DEMAND FOR LIFE ANNUITIES FROM MARRIED COUPLES WITH A BEQUEST MOTIVE[♦].

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ABSTRACT (April 21, 2005)

This paper will try to explain the "annuities puzzle" in greater depth by introducing the bequest motive, both strategic and altruistic. It will try to determine whether this motive really is a relevant feature influencing the demand for lifetime annuities from married couples. With this aim in mind, we develop an optimization model of the utility provided by purchasing a lifetime annuity with contingent survivor benefit or a joint survivor life annuity. Our model is based on that first put forward by Brown & Poterba (2000), then add to it elements from other models such as Friedman and Warshawsky's (1990) and Vidal & Lejárraga (2004) which include the bequest motive. This will enable us to calculate equivalent wealth in various contexts: the possibility of access to actuarially fair annuity markets, the inclusion of so-called market *imperfections*, and the assumption that couples already have part of their wealth in pre-existing lifetime annuities. Numerical results are presented for the case of Spain. The bequest motive is found not to be a significant factor influencing the demand for annuities from couples. Indeed very few couples would be willing to purchase them once we take into account the combined effects of market imperfections, the possibility of pre-existing annuities and the bequest motive. These findings have repercussions for policy-makers regulating defined contribution capitalization systems which are complementary to defined benefit systems.

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Key words: Capitalization, Pension Funds, Retirement, Utility.

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I. - INTRODUCTION.

Life annuities are a type of pension marketed by insurance firms. In exchange for an initial premium, these firms commit themselves to paying certain periodic amounts until the death of the policyholder, thereby taking on the annuitant's longevity risk. In the absence of annuities, retirees could reduce their annual consumption in response to the uncertainty surrounding their date of death. However, they would then run the risk of dying before they had consumed all their available wealth, and this represents a cost of missed opportunity for consumption. This cost will be less if the individuals value the wealth that remains at the time of their death as a bequest.

Longevity risk is the risk of needing more resources due to living longer than expected. There are two basic questions underlying any study of longevity risk: 1.-What does this risk depend on? Basically there are five elements to take into account: level of initial wealth, desired annual consumption, death probabilities, and the return on and volatility of the portfolio. 2.-Is this an important risk for the retiree? It would appear that it is. Indeed, it may be thought that the risk has been underestimated since, as a general rule in the papers that we will comment upon later, retirees are assumed to have excellent investment skills, transaction costs are not usually taken into account, the assumptions made regarding return and risk tend to be optimistic, and contradictions appear with the standard model of utility maximization.

Using parameters for Canadian mortality and capital markets, Milevsky & Robinson (2000) compute the lifetime and eventual probability of ruin for an individual who wishes to replicate a synthetic life annuity from an initial endowment invested in a portfolio earning a stochastic (lognormal) rate of return. They find that self-annuitization provides greater liquidity than voluntary annuitization; however, it does so at the cost of possibly outliving resources. Albrecht & Maurer (2002) arrive at similar conclusions in evaluating the risk for the German market. They calculate a personal probability of consumption shortfall and show that it is substantial, particularly for high-entry ages. Huang et al. (2004) compute the probability of lifetime ruin, this being the probability that a fixed retirement consumption strategy will lead to financial insolvency under stochastic investment return and lifetime distribution. They conclude that a 65-year-old retiree requires 30 times his desired annual real consumption to generate a 95% probability of sustainability –which is equivalent to a 5% probability of lifetime ruin- if the funds are invested in a well-diversified equity portfolio earning a real 7% per annum with a standard deviation of 20%. Finally, Schmeiser & Post (2004) follow the line taken by Kotlikoff & Spivak (1981), arguing that heirs might be willing to bear the shortfall risk of the retiree's self-annuitization since they might benefit from a bequest. Using German capital and annuity market data, they show that in many cases the family strategy offers enormous chance potential with low shortfall risk[1].

The "annuity puzzle" appears because empirical evidence shows the extreme rarity of voluntary private individual annuity contracts even though, according to Yaari (1965), individuals would be better off holding only annuitized assets in the absence of a bequest motive, or a portfolio of annuitized and traditional assets in the presence of a bequest motive. Davidoff et al. (2003) show that the conditions under which the purchase of annuities is optimal are not as demanding as those set out by Yaari (1965). If financial markets are complete, the only requirements are that no bequest motive exists and that the expected rate of return on annuities is greater than the return on a benchmark financial asset. Partial annuitization is optimal when the condition of there being a

[1] The "self-annuitization" papers mentioned are only a small sample of the interest that studying alternatives to annuitization has generated in the economic, financial and actuarial literature over the last few years. Apart from those cited, anyone interested in the subject should also see the papers by Young (2004), Gerrard et al. (2004), Milevsky et al. (2004), among many others.

complete insurance market is relaxed. Previous papers have not looked into the problem of annuities from the standpoint of couples with bequest motive.

The literature proposes several explanations for the "annuity puzzle" from the consumer's perspective:

1) The crowding-out effect of defined benefit systems is probably the most important reason. The presence of pre-annuitized wealth gives a certain level of protection against the risk of longevity.
2) The fact that the private annuity market is not "actuarially fair". The load factor is very high in some annuities markets.
3) The problem of adverse selection. The presence of asymmetric information in insurance markets implies that individuals self-select themselves on the basis of private information about their longevity.
4) Individuals use family self-insurance. This allows them to take advantage of the possibilities of joint consumption, sharing out financial resources among the members of the family, thereby reducing the attractiveness of annuities.
5) The perception of the accumulated retirement fund in the form of a lump sum allows the individual to face the appearance of unforeseen future expenditures.
6) The different ways in which different types of pension are taxed in some countries (not always to the benefit of the lifetime annuity option).
7) The fact that in some countries, Lopes (2003), annuities are very expensive compared to the total accumulated assets held by families. Many families cannot afford even to enter the annuities market.
8) It has been detected that consumers are not sufficiently educated to make welfare-maximization decisions.
9) The existence of any motivation to leave a bequest to the heirs, although there are some quite contradictory results in the literature on this aspect, as will be seen in the following section.

A number of papers have appeared recently seeking a plausible explanation for the annuity puzzle from the point of view of the supplier. Piggott et al. (2003) suggest that insurers appear reluctant to offer lifetime annuities, perhaps because of a perception within the industry that systematic risk, in the form of breakthrough life-prolonging technical innovation, may bankrupt an insurance company with a large life annuity portfolio. Likewise, Impavido et al. (2003) speculate that another reason why annuities fail to dominate over traditional assets may be related to the excessive risk borne by the providers of traditional annuity guarantees for a given institutional environment.

The aim of this paper is to shed more light on the "annuities puzzle" from the standpoint of couples. This will be done by introducing the bequest motive, both altruistic and strategic, and determining whether it really is a relevant factor influencing the theoretical decision to purchase lifetime annuities. With this aim in mind, we develop an optimization model of the utility provided by purchasing a lifetime annuity with contingent survivor benefit or a joint survivor life annuity Our model is based on that first put forward by Brown & Poterba (2000), then add to it elements from other models such as Friedman and Warshawsky's (1990) and Vidal & Lejárraga (2004) which include the bequest motive. So as to make the model as realistic as possible, we also take into account that the couple may already have a pre-existing annuity and that the annuity market may not be actuarially fair. Our main findings are that the bequest motive is not a significant factor influencing the demand for annuities from couples and that very few couples would be willing to purchase them once we take into account the combined effect of market imperfections, the possibility of pre-existing annuities and the bequest motive. These results have important repercussions for policy-makers regulating defined contribution capitalization systems which are complementary to defined benefit systems.

After this introduction, the paper is divided into four sections. In Section II we give a brief overview of the main papers connecting the bequest motive to the valuation of lifetime annuities. In Section III we develop a model for the optimal consumption flow which will maximize the couple's expected utility. This model takes into account the bequest motive, according to the couple's various preferences for consumption and perceptions of risk, for different types of

pension: a lifetime annuity with contingent survivor benefit and a joint survivor life annuity. In Section IV we analyze welfare by calculating equivalent wealth in different contexts: (i) the possibility of access to an actuarially fair annuities market (for indexed and non-indexed annuities); (ii) the incorporation of so-called market *imperfections*; and (iii) the hypothesis that the couple already have part of their wealth in pre-existing lifetime annuities. Numerical results are presented for the Spanish case. Section V presents a summary of our findings and considers options for future research.

II. – THE BEQUEST MOTIVE AND ANNUITY VALUATION.

One of the reasons why the existence of a bequest motive[2] has not normally been included in the analysis of welfare could well be for the sake of maintaining the analytical simplicity of the model. Also, however, Benitez-Silva (2003) points out that it could be due to the lack of consensus about the relevance of the bequest motive in an individual's decision as to whether to purchase annuities, and about how to model this bequest factor.

Yaari (1965) points out that in a life-cycle framework consumers will be better off holding only annuity assets if they have no bequest motives, whereas, when they have a bequest motive, they will hold a portfolio of annuities and bequeathable assets so that the marginal utility of bequests and consumption are the same. Lewis (1989) extends Yaari's life insurance framework to include explicitly the preferences of other household members. Life insurance is demanded by the dependents that face an income stream contingent on the breadwinner's lifetime. He concludes that any change in a beneficiary's expected bequest is almost fully offset by a change in life insurance holdings. This means that the future consumption of a breadwinner's surviving dependents is affected only marginally by the size of his bequest.

Brown (2001a) questions the importance of the bequest motive in influencing marginal annuity purchasing decisions – neither the presence of children nor the bequest motive are determinants in a life annuity purchasing decision. The author states that a simple life-cycle model without bequests gives predictions that are consistent with marginal annuity purchasing behavior, and is therefore a useful first approximation to behavior. These results coincide with those of Hurd (1987), who concludes that bequest motives have no significant influence on the marginal financial behavior of elderly individuals. This is further supported in a later work, Hurd (1989), which finds that many bequests are apparently accidental, resulting from uncertainty about the time of death, without there being any evidence of real bequest motives.

These findings contradict those of Bernheim (1991) and Laitner & Juster (1996), who claim that bequest motives do indeed influence the decision to purchase annuities. Also, Friedman & Warshawsky (1990) conclude that the presence of a bequest motive will reduce or eliminate the demand for annuities, even when the return on them exceeds the real market interest rate. In a previous work, Friedman & Warshawsky (1988), the same authors indicated that the interaction of a deliberate bequest motive and the actuarial unfairness of the annuities offered by insurance firms could dissuade individuals from purchasing them.

Jousten (2001) finds that the presence of a bequest motive has strong implications for the valuation of annuity contracts, and the previous literature has largely ignored the impact of bequest motives on the value of annuities. Lopes (2003) suggests that the bequest motive can significantly reduce the demand for annuities, and can be viewed as a possible explanation for the low demand observed in the US annuity market.

Brown (2001b) introduces the bequest motive into the single person model, without distinguishing between altruistic and strategic motives. The results indicate that in very few cases does the obligatory purchase of annuities lead to lower welfare than when no annuities are purchased, although the welfare gains are less than those obtained in the no-inheritance life-cycle model.

[2] A good summary of the bequest motive can be found in the paper by Impavido et al. (2003).

According to Bernheim (1991), if the retiree can generate savings external to the life annuities and/or purchase additional, annually renewable life insurance, then incorporating a bequest motive does not result in major differences in welfare gains.
Finally, Vidal & Lejárraga (2004) conclude that, in a model for an individual into which both altruistic and strategic bequest motives have been introduced, and where the effect of market *imperfections* and pre-existing annuities has been added, very few individuals in these conditions would be willing to purchase annuities. In many cases it would be best to allocate only a part of the wealth to the purchase of an annuity and/or postpone purchase until a later date.

III. - THE MODEL.

An important branch of the literature has analyzed the effect of annuitization on expected utility. An alternative approach, proposed by Milevsky (1998, 2001), has been to calculate the impact of deferring annuitization on expected returns, and the probability that deferral will leave the individual no worse off. In the most recent paper, Milevsky & Young (2003) locate a general optimal annuity purchasing policy under an open-market structure where individuals can annuitize a fraction of their wealth at different points in time. They find that an individual will initially annuitize a lump sum and then buy annuities in order to keep wealth to one side of a separating line in wealth-annuity space, a type of barrier control result. The papers mentioned above are presented from the standpoint of the individual and do not consider the bequest motive. Elsewhere, Benitez-Silva (2003) focuses on the decision to purchase annuities from the leisure-work perspective, including the bequest motive, but from the standpoint of the individual and without considering the survival probabilities.

This paper will analyze the effect of annuitization on expected utility. Joint utility functions need to be considered in order to measure the expected utility deriving from the optimization of consumption for a married couple. Our model is based on that first put forward by Brown & Poterba (2000), then add to it elements from other models such as Friedman and Warshawsky's [1990] and Vidal & Lejárraga (2004) which include the bequest motive. The paper concentrates on the case of a couple in which at least one partner is of retirement age and in which there are no children to be included as dependents when buying a pension. This is a realistic assumption since there will not normally be any dependent children by this age.

When the time comes to retire, the couple have to decide how to distribute their accumulated wealth over time in order to ensure they will be able to cover their future consumption needs. The basic assumptions are:

1) The couple can allocate their entire wealth to one of the life annuities described below.
2) There are bequest motives, but only the utility deriving from possible bequests once death occurs is considered. No account is taken of the fact that in certain cases the couple may want to make gifts of wealth while still alive, instead of waiting for the bequest to be made after they die[3].
3) No other type of uncertainty with regard to the interest rate, the evolution of mortality or the rate of inflation is considered.
4) Following Brown & Poterba (2000), it is assumed that the household utility function is a weighted sum of the utility functions of both members of the couple, U_m and U_f. Specifically:

$$U_c\left(C_t^m, C_{t-n}^f\right) = U_m\left(C_t^m + \lambda C_{t-n}^f\right) + \varphi U_f\left(C_{t-n}^f + \lambda C_t^m\right) \qquad \textbf{(1.)}$$

where parameter φ represents the relative weight of the wife's utility in the household utility aggregate. C_t^m and C_{t-n}^f denote the consumption of the husband and the wife respectively, at each

[3] Making a gift of money before death could come about because of the different tax treatment applied to the two situations, or simply because the couple would prefer their relatives to enjoy the goods (financial wealth) the couple can give them while alive as soon as possible.

age *t*, in which we consider the woman to be *n* years younger than the man, although the notation will also be valid in the case where she is older than her husband simply by allocating a negative value to *n*. U_c represents the couple's utility function when both of them are alive, while U_m and U_f are the utility functions for the man after his wife has died, and for the woman after her husband is dead, respectively.

The husband's utility function depends on $(C_t^m + \lambda C_{t-n}^f)$ and the wife's on $(C_{t-n}^f + \lambda C_t^m)$, where λ is the percentage of consumption that can be shared. When φ=1 and λ=1, the utility functions for men and women coincide for each period in which both members of the couple are alive.

III.1. - The couple do not have access to the lifetime annuities market

Assuming that the couple have no access to the annuities market and that they value the existence of a bequest to leave their heirs, the consumption optimization model is:

$$\max_C \sum_{t=e_r}^{\omega} \frac{\overbrace{U_c(C_t^m, C_{t-n}^f)_{t+1-e_r} P_{e_r}^m \cdot_{t-n+1-y} P_y^f}^{1} + \overbrace{U_m(C_t^m, 0)_{t+1-e_r} P_{e_r}^m \cdot_{/t-n+1-y} q_y^f}^{2}}{(1+\delta)^{t+1-e_r}} + \frac{\overbrace{U_f(0, C_{t-n}^f)_{t-n+1-y} P_y^f \cdot_{/t+1-e_r} q_{e_r}^m}^{3}}{(1+\delta)^{t+1-e_r}} + \overbrace{h_t \frac{V(W_{t+1})}{(1+\delta)^{t+1-e_r}} \cdot \underbrace{\left[{}_{t-e_r} P_{e_r}^m \cdot_{/t-n+1-y} q_y^f \cdot q_t^m + {}_{t-n-y} P_y^f \cdot_{/t+1-e_r} q_{e_r}^m \cdot q_{t-n}^f \right]}_{5}}^{4} \quad \textbf{(2.)}$$

$$\text{s.t. } C_t^m + C_{t-n}^f = W_t(1+r) - W_{t+1} \quad \textbf{(3.)}$$

$$W_t \geq 0, \quad \forall\, t \geq e_r \quad \textbf{(4.)}$$

where:

W_t : Wealth corresponding to age *t* for the man and age *t-n* for the woman.

y: The woman's age at the initial moment, which will be equal to e_r - *n*.

${}_{t+1-e_r} P_{e_r}^m$: Probability that a man aged e_r will live for another *t*+1- e_r years.

${}_{t-n+1-y} P_y^f$: Probability that a woman aged *y* will live for another *t-n*+1-*y* years.

${}_{/t+1-e_r} q_{e_r}^m$: Probability that a man aged e_r will die before living another *t*+1- e_r years.

${}_{/t-n+1-y} q_y^f$: Probability that a woman aged *y* will die before living another *t-n*+1-*y* years.

$V(W_t)$: The welfare function for the period corresponding to age *t*, defined over wealth (W_t).

δ: The pure rate of time preference, i.e. the classical exponential discount factor of future utility.

e_r: Retirement age.

ω: An individual's maximum lifespan.

r: Nominal expected risk-free rate (assumed constant over the retiree's lifetime).

h_t : Relative weight of the utility of the bequest considered by the individual at age *t* with respect to the expected utility corresponding to the flow of consumption at age *t*: C_t..

The three possible valid states for consumption are shown along with their associated probability in brackets 1, 2 and 3 in the numerator of Equation (2). The first state is the assumption that both are still alive; the second when only the man is alive; and finally the third when only the woman is alive.

Finally, the fourth summand evaluates the expected utility provided by the unconsumed wealth the couple have available to bequeath in the year when both are dead. This wealth will be bequeathed in the year both members of the couple die, which in terms of probability is expressed in Factor 5 of Equation (2). Constraint (3) shows the relation between consumption and wealth, while (4) refers to

the fact that the wealth cannot be negative. Here we consider exclusively the utility deriving from possible bequests once death has occurred.

One of the determining elements in the consumption optimization model when the bequest motive is included is parameter h_t, which indicates the couple's assessment of the amount of wealth that could be bequeathed at any given time to their heirs. Friedman & Warshawsky (1990) do not believe that this parameter depends on age, but that it is related to the bequest-consumption ratio corresponding to final period W_ω / $C_{\omega\text{-}1}$. Likewise Brown (2001b) and Jousten (1998) assume it to be constant over the life cycle. Other researchers such as Hurd (1989 & 1999) do not weight the utility of bequests relative to that of consumption, and apply the same valuation on the part of the individual to both.

Fischer (1973) and Yaari (1965) consider the parameter that reflects the value given to the possibility of leaving a bequest, h_t, to be a hump-shaped function due to the greater importance that individuals give the bequest in mid-life when family members have a greater dependence on them, then decreasing in retirement. This is valid in the case where the couple's motivations for leaving a bequest are altruistic. If, however, the retiree's ends are strategic in the sense that they are seeking to encourage family members to care for them in old age in exchange for the promise of a bequest, it would be more appropriate to consider that parameter h_t increases with age. Clearly the higher the value of parameter h_t the lower the level of consumption, the aim being to bequeath greater wealth, and therefore the valuation of welfare provided through the purchase of a life annuity will also be lower.

III.2. - The couple have access to the joint annuity with contingent survivor benefit or joint survivor annuity market

Two types of alternative lifetime annuities are analyzed, both of which enable the mortality risk of each member of the couple to be transferred: lifetime annuity with contingent survivor benefit and joint survivor life annuity.

1) ***Lifetime annuity with contingent survivor benefit[4].*** With this type of lifetime annuity a periodic payment is made to the primary annuitant, which he receives until his death. From this moment his wife, assuming she has survived until this date, will start to receive an amount calculated as a percentage of what the deceased annuitant was receiving. This percentage is set by the purchaser when the relevant insurance policy is bought.

2) ***Joint survivor life annuity.*** This is a contract whereby the insurance company undertakes to pay a periodic amount while both members of the couple are alive, and a fraction of this amount, ρ or η, when one of them has died, for as long as the other lives.

It should be noted that the difference between a lifetime annuity with contingent survivor benefit and a joint survivor life annuity is not only actuarial. They also have very different economic interpretations:

1) With a lifetime annuity with contingent survivor benefit, only one of the members of the couple has entitlement to a pension, while the other is a mere dependent.
2) With a joint survivor life annuity, both members have entitlement to a pension but, because there is a link between them, the pension is treated jointly.

When the couple decide to allocate part of their initial accumulated wealth to buying a joint survivor annuity or an annuity with contingent survivor benefit, the model for utility optimization can be expressed as:

[4] In the US pension practice (and in the ERISA legislation) is called a "qualified joint and survivor annuity".

$$\text{s.t. } C_t^m + C_{t-n}^f = W_t(1+r) - W_{t+1} + A_t \qquad \textbf{(5.)}$$

$$W_t \geq 0, \quad \forall\, t \geq e_r \qquad \textbf{(6.)}$$

where Term A_t, included in the budget constraint, represents the amount the couple will receive at moment *t*, deriving from the lifetime annuity contract. The value of this will be given by:

a) Equations (7) and (8) in the case of an annuity with contingent survivor benefit:

$$A_{e_r}^{CSB} = W_{Annuities} \Big/ \sum_{t=e_r}^{\infty} \frac{(1+\alpha)^{t-e_r} \cdot \theta \cdot \left[{}_{t+1-e_r}P_{e_r}^m + \gamma \; {}_{t-n+1-y}P_y^f \left(1 - {}_{t+1-e_r}P_{e_r}^m\right)\right]}{\left[(1+r)\right]^{t-(e_r-1)}} \qquad \textbf{(7.)}$$

$$A_t^{CSB} = A_{e_r}^{CSB}(1+\alpha)^{t-e_r}, \qquad \forall t \; > \; e_r \qquad \textbf{(8.)}$$

where:

γ: Percentage payable to the designated beneficiary.

θ: Represents the degree of actuarial fairness of the annuity in such a way that if $\theta = 1$, the annuity is actuarially fair. For values of $\theta < 1$, market *imperfections* are considered to exist. The joint effect of the different mortality and survival probabilities used by the insurance company when selling annuities, the charges applied deriving from management and administration costs, and the possible differences in the rate of interest guaranteed on the pension in comparison to the market rate mean that the conversion factor is reduced.

ρ:: Fraction of the annual amount of joint pension when one of them has died, for as long as the other lives. This fraction is normally 1, 2/3 or 1/2.

α : Annual accumulative growth of the pension. In the calculations made later, it will be shown that two hypotheses have been taken into account: α equal to zero, in which case the pension will be constant in nominal terms, and α equal to expected inflation (π), in which case the pension will be constant in real terms. Obviously the risk of inflation is not taken into account since it is not accepted that the real rate of inflation could be different from the expected rate.

b) Equations (9) and (10) in the case of joint survivor life annuity:

$$A_{e_r}^{JS} = W_{Annuities} \Big/ \sum_{t=e_r}^{\infty} \frac{(1+\alpha)^{t-e_r} \cdot \theta \cdot \left[{}_{t+1-e_r}P_{e_r}^m \cdot {}_{t-n+1-y}P_y^f + \rho \cdot {}_{t+1-e_r}P_{e_r}^m \left(1 - {}_{t-n+1-y}P_y^f\right) + \eta \cdot {}_{t-n+1-y}P_y^f \left(1 - {}_{t+1-e_r}P_{e_r}^m\right)\right]}{\left[(1+r)\right]^{t-(e_r-1)}} \qquad \textbf{(9.)}$$

$$A_t^{JS} = A_{e_r}^{JS}(1+\alpha)^{t-e_r}, \qquad \forall t > e_r \qquad \textbf{(10.)}$$

with η being the fraction of the annual amount of joint pension when the man has died and the woman is still alive.

III.3. - The couple have pre-existing lifetime annuities with contingent survivor benefit.

In the case of a couple who already have part of their wealth in a pre-existing lifetime annuity and decide to buy another one, the optimization model -assuming they do not have access to actuarially fair annuity markets- has the following constraints:

$$\text{s.t. } C_t^m + C_{t-n}^f = W_t(1+r) - W_{t+1} + R_t \qquad \textbf{(11.)}$$

$$W_t \geq 0, \quad \forall\, t \geq e_r \qquad \textbf{(12.)}$$

where $W_{er} = W_{NP}$, and $W_0 = W_{NP} + W_{PA}$, and:

W_{NP} : Level of initial wealth not allocated to pensions.

W_{PA} : Level of initial wealth in pre-existing lifetime annuities.
R_t is a lifetime annuity with a 50% survivor payout, payable in arrears, index-linked to the Retail Price Index (RPI), assumed to derive from a pre-existing public pension system, obtained as:

$$R_{e_r} = W_{PA} \Big/ \sum_{t=e_r}^{\infty} \frac{(1+\pi)^{t-e_r} \cdot \left[{}_{t+1-e_r}P_{e_r}^{m*} + 0.5 \cdot {}_{t-n+1-y}P_{y}^{f*}(1 - {}_{t+1-e_r}P_{e_r}^{m*}) \right]}{\left[(1+r)\right]^{t-(e_r-1)}} \quad \textbf{(13.)}$$

$$R_t = R_{e_r}(1+\pi)^{t-e_r}, \qquad \forall t \geq e_r \quad \textbf{(14.)}$$

${}_{t+1-e_r}P_{e_r}^{m*}$: Probability that a man aged e_r will live for another t+1- e_r years according to the mortality tables used by the insurer.
${}_{t-n+1-y}P_{y}^{f*}$: Probability that a woman aged e_r will live for another t+1- e_r years according to the mortality tables used by the insurer.

Assuming it is possible to buy actuarially fair annuities with contingent survivor or joint survivor payout, in this context where there is a pre-existing annuity the optimization model represented in Equation 3 would have the following constraints:

$$C_t^m + C_{t-n}^f = W_t(1+r) - W_{t+1} + R_t + A_t \quad \textbf{(15.)}$$

$$W_t \geq 0, \quad \forall\, t \geq e_r \quad \textbf{(16.)}$$

where W_{er} is equal to the initial wealth not invested in annuities with $W_{e_r} = W_{NP}$ and $W_0 = W_{NP} + W_{PA} + W_{ANNUITY}$. A_t is determined from Equation (7) or (9).

III.4. - The couple's equivalent wealth

One of the most usual ways of evaluating gains in welfare is to measure what level of wealth would be necessary for them to take on the same expected utility curve in any of the cases analyzed. This measure is equivalent wealth, given by μW_0, where the coefficient μ is defined as:

$$\mu = \frac{W_0 + \Delta W}{W_0} = 1 + \frac{\Delta W}{W_0} \quad \textbf{(17.)}$$

ΔW being the amount of additional wealth the couple would have to be given -following their optimal consumption path in any of the cases put forward- to enable them to reach the same level of utility they obtain when the consumption path is maximized in any of the others. The quotient ΔW/W0 determines the welfare gain for the couple, as a percentage of the level of wealth accumulated at the start.

This measure is aimed at determining by how much a couple, averse to risk, would value the possibility of buying a lifetime annuity and being able to protect themselves against the risk of excessive longevity in terms of the metric which the theory of utility supplies, in which both financial and psychological parameters such as their attitudes to risk and consumption are taken into account. It should be stressed that the evaluation will be an excess approach since it considers that there is no other family insurance implicit, and neither are there other investments -such as a

house- to offer another source of income not usually correlated. In addition, the equivalent wealth has some conceptual and measurement problems[5], but its validity is widely accepted.

Equivalent wealth will be given by percentage μ_{ij} which is:

$$UE_i(\mu_{ij}W_0) = UE_j(W_0), \qquad i, j = 1,....,5 \tag{18.}$$

where:

$UE_j(W)$: Expected utility, derived from the optimal consumption path the couple would choose in case j (1= with no access to the life annuities market, 2= purchase of a single premium non-indexed lifetime annuity with contingent survivor benefit, 3= purchase of a single premium indexed lifetime annuity with contingent survivor benefit, 4= purchase of a single premium non-indexed joint survivor life annuity, 5= purchase of a single premium indexed joint survivor life annuity), based on initial wealth W.

μ_{ij} represents the factor to be applied on the initial wealth of a couple who have chosen option i for them to obtain the same utility as an individual in situation j.

III.5. - Utility function and the solution of the model.

Just as is usually done in the case of an individual, we assume that the husband and wife have constant relative risk aversion (CRRA) utility functions. The analytical expressions of the utility functions used are the following:

$$U_m\left(C_t^m + \lambda C_{t-n}^f\right) = \left[\frac{\left(\frac{C_t^m + \lambda C_{t-n}^f}{(1+\pi)^{t+1-e_r}} \right)^{1-\beta} - 1}{1-\beta}, \text{if } \beta \neq 1 \right]; \left[\ln\left(\frac{C_t^m + \lambda C_{t-n}^f}{(1+\pi)^{t+1-e_r}} \right), \text{if } \beta = 1 \right] \tag{19.}$$

$$U_f\left(C_{t-n}^f + \lambda C_t^m\right) = \left[\frac{\left(\frac{C_{t-n}^f + \lambda C_t^m}{(1+\pi)^{t+1-e_r}} \right)^{1-\beta} - 1}{1-\beta}, \text{if } \beta \neq 1 \right]; \left[\ln\left(\frac{C_{t-n}^f + \lambda C_t^m}{(1+\pi)^{t+1-e_r}} \right), \text{if } \beta = 1 \right] \tag{20.}$$

where $\beta > 0$ represents the risk aversion coefficient[6] and the expected rate of inflation is π. As inflation is included, the consumption path remains nominal but is evaluated in real terms. This utility function belongs to the *isoelastic* family of functions, thus the problem remains invariable to the scale of wealth. The degree of concavity of the utility function reflects the individual's level of risk aversion.

The utility deriving from the bequest at each time t is given by the same isoelastic function, in which death has been assumed to occur halfway through the corresponding year (uniform distribution of deaths), so that the accumulated wealth at the date of death will be equal to that at the beginning of period t+1, discounted for one half year: $W_{t+1}((1+r))^{-1/2}$. Hence the function that gives the value of the bequest's utility is given by the expression:

[5] Following Petrova (2004), the conceptual problems stem from the assumptions of the "standard economic model" that people have time consistent preferences, that they are rational and their behavior can be characterized as the dynamic optimization problem. The measurement problems are also important because there are factors excluded from the equivalent wealth that matter for annuitization, and some issues related to the way the equivalent wealth is parameterized need to be revisited.

[6] Halek & Eisenhauer (2001), Powell & Ansic (1997) and Jianakoplos & Bernasek (1998) amongst others, have pointed out that women are significantly more risk averse than men, but we take the widespread assumption that both men and women have the same risk aversion coefficient.

$$V(\mathrm{W}_{t+1}) = \left[\frac{\left[\frac{W_{t+1} \cdot [(1+r)]^{-1/2}}{(1+\pi)^{t+1-e_r}} \right]^{1-\beta} - 1}{1-\beta}, \text{if } \beta \neq 1 \right]; \left[\ln \left[\frac{W_{t+1} \cdot [(1+r)]^{-1/2}}{(1+\pi)^{t+1-e_r}} \right], \text{if } \beta = 1 \right] \qquad \textbf{(21.)}$$

The mathematical models in this paper have been translated into LINGO® software programming language, and this program was used to obtain the numerical results shown in the various tables. For non-linear programming problems, the LINGO® package uses an algorithm based on the Generalized Reduced Gradient (GRG2) method. In addition to this, to help obtain a first feasible solution quickly it includes a recursive linear programming algorithm. GRG2 is based on Wolfe's reduced gradient method, later taken up by Abadie and Carpentier, in which the feasible improvement direction is not the generalized reduced gradient (GRG), but a second order approximation[7].

IV. - RESULTS.

The solution of the model yields the optimal consumption path that maximizes the couple's expected utility including a possible bequest. Using this as a point of departure, we can calculate equivalent wealth, which is the indicator used to evaluate welfare. The following assumptions and parameter values were used in the calculations:

1. The consumer's mortality risk is taken from GRMF-95 survival and mortality tables. These tables are the ones normally used by insurance companies operating in Spain and, in general terms, they show a life expectancy for any particular age that is greater than that given in the latest tables available for the population of Spain as a whole (mortality tables for the population of Spain 1998-1999 published by the National Institute of Statistics).

2. It is assumed that the husband's and the wife's probabilities of dying are independent. The importance of the effect of dependent mortality on annuity valuation is not very clear in the literature. Frees et al. (1996) find that annuity values are reduced by approximately 5% when dependent mortality models are used compared to the standard models that assume independence, whereas Brown & Poterba (2000) report only modest "broken heart"[8] effects on the annuity equivalence wealth measure. In Spain, as in most countries, standard insurance industry practice assumes independence of lives when valuing annuities where the promise to pay is based on more than one life.

3. The insurance company sells actuarially fair annuities ($\theta = 1$), which means simultaneously that:

a) It does not apply any type of charge on the purchase of an annuity with a single premium.
b) The survival probabilities that the insurer uses in setting the premium coincide with the consumer's probabilities (${}_{t+1-er}P^{*}_{er} = {}_{t+1-er}P_{er}$). Due to the way insurance companies currently classify risks -mainly by age and sex- it is practically impossible for this to come about. According to Brown & McDaid (2003), at least ten other important factors should be taken into account: race, level of education, wealth, employment, marital status, religion, lifestyle, weight, and smoking and drinking habits.
c) The nominal market interest rate, *r*, coincides with the annuity's technical interest rate, and is equal to 4.545%, approximately the long-term technical interest rate insurance companies have used when selling annuities in Spain over the last two years.

4. The insurance company sells annuities which are not actuarially fair, in which case the conversion factor is reduced by 15%, i.e. $\theta = 0.85$. As is widely accepted, Blake (1999),

[7] This is a well-known algorithm that can be seen in detail in the papers by Bazaraa et al. (1993).
[8] The tendency is for the mortality rates of surviving spouses to be somewhat higher for several years after their spouse's death than the mortality rates for similar individuals who have not lost a spouse.

annuities markets are not sufficiently well developed even in many of the more financially advanced countries, and so considering actuarially fair markets could therefore be thought too unreal an assumption[9]. Some authors disagree with the term *imperfection*, and simply attribute the decrease in the amount of the annuities on the market over the actuarially fair amount to the price that has to be paid to the company for assuming the financial risk and the risk of longevity.

5. Level of risk aversion β takes two values (0.7 and 2.9). Although there is no consensus in the literature as to which values should be used for the degree of risk aversion[10], for annuity valuation and CRRA utility functions, Feldstein & Ranguelova (2001) provide some qualitative arguments that the value of CRRA is less than 3 and probably even less than 2.

6. Retirement age is 65 for men; the woman is three years younger than the man.

7. The expected rate of inflation is equal to 1.5%, and so the real interest rate is 3%.

8. The preference rate according to the individual's level of impatience is given by the expression $\delta = \xi[(1+r)-1]$, where the values of parameter ξ (2, 1 and 0.25) classify the individuals as (A) very impatient, (B) impatient, and (C) very patient respectively. According to Yagi & Nishigaki (1993), the degree of the time discount rate is correlated to the degree of myopia, and this has an important effect on the demand for lifetime annuities. In most of the papers cited, the level of impatience is not usually emphasized. Seldom are impatient or very impatient couples considered, and this could be due to the fact that they are more difficult to calculate, but there is evidence, Rabin (1998), that people differ in their rates of time preference.

9. The type of pension bought is a lifetime annuity with contingent survivor benefit payable at 50%, or a joint survivor annuity where, on the death of either member of the couple, the surviving spouse would receive 50% of the pension payable when they were both alive (i.e. the value of $\rho=\eta=0.5$).

10. We have also considered the level of joint consumption (λ) to be equal to 0, and the weighting factor of the woman's utility function to be $\varphi=1$. According to Lejárraga (2003), both, the variation in the level of joint consumption (λ) and the variation in the weighting factor of the woman's utility function, have a small effect on the valuation of lifetime annuities.

11. In the case of a couple who already have part of their wealth in a pre-existing lifetime annuity and decide to buy another one, it is supposed that this wealth amounts to 50%, and is in the form of a lifetime annuity with a 50% survivor payout, payable in arrears, index-linked to the RPI, and assumed to derive from a pre-existing public or private pension system. This is a quite valid assumption in the case of Spain since the percentage payable to the beneficiary is around 50% and the State pension is indexed to real inflation.

12. Two alternatives have been taken into account for function $\mathbf{h}_t$. On the one hand, when there are altruistic motives for the bequest, the function was taken to decrease after age 65 since this is the individual's inactive phase. On the other hand, if there is a strategic interest in

[9] Many papers about market *imperfections* and adverse selection have been published. Anyone interested in this controversial subject should consult those by Friedman & Warshawsky (1990); Mitchell et al. (1999); James & D. Vittas (2000), James & Song (2001), Poterba (2001), Mitchell & McCarthy (2002a and 2002b), Mung (2002), Finkelstein & Poterba (2002), Brown (2003), Villeneuve (2003), Finkelstein & Poterba (2004), Cannon & Tonks (2004), Von Gaudeker & Weber (2004) and Yat & Chan (2004). The size of the *imperfection* and the intensity of the adverse selection vary considerably from one market to another.

[10] According to Halek and Eisenhauer (2001), there are differences in the degree of risk aversion across demographic groups based on age, gender, education, nationality, race, marital and parental status, religion, health and behavioral indicators, and employment status, income and wealth.

bequeathing wealth in exchange for possible assistance from the family in old age, function $\mathbf{h_t}$. was taken as increasing with age. In particular, the values of $\mathbf{h_t}$. are as follows:[11]:

a. Altruistic bequest motives:

$$h_t^A = h_{t-1}^A / 1.02 \quad ; \quad h_\omega^A = 2 \qquad , \forall\ t < \omega \tag{22.}$$

b. Strategic bequest motives:

$$h_t^S = h_{t-1}^S * 1.02 \quad ; \quad h_{e_r}^S = 2 \qquad , \forall\ t > e_r \tag{23.}$$

The bequest parameter in this simulation was chosen according to conservative criteria so as to avoid obtaining results in which the weight of the bequest motive in the utility function was possibly overvalued. In any case we are only trying to analyze the effect of considering altruistic or strategic motives for leaving a bequest compared to the decision to buy a life annuity, as opposed to not incorporating the life annuity in the model. Therefore the results presented below should be analyzed with the "determinant" that the intensity of the bequest motive is not empirically tested, although the values used to obtain certain general conclusions are in fact considered appropriate.

IV.1.- The couple's equivalent wealth with a bequest motive.

In this subsection we calculate the value of equivalent wealth for couples, making a distinction between whether the motivation for the bequest is strategic or altruistic.

Table 1 shows the results obtained from comparing the purchase of a lifetime annuity with a 50% contingent survivor benefit, both indexed and non-indexed, and a situation where the couple have no bequest motive. This is the situation which Milevsky & Young (2003) have dubbed "all or nothing", and appears to be rather restrictive.

The result shown in italics in Table 1 means that for the couple there would be no difference between one monetary unit allocated to annuities and *1.327* of wealth where nothing is allocated to annuities. In other words, the couple would be willing to give up *1.327* units of current wealth in order to have 1 monetary unit in the form of a lifetime annuity with a 50% contingent survivor benefit.

In all cases the equivalent wealth obtained is slightly less than that calculated in the model in the case of "without bequest motive"[12]. In other words the bequest motive makes lifetime annuities with contingent survivor benefit less attractive, although it does not increase the number of profiles of couples who would prefer not to purchase one. In all cases the model based on strategic rather than altruistic motives would be preferable. In the same way as happened with couples with no bequest motive, the welfare attained with lifetime annuities with contingent survivor benefit increases when aversion to risk becomes greater and impatience to consume becomes lower.

The profile distribution of couples preferring non-indexed to indexed lifetime annuities with contingent survivor benefit practically coincides with that for couples who have no bequest motive, whether altruistic or strategic. Indexed rather than non-indexed lifetime annuities with contingent

[11] As was mentioned above, there is no consensus in the literature on how to model bequest motives, and the values considered for the parameter weighting the utility of the bequest relative to the utility of consumption are quite disparate. Thus Brown (2001b) uses two different hypotheses: 0.5 and 1. Fischer (1973) considers values in a range of approximately 4.5-9.8 –or 28.2-120.8 with the rate of consumption preference hypothesis– starting from age 65. The bequest parameter applied in Jousten's model (1998) is equal to 5.5·10-5. Bequest motive is in the form of a linear bequest utility term. The parameter on the linear bequest utility term is $5.5*10^{-5}$. Friedman & Warshawsky (1990) determine the optimal percentage of wealth to annuitize assuming that the bequest parameter can vary between 0 and 100. Finally, Lopes (2003) solves her model with the bequest parameter equal to 7.

[12] It should also be remembered that going from the case of the individual to the case of couples already meant a reduction in the value of equivalent wealth.

survivor benefit are preferable for couples with little impatience for consumption and a high aversion to risk.

Table 1: Couple's equivalent wealth with a bequest motive

	Non-indexed lifetime annuity with a 50% contingent survivor benefit.						Indexed lifetime annuity with a 50% contingent survivor benefit.					
	Altruistic		Without bequest motive		Strategic		Altruistic		Without bequest motive		Strategic	
β→ δ↓	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9
A	**0.966**	1.242	**0.997**	1.281	**0.985**	1.249	**0.916**	1.181	**0.943**	1.225	**0.935**	1.19
B	1.118	*1.327*	1.193	1.36	1.152	1.33	1.104	1.313	1.176	1.359	1.143	1.32
C	1.189	1.371	1.312	1.397	1.222	1.371	1.21	1.405	1.35	1.449	1.255	1.409
A) With a 15% reduction factor.												
A	**0.821**	1.056	0.847	1.088	**0.837**	1.061	**0.778**	1.004	**0.801**	1.041	**0.794**	1.012
B	**0.95**	1.128	1.014	1.156	**0.979**	1.13	**0.939**	1.116	**0.999**	1.156	**0.971**	1.122
C	1.01	1.165	1.115	1.188	1.039	1.165	1.028	1.194	1.148	1.232	1.066	1.197
B) With 50% wealth in pre-existing annuities.												
A	**0.918**	**1.018**	**0.920**	1.042	**0.920**	1.026	**0.893**	**0.985**	**0.895**	1.011	**0.895**	**0.994**
B	**1.027**	**1.092**	1.038	1.116	1.037	1.098	1.018	1.079	1.058	1.107	1.036	1.086
C	1.080	1.138	1.124	1.156	1.100	1.141	1.090	1.147	1.141	1.174	1.115	1.152
C) With 50% wealth in pre-existing annuities and a 15% reduction factor												
A	**0.850**	**0.932**	**0.852**	**0.952**	**0.852**	**0.938**	**0.830**	**0.904**	**0.831**	**0.926**	**0.831**	**0.911**
B	**0.944**	**0.997**	**0.953**	1.018	**0.951**	1.003	**0.938**	**0.985**	**0.944**	1.010	**0.942**	**0.992**
C	**0.992**	1.038	1.029	1.054	1.009	1.041	1.004	1.045	1.043	1.069	1.022	1.050

Source: Authors and Vidal et al. (2003)

In all the tables the values for the equivalent wealth of the non-indexed (indexed) annuities that are greater than those for indexed (non-indexed) annuities have been underlined. Values less than 1, which mean that wealth without annuities is preferable, are shown in italics.

The introduction of market *imperfections* into Part A) of Table 1 brings about a decrease in equivalent wealth, as happened in the cases of an unmarried individual and of a couple with no bequest motive.

The preference for indexed annuities is maintained for practically the same profiles as in the case of couples with no bequest motive. In addition, in all cases the purchase of a lifetime annuity would result in greater welfare when the bequest is valued for strategic rather than altruistic motives, although the differences are reduced considerably when risk aversion is high and impatience for consumption is low.

Just as happens in the models for individuals and couples without a bequest motive, and also in the case of an unmarried individual who wants to leave a bequest to his heirs, the equivalent wealth varies in each case by exactly the same percentage that determines the difference between what the couple would consider to be "actuarially fair" and what the insurance company offers. In this case, the equivalent wealth that provides the same utility without purchasing annuities as allocating all the available initial wealth to a lifetime annuity is 15% less than would be needed if the annuity bought had no charges or supplements on the survival rates.

The percentage decrease in the value of the annuity over the value of the pension actuarially equivalent to the premium contributed in the terms mentioned above, which the couple would be willing to accept -given that it would still be more useful to them to purchase the annuity than not purchase it- is calculated in Table 2. All those cases where the maximum admissible percentage of

reduction in pension is less than 15% or negative are shown in bold in Table 2. In Part A) of Table 1, it corresponds to values of equivalent wealth less than 1.

Table 2: Maximum percentage of pension reduction for a lifetime annuity with a 50% contingent survivor benefit

	Non-indexed lifetime annuity						**Indexed lifetime annuity**					
	Altruistic		**WBM**		**Strategic**		**Altruistic**		**WBM**		**Strategic**	
β→ δ↓	**0.7**	**2.9**	**0.7**	**2.9**	**0.7**	**2.9**	**0.7**	**2.9**	**0.7**	**2.9**	**0.7**	**2.9**
A	**-3.5**	19.5	**-0.3**	21.9	**-1.5**	19.9	**-9.2**	15.3	**-6.0**	18.4	**-7.0**	16.0
B	**10.6**	24.6	16.2	26.5	**13.2**	24.8	**9.4**	23.8	15.0	26.4	**12.5**	24.2
C	15.9	27.1	23.8	28.4	18.2	27.1	17.4	28.8	25.9	31.0	20.3	29.0

Source: Authors

The inclusion of pre-existing annuities -in Part B) of Table 1- brings about a significant decrease in equivalent wealth for couples with a bequest motive. Only for couples with high risk aversion and little impatience to consume would there be virtually no difference between purchasing an indexed lifetime annuity with contingent survivor benefit and not purchasing annuities at all.

The results would be much more marked as regards the decrease in equivalent wealth if we were to suppose that the couple have more than 50% accumulated wealth[13] in annuities since in the case of Spain, given the high theoretical replacement rate supplied by the public pension system, it would be a hypothesis closer to reality for many couples.

The preference for indexed rather than non-indexed annuities is maintained on the same terms as in the case of not taking the bequest motive into account. The comparison between the altruistic and strategic bequest alternatives is more favorable to the latter, although the variations are very small.

Once market *imperfections* are introduced into the model -Part C) Table 1- along with pre-existing annuities, there is a definite decrease in equivalent wealth, as would be expected. The relations between the various combinations of cases remain similar to those occurring under previous hypotheses: the profiles of couples with a high level of impatience to consume and little aversion to risk prefer non-indexed to indexed annuities with contingent survivor benefit; and the welfare attained when strategic rather than altruistic bequest motives are taken into account is slightly greater, although the difference is very small. Most couples with a bequest motive should decide not to purchase annuities with contingent survivor benefit. Even those couples with no impatience to consume have little incentive to buy annuities, less than a 5% increase in welfare. Only those couples who are sufficiently educated to make welfare-maximization decisions would be capable of appreciating the gains in welfare.

If we take into account the combined effect of market imperfections, the possibility of pre-existing annuities and the bequest motive, equivalent wealth can decrease by up to 28% compared to what could be obtained if markets were actuarially fair, if there were no bequest motive or pre-existing annuities, for couples with greater risk aversion with little impatience for consumption. The bequest motive does not have a great impact on equivalent wealth, this being reduced in most cases by between 1% and 4%.

Another way of measuring the preference that couples show in accessing the annuity market is to calculate the maximum percentage of wealth accumulated at retirement that they are willing to annuitize. To determine this maximum percentage, it has to be established that the wealth allocated to annuities is a control variable of the problem set. It should not be considered as taking on a fixed value equal to the initial wealth the individual has available at the time of retirement; this

[13] This is the hypothesis most used in the literature, but according to Dushi & Webb (2004), it would not be very appropriate in the case of the USA since it would only be true for individuals situated in the highest decile of wealth; for all other individuals the level of wealth in the form of pre-existing life annuities would be much higher.

assumption is made to obtain the value of equivalent wealth. As was mentioned before, the mathematical models in this paper have been translated into LINGO® software programming language, and this program was used to obtain the numerical results shown in the various tables. This way of measuring welfare is to some extent comparable to what Milevsky & Young (2003) call the "open market".

Table 3: Optimal percentage of initial wealth allocated to a lifetime annuity.

Indexed lifetime annuity with a 50% contingent survivor benefit.

	Altruistic		**WBM**		**Strategic**	
$\beta\rightarrow$ $\delta\downarrow$	0.7	2.9	0.7	2.9	0.7	2.9
A	37%	79%	42%	82%	42%	80%
B	88%	93%	88%	96%	92%	94%
C	100%	100%	100%	100%	100%	100%

Source: Authors

The results shown in Table 3 are most revealing: some couples would prefer not to allocate all their wealth to the purchase of an actuarially fair life annuity, i.e. they would obtain greater gains in welfare by being able to choose what percentage to allocate to the purchase of an annuity. Only in the case of couples with very little impatience for consumption is it optimal to allocate 100% of the wealth to buying an annuity. Generally speaking a couple with strategic motives will allocate a slightly greater amount than those with altruistic motives.

IV.2. - The couple's equivalent wealth. Lifetime annuity with contingent survivor payout versus joint survivor annuity. Indexed annuities.

As can be seen in Table 4 -and despite the absolute differences in the values obtained- the list of values for the various profiles studied is similar for both types of annuity. As mentioned earlier, the difference between a lifetime annuity with contingent survivor benefit and a joint survivor annuity also has a very different economic interpretation, which makes it necessary to study them carefully. The annuity with contingent survivor benefit means that only one partner in the couple has entitlement to a pension, and that the other is a dependent. Joint survivor annuity means both members of the couple have entitlement.

The results are similar if the couple's choosing a joint survivor life annuity is considered in the model, in accordance with Expression (10), in which $\rho = \eta = 50\%$. For example, for risk aversion values equal to 0.7 and in case A -which is characterized by a high level of impatience for consumption-, equivalent wealth is 1.049 for couples with altruistic bequest motives and 1.064 for those with strategic motives. In a profile showing a neutral attitude towards consumption -case C-, with a degree of risk aversion equal to 2.9, the values are 1.378 and 1.382 respectively according to whether the bequest motive is altruistic or strategic. In other words, the relation between strategic bequest and altruistic bequest is maintained when equivalent wealth is slightly greater in the first case. In addition to this, in all cases the welfare attained through a joint survivor life annuity is noticeably greater than that obtained with the purchase of a lifetime annuity with contingent survivor benefit – between 1% and 15% more in the profiles analyzed. The reason for this is that the annual amount of the joint annuity is greater than that of the annuity with a survivor contingent given that in the former, on the death of either member of the couple, the annual pension is reduced to half rate, whereas with the annuity with contingent survivor benefit the pension is reduced to the percentage agreed only if the primary annuitant dies.

Table 4: Couple's equivalent wealth and optimal % of initial wealth allocated to a annuity. Lifetime annuity with a 50% contingent survivor benefit. (LACSB). Comparison with a joint survivor annuity (JSA). Indexed annuities.

	Altruistic				Strategic			
	JSA		LACSB		JSA		LACSB	
δ↓ β→	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9
A) Fair annuity								
A	1.049	1.278	**0.916**	1.181	1.064	1.282	**0.935**	1.190
	46%	82%	**37%**	79%	50%	83%	**42%**	80%
B	1.155	1.378	1.104	1.313	1.197	1.382	1.143	1.320
	94%	95%	88%	93%	96%	96%	92%	94%
C	1.234	1.424	1.21	1.405	1.309	1.423	1.255	1.409
	100%	100%	100%	100%	100%	100%	100%	100%
B) With 50% of wealth in pre-existing annuities, 15% reduction factor								
A	**0.847**	**0.924**	**0.830**	**0.904**	**0.850**	**0.931**	**0.831**	**0.911**
	0%	**14%**	**0%**	**7%**	**0%**	**16%**	**0%**	**9%**
B	**0.957**	1.008	**0.938**	**0.985**	**0.972**	1.015	**0.942**	**0.992**
	0%	33%	**0%**	**27%**	**12%**	35%	**0%**	**29%**
C	**1.022**	1.070	1.004	1.045	1.045	1.075	1.022	1.050
	50%	47%	28%	43%	50%	48%	50%	45%

Source: Authors

When market *imperfections* and the possibility of pre-existing annuities are introduced into the model -Part B) of Table 4- the results are very similar for both types of pension. In other words it is less favorable for the couple to purchase than directly allocate their resources without accessing the annuities market. On the other hand, joint survivor life annuities are always more valuable -from the point of view of the welfare they provide the couple- than the type of pension which becomes payable to the other person only when it is the primary annuitant that dies.

Table 4 also shows the optimal percentage of initial wealth allocated to a lifetime annuity. This information provides a valuable perspective because, following the criteria for equivalent wealth for the hypothesis in Part B of the table, some couples would refuse to allocate additional resources to annuities. The reality, however, is different; they would not allocate 50% of the wealth not already allocated to annuities. In most cases the "open market" policy would be best. Even for many of the cases analyzed in Part A of the table, mainly those couples who do not consider themselves impatient to consumption, the best strategy is not to allocate 100% of wealth to buying lifetime annuities but rather to allocate a lesser amount.

As we have just seen, a joint annuity provides better welfare for couples, but it is difficult to find companies that sell them in the Spanish market. This is due to possible legal and fiscal problems and also because the policyholder of the annuity, which up to now in Spain is generally the man, is not willing to give up part of the savings in favor of his wife or partner since this could have an adverse effect on him in the case of divorce or because the amount of his annuity would be reduced if his wife were to die first. The question of buying a joint annuity would create much fewer practical problems if both members of the couple accumulated financial resources in order to pay the premium.

IV.3.- The couple's equivalent wealth and retirement age.

An interesting question on the subject of retirement pensions is the age at which one may begin to receive it. According to Devesa & Vidal (2001), retirement pension systems financed by capitalization are usually also defined contribution systems, and therefore there is greater freedom when choosing the age to retire. The consumer's age has an influence similar to the behavior seen in the case of couples without bequest motives, as shown in the results obtained for equivalent wealth when retirement age is either brought forward or deferred (Table 5).

Early retirement brings about a decrease in welfare gains. This result is consistent with that obtained by Kotlikoff & Spivak (1981), for whom younger people do not place so much value on annuities given that a large part of the utility deriving from consumption is almost certain due to the fact that their probabilities of survival in the most immediate future are quite high. Dushi & Webb (2004) come to the same conclusions.

In all the suppositions there comes about a decrease in equivalent wealth in comparison to the case where no bequest motive is considered. It is generally greater when the bequest motive is strategic rather than altruistic. Most profiles would choose indexed rather than non-indexed pensions, which is the opposite of what would happen if the bequest motive were not considered.

When annuities which are indexed to inflation are purchased, greater increases in welfare are obtained by deferring retirement age than in the case of non-indexed annuities, the more impatient the couple are to consume and the less inclined they are towards risk.

In fact the effect of deferring retirement age is that the couple's probabilities of survival are reduced, hence the rate of return on the annuity is increased. This in turn stimulates demand given that it manifests itself as an increase of equivalent wealth, as was demonstrated under restrictive conditions by Yagi & Nishigaki (1993) for the case of the individual.

Table 5: Couple's equivalent wealth and retirement age.

	Altruistic			**WBM**			**Strategic**		
A) Non-indexed lifetime annuity with a 50% contingent survivor benefit.									
$e_r \rightarrow$ $\delta, \beta \downarrow$	**60**	**65**	**70**	**60**	**65**	**70**	**60**	**65**	**70**
A – 0.7	**0.94**	**0.966**	**0.995**	**0.959**	**0.997**	1.044	**0.954**	**0.985**	1.022
A – 2.9	1.193	1.242	1.304	1.222	1.281	1.358	1.198	1.249	1.314
B – 0.7	1.099	1.118	1.14	1.155	1.193	1.242	1.126	1.152	1.184
B – 2.9	1.268	1.327	1.4	1.291	1.36	1.448	1.27	1.33	1.406
C – 0.7	1.161	1.189	1.219	1.258	1.312	1.377	1.186	1.222	1.265
C – 2.9	1.302	1.371	1.456	1.32	1.397	1.495	1.301	1.371	1.458
B) Indexed lifetime annuity with a 50% contingent survivor benefit.									
A – 0.7	**0.882**	**0.916**	**0.952**	**0.898**	**0.943**	**0.997**	**0.895**	**0.935**	**0.979**
A – 2.9	1.127	1.181	1.247	1.16	1.225	1.308	1.134	1.19	1.26
B – 0.7	1.083	1.104	1.128	1.135	1.176	1.227	1.114	1.143	1.177
B – 2.9	1.257	1.313	1.383	1.292	1.359	1.445	1.263	1.32	1.393
C – 0.7	1.185	1.21	1.238	1.301	1.35	1.411	1.22	1.255	1.294
C – 2.9	1.342	1.405	1.482	1.375	1.449	1.543	1.344	1.409	1.489

Source: Authors and Vidal et al. (2003)

On the other hand, deferring retirement age increases the value couples give lifetime annuities with contingent survivor benefit. The most important implication in what is described above is that the couple's purchase of lifetime annuities should not be considered for a single moment in time; it may not be the optimal choice at a particular age -ordinary retirement age- but it may be optimal a few years later

V.- CONCLUSIONS AND FUTURE RESEARCH.

This paper aims to contribute to a more detailed clarification of the "annuities puzzle" by introducing the bequest motive. With this aim in mind, a wide set of valued suppositions with regard to couples have been presented which have barely been developed in the economic literature. Light is also shed on whether the bequest motive in itself is really a relevant factor influencing the theoretical decision as to whether to purchase annuities for couples.

The consideration of the basic model without market *imperfections* and the possibility of pre-existing annuities is the one that best measures the true impact of the bequest motive on the decision to purchase annuities. Because no other characteristics interfere, it appears to indicate that the bequest motive by itself, isolated, is not a truly relevant factor, although it must be stressed that:

1) Lifetime annuities with contingent survivor benefit become less attractive, although the number of profiles of individuals who would prefer not to purchase such annuities does not increase. In all cases the model based on strategic rather than altruistic motives would be preferable.

2) The profile distribution for couples who prefer non-indexed to indexed lifetime annuities with contingent survivor benefit practically coincides with that of couples with no bequest motive, whether altruistic or strategic. Indexed rather than non-indexed lifetime annuities with contingent survivor benefit are preferable for couples with little impatience to consume and a high aversion to risk.

3) The welfare obtained with a joint survivor life annuity is noticeably higher than that obtained by purchasing a lifetime annuity with contingent survivor benefit. Despite the absolute differences in the results obtained, the relation of the values for the various profiles studied is similar for both types of annuity.

The most extended model with all the characteristics incorporated -market *imperfections* and the possibility of pre-existing annuities, valued at retirement age and assuming that 100% of the wealth not tied up in annuities should be allocated to the purchase of an additional annuity- practically solves the so-called "annuities puzzle" given that very few couples would be willing to purchase them if these conditions applied. This is what normally happens in reality when they are given freedom of choice. It is important to highlight that:

a) Most couples with a bequest motive should decide not to purchase annuities with contingent survivor benefit. Even those couples with no impatience to consume have little incentive to buy annuities, less than a 5% increase in welfare. Only those couples who are sufficiently educated to make welfare-maximization decisions would be capable of appreciating the gains in welfare.

b) A good strategy could be to defer the purchase until later and/or allocate only part of the wealth owned to the purchase of an annuity since, as shown above, the couple would attain greater welfare.

Finally it should be stressed that the results shown strengthen the conclusions put forward in previous papers, in so far as it appears better for the regulations governing different types of annuity to have a certain amount of flexibility in order to accommodate individual circumstances and the aims of public policy. In defined contribution capitalization systems which are complementary to defined benefit systems, in which couples and individuals already have a large part of their wealth in the form of annuities, there should be full freedom of choice with no obligation for people to allocate any amount at all to the purchase of additional annuities.

There are at least three new aspects that the authors have identified for future research:

1) the task of incorporating into the model other sources of uncertainty apart from the date on which the death of the couple will occur. One aspect that could increase the value of annuities

for couples or individuals is the fact that a lifetime annuity is the only way of ensuring a fixed return compared to the variability of the interest rates offered by the market over time. In the model, as is normal in the referenced literature, it has been considered that investment in current wealth at the market interest rate can provide a fixed return equal to that of the annuity, a hypothesis which is not very realistic and which should be of concern to the individual or couple who are averse to risk.

2) to analyze the demand for annuities that contain period certain or refund options, since this type of annuity could at least in theory be valued more highly by couples with a bequest motive.

3) to validate the results for other utility functions. The CRRA utility function has become the most widely used assumption in the financial (and even the macroeconomic) literature in the intertemporal context, but in practice the aversion coefficient and the consumption substitution elasticity do not have to be inversely related nor even necessarily linked. There is currently, Davidoff et al. (2003), Ponzetto (2003) and Rabin (1998) amongst others, a trend towards the revision of this concept.

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