A Two-Dimensional Criterion for Tax Policy Evaluation. A Primer from the Reform of Personal Income Taxation in Russia.

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Abstract.

The comparison of different tax schedules is a problem of high dimensionality. In the current paper it is proved that under very general assumptions a utilitarian welfare function and an inequality index compose a sufficient statistic for the after-tax welfare distribution, reducing significantly the dimensionality of the decision problem. The criterion was used to compare tax schedules before and after the reform of the Personal Income Tax and social taxation in Russian Federation.

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Introduction. The evaluation of different tax systems per se is a very complicated problem. For example, if Pareto – criterion is used for this purpose the dimensionality of the problem becomes so big that it is not solvable even using modern technical tools. However this comparison is essential and experts are often relying on some verbal indicators to evaluate a tax reform proposal.

We will concentrate here on the analysis of the policy, maximizing consumer's welfare. Real policy may pursue another aims but in that case it would distort consumers behavior leading to welfare loss. Ideally a policymaker would use the welfare function of the society for such a comparison. However, to use the welfare function one needs to aggregate individual preferences which can also be unknown. Thus in most cases one it possible only to assume that the welfare function exists. One can compare different functional forms for the welfare function, but in this case a policymaker does not have some consistent criterion for the evaluation of the given proposals. Moreover given certain specification it remains unclear how close this specification is to the true welfare function.

In this paper it is shown that for a quite wide variety of welfare functions given some distribution of wealth in the population utilitarian welfare function with equal weight and an inequality index compose a sufficient statistic. Thus, the criterion for comparison is two dimensional. Ranking of welfare functions is already a difficult task (see Shorrocks, (1987) and Sen, (1973)) so ranking of two-dimensional characteristic is more difficult. The possible solution is to compare the characteristics with some standard characteristics. One can find it convenient to compare the situations with an imaginary no-tax case. This comparison will be incorrect because that will be an attempt to compare unconstraint Pareto – optimum with a situation which at best can be a constraint Pareto – optimum. Therefore the result will depend on a particular welfare specification. The possible solution to this problem is to compare a current tax system with a system, maximizing a particular welfare function given tax revenue constraint. This is done empirically in this paper.

Statistical analysis of welfare functions. In this paper we consider a class of welfare functions which are mapping from the space of distributions of income to real numbers. In other words welfare functions assign a number to each particular income distribution function. The overview of some properties of such functions can be found in Myles (1995).

Consider an integrable distribution function f. Then the aggregate welfare function $W[f]$ can be taken to be positive. In reality however W is unknown. To uncover some of its properties from economic data we need to impose more properties on it. Suppose that $W[\bullet]$ has Gateaux derivatives up to the third order and denote them G_1, G_2, G_3. Suppose also that the empirical distribution of incomes $\hat{f}$, for instance taken from a survey is known. Then we could expand it as $W[\hat{f}]=W[f]+G_1\|f-\hat{f}\|_1+G_2\|f-\hat{f}\|_2+\delta(G_3, \|f-\hat{f}\|_3)$. Here $\|f-\hat{f}\|$ is some norm for the distance between the actual and the empirical distribution function. Assume also that $\hat{f}$ is uniformly converging to f. This makes $\delta(G_3, \|f-\hat{f}\|_3)$ be converging to zero faster than other terms and leaves only the first two components in the expansion.

Note that the empirical distribution function has a random character and so does the chosen norm. Now note that the corresponding Gateaux derivatives are non-random. The performed expansion allowed us to represent the stochastic welfare function as a linear combination of the measures of distribution.

The notion of sufficient statistic has become common in the statistics literature. The relevant references include Lehman, (1959). Now consider the transition from the random variable $W[\hat{f}]$ to random variable $\|f-\hat{f}\|$. Due to presumed expansion the connection between them is quadratic. The corresponding Jacobian is linear. But this also makes this linear combination factor out of the probability distribution function, so that if the density function for

$\|f-\hat{f}\|$ is φ, then the density for $W[\hat{f}]$ can be represented (in the limit) as $\varphi(\|f-\hat{f}\|)(G_1+G_2\|f-\hat{f}\|_1)$. This means that the linear combination is factored out from the density function for the welfare. By factorization theorem we can argue that the combination $G_1+G_2\|f-\hat{f}\|_1$ is a sufficient statistic for the welfare function.

Literally speaking sufficiency implies that the whole information about the distribution is captured by those variables. This means that the represented combination of the norm of the empirical density function if it is uniformly converging in probability to the true density function.

Take the norm $\|\bullet\|$ be of an integral form that is $\|f\| = \int_{-\infty}^{+\infty} \varphi(f(t))f(t)dt$. Then the norm $\|f-\hat{f}\|_1$ can be chosen to represent the weighted average of the distribution, while the norm $\|f-\hat{f}\|_2$ can be chosen to represent Gini coefficient. Thus the combination of the weighed mean of income and the inequality index represent the sufficient statistic for the consumer's welfare.

Ranking the welfare. Statistical analysis of welfare functions lead us to the conclusion that the sufficient statistic for capturing the welfare is two-dimensional. In this case two different after tax income distributions of the population can be incomparable because to be able to rank two distributions one needs to establish strict Lorenz domination of one distribution by the other (see Kakwani, (1956)). The proposed way of resolving this problem is to build classes of equivalent distributions. In this case the policymaker in the "worst" case of incomparable distributions can at least say that the one he or she is choosing is not the inferior one. The possible way of building these classes is to compare the current distribution of incomes with the one that arises when optimal taxation is in effect. The optimum taxation here is presumed to maximize the welfare function given the revenue constraint for the government budget.

The formal theory of income taxation was introduced in the paper by Mirrles, (1977). Mirrles managed to reduce the welfare maximization problem to the problem of optimal control. The derivations lead to the conclusion that the optimal marginal income tax rate is increasing as the elasticity of labor supply increases. Moreover, marginal tax rate increases as the productivity of workers is decreasing as well as the density of individuals with given income level is decreasing. Formal analysis leads us to the expression for the optimum income tax rate given the parameters of the consumers. Such analysis was performed for the United States by Saez, (1999). Therefore to be able to construct reference level for welfare comparison we need to estimate the parameters of consumers labor supply.

Micro-analysis of labor supply. Analysis of labor supply in Russia has been thoroughly made since the extensive databases such as RLMS became publicly available. Relevant literature include Stillman, (2000), Sabirianova, (2000), Roshin and Razumova, (2002) and others. In these papers RLMS database is found to be extensive and giving reasonable estimates for the various individual parameters. In this paper we obtain the characteristics of the labor supply for different population groups to get the elasticities of labor supply with respect to post tax wage. To get this dependence an hurly wage for RLMS respondents was calculated which was augmented with the aid of Goskomstat data on regional price indices. The original sample was split into 4 and 20 parts according to the quantiles of hourly wage distribution.

The rounds of RLMS were chosen to capture the effect of the reform of income taxation in the Russian Federation so only ninth and tenth rounds were used. The basic result of the reform is that the structure of labor supply has changed. As one can see from Table 1, labor force participation has decreased in the year 2001.

Table 1.

The structure of employment in the rounds 9 and 10 in the RLMS sample.

Year	Fraction of respondents,	Fraction of female	Fraction of male

	having a job	respondents having a job	respondents having a job
2000	49.94%	45.40%	55.91%
2001	49.52%	45.94%	54.39%

The discrepancy in the figures is quite small and can be caused by some randome increment in the sample. An important result is that the tendencies for labor force participation for males and females are different. While the participation of females has risen by 0.54% the participation among males has dropped by 0.48%.

The model for estimation was taken to be a combination of linear and nonlinear model. The main dependence for estimation is the dependence of the weekly hours of work from the hourly wage and household's individual parameters. The hourly wage in turn is taken from the supporting equation which is a standard Mincer wage equation with independent variables containing age, age squared and experience level. In addition the first step equation included binary variables representing having children younger than 5 years and between 5 and 17 years old, and the quality of parents' education.

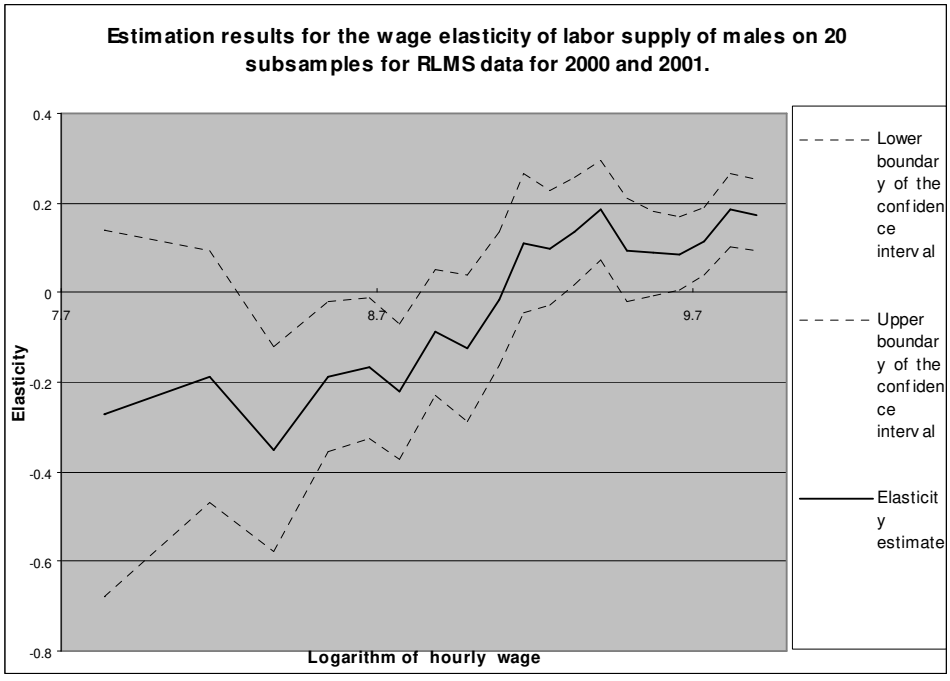
The main equation of the hours worked monthly was estimated in logarithms:

$\ln(H) = \alpha + \beta \ln(\hat{w}) + \zeta\, \text{Faminc} + e$, while wag equation takes the form:

$\ln(w) = \gamma + \delta\, \text{AGE} + \eta\, \text{AGE}^2 + \theta\, \text{EXP} + \lambda\, \text{INS} + \nu M + \mu$, where $\alpha - \lambda$ – coefficients, e, μ – random increments, $\hat{w}$ - wage, predicted from the first step, Faminc - family income variable, AGE – age of RLMS respondent, EXP – work experience, INS - the variable, reflecting the possession of higher education, M – the variable, reflecting marriage status. The first step equation can be motivated by the theory of human capital so that age and age squared reflect the return to human capital and diminishing returns of scale. The family and education variable reflect the dynamics of the human capital over time.

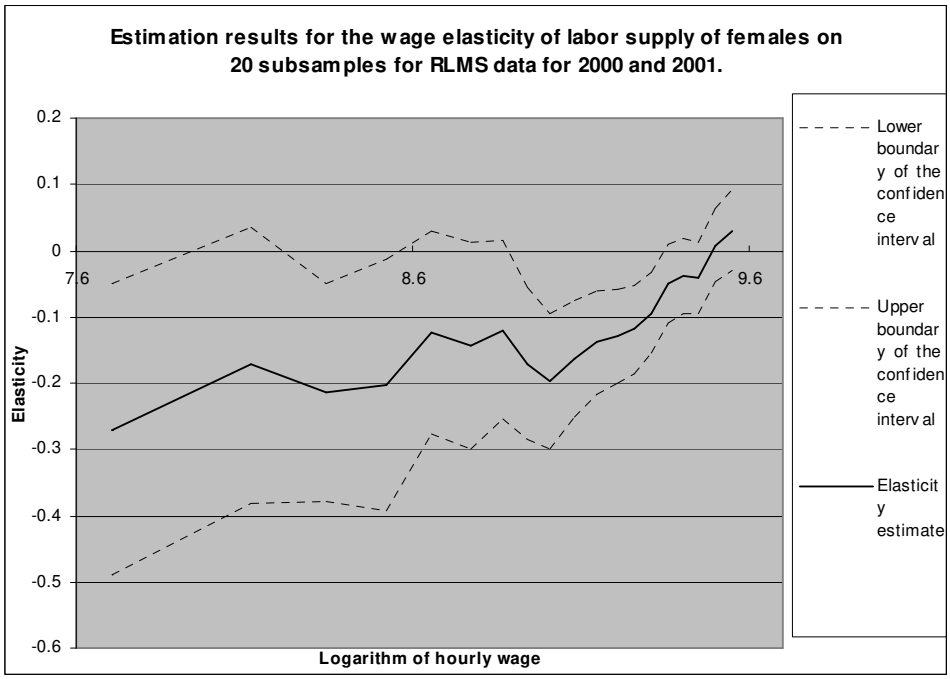
The sample was split into 20 parts and for each labor supply equation was estimated.

The presented graph represents the elasticity of labor supply as the logarithm of hourly wage is changing



One can see that the elasticity of labor supply tends to increase on the interval of observation but it is statistically insignificant for low wage levels. Therefore labor supply for the high income males is the most elastic to wage changes.

The labor supply of females is overall less elastic than the labor supply of males.



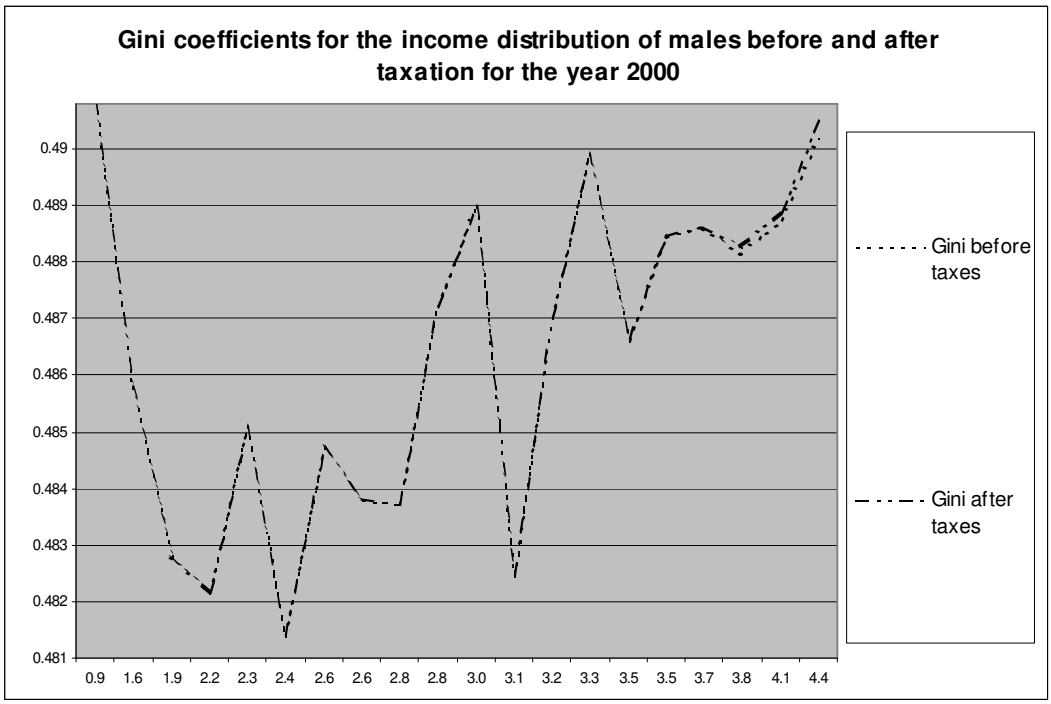
While this graph shows that the elasticity of labor supply tends to grow as the wage grows. However in most cases the elasticity estimate is not significant which can imply that the labor supply of females is less sensitive with respect to real wage.

Therefore it can be concluded that the labor supply of males and females has the property of increasing elasticity as the real wage increases. One can also notice very low estimates for the labor supply elasticity which can be caused by the legal constraints on the working day. This effect can be reduced if one considers secondary employment as well, but then some complications from taxation issues and interpretation of hourly wage can occur.

Inequality analysis. To analyze the connection between distributional and welfare impact of taxation it is necessary to determine the distributional characteristics of the taxation. It was shown above that Gini index is a part of the sufficient statistic for the welfare function so it can be used to measure this distributional impact.

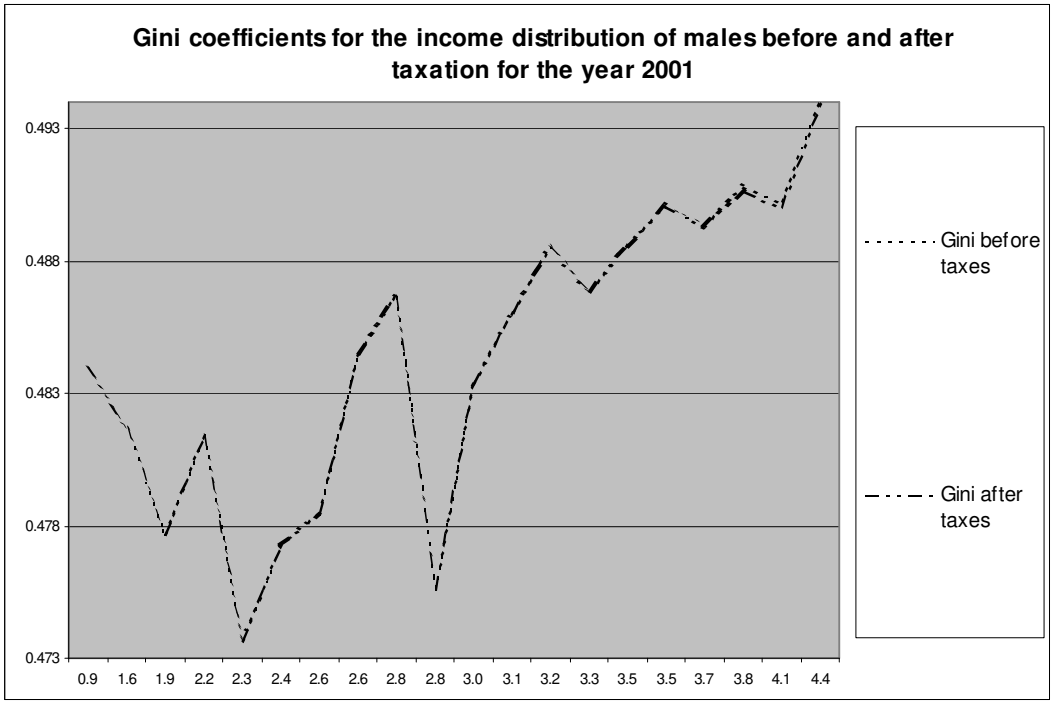
For the construction of distributional impact the panel of RLMS respondents was matched with their individual characteristics relevant for the tax liabilities calculation. The built array of data was used to calculate the income of the respondents before taxes (assuming that the respondents were reporting their true post-tax income and all possible allowenses and deductions were fully utilized for their actual tax payments). The considered taxes included the personal income tax and the payments to the non-budgetary funds which were supposed to be fully shifted towards the workers. This helped to calculate after-tax and post-tax wages for all working individuals.

Consider the trends in inequality indices on the basis of 20 subsamples of the original RLMS sample. On the graph below one can see the dependence of the Gini coefficient from the logarithm of hourly wage of RLMS male respondents for the year 2000.



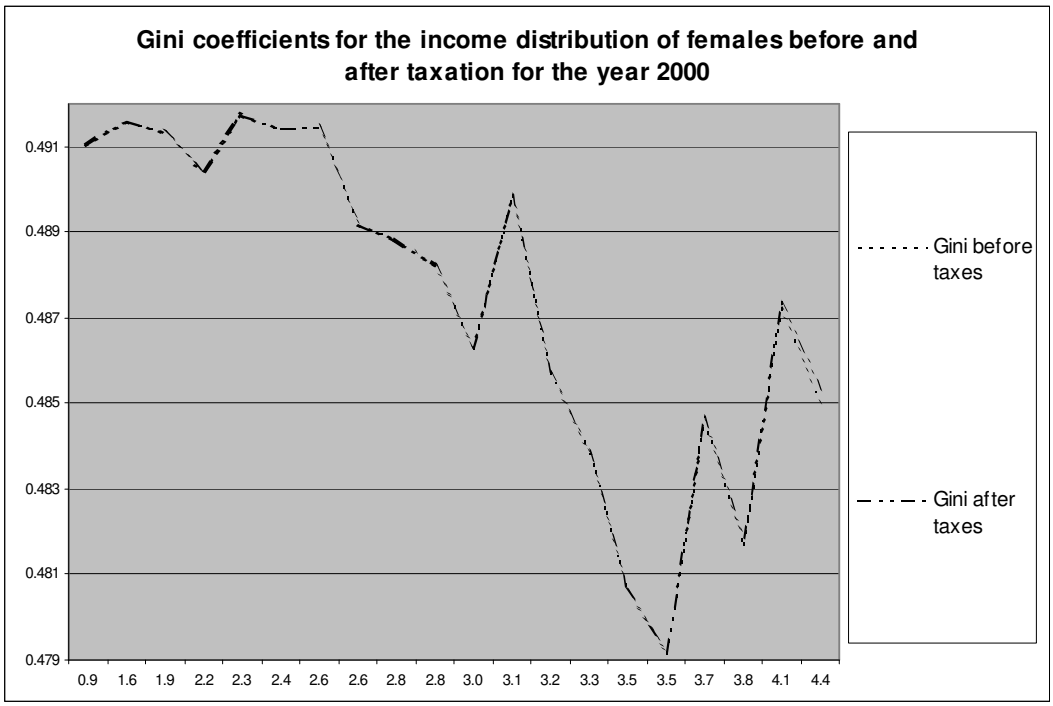
Inequality indices are close up to the groups of high income males. One can also notice that as a wage becomes higher the inequality of wages becomes higher as well with the exception of the males with the lowest incomes.

This tendency remains in effect for the sample of males in the year 2001.



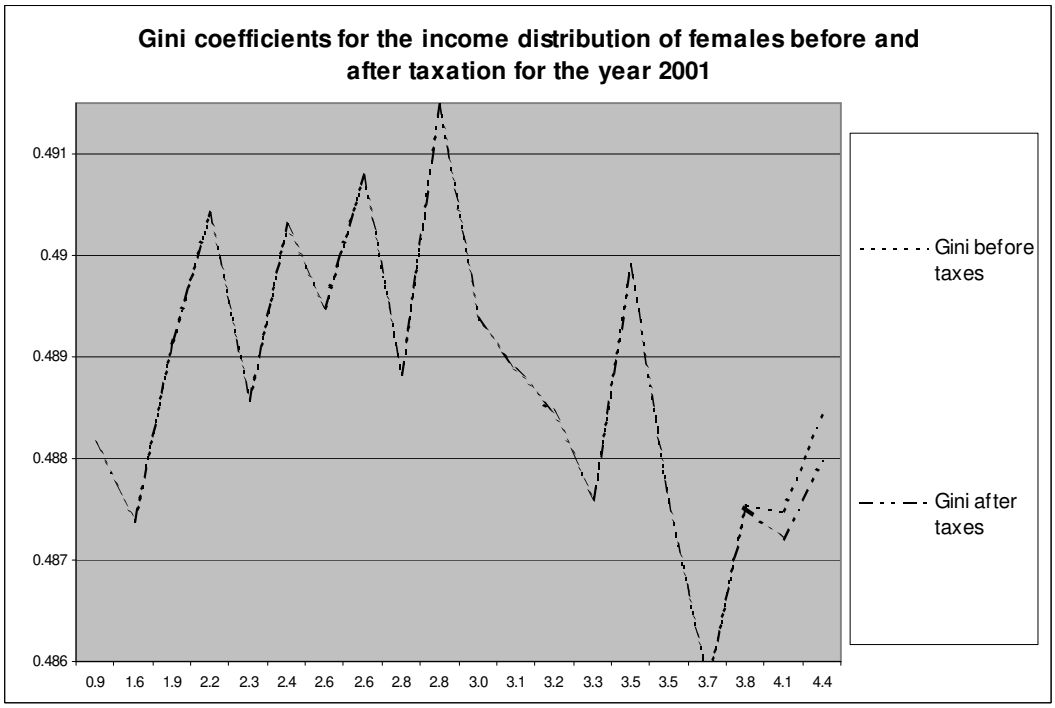
In this case no significant decrease in inequality for the low income males is observed.

The trend in inequality for the females is the opposite from that of the males.



As one can see from the graph a steep decrease in inequality when the wage increases form the lowest to the median is followed by a small increase in inequality for the wages of the women with the highest wages. It is also possible to see that the difference between the inequality indices before and after taxes is small.

In the year 2001 the decreasing trend in wage inequality for female subsamle is not observed.



While it is possible to argue that the inequality in the subsamples of females with higher wages is lower on average, this trend is unstable over the time. The inequality is smoothly increasing from the lowest to the average wage and is increasing when the wage is higher than average.

The construction of welfare – maximizing marginal income tax rates. To construct reference level for the two-dimensional welfare comparisons we need to establish the parameters of the welfare-maximizing tax system. In this paper Mirrles calculations were used to find the optimal marginal income tax rates. To simplify the computational procedure the utility functions of individuals were chosen to be separable with respect to consumption and leisure. This lead us to the parameters of the optimal tax system.

Mirrles result implies that the marginal tax rates for the highest and the lowest wage individual should be zero. On the other hand, as elasticity of labor supply goes down to zero the optimal tax rate can be set equal to 1 (as the individual is not distorted by high marginal tax rates). Thus according to our estimates of labor supply elasticities, the optimal tax system for Russia should be based on complete smoothing of incomes for the most of the income distribution (i.e. withdrawing all income beyond the certain level).

This result of course cannot be taken as a guide for the tax reform. Firstly, it is obtained assuming perfect tax administration without taking into account the behavioral response of individuals on the change in the tax rates by increased tax evasion. Secondly, administrative costs of collecting income tax with high marginal tax rates are not taken into account. Lastly, individual preferences over the level and quantity of offered public goods are not incorporated into considered utility functions. In particular, high marginal tax rates can lead to overproduction of public goods and will not be optimal.

The constructed optimal income tax schedule was applied to the calculated pre-tax incomes of individuals in the RLMS sample.

Calculation of welfare change due to taxation. In the previous paragraph the construction of inequality indices was discussed. The other component of the two – dimensional criterion under consideration is the weighted average of individual welfares which represents the utilitarian welfare function. In this paper it is assumed that the individual utility is separable with respect to leisure and consumption. As a result the elasticity of compensated labor supply coincides with the uncompensated elasticity. This means that the estimates of the uncompensated elasticities obtained above remain applicable here. Standard ways of calculation the welfare change are appropriate here as appropriate normalization leads to equivalence of leisure and consumption. The general task for the welfare change is to construct a reasonable estimate of the utility change for the economic agents. The value of the utility function itself has no meaning so appropriate normalization is needed to make it comparable across individuals. Such normalization can be represented by the money-metric utility function.

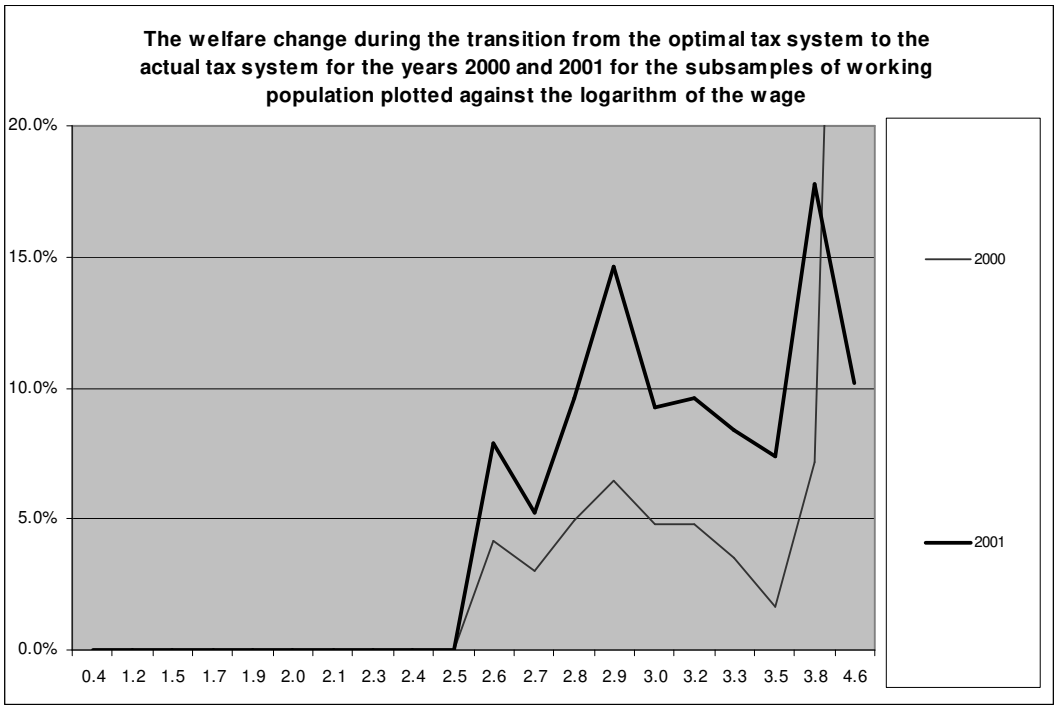
To obtain the appropriate welfare change for the simplest case consider the individual utility as a function of consumption (equal to wage income wl for simplicity) and utility of labor l : $U=wl+\varphi(1-l)$, then the change in the wage leads to the following change in the utility: $\delta U=\{[w-\varphi'(1-l)]l'+l\}\delta w+\frac{1}{2}\{[w-\varphi'(1-l)]l''+l'+\varphi''l'^2\}(\delta w)^2$. Using the envelope theorem one can reduce this expression to: $\delta U=l\delta w+\frac{1}{2}\{2l'+\varphi''l'^2\}(\delta w)^2$.

If we denote the income of the individual by y and elasticity of labor supply with respect to the wage by $\left|\varepsilon_l^w\right|$, the following expression can be obtained: $\delta U=l\delta w+\frac{1}{2}y\left|\varepsilon_l^w\right|(\frac{\delta w}{w})^2$.

The first term in this expansion represents the change in the utility level due to the change in individual income. The second term shows the change due to the change in the labor-leisure choice. Particularly, if the change in the wage occurs due to the change in the tax rate then the second term can be rewritten as: $\delta U^{(2)}=\frac{1}{2}y\left|\varepsilon_l^w\right|(\frac{\delta\tau}{1+\tau})^2$.

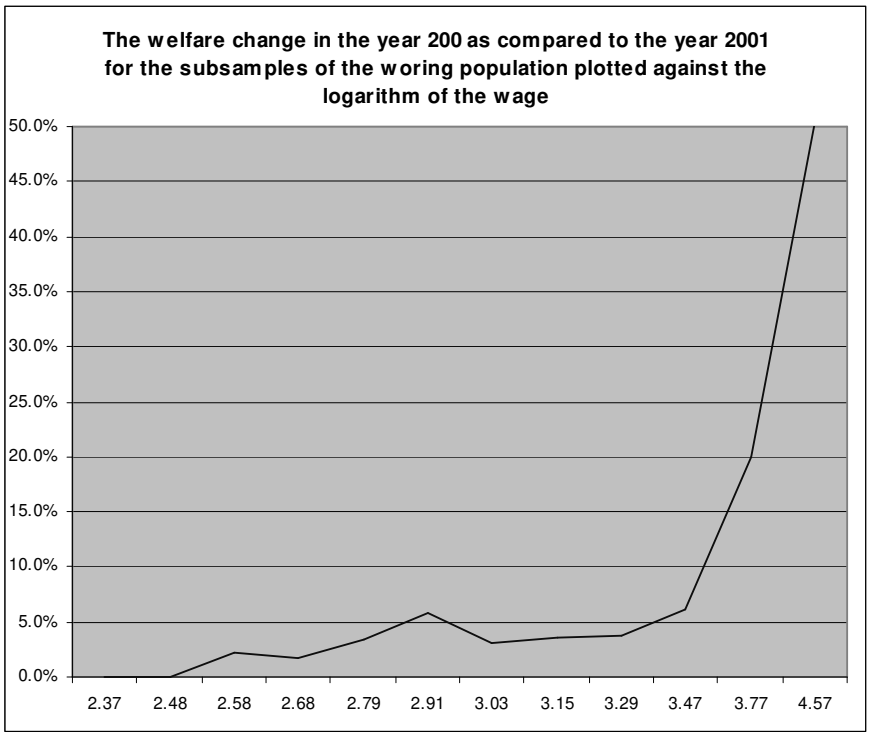
The analog of this formula was used for the calculation of the welfare change for the transition from the optimal income taxation to the actual income taxation.

The graph below represents the results of calculations of the welfare change in the sample during the transition from the optimal income taxation to the actual income taxation.



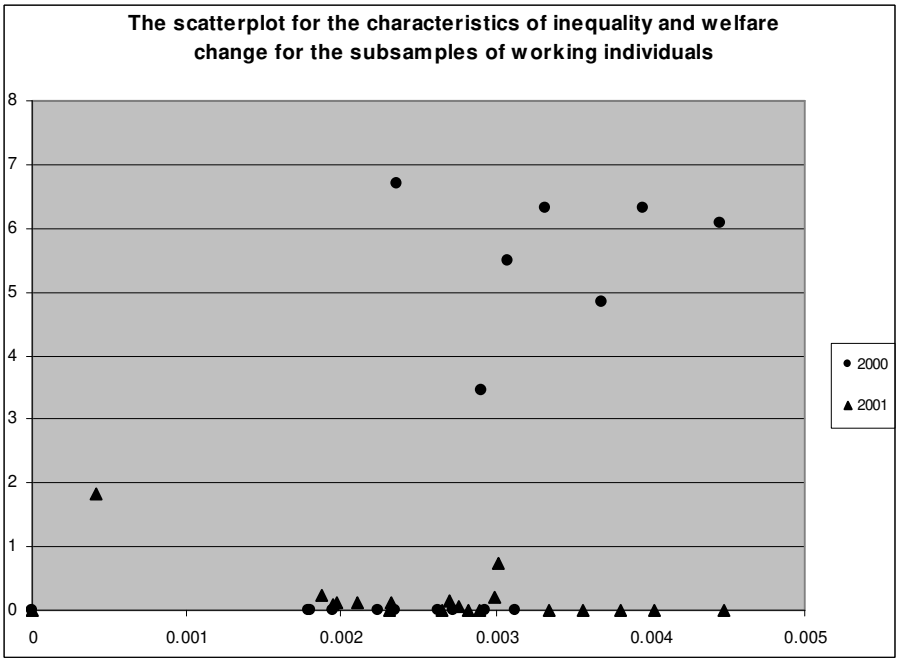
As one can see the distribution of welfare change is such that the main change occurs on the high income individuals. This fractions tend to grow with the growth of income both in the year 2000 and the year 2001. One can see from the graph that in the year 2000 the wealthiest individuals had relatively bigger welfare change then those in the year 2001. At the same time the fractions of welfare change for the lower income individuals increases in the year 2001.

Aside from the rather abstract consideration of the welfare at the current tax schedule and the optimal tax schedule it is possible to look at the welfare change during the tax reform in the year 2000. The graph below shows that the highest changes in the welfare due to the reform in the year 2000 are observed for the highest income individuals. The reform, on the other hand, resulted in the decrease in the marginal tax rates for the high wealth individuals which leads us to the conclusion that the reform was welfare – improving with respect to the utilitarian welfare criterion. Moreover the welfare increase is higher for the higher wage individuals.



Thus this analysis points out that without regard to the incomparability of the welfare change distributions for the two years the tax reform has lead to the increase in the welfare if measured with the aid of the utilitarian criterion.

The application of the two-dimensional criterion. To compare the situations before and after the tax reform in the year 2000 two indicators – welfare change value (as compared to the optimal tax system) and the inequality index was assigned to each of the twenty subsamples. These characteristics correspond to the coordinates on the graph below.



One can notice that the points for the year 2000 on average are located far from the origin (which corresponds to the optimal tax system[1]). On the other hand the points for the year 2001 are located closer to the origin.

To find out the difference between the two locations significantly the test for the mean comparison was performed. It shows that the locus of the points for the year 2001 is statistically closer to the origin than that of the points for the year 2000. This leads to the conclusion that the tax reform has led to the increase in the welfare of Russian working individuals according to the presented two-dimensional criterion.

Conclusion and policy recommendations. This paper considered one way of the reducing the dimensionality of the problem of welfare comparisons for different tax systems. It was proved that a utilitarian welfare function and an inequality index represent the sufficient statistic for the wide variety of welfare functions. Further it was argued that the comparison of the current tax system with the situation without taxes is not completely justified because it leads to the comparison of a constrained Pareto-optimum with an unconstrained optimum. The better criterion can be built by the comparison of the current tax schedule with the tax schedule. Leading to the constrained Pareto – optimum. Such schedule was built on the basis of Mirrles theory of optimal taxation.

Implementation of this criterion to the situation of the Russian tax reform in the year 2000 has led us to the conclusion that the tax reform was welfare – improving with respect to the built criterion.

The constructed criterion can appear to be useful in the policy analysis because it allows to reduce significantly the arbitrary character of the choice between the "almost equivalent" tax rules. This criterion is applicable to the situations when the aim of the tax policy is the increase in the welfare individuals and it can be not relevant in case of other policy objectives. However as the maximization of the aggregate welfare is important in most cases it can be used as one of the additional characteristics for the analysis of policy change.

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[1] Note that some points coincide with the origin. This happens because a part of estimated elasticities appeared to be unsignificant. In this case the change in the marginal tax rates does not lead to the change in the tax burden. Therefore, any marginal tax rate (between 0 and 1) is equivalent to the optimal tax rate.

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