The inefficiency of firm-augmenting public input

vs.

The inapplicability of provision rules

by

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Summary

The paper "The inefficiency of firm-augmenting public input vs. the inapplicability of provision rules" contributes to the debate about the appropriate efficiency rule for the provision of a firm-augmenting public input. The debate about an appropriate efficiency rule is due to the dissatisfaction of Kaizuka-rule, i.e. a Samuelson-type condition for public inputs, in the long run (e.g. Richter 1994). Whereas most authors obliterate the difference between short and long run by fixing the number of firms (Hillman 1978, McMillan 1979, Feehan 1989, Richter 1994, Matsumoto 2000) Boadway (1973) and Henderson (1974) claim that the Kaizuka-rule is not applicable in the long run.

According to Boadway's (1973) graphical based reasoning increasing the level of firm-augmenting public input increases social surplus in the long term. Thus alternatively to the Kaizuka-rule, the apt rule for the provision of firm-augmenting public input is the equality of the rise in social surplus to the climb of cost of public input provision at the margin – i.e. the Boadway-rule. But by deriving the Boadway-rule for the first time analytically the author shows that a rise of firm-augmenting public input has no effect on social surplus. Thus, an exogenous change in firm-augmenting public input is neutral to welfare. Only in the short run there is a benefit of firm-augmenting public input, which corresponds to the sum of its marginal revenues to firms. As this sum is equated to the marginal cost of public input provision the Boadway-rule is essentially the Kaizuka-rule. Since in the long term the sum of marginal revenues is zero both rules are not met.

The latter is due the concept of firm-augmenting public input itself. As firm-augmenting public input is a produced good a long-term efficient provision requires positive marginal productivities to firms because marginal costs of firm-augmenting public input are positive. At the same time firm-augmenting public input is rival between private factors within a firm and non-rival among firms. This vertical mixture of goods properties gives rise to a zero marginal

productivity in a long run competitive equilibrium. Thus, the vertical mixture of firm-augmenting public input and the assumption of firm-augmenting public input cause an a priori exclusion of a long-term efficient equilibrium.

As a consequence this analysis confirms empirical observations from authors that say no convincing examples for firm-augmenting public input exist (Hillman 1978, McMillan 1979, Feehan 1989), and backs up Feehan's assumption (1998), which says that alone factor-augmenting public input as a public intermediate product is theoretically justified. Finally, in view of the results of this paper models that use firm-augmenting public input, such as that of fiscal competition (e.g. Richter 1994, Matsumoto 2000) and of endogenous growth (e.g. Barro/Sala-i-Martin 1992, Turnovsky 1996, Ott 2000), should be reconsidered.

Abstract

This paper contributes to the debate about the appropriate efficiency rule for the provision of a firm-augmenting public input. This debate is caused by the dissatisfaction of Kaizuka-rule, i.e. a Samuelson-type condition for public inputs, in the long run. Therefore the applicability of Kaizuka-rule has been questioned. By developing an alternative efficiency rule this paper shows that firm-augmenting public input cannot be provided efficiently. The latter is due to the goods' properties of firm-augmenting public input along with the assumption of firm-augmenting public input as an intermediate good a long term efficient equilibrium is excluded a priori. Consequently firm-augementing public input is unsuited for depicting public intermediate goods in economic models. Thus models, which use firm-augmenting public input, such as that of fiscal competition and of endogenous growth, should be reconsidered.

1 Introduction*

This paper contributes to the debate about the appropriate efficiency rule for the provision of a firm-augmenting public input. This debate is due to the fact that the Kaizuka-rule is not met in a long-run competitive equilibrium in cases of firm-augmenting public input (e.g. Richter 1994). The Kaizuka-rule is the counterpart to the Samuelson condition for public consumer goods. Whereas most authors obliterate the difference between short and long run by fixing the number of firms (e.g. Hillman 1978, McMillan 1979, Feehan 1989, Richter 1994, Matsumoto 2000), Boadway (1973) and Henderson (1974) claim that the Kaizuka-rule is not applicable in the long run. By developing an alternative efficiency rule to the Kaizuka-rule – the Boadway-rule – this paper gives new insights into the causes of this long-run inefficiency. Moreover, the findings of this paper influence the results of analyses in modern theories of endogenous growth and fiscal competition, where the concept of firm-augmenting public input is used (e.g. Richter 1994, Turnovsky 1996, Matsumoto 2000). As examples for firm-augmenting public input, legal and security services are usually cited (e.g. Feehan 1989).

From the mentioned debate the following proposition arises: If the Kaizuka-rule is not applicable, another efficiency-rule, e.g. the Boadway-rule, should be. Otherwise it follows that firm-augmenting public input cannot be provided efficiently in the long run. This proposition is examined in the following with respect to the Boadway-rule.

For this, the Boadway-rule is derived in sections 3 and 4 and compared to the Kaizuka-rule in section 4. Section 5 analyses the causes of the long run inefficiency of the firm-augmenting public input. Final comments are given in Section 6. First, however, the model, which will be used to develop the Boadway-rule, is presented in section 2.

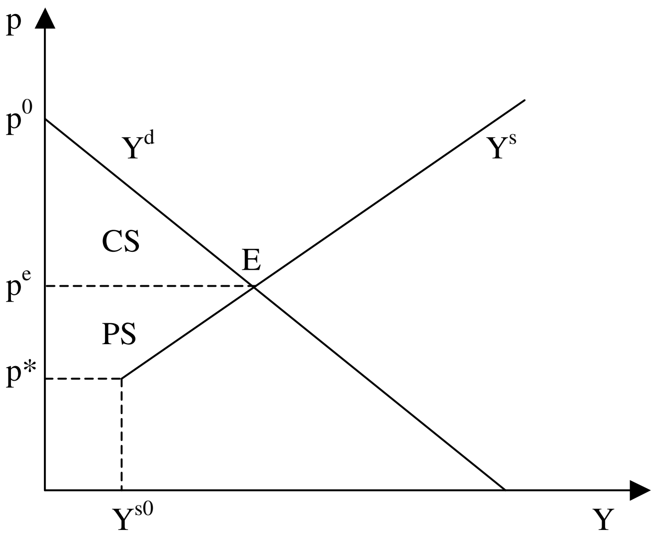
2 Model

This analysis proceeds on the assumptions made in Boadway`s diagrammatic study (1973, section III.B) because the Boadway-rule traces back to the latter. Analogous to Boadway's (1973) model, we are looking at a single consumer goods industry (Y), to which the wage rate (w) is given. A governmental institution provides the firm-augmenting public input to the firms of this industry free of charge. For the sake of simplicity, public input is assumed to be financed by a lump sum tax. Furthermore, a governmental institution has knowledge of the production technology of the private firms and has no self-interests. The amount of firm-augmenting public input is set from the firms' point of view. Due to the firm-augmenting public input's non-rivalry between firms in the industry Y, each firm i can use the same amount simultaneously. However, the firm-augmenting public input must be divided between the workers within the firm. This is due to the assumption of linearly homogeneous production technology in labour (L_i) and firm-augmenting public input (G) (see McMillan 1979, pg. 279) to which each of the firms i in industry Y has free access (see appendix, ad (1)):[1]

$$Y_i = f(L_i; G) \qquad \text{with: } i = 1,...,n. \tag{1}$$

As firm-augmenting public input is provided to the firms free of charge, their costs are solely those of labour. Both the consumption goods market as well as the labour market is in a state of perfect competition. The consumer side of the goods market is represented by the Marshall demand function $Y^d(p, \overline{w})$.[2]

Using the following illustration shows the well-known concept of social surplus:[3,4]



Y^d: = market demand function
Y^s:= market supply function for given number of firms
Y^{s0}: = market supply amount for p=p*
p: = market price for good Y
Y: = amount of good Y
CS: = consumers' surplus
PS: = producers' surplus

Illustration 1: Social surplus for the market of good Y.

Given the firm-augmenting public input, the wage rate and the number of firms as exogenous, the social surplus is determined by the equilibrium market price (p^e). As can be seen from illustration 1, social surplus complies with the sum of consumers' surplus, measured by the triangle p^eEp^0, and producers' surplus, which is equal to the area p^eEp^*B:

$$\overline{S} = \underbrace{\int_{p^e}^{p^0} Y^d(\psi;\overline{w})d\psi}_{=CS} + \underbrace{\overline{n}(p^e Y_i^s(\hat{L}_i(p^e;\overline{w};\overline{G});\overline{G}) - \overline{w}\hat{L}_i(p^e;\overline{w};\overline{G}))}_{=PS} \qquad (2)$$

with: ψ := integration variable,

$\hat{L}_i$:= profit maximising amount of labour input for firm i,

$\hat{L}_i(p;\overline{w};\overline{G})$:= labour demand function for firm i; $\frac{\partial \hat{L}_i}{\partial p} > 0, \frac{\partial \hat{L}_i}{\partial G} > 0, \frac{\partial \hat{L}_i}{\partial w} < 0$,

Y_i^s := supply amount for firm i,

$Y_i^s(\hat{L}_i(p;\overline{w};\overline{G});\overline{G})$:= supply function for firm i; $\frac{\partial Y_i^s}{\partial G} > 0$, $\frac{\partial Y_i^s}{\partial \hat{L}_i} > 0$,

$\overline{n}Y_i^s = Y^s(p;\overline{n};\overline{w};\overline{G})$:= market supply function.

Since the number of firms is exogenous the social surplus (2) measures c.p. the social welfare for the short-term market equilibrium (E).

3 Cost and welfare effects of an exogenous firm-augmenting public input

This section shows that contrary to what has been stated in relevant literature since Boadway (1973), an increase of firm-augmenting public input does not cause a decreasing minimum average cost to a single firm. At the same time this is a first step to deriving the Boadway-rule. As the focus in this section is on private costs, the cost and welfare effects of an *exogenous* increase of firm-augmenting public input will be described (see section 3.2). At first it will be shown, that with given exogenous firm-augmenting public input the long term market equilibrium for the consumer good is Pareto efficient (c.p.) (see section 3.1).

3.1 Long-term market equilibrium

In order to ascertain the conditions for a long-term market equilibrium for which social surplus is c.p. at a maximum, social surplus will be optimised with respect to the number of firms.

For this the market equilibrium condition has to be taken into account:

$$Y^d(p^e;\overline{w}) = nY_i^s(p^e;\overline{w};\overline{G}) \tag{3}$$

By using the implicit-function theorem, it is gathered from condition (3), that the equilibrium market price is a function of the number of firms, firm-augmenting public input and wage rate:

$$p^e = p^e(n,G,w) \qquad \text{with: } \frac{\partial p^e}{\partial n} < 0,\ \frac{\partial p^e}{\partial G} < 0,\ \frac{\partial p^e}{\partial w} > 0. \tag{4}$$

Hence, by adding equation (4) into the social surplus function (2), the following is true:

$$\begin{aligned} S = & \int_{p^e(n,\overline{G},\overline{w})}^{p^0} Y^d(\psi;\overline{w})d\psi \\ & + n(p^e(n,\overline{G},\overline{w})Y_i^s(\hat{L}_i(p^e(n,\overline{G},\overline{w});\overline{w};\overline{G});\overline{G}) \\ & - \overline{w}\hat{L}_i(p^e(n,\overline{G},\overline{w});\overline{w};\overline{G})) \qquad \rightarrow \max_n. \end{aligned} \tag{5}$$

The efficiency condition for maximum social surplus is:

$$p^e = \frac{\overline{w}\hat{L}_i}{Y_i^s}. \tag{6}$$

In accordance with condition (6), the equilibrium price must be equal to a firm's average costs for maximum social surplus.[5] As the firms are producing at maximum profit, (6) implies, that average costs are equal to marginal costs:

$$\frac{\overline{w}\hat{L}_i}{Y_i^s} = \frac{\overline{w}}{\frac{\partial f}{\partial L_i}}. \tag{7}$$

In market equilibrium with maximum social surplus, firms will make no profits (see (6) and (7)) so that no producers' surplus will arise (see illustration 2b). Moreover, condition (7) corresponds to the necessary condition for minimum average cost. As firms will enter the market until no more profits can be made, the market equilibrium at maximum social surplus is in accordance with the long-term competitive equilibrium.

Thus, the long term market equilibrium, given an exogenous firm-augmenting public input, is Pareto efficient, c.p. Consequently, the number of firms (n*) determined by the market's allocation mechanism is welfare maximising. The latter can be clarified by looking at the condition for long-term market equilibrium. For this the long-term supply function (Y_l^s) has to be ascertained. As the latter is only valid for the long-term equilibrium market price, which in turn corresponds to the minimum average costs (p^e=p*), the long-term supply function is horizontal (see illustration 2b):[6]

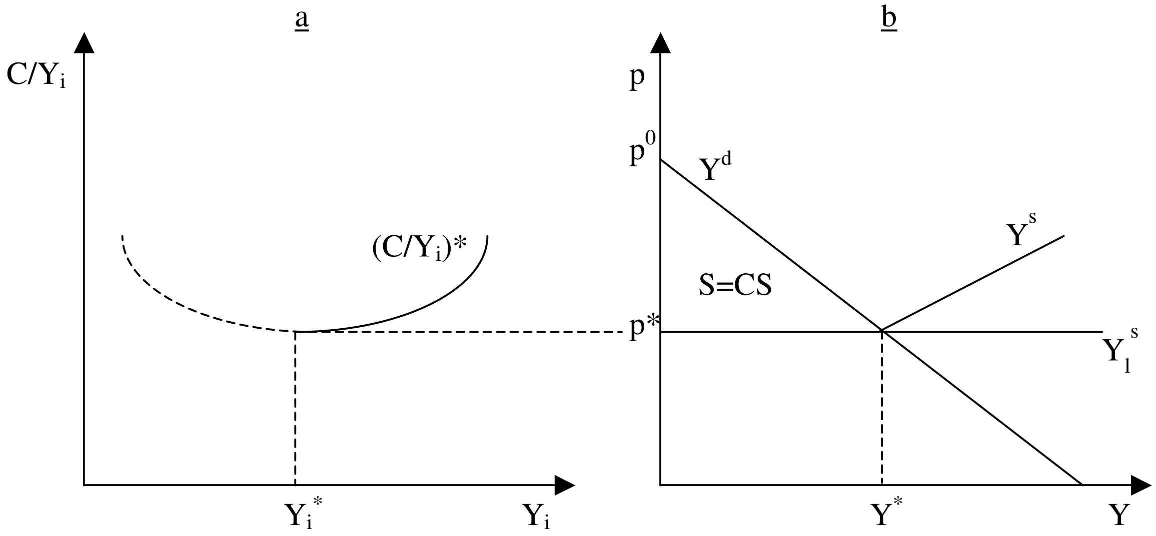
$$Y_l^s = Y_l^s(n, p^*; \overline{G}, \overline{w}) = nY_i^s(p^*; \overline{G}; \overline{w}). \tag{8}$$

Inserting the long-term supply function (8) into the market equilibrium condition (3) will result in the condition for long-term market equilibrium:

$$Y^d(p^e; \overline{w}) = Y_l^s(p^*, n; \overline{G}; \overline{w}) \quad \Rightarrow \quad p^e = p^* \ \wedge \ n = n^* \tag{9}$$

Since the long-term market equilibrium price (p^e=p*) is determined by the supply function (8), the equilibrium condition (9) assign the welfare maximising number of firms (n*).

As an interim result it can be seen that social surplus in long-term competition equilibrium, given exogenous firm-augmenting public input and an endogenous number of firms, is at a maximum.



with: $Y^s = n^* Y_i^s(p;\overline{G};\overline{w})$ and $Y_l^s = n\, Y_i^s(p^*;\overline{G};\overline{w})$.

Illustration 2: Firm and market equilibrium at maximum social surplus.[7]

3.2 Exogenous increase of firm-augmenting public input

Assuming long term market equilibrium (p^*,Y^*) and firm equilibrium Y_i^*, the effect of an exogenous increase in firm-augmenting public input on minimum average costs and social surplus will now be examined. The analysis in this section shows, that Boadway`s (1973, 254) intuitively plausible result, which will be explained in the following, cannot be true.

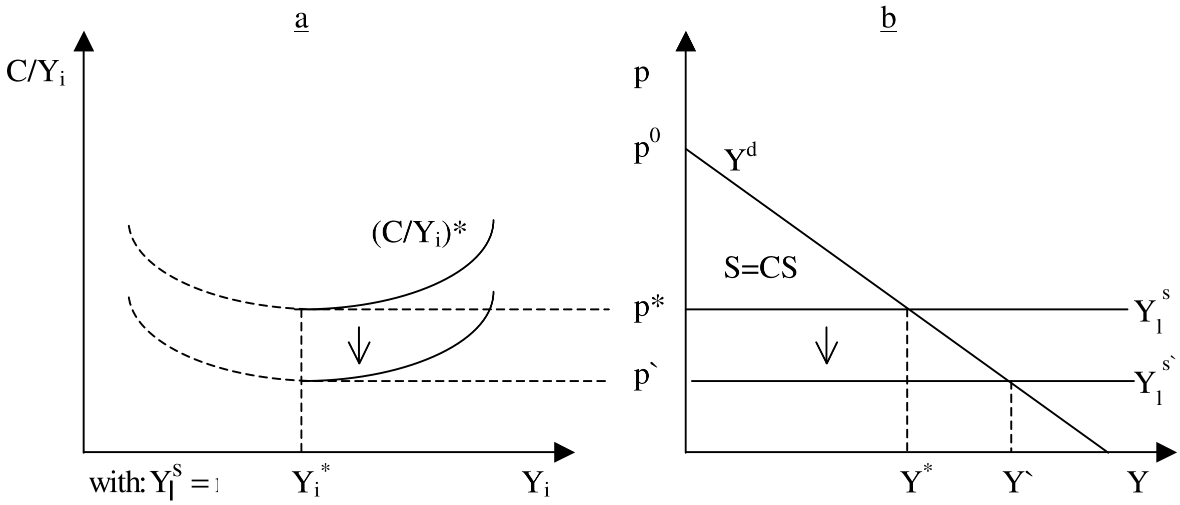


Illustration 3: Shifting the average cost curve ($\frac{C}{Y_i}$) and the supply function Y_l^s through dG>0 in accordance with Boadway (1973, 254 III. 3a and 3b).

If a governmental institution increases the output of firm-augmenting public input, each firm's average cost curve will, according to Boadway (1973, 253), shift vertically downwards, without affecting the firm's output (Y_i*). Accordingly, the horizontal supply curve Y_l^s will move down, which, with a given demand curve (Y^d), will cause the market price to sink from p* to p'. The equilibrium amount will now rise from Y* to Y', so that with firm output remaining constant (Y_i*), the number of firms must have risen. The rise in the number of firms can be explained by the fact that the established firms can make profits after a rise in G because the average costs have sunk. The prospects of making a profit are an incentive for more firms to enter the market until no more profits can be made. According to Boadway's analysis, the social surplus will grow by an increase in firm-augmenting public input (see illustration 3b). In order to achieve this result, it must be true that the firms' minimum average costs can be reduced through an increase in firm-augmenting public input. This will now be examined on the basis of the production function assumed by Boadway (1973).

The assumption of a linearly homogeneous production function causes the average cost function to be dependent only on the factor ratio (g) between the firm-augmenting public input and the amount of labour input per firm. Due to the assumption of linear homogeneity, the production function (1) can be formulated as follows (see appendix (A1)):

$$Y_i = L_i z(g) \qquad \text{with: } g = \frac{G}{L_i}. \tag{10}$$

Consideration of the production function (11) leads to the following expression for average costs:

$$\frac{C}{Y_i} = \frac{\overline{w}}{z(g)} \;\rightarrow\; \min_g . \tag{11}$$

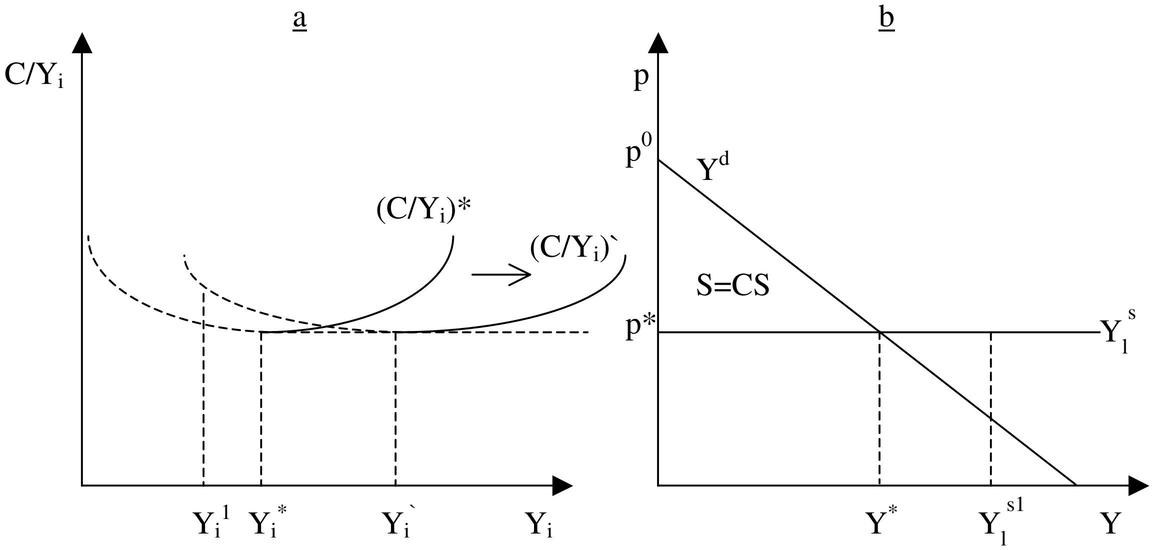
Minimising the average costs over g results in the following condition:

$$\frac{d\left(\frac{C}{Y_i}\right)}{dg} = -\frac{\overline{w}z'(g)}{z(g)^2} = 0 \;\Rightarrow\; z'(g) = 0 . \tag{12}$$

In addition, the average cost function's second derivative is positive,[8] so that *a minimum of average costs exists only for a single factor input ratio (g=g*).* Thus the minimum of average costs is unique. As a result, the amount of labour input per firm must rise proportionally to G and the firm's costs proportionally to their output, so that after an increase in firm-augmenting public input the firms can produce at minimum average costs again. *Thus, contrary to Boadway's depiction (see illustration 3a), an increase in firm-augmenting public input will not reduce the minimum average costs. Therefore the average cost curve of each firm will shift not down, but to the right (see illustration 4a).[9] As the firms must at least cover their average costs in order to stay in the market the long-term supply function can also not shift downwards. (see illustration 4b).* Therefore it is not possible to lower the long-term equilibrium price (p^e=p*) by raising the firm-augmenting public input, which is mirrored in the condition for long-term market equilibrium:

$$Y^d(p^e; \overline{w}) = Y_l^s(p^*, n; G; \overline{w}) \;\Rightarrow\; p^e = p^* . \tag{13}$$

Thus the long-term market equilibrium will not alter if a governmental institution raises firm-augmenting public input. *Consequently, contrary to Boadway's results, social surplus will stay constant, although the same assumptions as in Boadway's model are made. Therefore, an exogenous variation of firm-augmenting public input is neutral to welfare.*



with: $Y_l^S = nY_i^S(p^*, G; \overline{w})$.

Illustration 4: Modification of illustration 3 corresponding to the assumption of a linearly homogeneous production function.

4 Equivalence of Boadway- and Kaizuka-rule

This section shows that the Boadway-rule is the partial analytical equivalent of the Kaizuka-rule. The latter was developed within a general equilibrium model (Kaizuka 1965).

Since in this section firm-augmenting public input is regarded as an endogenous variable, the costs of producing firm-augmenting public input have to be considered. Therefore, the difference between social surplus (S) and the costs of production of firm-augmenting public input (C_G) is chosen as a measure of welfare. This difference shall be called net surplus (N).[10]

$$N = S(p^e(n, G, \overline{w}), n, G; \overline{w}) - C_G(G) \quad \rightarrow \quad \max_{n,G}. \tag{14}$$

Given a fixed wage rate, the social welfare is at a maximum if a governmental institution can maximise net surplus (N) by the number of firms (n) and firm-augmenting public input (G). In order for this to be the case, apart from the efficiency condition (6), which is brought about by the partial differentiation of N with respect to n, a second condition must be true. Taking into account the social surplus function (see (5)), the first-order condition can be formulated as follows (see appendix, ad (15)):

$$p^e n \frac{\partial f}{\partial G} = \frac{dC_G}{dG}. \tag{15}$$

In accordance with (15), a governmental institution must provide firm-augmenting public input in such a way that the sum of marginal revenue of firm-augmenting public input using the market price p^e (left side of (15)) corresponds to the marginal costs of producing firm-augmenting public input (right side of (15)). Condition (15) contains the same statement as the Kaizuka-rule (see Kaizuka 1965, 120), so that it is the partially analytical equivalent of the Kaizuka-rule. According to Boadway (1973, 253), however, the Kaizuka-rule ((15) correspondingly) is only applicable if the number of firms is exogenous. Therefore, in considering Boadway (1973), a governmental institution should, given an endogenous number of firms, use a different rule, namely the Boadway-rule.

To develop the Boadway-rule, Boadway's (1973, 252) procedure will be used. Since a long-term market equilibrium is presupposed, the following will be true for the equilibrium market price:

$$p^* = p^e(n, G, \overline{w}). \tag{16}$$

As only a unique equilibrium price exists in the long run (see section 3.2), the number of firms will be a function of firm-augmenting public input in the long term:[11]

$$n^* = n^*(G) \qquad \text{with: } \frac{dn^*}{dG} < 0. \tag{17}$$

Therefore - as was assumed before (see (14)) - it is still possible for a governmental institution to influence the number of firms, but now indirectly, through the variation of firm-augmenting public input. As the latter is an incentive or disincentive respectively to set up new firms (see (17)), the number of firms finally results from the decisions of market participants.

In order to ascertain the influence an increase of G will have on social welfare, presupposing a long term market equilibrium, equations (16) and (17) have to be inserted into the net surplus function:

$$N = S(p^*, n^*(G), G; \overline{w}) - C_G(G) \quad \rightarrow \quad \max_G . \tag{18}$$

Now, in order to maximise social welfare, a governmental institution has to consider the Boadway-rule, which corresponds to the first-order condition of (18) (see appendix, ad (19)):

$$(p^* Y_i^s - \overline{w}\hat{L}_i)\frac{dn^*}{dG} + p^* n^*(G)\frac{\partial f}{\partial G} = \frac{dC_G}{dG} . \tag{19}$$

According to (19) a governmental institution should obviously consider that a change in firm-augmenting public input will influence the number of firms on the goods` market. Thus it is clear what Boadway's differentiation of the efficiency rules into exogenous and endogenous number of firms is based on. However, the differentiation has to be stated in another way: in the case of the Kaizuka-rule, a governmental institution varies the number of firms directly by observing equation (6) (see (14)), whereas in the case of the Boadway-rule the number of firms finally results from the allocation mechanism of the market (see (17) and (18)). If the latter did not apply, $\frac{dn^*}{dG} = 0$ would be true. Having this in mind and looking at equation (19), one can easily see that the Boadway-rule is the equivalent to the Kaizuka-rule for a market-determined number of firms.

Even more important, as free market entry is presupposed, firms will make no profits in the long-term equilibrium, so that the term in brackets of the first summand of (19) will have to equal zero (see (6)). Due to variable output elasticities and linear homogeneity in the case of firm-augmenting public input $\frac{\partial f}{\partial G} = 0$ is true. Hence, the second summand of (19) also vanishes. Therefore, not only the Kaizuka-rule but also the Boadway-rule cannot be fulfilled in the long-term equilibrium.[12] Consequently the long-run inefficiency is *not* caused by the inapplicability of the Kaizuka-rule. In fact this long run inefficiency is due to the concept of firm-augmenting public input itself, which the next section will clarify.

5 Firm-augmenting public input rules out long-term efficiency

In this section it will be shown why the specification of firm-augmenting public input makes it impossible to determine a long run efficient solution.

From a firm's point of view, the amount of firm-augmenting public input is given, so that due to variable elasticities of production (see appendix, ad (1)), a factor input ratio that is efficient internally must be present. This is achieved at (g*), where average costs are minimal. By using the industry's production function for firm-augmenting public input, it can now be shown that maximum industry output with an endogenous number of firms, given a labour input amount specific to the industry, as well as firm-augmenting public input, is achieved exactly when each firm produces at average cost minimum.

$$Y=F(L;\, nG)=nf(L/n;G) \qquad \text{with: } L=nL_i. \tag{20}$$

For maximum industry output the following is true:

$$\frac{\partial Y}{\partial n}=Y_i - L_i\frac{\partial f}{\partial L_i}=0 \qquad \text{with: } dL=dG=0. \tag{21}$$

The necessary condition for maximum industry output (21) is equivalent to the necessary condition for an average costs minimum (7), so that at maximum industry output each firm will produce at average cost minimum.

The firms' production function's linear homogeneity implies rivalry of firm-augmenting public input within the firm, i.e. on the factor level. As the industry's production, specified in (20), also takes non-rivalry among firms into consideration, the efficiency condition (21) is finally the outcome of the goods' properties of firm-augmenting public input. *In this way the firm's marginal productivity of firm-augmenting public input is zero at an efficient number (and* *size) of firms, because firm-augmenting public input is rival on the factor level and non-rival on the firm level and, therefore, can be called a vertical mixed good.* Thus efficient use of firm-augmenting public input within an industry implies zero marginal productivity of firm-augmenting public input.

According to the Boadway- and Kaizuka-rule respectively, an efficient provision of firm-augmenting public input will require that the sum of firm's marginal revenues equals marginal costs of firm-augmenting public input (see (15) and (19)). Since the efficient use of firm-augmenting public inputs includes a zero marginal productivity of firm-augmenting public input the provision rules could only be fulfilled if marginal costs of production of firm-augmenting public input vanish also. However, as the assumption of minimum or maximum of a cost function has no economic meaning marginal costs of production can merely equal zero if total costs have an inflection point. Since marginal costs are minimal at the inflection point of total costs ($C_G(G)$) zero marginal costs cannot coincide with minimum average costs and thus with a zero marginal productivity of firm-augmenting public input ($\frac{\partial f}{\partial G}$). As a consequence the provision rules are also not fulfilled and firm-augmenting public input is produced inefficiently.

However, if a cost function ($C_G(G)$) without an inflection point, e.g. a linear homogeneous function, is assumed marginal costs have to be positive. Consequently Kaizuka- and Boadway-rule cannot be fulfilled. The provision rules could only be met if no resources for the provision of firm-augmenting public input were needed. A firm-augmenting public input would be similar to a free good. In that case a long run efficient equilibrium could be attained. But according to the concept of firm-augmenting public input the latter is assumed as an intermediate good and thus marginal costs are positive. *This along with the goods' properties of firm-augmenting public input, i.e. rivalry within a firm and non-rivalry between firms, cause an a priori exclusion of a long-term efficient equilibrium.* Therefore using a firm-augmenting public input makes it impossible to answer the crucial question how a governmental institution should provide public intermediate goods in the long run. *Consequently the specification of firm-augmenting public input is unsuited for depicting public intermediate goods in microeconomic-based models.*

6. Conclusion

This paper has shown that, under the benchmark-case of perfect competition and a welfare-maximising government, the firm-augmenting public input cannot be provided efficiently in the long term because firm-augmenting public input is a vertically mixed good and is assumed as an intermediate product. Correspondingly, the long-run inefficiency of firm-augmenting public input is not due to the inapplicability of the Kaizuka- and the Boadway-rule. In fact both are applicable. Furthermore, the analytical derivation of the Boadway-rule shows the equivalent to the Kaizuka-rule.

The conceptional problems of firm-augmenting public input are reflected empirically by observations from authors that say no convincing examples for firm-augmenting public input exist (Hillman 1978, McMillan 1979, Feehan 1989). Moreover, Feehan's assumption (1998), which says that alone factor-augmenting public input as a public intermediate product is theoretically justified, is backed up.

Finally, in view of the outcome of this paper models that use firm-augmenting public input, such as that of fiscal competition (e.g. Richter 1994, Matsumoto 2000) and of endogenous growth (e.g. Barro/Sala-i-Martin 1992, Turnovsky 1996, Ott 2000), should be reconsidered.

Appendix[13]

ad (1):

Due to linear homogeneity the production function (1) can be expressed as follows:

$$Y_i = f(L_i, G) = L_i z\left(\frac{G}{L_i}\right) = L_i z(g) \qquad \text{with: } g = \frac{G}{L_i} > 0. \qquad \text{(A1)}$$

A production function contains the set of technically efficient production processes by which negative marginal productivities of L_i and G are impossible (see Ferguson 1969, 115f). According to the specification of the production function with firm-augmenting public input, a point g=g* must exist, for which the following is true (see Henderson 1974, 324ff):

$$\frac{\partial Y_i}{\partial G} = \frac{\partial f}{\partial G}(g^*) = z'(g^*) = 0. \qquad \text{(A2)}$$

If the marginal productivity of firm-augmenting public input becomes zero (see (A2)), firms will carry on producing because the firm-augmenting public input is available to the firms free of charge. If (A2) is true, due to the linear homogeneity of the production function, a point $g=g_u$ must also exist, for which is true (see e.g. Ferguson 1969, chap. 5.3.):

$$\frac{\partial Y_i}{\partial L_i} = \frac{\partial f}{\partial L_i}(g_u) = z(g_u) - g_u z'(g_u) = 0. \qquad \text{(A3)}$$

Although the production processes (bG_u, bL_{iu}, bY_{iu}) (with: $g_u = \frac{bG_u}{bL_{iu}}$, b=constant and b>0) in $g=g_u$ are technically efficient, due to the positive real wage, they are not relevant economically. Thus, only technically efficient production processes in the interval $]g_u,g^*]$ are considered.

The first and second partial derivations of the linearly homogeneous production function (1) have the following signs in the interval $]g_u,g^*]$:

$$\frac{\partial Y_i}{\partial L_i} = \frac{\partial f}{\partial L_i}(g) = z(g) - gz'(g) > 0, \qquad \text{(A4)}$$

$$\frac{\partial Y_i}{\partial G} = \frac{\partial f}{\partial G} = z'(g) \geq 0, \qquad \text{(A5)}$$

$$\frac{\partial^2 Y_i}{\partial^2 L_i} = \frac{\partial^2 f}{\partial^2 L_i} = \frac{g^2}{L_i} z''(g) < 0, \tag{A6}$$

$$\frac{\partial^2 Y_i}{\partial^2 G} = \frac{\partial^2 f}{\partial^2 G} = \frac{z''(g)}{L_i} < 0, \tag{A7}$$

$$\frac{\partial^2 Y_i}{\partial L_i \partial G} = \frac{\partial^2 f}{\partial L_i \partial G} = -\frac{g}{L_i} z''(g) > 0. \tag{A8}$$

As the first partial derivatives with dependence on the factor input ratios g could change their signs (see (A2)-(A5)), and as they, like the average revenues (Y_i/G) and (Y_i/L_i) are continuous in g, the elasticities of production for labour (L_i) and firm-augmenting public input (G) must be variable. Taking into account (A1) and (A5), the elasticity of production for firm-augmenting public input (β) can be written as follows:

$$\beta(g) = \frac{\frac{\partial f}{\partial G}(g)}{\frac{Y_i}{G}} = \frac{gz'(g)}{z(g)} \geq 0 \quad \text{with} : \beta(g) = \begin{cases}]0,1[\text{ for }]g_u, g^*[, \\ 0 \quad \text{for } g = g^*. \end{cases} \tag{A9}$$

Thereby the elasticity of production for labour (α) is also dependent on g:

$$\alpha = 1 - \beta(g) \quad \Rightarrow \quad \alpha = \alpha(g) \quad \text{with} : \alpha(g) = \begin{cases}]0,1[\text{ for }]g_u, g^*[, \\ 1 \quad \text{for } g = g^*. \end{cases} \tag{A10}$$

ad (15):

Inserting the function of social surplus (5) into the equation of net surplus (see (14)) leads to:

$$\begin{aligned} N = & \int_{p^e(n,G,w)}^{p^0} Y^d(\psi; w)d\psi \\ & + n(p^e(n,G,w)Y_i^s(\hat{L}_i(p^e(n,G,w),G,w),G) \\ & - w\hat{L}_i(p^e(n,G,w),G,w)) + C_G(G) \\ & \rightarrow \max_{n,G}. \end{aligned} \tag{A11}$$

Partially differentiating from N to n and G will result in the necessary conditions for a maximum of net surplus (A11):

$$\frac{\partial N}{\partial n} = \frac{\partial S}{\partial n} = p^e Y_i^s - w\hat{L}_i = 0, \qquad \text{(A12)}$$

$$\begin{aligned}\frac{\partial N}{\partial G} &= \frac{\partial p^e}{\partial G}\underbrace{(-Y^d + nY_i^s)}_{=0} \\ &\quad + n\left(\frac{\partial \hat{L}_i}{\partial p^e}\frac{\partial p^e}{\partial G} + \frac{\partial \hat{L}_i}{\partial G}\right)\underbrace{\left(p^e\frac{\partial Y_i^s}{\partial \hat{L}_i} - w\right)}_{=0} \\ &\quad + np^e\frac{\partial f}{\partial G}(\cdot) - \frac{dC_G}{dG} \\ &= 0.\end{aligned} \qquad \text{(A13)}$$

As market equilibrium is presupposed for the concept of social surplus, and firms produce profits maximisingly, it must be so that the terms in the marked brackets will have the value zero for *any value of n and G*. Out of that condition (15) follows.

ad (19):

Inserting the function of social surplus (5) into equation (18) of net surplus results in:

$$\begin{aligned}N &= \int_{p^*}^{p^0} Y^d(\psi, w)\,d\psi \\ &\quad + n^*(G)(p^* Y_i^S(\hat{L}_i(p^*, G, w), G) \\ &\quad - w\hat{L}_i(p^*, G, w)) - C_G(G) \\ &\rightarrow \max_G.\end{aligned} \qquad \text{(A14)}$$

The necessary condition for a maximum of net surplus (A14) is:

$$\frac{\partial N}{\partial G} =$$

$$(p^* Y_i^s - w\hat{L}_i)\frac{dn^*}{dG} + p^* n^* \frac{\partial f}{\partial G}(\cdot) + n^* \frac{\partial \hat{L}_i}{\partial G}\underbrace{\left(p^* \frac{\partial Y_i^s}{\partial \hat{L}_i} - w\right)}_{=0} - \frac{dC_G}{dG} = 0. \qquad \text{(A15)}$$

As firms are producing at profit maximum, the term in brackets of the last part of (A15) for *any* G must be zero. This leads to condition (19).

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1 An example for a specification of production function (1) is: $Y_i = \frac{b_1 L_i^2 G^2}{b_2 L_i^3 + b_3 G^3}$ with: $b_1, b_2, b_3 > 0$.

2 For Y^d the following is true: $\frac{\partial Y^d}{\partial p} < 0$.

3 For purposes of clarification it must be said that the supply and demand curves in the illustrations of this paper are presented in linearised form.

4 At the point p=p* each firm produces at average cost minimum (see illustration 1). Therefore they will not offer goods for p<p* and will produce the amount Y^{s0} at the point p=p*.

5 Since $\frac{d^2 S}{d^2 n} = \frac{\partial p^e}{\partial n} Y_i^s < 0$ is true, maximum social surplus is unique.

6 Each firm i produces a positive output (Y_i*) in long-term equilibrium (see illustration 2a). Thus, the long-term market supply function (Y_l^s) will not contain the value zero. In order to simplify the graphics representation, however, the long-term market supply function in illustration 2b and the following illustrations pass through the zero point. This implies that the firm output i is small in comparison to the industry output and that the number of firms in market Y is relatively large.

7 Two things, which are a byproduct of this analysis, must be pointed out concerning illustration 2:

(i) In illustration 2b, not only the supply function (Y_l^s) is shown, but also the short term supply function (Y^s), which is given for the efficient number of firms (n*). At equilibrium price p* the short term and long term supply functions meet.

(ii) Furthermore, in illustration 2a, contrary to Boadway's depiction (1973, 254, illustration 3a), the falling average cost curve is mapped as a dashed line because the average cost curve $(C/Y_i)^*$ in the area $Y_i < Y_i^*$ is not compatible with neoclassical production theory. This argument is explained in the following.

Based on the specification of production function (1), the following is true for the average cost function of firm i: $\frac{C}{Y_i} = \frac{\overline{w} L_i(Y_i; \overline{G})}{Y_i}$ with: $\frac{\partial L_i}{\partial Y_i} > 0; \frac{\partial L_i}{\partial G} \leq 0.$ The derivation of (C/Y_i) with respect to Y_i shows that the average costs fall strictly monotonous in $Y_i < Y_i^*$:

$$\frac{\partial\left(\frac{C}{Y_i}\right)}{\partial Y_i} = \frac{w}{Y_i}\left(\frac{1}{\frac{\partial f}{\partial L_i}} - \frac{L_i}{Y_i}\right) < 0 \Leftrightarrow \frac{\partial f}{\partial L_i} > \frac{Y_i}{L_i} \qquad \text{for } Y_i < Y_i^*.$$

Falling average costs correspond to a value for production elasticity of labour of greater than one ($\alpha > 1$) (right equation in the above formula). This implies, assuming a linearly homogeneous production function ($\alpha + \beta = 1$) (see (1)), a negative production elasticity of firm-augmenting public input ($\beta < 0$). As $G > 0$ and $Y_i > 0$, the marginal productivity of firm-augmenting public input is negative ($\frac{\partial f}{\partial G} < 0$), which will result in a technically inefficient production process (see appendix, ad (1)). However, the concept of a production function does not capture any technically inefficient process of production, which means that the dashed part of the curve in illustration 2a is not defined.

[8] It is true that: $\frac{d^2\left(\frac{C}{Y_i}\right)}{d^2 g} = \frac{\overline{w}(2z'(g)^2 - z''(g)z(g))}{z(g)^3} > 0$ (see also appendix, ad (1)).

[9] In addition, the curvature of the average cost curve $(C/Y_i)'$ must be less than that of the average cost curve $(C/Y_i)^*$, because the partial elasticity of the average cost curve for Y_i, $\frac{\partial\left(\frac{C}{Y_i}\right)}{\partial Y_i}\frac{Y_i}{\left(\frac{C}{Y_i}\right)} = \frac{1}{\alpha(g)} - 1$, and the average cost curve alone, are dependent on the factor input ratio (g) (see (11)). Thus, each factor input ratio, e.g. g_1, has a certain amount of average costs and elasticity of average costs assigned to it, so that an increase in firm-augmenting public input will not only cause the average cost curve to shift to the right but also reduce the curvature. In economic terms this is due to the fact that with more G, a firm's cost increase will become less during an increase of output.

[10] The firm-augmenting public input is assumed to be produced with labour. The governmental institution is a price taker in the labour market. A more detailed discussion of $C_G(G)$ is given in section 5.

[11] Through equation (18), Hillman's statement (1978, 274) is also proven to be wrong, as he, like Boadway, assumes that the equilibrium number of firms (n*) will rise as firm-augmenting public input increases.

[12] In section 5 we will see that there is an exception.

[13] Contrary to the main text, exogenous variables are not marked with a horizontal line, as the emphasis is on the mathematical analysis.

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