[1] After the "multilevel marketing" pioneer Carlo Ponzi (Parma, 1882; Rio de Janeiro, 1949).

parameters we obtain empirical sustainability indicators for eight countries of the EMU area, observe their evolution over the last decade and analyze their implications for fiscal rules in the EMU (in particular, the well-known 3% rule).

The remainder of the paper falls into five paragraphs. Paragraph 1 criticizes the approach to sustainability testing based on the intertemporal budget constraint. Paragraph 2 describes the sustainability thresholds derived by Zee (1988) in the neoclassical overlapping generation model. Paragraph 3 derives a Keynesian sustainability indicator by exploiting the stability conditions of the dynamic Keynesian model of Tobin and Buiter (1976). In paragraph 4 the two indicators are estimated for eight countries of the EMU area (the twelve member countries less Austria, Finland, Greece and Luxembourg[2]) and their evolution over the last decade is observed. Paragraph 5 draws the main conclusions. A detailed account of the data sources and calculations is available on http://bagnai.org.

1. Empirical Assessment of Sustainability: Solvency Constraint vs. Dynamic Stability

In this paragraph we briefly expound some reasons of dissatisfaction with the sustainability definitions based on the solvency constraint. These criticisms motivate the interest for an alternative approach to public debt sustainability testing.

In order to define the solvency constraint, let us express the government budget identity in discrete time as a ratio to GDP as follows

$$\frac{B_t}{y_t} = \frac{1+r}{1+n}\frac{B_{t-1}}{y_{t-1}} - a_t \tag{1}$$

where B_t is the real stock of bond issued by the government evaluated at the end of time t, y_t is real GDP, r is the real interest rate on public debt prevailing between t-1 and t, n is the real rate of growth, and a_t is the primary surplus/GDP ratio

[2] These countries were excluded due to lack of data. See the statistical appendix on http://bagnai.org.

(including seigniorage). We assume constant interest and growth rates in order to keep the notation as simple as possible. Although some authors dub equation (1) as "uniperiodal" or "static" budget constraint (see for instance Chalk and Hemming (2000)), this identity does simply express the fact that current expenditures (including the service of debt) can be financed by levying taxes and/or issuing new debt, and as such does not put any constraint whatever on the current or future value of public debt (see Haliassos and Tobin (1990)).

This is best seen by leading identity (1), solving it with respect to the current value of B_t/y_t, and iterating it forward

$$\frac{B_t}{y_t} = \sum_{j=1}^{\infty} \left(\frac{1+n}{1+r} \right)^j E_t \left[a_{t+j} \right] + \lim_{j \to +\infty} \left\{ \left(\frac{1+n}{1+r} \right)^j E_t \left[\frac{B_{t+j}}{y_{t+j}} \right] \right\} \tag{2}$$

Since the debt/GDP ratio can in principle take any value in the limit, equation (2) clarifies that equation (1) does not constrain the current value of B_t/y_t for any given path of a_t.

The intertemporal budget constraint (or solvency constraint, or NPG) is set by imposing that

$$\lim_{j \to +\infty} \left\{ \left(\frac{1+n}{1+r} \right)^j E_t \left[\frac{B_{t+j}}{y_{t+j}} \right] \right\} = 0 \tag{3}$$

For the constraint (3) to be verified it suffices that B_t/y_t does not grow at a rate exceeding $(1+r)/(1+n) - 1 \approx r - n$; this happens when the service of the debt is not covered entirely by issuing new debt. If we rule out this perpetual rollover case (Ponzi game), equation (2) establishes that the current value of the debt is equal to the expected present value of the infinite stream of future surpluses, which means that when the constraint is in force the debt must eventually be repaid, i.e., that the government must realize in the future at least one primary surplus (if the current value of the debt stock is positive, there must be at least one positive value of a_t in the summation for the constraint to hold).

Starting from Hamilton and Flavin (1986), most recent empirical work on public debt sustainability defines sustainability as the respect of the intertemporal constraint (3). As stated before, we find that this definition is unsatisfactory on both operational and theoretical grounds.

From the operational point of view, it is well known that the intertemporal budget constraint is respected even by paths of the public debt/GDP ratio that grow exponentially, provided their rate of growth is less than the spread between the real interest and the real growth rate (see equation (3) above). Needless to say, this is an extremely unpalatable conclusion.

From the economic point of view, the most striking feature of the sustainability definition based on the intertemporal budget constraint is its internal inconsistency. The reasoning underlying this definition is as follows: if we observe somehow in the data a violation of the intertemporal budget constraint, this means that sooner or later the government will have to change its policies in order to meet the solvency requirements, hence, the current policies are not "sustainable". This argument ignores that the NPG is not a bare fact of nature: rather, it is a constraint imposed on the behaviour of debtors by the rational behaviour of creditors in a well defined class of intertemporal equilibrium models. The solvency constraint stems from the fact that in these equilibrium models nobody *wants* to be the creditor of an insolvent debtor (i.e., nobody wants to hold a net creditor position in expected present value), and therefore nobody (be it the government or another economic agent) *can* be insolvent (i.e., nobody can find itself in a net debtor position).

Therefore, if the model on which the constraint is based is true, "unsustainable" debt paths will never be observed (thus thwarting any possible empirical test). This is because, as stated before, the solvency constraint *must* be respected in equilibrium: therefore, *observed* violations of the constraint *must* be apparent and temporary, hence irrelevant as far as solvency (which is an asymptotic requirement, as shown by (3)) is concerned. In fact, if we were willing to admit for argument's sake that the observed deviation were persistent, and therefore that restoration of the equilibrium called actually for a structural change (as Wilcox (1989) appears to believe), it is unclear how rational agents could postpone it, thereby bearing the cost of a persistent deviation from their equilibrium path. This means that if we agree with the equilibrium approach on which it is grounded, *the intertemporal budget constraint may appear to be violated only in those economies in which it is not binding* (and therefore cannot be utilized to define sustainability), either because economic agents are irrational in some way, or because the structure of the economy does not request that a solvency constraint be respected in equilibrium, as it happens in dynamically inefficient economies, but also in dynamically efficient open economies (see for instance Persson, 1985), and in other cases of "rational Ponzi games" (see O'Connel and Zeldes, 1988).

The argument developed so far in this paragraph is not new. On the contrary, it is precisely that followed by Hamilton and Flavin (1986) in their pioneering work on sustainability. For these authors the respect of the intertemporal budget constraint in the data means that this constraint is in force and that the public expects it to hold. Therefore, if the solvency constraint appears to be respected by the data *and* the government is running a policy of persistent primary deficits, then the government will have to reverse its policy sooner or later (i.e., this policy is unsustainable). In other words, *the respect, not the violation*, of the constraint is an indicator of the unsustainability of "Ponzi policies".

It is unclear why this argument, which is perfectly consistent with the neoclassical premises of the model underlying the constraint (3), was completely reversed by the subsequent work in this area, in favour of the inconsistent reasoning expounded above (initiated by Wilcox, 1989). This is probably due to the fact that the implications Hamilton and Flavin's approach are "too much unpalatable", because they lead to the Panglossian conclusion that the observed path of the public debt only explodes when it can safely explode without compromising the equilibrium of economic agents. Summing up, the intertemporal constraint approach leads to absurd conclusions when its application is inconsistent with the underlying model, and to even more absurd conclusions when it is consistently applied.

Fortunately, the intertemporal budget constraint is not the only analytical tool available for sustainability testing. A well established, although presently a bit underrated, strand of the literature identifies sustainability with the dynamic stability of the public debt/GDP ratio around a constant steady state (Domar (1944), Masson (1985), Tobin (1986), Spaventa (1987), Zee (1988), Blanchard *et al.* (1990), Pasinetti (1998), Heise (2002)). By ruling out explosive paths of the public debt/GDP ratio, this definition has a lot more intuitive appeal than the solvency requirement. Nevertheless, the dynamic stability approach was sometimes criticized because in its simplest form it implies that *any* constant path of the public debt/GDP ratio is sustainable. This is clearly an absurd proposition (although far more acceptable than the "ever exploding" debt paths allowed by the solvency requirement).

However, if we leave the simple analysis of the public debt stock/flow identity (1) and consider more detailed models of the economy, including a representation of the crowding-out mechanism, we find that the dynamic stability conditions do generally depend on the size of the public debt/GDP ratio. This allows to define sustainability "thresholds" for the value of the public debt/GDP ratio, i.e., the values of this ratio beyond which the stability conditions are violated (and therefore every perturbation can set the economic system on a divergent debt spiral). In this case the sustainability definition based on dynamic stability gains a lot of intuitive appeal (besides being based on rigorous economic analysis): a sustainable public debt/GDP ratio is one that is constant *and* below given thresholds.

These thresholds depend in an economically meaningful way on some key parameters of the economic system. Moreover, they can be used to measure the evolution of sustainability over time, in relationship to the evolution of the macroeconomic framework as captured by these key parameters, and are therefore much more informative than the results of the so called "solvency" tests. These tests, in fact, only say whether or not the public debt is "sustainable" today, without saying why it is so, and without indicating whether the evolution of the macroeconomic framework is likely to impair further or to restore sustainability.

Of course, these thresholds are model specific, being based on the stability analysis of a given model. However, it should be clear from the above discussion that this limit is shared by the intertemporal constraint approach, which is also based on a specific class of models (although in a less transparent manner, which allows most authors to present it as an "atheoretical" requirement which does not need to be discussed). In this respect, the advantage of the dynamic stability approach is that the researcher explicitly chooses a model in which he believes (and can compare, if he wishes, the conclusions of this model with those of competing explanations of the world)! In this paper we present two different indicators, based respectively on a neoclassical and on a Keynesian model. Their derivation is discussed in the next two paragraphs, starting from the neoclassical indicator, and their application is illustrated in paragraph 4.

2. Sustainability Thresholds in a Neoclassical Overlapping Generations Model

Zee (1988) applies to an overlapping generation model à la Diamond (1965) the definition of sustainability as dynamic stability of the debt/GDP ratio, thereby obtaining sustainability thresholds for the public debt. We describe briefly his model in order to introduce the notation that will be used in the empirical section of this paper.

In Zee's model the labour force, that expands at the exogenous rate n (which coincides with the real growth rate of the economy), is composed of individuals that live two periods. The representative agent works when young, earning the net real wage $(1-\tau)w_t$, where τ is the tax rate on income. The share s_t of lifetime income that is not consumed in the first period (the saving rate) is invested at the net rate of interest $r_{t+1}(1-\tau)$, where r_{t+1} is the gross rate of return earned in t+1 for postponing a unit of consumption in t, in order to finance consumption when old. As usual, the representative individual sets his lifetime consumption path (hence, the size of s_t), by maximizing a biperiodal utility function, given the factor rewards determined by standard neoclassical equilibrium in the supply side of the model. This standard model is augmented with a government sector, whose budget constraint, expressed in per capita terms, is

$$b_t = \frac{1+r_t(1-\tau)}{1+n} b_{t-1} + g_t - \tau x_t \tag{4}$$

where b_t is public debt (consisting of one-period bonds paying the equilibrium rate of return), g_t is current expenditure and x_t is per capita output (hence, τx_t is per capita income tax revenue). In other words, the government finances current expenditure and the service of debt by levying income taxes and issuing new debt. Equation (4), together with the asset market equilibrium condition, constitute a second order nonlinear difference equation system in b_t and r_t whose (local) stability conditions are obtained after linearization around the steady state.

A necessary condition for stability in the model is that the difference equation (4) has a stable root, namely, that $n > r_{t+1}(1-\tau)$. In the standard dynamic analysis of the burden of debt, which takes the interest rate as exogenous, equation (4) would be a constant coefficients linear equation and this condition would also be sufficient for the stability of b_t. In Zee's model, however, the interest rate cannot be taken as given, because any shift from tax to debt financing crowds out physical capital and determines an increase in the real rate of interest.

Zee shows that in the presence of crowding-out the sufficient conditions for dynamic stability are

$$\frac{B}{y} < \tilde{b} = k\left[\frac{\varepsilon(1+n)\tau}{1-\tau} - \{n - r(1-\tau)\}\Phi\right] \tag{5}$$

and

$$\Phi \equiv [1 - \eta(1\text{-}s)] + \frac{(1+n)\varepsilon}{r(1-\tau)} - \frac{s\delta(1-\varphi)}{\varphi} < 0 \tag{6}$$

where

δ	interest rate elasticity of saving
ε<0	interest rate elasticity of investments
η>0	income elasticity of consumption
k	capital/output ratio
φ	capital elasticity of output

(all the variables are undated because they are evaluated at the steady state).

Equation (5) defines a sustainability threshold $\tilde{b}$ for the public debt/output ratio $b/x \equiv B/y$. If the debt exceeds this threshold, the economic system is dynamically unstable, and the debt will respond to any exogenous shock by turning into an explosive trajectory.

The parameters in the above equations (5) and (6) are readily available and their economic meaning is transparent and can be easily traced back to the transmission mechanisms featuring in the model. A sensitivity analysis shows that the key parameters that influence the size of $\tilde{b}$ are n, r and τ. Their effects on the sustainability thresholds can be easily understood with reference to equation (4): an increase in n and τ will increase the threshold, while an increase in r will lower it. Moreover, the threshold increases linearly with the capital/output ratio (an economy with a larger capital stock can afford a larger stock of debt) and with the absolute values of the interest rate elasticities δ and ε (the more responsive are these flows to the interest rate, the smaller the increase of interest rate which is needed to restore the equilibrium between saving and investment after a shift from taxes to debt, hence the less impaired the system dynamic stability).

When the biperiodal utility function is of the Cobb-Douglas type, the income elasticity of consumption is unit (η=1) and the interest elasticity of saving is zero (δ =0), so that equation (6) simplifies to

$$\Phi = s + \frac{(1+n)\varepsilon}{r(1-\tau)}$$

Since the parameter φ is multiplicated by δ, it also disappears from equation (6), leading to six the number of the parameters involved in the indicator (5). In the applications of (5) we will assume that both the utility and the production function are Cobb-Douglas. [3]

3. Sustainability Thresholds in a Keynesian Model

The dynamic Keynesian model proposed by Tobin and Buiter (1976, par. 4), building on the work of Blinder and Solow (1973), is obtained by augmenting a standard Keynesian model with the government budget identity; this identity, together with the investment function, builds up a second order nonlinear dynamic system that describes the evolution of the stocks of public debt and physical capital. The stability of the equilibrium depends on the fiscal and monetary policy rules. When the coefficient of monetization of the deficit is zero and public expenditure varies endogenously with the burden of debt, then a necessary condition for dynamic equilibrium is

$$\left[\tau - r(1-\tau)w + \alpha(1-\tau)\right] + \frac{L}{L_1 y}\left[B + y\frac{\alpha}{r}\right] > 0 \qquad (7)$$

where, in addition to the symbols already introduced, we have

α share of capital income on total income

L desired share of money in the consumers' portfolio

L_1 first derivative of money demand w.r.t. the real interest rate

w wealth/output ratio

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[3] It should be noted that almost every piece of empirical work on the consumption function either finds or imposes a unit income elasticity, while the results on the interest elasticity of saving (or of consumption) are rather mixed. Therefore the implications of our simplifying assumption appear to be supported by the empirical evidence.

(all variables are evaluated at the steady-state).[4] The stability condition (7) depends on the stock of public debt in real terms B and can be expressed as follows

$$\frac{B}{y} < b^* \equiv -\frac{\left[\frac{\tau}{1-\tau} - rw + \alpha\right]\psi + \alpha}{r} \tag{8}$$

where $\psi = (1\text{-}\tau)L_1 r/L < 0$ is the net interest elasticity of money demand.

The threshold (8) is directly proportional to τ and inversely proportional to r, as it is the threshold (5), and for basically the same reasons.[5] Moreover, the threshold (8) is also directly proportional to the absolute value of ψ. This feature is related to the crowding-out mechanism represented in the model. When an expansionary fiscal policy is financed by deficit, the interest rate needs to increase in order to induce the economic agent to reallocate their equilibrium portfolio from money to debt. The larger the responsiveness of money demand to the interest rate (as measured by the absolute value of ψ), the smaller the increase in r needed for restoring the portfolio equilibrium allocation. Therefore, a larger value of $|\psi|$ means *ceteris paribus* a lower increase in r, with smaller effects on the burden of debt.

It should be stressed that the stability condition (7), and therefore equation (8), is only a necessary condition: the necessary and sufficient conditions are met when equation (7) is satisfied together with two other inequalities, both depending on B/y. It can be shown that when the model parameters fall in a plausible range one among these two additional conditions is less restrictive than equation (7) (i.e., it is automatically satisfied if (7) holds), while the other can be more restrictive.

[4] Equation (7) is obtained by substituting the equations (5), (8) and (12) of Tobin and Buiter (1976) in their equation (25).

[5] An anonymous referee pointed out that the positive relation between the tax rate and the sustainability threshold is questionable, because economic agents are unwilling to accept "excessive" levels of taxation. In fact, the models from which the stability conditions are derived are quite stylized and their features do not allow us to give an empirical content to the concept of "excessive" taxation. However, in a more realistic setting, increasing taxation without restraints cannot be seen as a viable strategy for enhancing public debt sustainability.

However, equation (7) suits our needs, because we are interested in finding the values beyond which the debt determines the dynamic instability of the system (which are defined by the necessary conditions for stability).

	$\tilde{b}$ (neoclassical) Eq. (5)	b^* (Keynesian) Eq. (8)	b_t (actual value)
BEL	-1.93	-4.67	1.02
DEU	-0.23	1.94	0.65
ESP	0.41	-15.39	0.64
FRA	-0.83	3.50	0.70
IRE	18.40	-12.67	0.32
ITA	-0.32	1.38	1.20
NLD	-0.48	-5.93	0.52
PRT	0.04	1.71	0.59
EMU	-0.44	1.50	0.72

Table 1 – Sustainability indicators and current value of the public debt/GDP ratio for eight member countries and the EMU area.

4. Empirical Evidence for EMU countries

In this section we evaluate the sustainability thresholds (5) and (8) using data from eight countries of the Euro area; we also evaluate the indicators using area-wide aggregate data for the EMU. Summing up, the parameters needed in order to calculate the neoclassical threshold (5), under the simplifying assumption of Cobb-Douglas utility and technology, are six: ε, k, n, r, s and τ, while for the Keynesian threshold (8) we need only five parameters: α, r, τ, ψ and w. The two thresholds have two parameters in common: r and τ. Only two among the nine parameters required for the calculations need to be estimated by econometric methods: the interest elasticities of money demand, ψ, and investments, ε. The others can be constructed simply as ratios or growth rates of variables that are easily found in the published national account statistics.[6] This reduces the degree of arbitrariness in the evaluation of the indicators.

4.1Sustainability in EMU member countries

Table 1 reports the Keynesian and neoclassical sustainability indicators for the eleven countries considered and for the EMU, evaluated using the sample average of the parameters over the last thirty years (see the appendix for a detailed description of the calculations). A negative value of the indicator is clearly a signal of unsustainability, as it means that dynamic stability cannot be achieved for any positive value of the public debt/GDP ratio. As the two indicators are based on different models, we should not expect them to agree in their conclusions. However, in three out of nine cases, namely for Belgium, Spain, and the Netherlands, both indicators agree that public debt is unsustainable. In another case, Ireland, the neoclassical indicator is in favour of sustainability, while the Keynesian is not. On the contrary, in the remaining cases (Germany, France, Italy, Portugal, and the EMU area) the Keynesian indicator supports sustainability, while the neoclassical does not. Summing up, the neoclassical indicator gives a somewhat more pessimistic view, while the Keynesian indicator is less stringent.

Table 2 compares these results with those of the most recent studies based on the solvency constraint. These studies do not always agree on their conclusions, with some exceptions: Germany (where sustainability is supported in three out of four studies), Belgium, Ireland, Italy and Portugal (where the public debt is generally found to be unsustainable). Since the Nineties have witnessed a great effort of fiscal retrenchment in most of the countries considered, differences in the results between the earliest and the latest applications of the solvency tests may well depend on differences in the sample considered (with the most recent tests being more favourable to solvency).

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[6] A detailed indication of the data sources is reported in the statistical appendix available on http://bagnai.org.

	Sustainability indicators		Solvency constraint tests			
	$\tilde{b}$ (neoclassical) Eq. (5)	b^* (Keynesian) Eq. (8)	Corsetti and Rubini (1991)	Artis and Marcellino (1998)	Payne (1997)	Bravo and Silvestre (2002)
BEL					NA	
DEU		*	*		*	*
ESP			*		NA	
FRA		*				*
IRE	*				NA	
ITA		*				
NLD					NA	*
PRT		*	NA		NA	
EMU		*	NA	NA	NA	NA

Table 2 – Empirical results on public debt sustainability in EMU member countries. A star indicates that the indicator/test is in favour of the sustainability of the public debt.

While the solvency constraint tests end up in a dichotomous answer (sustainable/unsustainable), the dynamic stability indicators are more informative. In particular, they allow the distance of a given country from unsustainability to be measured. For instance, while according to the Keynesian indicator both Germany and Italy have a sustainable debt level, it is easily verified that the Italian public debt is closer than the German one to "unsustainability". On the other hand, although according to the neoclassical indicator the current level of debt is unsustainable in both Belgium and Portugal (a result on which the solvency tests are unanimous), it is apparent that Portugal (with a positive, even though small threshold) is much closer to sustainability than Belgium.

	b^* (Keynesian) Eq. (8)	γ (sample average)	$\bar{f} = \gamma b^*$
DEU	1.94	0.043	0.084
FRA	3.50	0.036	0.124
ITA	1.38	0.055	0.075
PRT	1.71	0.083	0.141
EMU	1.50	0.043	0.034

Table 3 – Deficit/GDP ceilings consistent with dynamic stability of the public debt/GDP ratio according to the Keynesian indicator.

More generally, the results in Table 1 show large differences between EMU member countries as far as the dynamic stability of their public debt is concerned. This raises the issue of the opportunity of imposing on all these countries the same deficit/GDP ceilings, as envisaged by the Maastricht treaty and the SGP. Since the steady state levels of debt and deficit are related, we can easily derive from Table 1 deficit ceilings grounded on economic theory. For instance, using the continuous time approximation of the debt stock/flow identity $\dot{b} = f - \gamma b$, where a dot indicates the time derivative, f is the deficit/GDP ratio and γ the nominal growth rate, the long-run deficit/GDP ratio $\overline{f}$ consistent with a steady state debt/GDP ratio of $\overline{b}$ is equal to

$$\overline{f} = \gamma\overline{b} \tag{9}$$

Using (9) and the sample average of γ in the post-Maastricht sample (1990-2003) we find for instance that in the countries where public debt is sustainable according to the Keynesian indicators the upper bounds of the deficit/GDP ratio consistent with dynamic stability are those reported in Table 3. An interesting finding of Table 3 is that the Keynesian indicator implies a 3.4% deficit ceiling for the aggregate EMU economy. Since area-wide aggregate parameters are in fact weighed averages of the corresponding country specific parameters, this result justifies the 3% deficit ceiling envisaged by the SGP in terms of dynamic stability of an "average" EMU member economy. However, the dispersion around this average is substantial, and in most cases (including Italy) the Keynesian sustainable deficit/GDP ratios are well above the 3% ceiling.[7]

[7] An anonymous referee has observed that some ceilings evaluated in Table 3 are quite large and therefore can hardly be seen as sustainable in the long run. In fact, the sustainability thresholds are necessarily approximated and should be interpreted with caution, being derived from the linearization of very simple dynamic models. We believe nevertheless that they provide useful insights into the sustainability stance of the countries considered.

4.2 Cross-country comparisons

Since the indicators depend on a restricted number of parameters, it is easy to compare the situation of different countries using "diamond" graphs where each parameter is plotted on a separate axis. The figures from 1 to 6 present such an analysis of the Keynesian indicator for a selected number of countries. The units of measurement are chosen in such a way that sustainability increases with the distance from the origin of the axes and scaled so that the origin coincides with the lowest value observed in the sample. Therefore, the real rate of interest r and the capital share on output α *decrease* with the distance from the origin (since lower values of these variables enhance sustainability, as results from Table 5), while the interest elasticity of money is taken in absolute value and increases with the distance from the origin. In other words, public debt is the less sustainable the more the "diamond" graph collapses toward the origin. This allows for quick comparisons between the sustainability positions of different countries. In Figures 1-6 we report as a benchmark the "diamond" constructed using the average EMU parameters.

Some features emerging from Figures 1-6 are worth noting. For instance, while the parameters of France and Germany are close to the EMU average (as expected, given that these two countries account for more than a half of EMU area in terms of GDP), the higher sustainability thresholds of France is explained almost exclusively by a higher tax ratio.

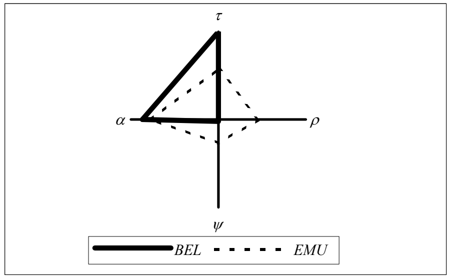

Figure 1 – Sustainability parameters in Belgium and the EMU.

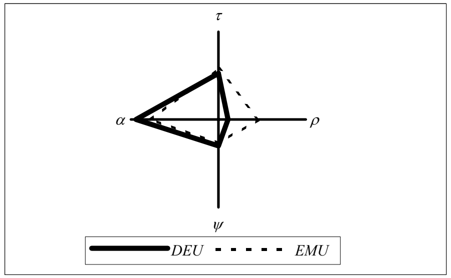

Figure 2 – Sustainability parameters in Germany and the EMU.

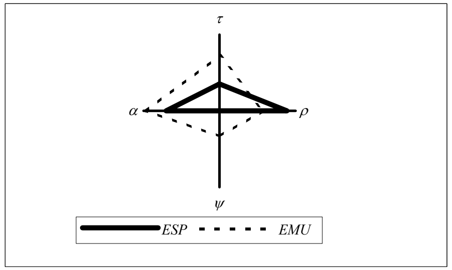

Figure 3 – Sustainability parameters in Spain and the EMU.

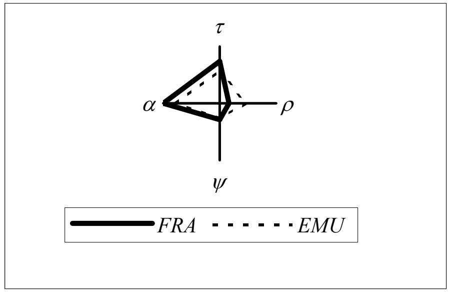

Figure 4 – Sustainability parameters in France and the EMU.

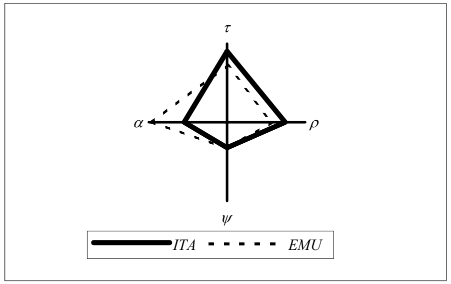

Figure 5 – Sustainability parameters in Italy and the EMU.

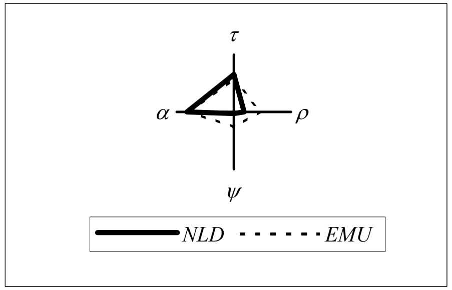

Figure 6 – Sustainability parameters in the Netherlands and the EMU.

Figures 1-6 – "Diamond" graphs of the parameters that affect the Keynesian sustainability indicator (ρ indicates the real interest rate). The parameters are scaled and measured in such a way that the sustainability thresholds increase with the distance from the origin of the axes. Each graph reports the average EMU parameters (dotted "diamond").

On the contrary, the lower sustainability threshold of Italy depends on a capital share on output higher than the European average, even though the long-run real interest rate is lower and the tax ratio is higher in Italy than in the average EMU country. The unsustainability of the Belgian public debt is determined instead by a real interest rate above the European average, joined with an interest elasticity of money below the European average: these two parameters lead to a negative sustainability threshold even in the presence of the highest tax ratios among the country considered.

4.3 The evolution of sustainability over time

The thresholds in Table 1 are evaluated using sample averages over the last thirty years, taken as an approximation of the steady state (or lung-run) values of the corresponding parameters. However, most parameters entering the sustainability indicators have shown persistent variations over the last decades. It is therefore of some interest to observe how these shifts in the parameters have affected the sustainability thresholds.

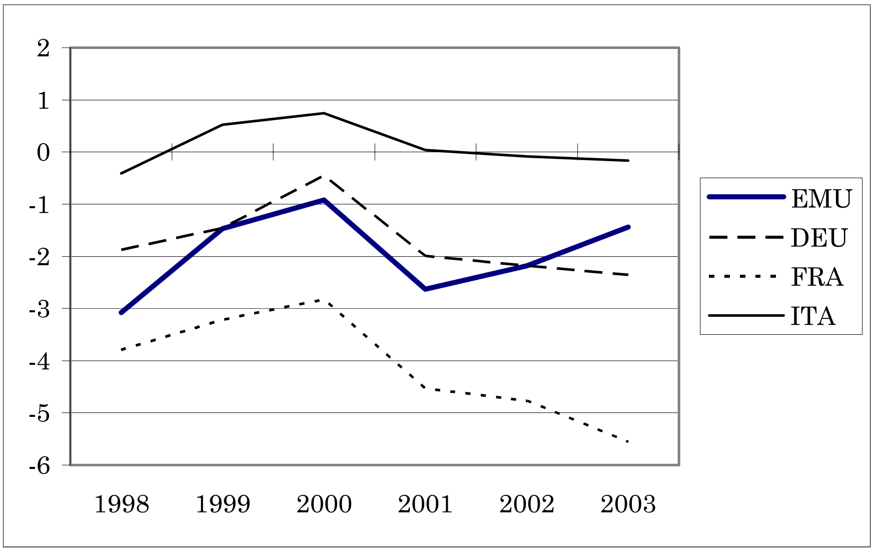

Figure 7 – The spread between the current value of the public debt/GDP ratio and the Keynesian sustainability threshold in some EMU member countries. Positive values indicate that public debt is above the sustainability threshold.

As an illustration of this kind of analysis, consider Figure 7, reporting the spread between the current value of the public debt/GDP ratio and the Keynesian sustainability threshold (8) evaluated with the current values of r, τ, α and w. A positive spread indicates that the public debt is above its sustainable level. Figure 7 shows that in the first two years since the adoption of the Euro Italy has experienced sustainability problems; this applies especially to 2000, a year in which rising real interest rates determined a decrease in the sustainable level of public debt throughout the EMU. Since then the favourable evolution of the real interest rate has brought Italy below its sustainable threshold.

A similar analysis performed with the neoclassical indicator would show that the sustainability position of most countries deteriorated with the inception of EMU, mostly because of a fall in real growth rates.

5. Conclusions

As stated above, the purposes of this paper were twofold: on the methodological side, it aimed at critically evaluating the solvency criterion, pioneered by Hamilton and Flavin (1986), which is nowadays almost hegemonic in the analysis of public debt sustainability, and at illustrating alternative measures of sustainability based on the dynamic stability approach originated by Domar (1944); on the applicative side, it looked at sustainability in EMU member countries, with particular attention given to the relations between sustainability and the design of fiscal rules.

The alternative approach proposed and applied in this paper leads to the definition of a sustainability threshold for the public debt, and therefore measures how far from unsustainability a given country is, which the solvency approach does not; it allows us to cast the debate on sustainability in terms of meaningful economic parameters, which the solvency approach does not; in particular, it assesses the effectiveness of fiscal rules expressed in terms of government deficit/GDP ratio, which the solvency approach does not; finally, it allows us to examine the evolution of sustainability over time, which the solvency approach does not.

While improving over the solvency criterion for the reasons listed above, the dynamic stability approach does not fully overcome some of its limitations.

First, the stability conditions are obviously model specific, as is the solvency criterion (which was recalled before to feature only in a well specified class of intertemporal models). On the one hand, this questions the general validity of the results obtained, although we have seen that some key parameters play quite similar roles in indicators stemming from very different models, thus mitigating the "model specificity" problem. On the other hand, however, the dynamic stability approach has, in our view, the merit of being intrinsically neutral, because it can be applied to models grounded on different economic theories, as shown in this paper. This does not apply to the solvency criterion.

Secondly, the stability conditions are analytically manageable only in rather stylized dynamic models. This confines the empirical applications of the stability criterion to a relatively narrow class of economic models. In fact, using the words of Zee (1988, p. 669), the sustainability indicators should be seen only as "first order approximations of the margin between an economy's existing debt level, and the level at which it can be sustained, given unchanged existing economic conditions". The most relevant feature that the sustainability indicators fail to take into account is probably the impact of demographic evolution on the sustainability of fiscal policies. It should be stressed, however, that the same applies to the solvency tests. We believe in this respect that a reliable assessment of the sustainability of a country's public debt can be carried out only within a full structural model of the economy, linked with a submodel representing the medium- to long-run demographic tendencies.

Keeping in mind these limitations, the indicators applied in this paper confirm the rather gloomy picture emerging from previous empirical analysis of sustainability in the EMU. In particular, the neoclassical indicator leads to more pessimistic results (in line with those of the earliest study based on the solvency constraint), while the Keynesian criterion finds sustainability in five out of the eight countries considered, as well as in the EMU area taken as a whole.

The indicators, however, besides confirming (or refuting) the results of previous studies, give a number of other useful insights. For instance, both indicators point out that as far as sustainability is concerned, EMU member countries find themselves in very different positions from each other. This questions the validity of uniform deficit caps such as those envisaged by the SGP. More specifically, by applying the Keynesian criterion it was shown that a 3% deficit cap can in fact be justified as a sustainability requirement for an "average" EMU member country. At the same time, it was also shown that dispersion around this "average" is very large, and that as far as sustainability is concerned this ceiling is too low for the three major member countries (while being too permissive for other countries). In other words, if we believe that the Keynesian dynamic model provides a valid representation of the economic reality, we must conclude that the recent decision of France and Germany not to comply with the provisions of the SGP is unobjectable on sustainability grounds.

As far as Italy is concerned, an application of the Keynesian criterion shows that its public debt is very close to the sustainability ceiling and even exceeded it in the first years if the EMU, as a consequence of the adverse evolution of the real interest rates. The relatively low sustainability threshold of Italy was shown to depend on a relatively high capital share on income (hence a relatively low long-run fiscal multiplier), in the presence of a real interest rate below, and a tax ratio above, the EMU average. Bringing the capital share on income in line with the EMU average would raise the sustainable level of debt to 280% of GDP.

Finally, the purpose of this paper was obviously not that of saying an ultimate word on the topic, but rather that of pointing out that "another sustainability is possible": beyond (and before) the intertemporally glamorous solvency constraint, there is a strand of literature which needs and deserves to be further pursued.

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