Social Security Reform and Intergenerational Trade: Is there Scope for a Pareto-Improvement?*

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Abstract

In earlier literature, the suggested Pareto improvements in pay-as-you-go (PAYG) systems have relied on the presence of externalities or the possibility of intragenerational redistribution. We show that neither assumption is necessary in an economy with intergenerational trade in a fixed factor of production, here labeled as land. Reducing the social security tax rate encourages investment in complementary human capital. Future efficiency gains accruing to land are capitalized in its value which compensates the land-owning pensioners for reduced benefits. We also explain why the PAYG system may have lost its appeal even for pensioners after its introduction.

Keywords: Social Security Reform; Fixed Factor; Pay-As-You-Go System, Capital Gains Taxation

JEL Codes: H55, H21, I38

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*A previous version of the paper circulated under the title "Social Security Reform with a Fixed Factor". We are grateful for useful comments from Clemens Fuest, Ulrich Hange, Roisin Hearn, Katarina Keller, Christian Keuschnigg, Etienne Lehmann, J. Michael Orszag, Pierre Pestieau, Ismo Risku, Lars Siemers, Hans-Werner Sinn, Peter B. Sørensen, Bas van Groezen, Harrie Verbon, Matthias Wrede and participants in seminars at CES in June 2001 and VATT in September 2001, the European Public Choice Society Annual Meeting in Belgirate, Italy, April 4-7, 2002, the Spring Meeting of Young Economists in Paris, April 18-20, 2002, the EEA Annual Meeting in Venice, Italy, August 22-24, 2002, the IIPF Annual Conference in Helsinki, Finland, August 26-29, 2002 and the CEBR conference in Copenhagen, June 2003. Earlier versions of the paper were written while Poutvaara was employed first at the Department of Economics, University of Helsinki, and then at the Government Institute for Economic Research (VATT). Financial support from the Danish Ministry of Economic and Business Affairs is gratefully acknowledged, while none of our employers are responsible for our conclusions.

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Introduction

In most, if not all, Western countries participation in the pay-as-you-go (PAYG) social security system does not seem attractive for the current younger generations. The expected rate of return on social security contributions is considerably lower than the expected rate of return on financial investments. Lower and even negative population growth only aggravates the burden that a PAYG system imposes on younger generations. This has stimulated a considerable interest in a social security reform in which the PAYG system would be replaced, at least partially, by a funded component taking advantage of a higher market rate of return.[1]

In the absence of altruism such a reform creates an intergenerational conflict. Although future generations benefit from a transition to a fully-funded system, existing pensioners would have to accept reductions in their benefits unless they receive compensation financed by the beneficiary future generations. Establishing such an intergenerational transfer institution is difficult even from a theoretical perspective. A natural but in this respect ineffective compensating transfer institution is public debt. If the PAYG system is free from inefficiencies not related to the inherent intergenerational redistribution[2], the gains of future generations are neutralized by higher future taxation needed to service public debt (Fenge, 1995 and Brunner, 1996).[3]

In this paper, we introduce a different intergenerational transfer institution through which the elderly participate in the future efficiency gains created by a decrease in the PAYG tax rate. This mechanism consists of voluntary market transactions in the fixed factor, labeled here as land for

[1] Most prominently, Feldstein (1996) advocates replacing a PAYG system by a funded one based on this argument. In contrast, see Diamond (1996) for a critical evaluation of the rate-of-return argument in restructuring social security.

[2] Intragenerational redistribution is part of the social security system in many countries, but the implied inefficiencies cannot be attributed to the existence of a PAYG system which inherently only induces intergenerational redistribution.

[3] The Pareto-efficiency of the PAYG system was first demonstrated in Breyer (1989) and Verbon (1989). In contrast to Fenge and Brunner, Breyer and Verbon assume an exogenous labor supply and a PAYG system with lump-sum contributions and pension benefits.

convenience.[4] The presence of land has two implications in our model. First, the economy is dynamically efficient, i.e. the interest rate exceeds the output growth rate (Homburg, 1991 and Rhee, 1991). As the output growth rate is the rate of return on PAYG contributions, the PAYG system is fiscally unattractive for future generations. Second, the part of the future efficiency gains which improves land productivity is immediately capitalized in its current market value. Both implications form the basis for the contribution of this paper: If sufficiently high, the reform-induced increase in land value outweighs the loss from cutting social security benefits for the old land owners. The PAYG reform thus provides a welfare improvement for all generations alive and for generations to be born into all future periods.

We also suggest alternative reasons for why a currently inefficient social security system was installed in the first place. Typically, social security systems were introduced and expanded in the 19th century and early 20th century, when other public expenditures were much smaller. Our analysis suggests that an increase in the wage tax rate collected to finance other public expenditures could alone render a social security system, which initially benefited the elderly, so costly that currently even they might benefit from its partial or even full dismantling. In addition, changes in the demographics, productivity growth or interest rates may render a system unattractive even for the elderly, who at least previously had supported it.

We analyze an overlapping generations model in which each cohort lives for three periods. During the first period of life, the members of the cohort invest privately in education. In the

[4]Evaluating the quantitative importance of the fixed factor, Laitner (2000) calculates the ratio of non-reproducible capital (which approximates the capitalized value of inelastically supplied factors) to reproducible capital in the U.S. This centers around 15-25 percent over the second half of the 20th century. The stake that private households have in the productivity changes of these fixed factors appears to have increased due to at least two dramatic changes in U.S. retirement saving during the last two decades. These are the transition from defined benefit plans to defined contribution plans, and the associated huge increase in the value of pension assets. Poterba et al. (2001) report that 59 percent of private retirement savings in 1980 were in employer-based defined benefit plans, while currently 85 percent are defined contribution plans in which individuals decide on the level and investment of their contributions. At the same time, the ratio of all private retirement assets to wage and salary earnings quadrupled.

second period of life, they supply labor services equal to their human capital to production and purchase land from the older generation. Social security contributions are collected. During the third period of life, they receive social security benefits (indexed to past contributions) as retirees, land rents as land owners, and sell the land to the next generation.

In addition to the presence of the capitalization effect as a compensation mechanism, our model differs from the existing literature on social security reform in several important respects. First of all, we prove the possibility of a Pareto-improvement in case where there is only intergenerational redistribution through social security.[5] Identical for all individuals, pension benefits are a constant fraction of former contributions.[6] Secondly, there are neither externalities, distortions arising from early retirement provisions, nor labour market imperfections in our model. These distortions, or intragenerational redistribution, are critical sources of welfare gains in existing literature.[7] In that respect, we adopt a framework in which the existence of a Pareto-improving pension reform has not yet been proven. Thirdly, according to earlier literature opening the economy is found to be detrimental for the domestic welfare gains of a social security reform. An increase in private savings following a reduction in the PAYG pension pillar is distributed worldwide via the international

[5] Any existing PAYG system can be divided into two parts; one implementing intergenerational redistribution and the other intragenerational redistribution. We analyze a system which redistributes only intergenerationally since this part is inherently related to a PAYG system. Inefficiencies in the intragenerational tax-transfer scheme can be addressed without affecting the "core" of the PAYG system, namely intergenerational redistribution.

[6] This corresponds to social security systems in Austria, Finland, Germany, Italy and Portugal, in which the difference in the gross replacement rate between high-income and low-income earners is at most 3 percentage points. In the U.S., the gross replacement rate decreases from 71 percent for those with a final salary of $\$20,000$ to 45 percent for those with a final salary of $\$50,000$ (Miles and Timmermann 1999).

[7] Homburg (1990), Feldstein and Samwick (1998), Kotlikoff (1998), and Cooley and Soares (1999b) suggest a Pareto-improving reform analyzing a PAYG system which allows for intragenerational redistribution. Their analysis is consistent with the theoretical result in Fenge (1995) since the simulated PAYG systems do not exhibit intragenerational fairness. In this case pension benefits are only loosely linked to contributions, making the system highly distortionary. Belan et al. (1998), Corsetti and Schmidt-Hebbe (1997), Belan and Pestieau (1999), Pemberton (2000), and Gyárfás and Marquardt (2001) each identify a reform which relies on positive externalities from physical capital. Such externalities can be addressed by a savings subsidy without reforming the social security system (Sinn, 2000). Analogously, efficient retirement decisions can separately be achieved by eliminating incentives pushing for early retirement (Cremer and Pestieau, 2003). Pension reform in the presence of imperfect labour markets is considered in Demmel and Keuschnigg (2000). Besides a reduction in unemployment the efficiency gains of the reform originate from intragenerational redistribution. Related to our paper, reform induced windfall profits accrue to pensioners due to a re-evaluation of existing physical capital.

capital market while in a closed economy it solely translates into a higher capital stock (Pemberton, 2000). Assuming a small open economy, in which capital accumulation is unaffected by a reform-induced increase in domestic savings, we show that incentives to reform the PAYG system may still exist.

Related literature includes Laitner (2000) who analyzes social security reform when future productivity gains are incorporated into stock prices. Firms have two assets: reproducible physical capital and capitalized value of patents. Contrary to our analysis, Laitner does not consider the potential of a Pareto-improving transition. Since labor supply is fixed and there is no human capital formation, PAYG contributions and benefits are lump-sum. In this framework, the PAYG system is Pareto-efficient, clearly leaving no scope for analyzing the feasibility of a Pareto-improving social security reform; see Hange (2003) for a formal analysis. İmrohoroğlu et al. (1999) similarly consider pension reform in an economy with land. Assuming inelastic labor supply, they analyze the trade-off between the risk-sharing benefit (due to incomplete capital and insurance markets) and the negative effects on capital accumulation of the PAYG system. They argue that steady state generations are still better off when abolishing the PAYG system, leaving aside the transition problem. However, as a caveat, even in the presence of perfect markets, which compensate for incomplete risk-sharing under a fully-funded system, a Pareto-improvement for steady state and transition generations cannot be obtained for the reasons given above.

Intergenerational trade in land has important implications for the political process. Due to population aging, pensioners receive a larger representation in the political process which tends to preserve a generous pension system rather than allowing for a restrictive pension reform (Boadway and Wildasin, 1989). Cooley and Soares (1999a) show that the interests of the working generation near retirement and pensioners are sufficiently closely aligned which gives rise to a wide political majority against privatization of social security. Hansson and Stuart (1989) even argue that social security is an implicit contract among living generations in which the old have veto power. Pension reform thus requires an intergenerational consensus. We show that intergenerational trade is one mechanism to moderate if not resolve intergenerational conflict among living non-altruistic generations and, interestingly, to indirectly represent future generations' interests in the contemporary political process. Relatedly, Rangel (2002) emphasizes the potential of land to act as a "voice" mechanism of future generations in the current political process. In his paper, land taxation is critical for establishing a link across generations. In our paper, an intergenerational consensus may arise even without land taxes.

Intergenerational trade aligns intergenerational interest not only with respect to social security reform but also with respect to general public sector reforms. For instance, the mechanism could equally be involved in tax policy reform which entails a reduction in the level of future taxation. The induced capital gains on land value, non-distortively taxed away by the government, can build a fund out of which the budget balance in each future period is restored.[8] There are two theoretical reasons why the intergenerational transfer mechanism is particularly applicable to social security reform. Firstly, the mechanism is present without allowing the government access to a wider range of tax instruments than wage taxation. If the asset ownership is distributed in correspondence to benefits there would be no need for the government to be able to unexpectedly tax capital gains in order to compensate all pensioners. Secondly, the mechanism does not require that the government is able to commit to maintaining a once-established fund in all future periods (Ramsey tax planner), neither does it require "reputational" political mechanisms to be operative in infinite times in the absence of a commitment capacity. If however the ambitious assumptions are met, the capitalization effect in combination with capital gains taxation may also be applied

[8] We are indebted to Clemens Fuest and Pierre Pestieau for suggesting this wider applicability of the mechanism we have identified.

to tax policy reform. Finally, the presence of land also widens the scope for intergenerational transfers to the younger generations. Applied to education policy, Poutvaara (2003) shows that non-altruistic land owners may gain by providing public education to the young even when the land owners cannot tax the young and when the latter face no credit constraints.

The paper is organized as follows. In Section I, we present the model of an economy with pay-as-you-go social security. In Section II, we introduce social security reform and consider the potential for Pareto-improvements. In Section III, we present possible explanations for the rise and fall of the PAYG system. Section IV concludes.

I Model

Production

Production in any given period t depends on the available technology and factors of production. We assume that there are three factors of production: physical capital, human capital and land. The amount of land is normalized to unity. Aggregate human and physical capital in the economy in period t is denoted by H_t and K_t, respectively. There is no migration of human capital, while physical capital is internationally mobile. Therefore, domestic capital stock need not equal domestic aggregate saving. The production function is Cobb-Douglas with A_t reflecting the state of technology in period t. Thus,

$$Y_t = A_t H_t^{\alpha_H} K_t^{\alpha_K},$$

where $0 < \alpha_H, \alpha_K < 1$ and $\alpha_H + \alpha_K < 1$. Without loss of generality, capital does not depreciate. All markets are competitive, and therefore profit maximization implies that:

$$w_t = \alpha_H A_t H_t^{\alpha_H - 1} K_t^{\alpha_K}, \quad r = \alpha_K A_t H_t^{\alpha_H} K_t^{\alpha_K - 1}. \tag{1}$$

w_t denotes the wage rate per unit of human capital in period t and r is the exogenous world interest rate. The land rent in period t, R_t, is given as residual

$$R_t = (1 - \alpha_H - \alpha_K) A_t H_t^{\alpha_H} K_t^{\alpha_K}.$$

The production function for human capital is

$$h_t = e_{t-1}^{\beta}, \tag{2}$$

where β, $0 < \beta < 1$, is the elasticity of human capital supply with respect to investment in education. Individual human capital stock in period t depends on investment in education in the former period, e_{t-1}. The marginal productivity of education is diminishing and the unit cost of education is 1. The costs of education should be interpreted broadly in the sense that they might also include the monetarized value of effort cost. We do not consider opportunity costs explicitly, as they are effectively tax deductible with proportional taxation.[9] The aggregate stock of human capital is the product of the stock per worker, h_t, and the number of workers, N_t,

$$H_t = N_t h_t. \tag{3}$$

We assume a constant population growth rate, so that the size of the cohort *working* in period t is given by

$$N_t = N_0 (1 + n)^t, \tag{4}$$

where $n \geqq 0$ is the growth rate of the population per cohort. While our model could be solved

[9]This results as with proportional taxation opportunity costs of lost earnings are reduced by the same proportion as benefits of higher wages due to educational investment.

also for $n < 0$, we restrict attention to non-negative growth rates. Production depends also on technology parameter

$$A_t = A_0(1+g)^t,$$

where $g \geqq 0$ denotes technological progress. We assume identical individuals, extending the results in the Appendix to the case of heterogeneous individuals. Furthermore, there is no uncertainty.

Individuals can invest their savings in the international capital market or the national land market. Foreigners do not invest in the national land market. Even with integrated capital markets, full domestic land ownership could be guaranteed by foreigners facing a small transaction cost if they were to buy domestic land, whereas there would be no transaction cost in an international loan market. Transaction cost in foreign land acquisition might arise due to asymmetric information on the part of investors (Gordon and Bovenberg, 1996) which tends to play a diminished role in international loan markets.[10] The economy produces a composite good, which is a perfect substitute for that produced abroad. By arbitrage, land value in period t, V_t, is given by[11]

$$(1+r)V_t = R_{t+1} + V_{t+1}. \tag{5}$$

Recursive substitution yields:

$$V_t = \sum_{i=1}^{\infty} \frac{R_{t+i}}{(1+r)^i}. \tag{6}$$

[10] Even with the presence of transaction costs faced by foreigners, Eq. (5) would hold because a strict inequality as any difference in the rate of return between the two assets would be eliminated by trade in land by domestic citizens, financed by international borrowing.

[11] Though economic agents have a finite horizon, speculative bubbles are not considered as a component of the land price. Given the presence of a fixed factor, the economy turns out to be dynamically efficient which rules out the existence of bubbles (Tirole, 1985).

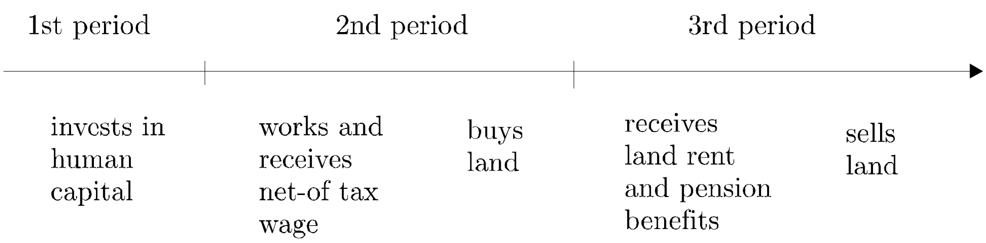

Figure 1: Timing of individual actions over the life-cycle.

Individual Maximization

We analyze an overlapping generations model in which each cohort lives for three periods. The timing of individual actions, apart from consumption, saving and borrowing, is depicted in Figure 1. In the first period of their life individuals choose their education. Human capital is supplied to the labor market in the second period.[12]

The government collects social security contributions and wage taxes at a rate τ^s and τ^w, respectively, and individuals receive a net-of-tax wage income, $[1-(\tau^s+\tau^w)]w_t h_t$. Social security contributions are used to finance benefits for the current old generation whereas wage taxes finance public consumption. The middle-aged generation invests its savings by buying land from the older generation and by participating in the international financial market. In the third period, individuals receive social security benefits. Formally, social security benefits in period $t+1$, b_{t+1}, depend on contributions made in period t, $c_t = \tau^s w_t h_t$, according to the formula

$$b_{t+1} = (1+x)c_t, \tag{7}$$

where x is the rate of return on contributions.[13] The rate of return provided by the PAYG system x is exogenous for each individual, but endogenous for the economy as shown in the next

[12] What is essential is that effective labor supply is endogenous in the second period of life. As an alternative to human capital formation we could assume endogenous time allocation between work and leisure.

[13] Without loss of generality we assume that pensions are not taxed. If pensions were taxed, then b would be replaced by an after-tax pension.

section. It is important to notice that a link between past contributions and benefits does not imply a funded social security system. In this setting a funded system is equivalent to private savings where the rate-of-return equals the market interest rate, while a PAYG system offers a lower rate-of-return as shown below. Finally, the older generation sells land to the current middle-aged generation.

Denote by C_t^j consumption in period t by an individual living his or her period j, where $j = 1$ for the young, $j = 2$ for the middle-aged and $j = 3$ for the elderly. Furthermore, denote savings in the capital market of a member of age group j at the end of period t by S_t^j, and the amount of land bought when being middle-aged by L_t^2. Then individual savings used for land acquisition is $L_t^2 V_t$. As all land is purchased by the middle-aged, $N_t L_t^2 = 1$. The value of land sold by old people is given by $L_t^2 V_{t+1}$, and land rent received in the third period of life is $L_t^2 R_{t+1}$.

Note that with population growth, $N_t L_t^2 = N_{t-1} L_t^3$. G_t denotes the level of the pure public good provided in period t.[14] We assume a well-behaved utility function defined over private and public consumption. The individual lifetime utility maximization problem facing the members of a generation working in period t is:

$$\max_{e_{t-1}, C_{t-1}^1, C_t^2, C_{t+1}^3, S_{t-1}^1, S_t^2, L_t^2} U(C_{t-1}^1, C_t^2, C_{t+1}^3, G_{t-1}, G_t, G_{t+1}), \tag{8}$$

subject to the budget constraints

[14] The labor tax rate, τ^w, is held constant throughout the analysis. However, public consumption, G_t, may change over time e.g. due to reform induced adjustments in human capital investments.

$$
\begin{aligned}
-e_{t-1} - C_{t-1}^1 - S_{t-1}^1 &= 0 \\
S_{t-1}^1(1+r) + (1-\tau^s-\tau^w)e_{t-1}^\beta w_t - C_t^2 - S_t^2 - L_t^2 V_t &= 0 \\
S_t^2(1+r) + L_t^2 R_{t+1} + L_t^2 V_{t+1} + (1+x)\tau^s e_{t-1}^\beta w_t - C_{t+1}^3 &= 0.
\end{aligned}
$$

Notice that non-negativity of consumption and human capital investment implies that $S_{t-1}^1 < 0$, so that consumption and investment in human capital in the first period is financed by borrowing. All individuals can save and borrow freely at the exogenous interest rate r, determined by the international capital market, to smooth their consumption over their lifetime. Therefore, following the Fisher Separation Theorem, optimal individual choices can be characterized by a two-step optimization problem: one where individuals choose educational investment to maximize discounted net-of-tax lifetime income and a second one where, for a given lifetime income, individuals choose their utility-maximizing intertemporal consumption profile by borrowing and lending in the perfect capital market.[15] As individuals rationally expect that $(1+r)L_t^2 V_t = L_t^2 R_{t+1} + L_t^2 V_{t+1}$, investment in the land market cancels out in the maximization of the net present value of lifetime income. Therefore, the individual maximization problem facing the members of a generation working in period t is:[16]

$$
\max_{e_{t-1}} \left(-e_{t-1} + \frac{1}{1+r}(1-\tau^s-\tau^w)e_{t-1}^\beta w_t + \frac{1}{(1+r)^2}\left[(1+x)\tau^s e_{t-1}^\beta w_t\right]\right). \tag{9}
$$

[15] Thus, welfare gains of a transition to a fully-funded system cannot originate from capital market imperfections as is the case, e.g., in Börsch-Supan and Winter (2001).

[16] To simplify the exposition, trade in land does not enter the maximization problem. This is justified by the fact that land value is exogenous from each individual's perspective and, thus, does not affect educational investment. Furthermore, given the Fisher Separation Theorem, the saving decision does not have to be made explicit when analyzing optimal educational investment. It exclusively serves to implement the optimal life-cycle consumption plan. The investment of savings in the international capital market and land market, in turn, follows from the arbitrage condition (5).

The first term in brackets is the private cost of educational investment in period $t-1$, the second term is the after-tax wage income in period t discounted to period $t-1$, and the third term is the social security benefit in period $t+1$, discounted to period $t-1$. The first-order condition becomes:

$$-1+\frac{1}{1+r}\left(1-\tau^s\frac{r-x}{1+r}-\tau^w\right)\beta e_{t-1}^{\beta-1}w_t=0,$$

which gives individually optimal educational investment in period $t-1$ as a function of wages. Using Eqs. (1) - (3) allows us to derive the level of investment in general equilibrium:

$$\widehat{e}_{t-1}=\left(\frac{1}{1+r}(1-\tau^s\frac{r-x}{1+r}-\tau^w)\beta\alpha_H A_t^{\frac{1}{1-\alpha_K}}N_t^{\frac{\alpha_H+\alpha_K-1}{1-\alpha_K}}\left(\frac{\alpha_K}{r}\right)^{\frac{\alpha_K}{1-\alpha_K}}\right)^{\frac{1-\alpha_K}{1-\alpha_K-\beta\alpha_H}}. \tag{10}$$

Straightforward comparative statics yields:

$$\left.\frac{\partial\widehat{e}_{t-1}(\cdot)}{\partial\tau^s}\right|_{r>x}<0.$$

The social security system discourages human capital investment if the rate of return under the PAYG system is lower than the interest rate. The rationale for this distortion is that lending in the international capital market (as well as investment in the domestic land market) for one period yields a rate of return r while compulsory savings under the PAYG system earn a rate of return x. If the latter falls below the former, the PAYG system imposes an implicit tax on contributions equal to

$$\tau^s\frac{r-x}{1+r}. \tag{11}$$

Therefore, the combined tax burden imposed by the wage tax and the implicit social security tax reads $\tau:=\tau^w+\tau^s\frac{r-x}{1+r}$. Its magnitude determines the downward distortion in educational investment and the Harberger Triangle the economy incurs.

Balanced Growth Path

Human capital investment is independent of land price which gives Lemma 1.

Lemma 1 *For any sequence of land prices* $\{V_t\}_{t=0}^{\infty}$ *Eqs. (1), (2), (3), and (10) imply*

$$\begin{aligned} \frac{h_t}{h_{t-1}} &= (1+g)^{\beta\gamma}(1+n)^{-\beta(1-\alpha_K-\alpha_H)\gamma}, \ \frac{H_t}{H_{t-1}} = (1+g)^{\beta\gamma}(1+n)^{(1-\alpha_K)(1-\beta)\gamma} \ \textit{and} \\ \frac{R_t}{R_{t-1}} &= \frac{Y_t}{Y_{t-1}} = 1+q \end{aligned} \tag{12}$$

where

$$1+q := (1+g)^{\gamma}(1+n)^{\alpha_H(1-\beta)\gamma} \ \textit{and} \ \gamma := \frac{1}{1-\alpha_K-\beta\alpha_H} > 0. \tag{13}$$

The growth rate of production, q, depends only on the parameters related to technological progress, population growth and technology for production and human capital formation. It is independent of the wage tax rate, social security contribution rate or interest rate, which affect only the level of production but not its growth rate. Given by Eq. (12), individual human capital stock increases over time if $g \geq n$, or if g is not much below n. If $g << n$, then $h_t < h_{t-1}$ and human capital stock per worker would decrease over time. However, even in such an economy aggregate human capital and output would be still increasing.

Land price dynamics are captured by Eq. (5). Rearranging and using Lemma 1, all "price-dividend" ratios consistent with arbitrage behavior must satisfy

$$\frac{1+r}{1+q}\frac{V_t}{R_t} = 1 + \frac{V_{t+1}}{R_{t+1}}. \tag{14}$$

Equation (14) defines a function $\frac{V_{t+1}}{R_{t+1}} = \phi\left(\frac{V_t}{R_t}\right)$ with $\phi' > 1$ if $r > q$. Lemma 2 summarizes the implications for the balanced growth path.

Lemma 2 *A unique balanced growth path exists if and only if $r > q$. The balanced growth path exhibits point stability.*

As illustrated in Figure 2, a unique balanced growth path ratio $\frac{V^*}{R^*}$ exists if and only if $\phi' > 1$ which exclusively holds if the interest rate exceeds the output growth rate, i.e. $r > q$. A positive and finite ratio of land price to land rent thus exists if the economy exhibits dynamic efficiency (Homburg, 1991 and Rhee, 1991).[17] As the economy is a price taker in the international capital market and growth is exogenously driven, the requirement of dynamic efficiency is a necessary and sufficient assumption for a balanced growth path to exist. The balanced growth path exhibits point stability since for any value of $\frac{V_t}{R_t} \neq \frac{V^*}{R^*}$, the ratio $\frac{V_t}{R_t}$ does not converge to $\frac{V^*}{R^*}$. Therefore, the only adjustment process consistent with perfect foresight is a jump to $\frac{V^*}{R^*}$ in the period following $\frac{V_t}{R_t} \neq \frac{V^*}{R^*}$.

In a PAYG system $N_{t-1}b_t = N_t c_t$. Since $N_t c_t = \tau^s \alpha_H Y_t$, Eqs. (7) and (12) imply

$$\begin{aligned} x &= \frac{b_t}{c_{t-1}} - 1 \\ &= (1+q) - 1. \end{aligned}$$

In a purely earnings-related system the rate of return on contributions under the PAYG system equals the economy's growth rate q (Aaron, 1966). Following Eq. (11) and dynamic efficiency, the earnings-related PAYG system imposes an implicit tax on contributions equal to $\tau^s \frac{r-q}{1+r} > 0$. Given the link between c_{t-1} and b_t, only a fraction $\frac{r-q}{1+r}$ of the contribution rate τ^s is a tax.

[17] The result is not specific to a Cobb-Douglas production function. With a more general production technology land continues to preclude dynamic inefficiency if land is essential, meaning that the income share of land does not vanish asymptotically (Rhee, 1991).

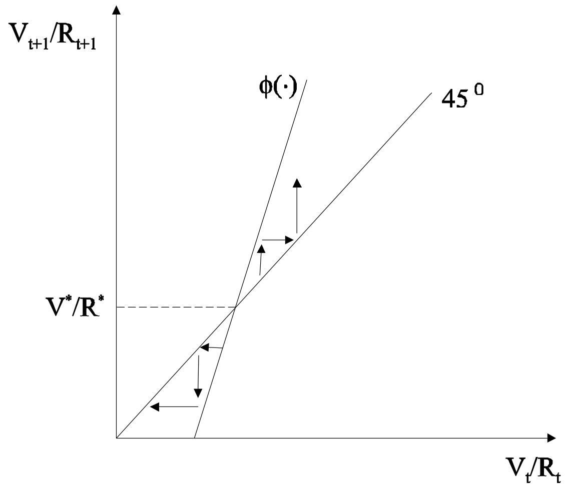

Figure 2: Steady State for $r > q$.

Using the land price equation (6) and the growth factor of land rents, the time path of land value is characterized by

$$V_t = R_{t+1} \frac{1}{r-q}. \tag{15}$$

The factor $(r-q)^{-1}$ captures the effect of future output growth and discounting on current land value and is independent of the social security system. Any change in land value following a security security reform in period t is captured by a jump in land rents in the subsequent period.

Finally, the financial position of the country vis-a-vis the rest of the world must satisfy the transversality condition. When the net foreign assets of the economy in period t are denoted by F_t, the transversality condition requires that $\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T F_{t+T+1} = 0$ (Obstfeld and Rogoff, 1996).[18] Here, the transversality condition is satisfied as the budget constraint is satisfied for each generation over its lifetime, and the growth rate of production as well as of land value is less than the interest rate.

[18] As saving and investment in the following period's capital stock takes place at the end of the period, it follows that $F_t = N_{t+1}S_t^1 + N_t S_t^2 - K_{t+1}$.

II. Social Security Reform

We assume that social security tax rates are cut by proportion ω, $\omega \in [0,1]$, from τ^s to $(1-\omega)\tau^s$. No reform occurs if $\omega = 0$ whereas $\omega = 1$ indicates a complete transition to a fully-funded system. The policy reform is announced and implemented at the beginning of the period t^* before the current younger generation has decided on educational investment and the elderly have sold land to the middle-aged generation.[19] As we analyze a small open economy, the transition to a new steady-state takes place in two periods. The investment in human capital adjusts fully to the path corresponding the new steady-state after the reform has been announced, implying that the aggregate supply of labor services and the stock of physical capital adjust in the subsequent period.[20]

Both the young and middle-aged benefit from social security reform. While the middle-aged only enjoy a lower implicit tax on their contributions, the young also reap the benefits from less distorted educational investments. The welfare effects of the elderly are shaped by two conflicting forces. On the one hand the elderly lose out due to the cut in social security benefits. On the other hand, they receive an unexpected wealth increase originating from a direct and indirect effect on educational investments. The reduced tax distortion directly increases human capital investments of the current young and future young generations, which increases land rents in all subsequent periods (starting in period $t^* + 1$). Since production factors are complements, a higher stock of human capital induces an inflow of physical capital and a complementary rise in the wage rate. As an indirect effect of the reduced tax distortion educational investments and land rents increase even further. Following Eq. (15), higher land rents generate an immediate jump in land

[19] Important for our result is that land is owned by the elderly at the time of policy announcement and implementation, which implies that both capital gains and reduced pension benefits accrue to current pensioners. Remarks on alternative timings of policy reform are offered in the Conclusion.

[20] In cases where distortions would arise from labor-leisure choice, instead of distorted investment in human capital, production and land rents would adjust already in the period of reform announcement and implementation.

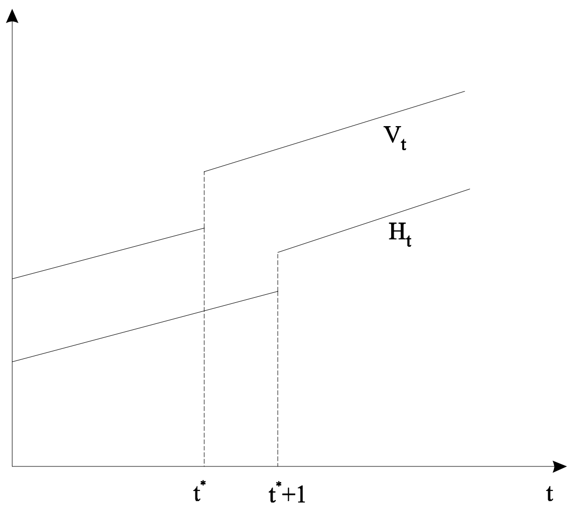

Figure 3: Effect of social security reform on H_t and V_t.

value accruing to land owners; see Figure 3 assuming equal slopes of H_t and V_t for expositional simplicity. As the middle-aged have already made their investments in human capital, the value of production and wage tax revenue in the period t^* does not change, neither does the land rent accruing to the elderly before they sell the land. Denoting the social security budget in period t by B_t, $B_t = \tau^s \alpha_H Y_t$, the social security reform is favored by the older generation if and only if the increase in land value, ΔV_{t^*}, exceeds the drop in pension benefits ΔB_{t^*}. Measuring the changes in relation to GDP, a pension reform is Pareto-improving if and only if

$$\frac{\Delta B_{t^*}(\tau^s, \tau^w, \omega, \alpha_H, \alpha_K, \beta, g, n, r)}{Y_{t^*}} + \frac{\Delta V_{t^*}(\tau^s, \tau^w, \omega, \alpha_H, \alpha_K, \beta, g, n, r)}{Y_{t^*}} > 0. \tag{16}$$

We define

$$z := \frac{1+q}{1+r}.$$

z is the ratio between the gross rate of return offered by the PAYG system and the gross rate of return offered by the financial market. With dynamic efficiency ($q < r$), $z < 1$. To understand

Proposition 1 and 2 it is helpful to note that z depends on the output growth rate q and the interest rate r both being exogenous. Any value $z < 1$ can be achieved by choosing a sufficiently high value of r. Proposition 1 presents the main finding of the paper.

Proposition 1 *Consider a social security reform where τ^s is marginally reduced. For any values $0 < \alpha_H, \alpha_K, \alpha_H + \alpha_K < 1$ and $0 < \tau^s, \tau^w < 1$, a sufficiently high value of z, $0 < z < 1$, and an induced value of $\beta^*(z)$, $0 < \beta^*(z) < 1$, always exists for which the reduction in benefits equals the increase in land value. Let the set of z-values which induce a $\beta^*(z)$ be denoted by Γ (which is always non-empty). For all combinations of $z \in \Gamma$ and β, $\beta^*(z) < \beta < 1$, the social security reform is intergenerationally Pareto-improving.*

Proof: See Appendix 2.

A Pareto-improving reform with intergenerational trade is feasible. To illustrate Proposition 1 thoroughly, Figure 4 displays $\beta^*(z)$ and the set Γ for three parameter combinations which involve a varying τ^w. If for the sake of illustration $\tau^w = 0.5$, the set $\Gamma = (0.38, 1)$. It is only for these z-values that a β-value < 1 exists which in combination with the corresponding z-value satisfies condition (16) as an equality. Given $z \in (0.38, 1)$ an intergenerational Pareto-improvement holds for all $\beta > \beta^*(z)$ as a higher elasticity of human capital supply magnifies the efficiency gains of the pension reform. The most stringent requirement of Proposition 1 is that the general wage tax must be strictly positive. The intuition is straightforward. The wage tax adds to the inefficiency in educational investment due to the implicit tax rate in the PAYG system. The distortion in educational investment is convex in the overall tax rate. Therefore, the efficiency gain of a pension reform increases over-proportionally as the initially prevailing overall tax rate rises. Figure 4 readily reveals that τ^w is critical not only qualitatively but also quantitatively.

Proposition 1 requires that z be sufficiently high, i.e. the rate of return offered in the PAYG

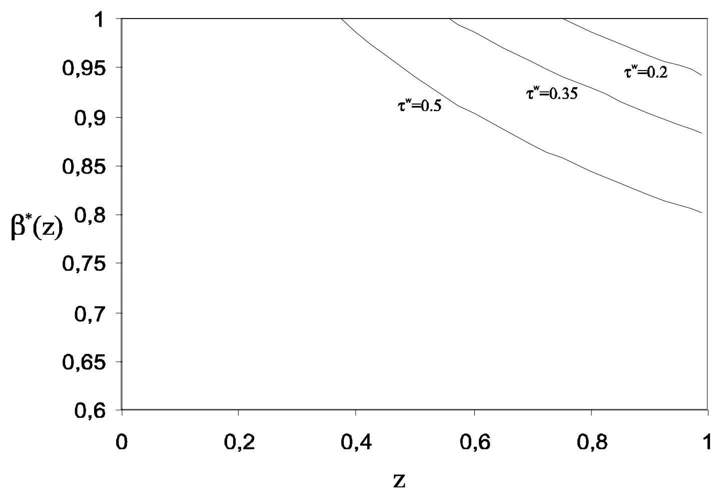

Figure 4: $\beta^*(z)$ for varying values of τ^w ($\alpha_K = 0.2$, $\alpha_H = 0.6$, and $\tau^s = 0.2$)

system and the fully-funded system must be sufficiently aligned. This is a surprising finding, as it implies that a reduction in the population growth rate (lower q and thus lower z), and therefore an increase in the dependency ratio, would actually reduce the scope for a Pareto-improving reduction in the social security tax rate. On the other hand, reduced population growth rate may undermine the sustainability of a PAYG system since the dependency ratio goes up. Therefore, there seems to be a conflict between the implications of the demographic structure for the potential for Pareto improvements and for the sustainability of the system. The findings can be reconciled by noting that the implicit tax rate increases as z reduces, but at the same time the future productivity effects are discounted more strongly (Eq. (15)). The latter effect dominates which diminishes the scope of the asset price effects to compensate the elderly.

In this framework demographic change affects asset prices in a different way to that which previous literature has focused on. Therein the higher rate of return under a fully-funded system might partially be eroded by population aging since future pensioners have to sell financial assets to a smaller generation of young investors when liquidating their portfolios; see Abel (2001) and Poterba (2001) for a critical evaluation of the "meltdown" hypothesis. In the present paper, equilibrium land price in each period is not directly affected by a change in the ratio of the number of suppliers to demanders. Instead, negative demographic shocks reduce asset returns due to lower steady state growth which immediately capitalizes in asset prices.

Proposition 2 presents comparative static results.

Proposition 2 *(i)* $\beta^*(z)$ *is decreasing in* z, $z \in \Gamma$.

(ii) The infimum of Γ *is increasing in* α_K *and decreasing in* α_H *and* τ^w. *Furthermore, keeping* z, $z \in \Gamma$, *constant,* $\beta^*(z)$ *is increasing in* α_K *and decreasing in* α_H *and* τ^w.

Proof: See Appendix 2.

Part (i) reveals that z and β can be imperfectly substituted in order to sustain a Pareto-improvement. Higher discounting of future productivity gains is neutralized by a higher elasticity of human capital investment which magnifies the response of educational investment to a reduction in the implicit tax rate. The range of z-values and the critical level $\beta^*(z)$ are influenced by the specifics of the economy.

It is a priori unclear how a change in α_K and α_H should be expected to affect the infimum of Γ. The first effect is that an increase in either of them shrinks the income share of land, which would suggest that the infimum of Γ should increase: As productivity increases accrue to a lower extent to land, it is reasonable to expect that this requires a lower discount rate z to keep the asset price adjustment at the same order of magnitude. The second effect is that an increase in either α_K or α_H increases the responsiveness of total production to changes in the factor whose share increases, which would suggest that the infimum of Γ should decrease. We find that for an increase in α_K, the first effect dominates, while for α_H, the second one is more important. This is in line with changes in adjustable physical capital stock being indirect effects, while changes in the stock of human capital are direct effects, magnified by indirectly induced effects of physical capital. A higher wage tax magnifies the distortions in educational investment which are partially eliminated by the pension reform. Discounting of productivity effects can thus be more severe, i.e. a lower value of z, and still allow for a Pareto-improvement. Applying an analogous line of argument the threshold $\beta^*(z)$ responds similarly to the considered parameter changes as a change in z does.

At this point it is informative to relate Propositions 1 and 2 to the literature on PAYG reform using public debt to compensate the elderly. In an earnings-related PAYG system the issuance of public debt does not yield a Pareto-improvement irrespective of the pre-existing level of general wage taxation; see Fenge (1995). Intuitively, the reduction in the implicit tax rate is neutralized by the increase in the wage tax necessary to service the public debt. With intergenerational trade in land, however, a lower implicit tax rate does not require adjustments in the explicit tax rate levied on future generations. The efficiency gain is permanent and a fraction of it capitalizes in the land price. The capitalization mechanism can be interpreted as an intergenerational transfer institution which transfers part of the future efficiency gains to the elderly in a lump-sum fashion.

Finally, we should notice that there is a further welfare gain from increased future public expenditures, see Eq. (8). As the general wage tax rate is kept constant, increased human capital production implies increased wage tax revenue as a result of a decreasing social security tax rate. If instead the revenue requirement for public expenditures were to stay constant, the implied future decrease in general wage taxation would further boost human capital investment and land value in the period a reform is announced. Additionally, private consumption would increase following a reduction in the tax burden.

III. The Rise and Fall of the PAYG System

Our analysis in the previous section raises a legitimate question: If the elimination of a PAYG system could be a Pareto improvement, why would such a system exist? It is not plausible that a PAYG system would have been implemented in the first place if it did not benefit the older generation at that time. The creation of a PAYG system could be explained, outside our model, by arguments such as the inability of the poor to save for their retirement in the 19th century. However, even our basic model captures some obvious candidates for why the PAYG system could have benefited the elderly in the first place, but lost its appeal subsequently. One such candidate is unequal distribution of land ownership. The poor, older citizens without assets would favor a PAYG system, as they are not hurt by the efficiency loss capitalized in land value. Therefore, an increasing middle-class with widespread stock ownership might contribute to the eroded popularity of the PAYG system among the elderly. Another obvious candidate is the secular increase in other public expenditures. An increased tax burden implies higher distortions, and increases the cost of maintaining a PAYG system. As the increase in the welfare state during the 20th century was not anticipated when the social security system was introduced, the elderly could have supported its introduction then, but now benefit from its abolition even if nothing other than the general wage tax rate has changed. We summarize this argument as

Proposition 3 *If the elderly are indifferent between* τ^s *and* $(1-\omega)\tau^s$*,* $\omega > 0$*, with a given wage tax rate* $\widetilde{\tau}^w$*, then they would strictly prefer* $(1-\omega)\tau^s$ *with all* $\tau^w > \widetilde{\tau}^w$*.*

Proof. The elderly being indifferent to a reduction of social security tax rate by $\omega\tau^s, \omega > 0$, implies that $\theta > 0$, as otherwise the elderly would always strictly prefer a higher social security tax rate. With wage tax rate $\widetilde{\tau}^w$,

$$\theta(1-\alpha_H-\alpha_K)\frac{z}{1-z}\left[\left(\frac{(1-\tau^s(1-\omega)(1-z)-\tau^w)}{(1-\tau^s(1-z)-\tau^w)}\right)^{\gamma\beta\alpha_H}-1\right]-\omega\tau^s\alpha_H=0.$$

Differentiation of the left-hand side with respect to τ^w yields

$$\theta\gamma\beta\alpha_H(1-\alpha_H-\alpha_K)\frac{z}{1-z}\left(\frac{(1-\tau^s(1-\omega)(1-z)-\tau^w)}{(1-\tau^s(1-z)-\tau^w)}\right)^{\gamma\beta\alpha_H-1}\frac{\tau^s\omega(1-z)}{(1-\tau^s(1-z)-\tau^w)^2}.$$

As this is positive with any $\omega > 0$, an increase in the wage tax rate is sufficient to induce the elderly to favor a cut in the social security tax rate. ■

As a corollary, Proposition 3 implies:

Corollary 1 *If the elderly are currently indifferent between maintaining the current social security system or eliminating it, they would have favored its establishment with any given lower level of wage tax rate.*

The proposed explanation for the growing concern for social security reform is supported by the steady increase in the labor tax rates (besides other taxes) in major industrialized countries over the second half of the 20th century; see Mendoza et al. (1994).[21] Daveri and Tabellini (2000) provide empirical evidence that this increase in tax ratios has lowered economic growth and, therefore, the rate of return offered by a PAYG system which further undermines its fiscal attractiveness. Undoubtedly, the explanation will not exclusively account for the intended or already implemented reforms in a variety of countries, but it will nevertheless complement other forces such as population aging.

IV. Conclusion

In this paper, we demonstrate that the scope for Pareto improvements in social security systems is wider than previous literature suggests. In earlier literature, the Pareto improvements in PAYG

[21] For instance, the macroeconomic U.S. labor income tax rate rose from roughly 18 percent in 1965 to 29 percent in 1988. Over the same time span, labor income tax rates steadily increased from 15 to 26 percent in Japan, from 20 to nearly 27 percent in the U.K., from almost 13 to 28 percent in Canada, and from 30 to 41 percent in Germany (only West Germany); see Figure 2 in Mendoza et al. (1994).

systems rely on externalities or intragenerational redistribution; neither being inherently related to a PAYG system. Both absent, we show that a Pareto-improving transition is still feasible in an economy with a fixed factor of production, here labeled as land. As land value captures future land rents, intergenerational trade in land allows the pensioners of the transition generation to participate in the benefits of reducing the social security tax rate. We also provide an explanation for why the PAYG system may have lost its appeal even for pensioners after its introduction.

Even if a decrease in social security benefits were to result in a larger capital gain in land value in the aggregate level, such reduction need not be Pareto-improving alone. If land is not fully owned by domestic pensioners, then capital gains taxation may be needed to implement a Pareto-improvement. The government could finance social security benefits for the elderly of the transition period by unexpectedly taxing capital gains resulting from the reduction in the social security tax rate. Demand for capital gains taxation also arises if the social security reform is announced prior to its implementation. Otherwise, benefits and costs of the reform accrue to different generations.

There are important caveats when drawing policy implications. The simulations are suggestive and are not meant to be calibrated to a specific economy. Thus, the results are of a theoretical nature. The compensation mechanism that we identify may, nonetheless, constitute a part of a partial social security reform. A policy-oriented analysis should calibrate the factor shares of fixed and adjustable physical capital, as well as the production function for human capital. As the effects of the PAYG system may depend crucially on other taxes, it would be desirable to replicate the essential features of the tax system in numerical analysis. This exercise, as well as modeling asset price effects through a revaluation of existing physical capital (as common in tax reform analysis; see e.g. Altig et al., 2001) is left to future research. Applied to social security reform, the last extension is likely to strengthen our findings since it opens up an additional intergenerational link through which the current elderly have a stake in future productivity gains.

Appendix A1: Social Security Reform with Heterogeneous Population

In this appendix, we analyze the conditions under which the results from Section II hold in an economy in which people differ in their ability and in their asset ownership. Let us divide the working population in period t into I groups $(i = 1, .., I)$, so that individuals in any group (t, i) share the same ability and asset ownership characteristics. We define ability as individual ability to absorb and effectively utilize investment in human capital. A member of group (t, i) being a worker in period t has ability $\infty > a_{t,i} > 0$. Instead of Eq. (2), the production function for human capital is now

$$h_{t,i} = a_{t,i}e^{\beta}_{t-1,i},$$

in which $e_{t-1,i}$ is investment in human capital in period $t-1$ by individuals belonging to the ability group (t, i). To reflect the empirical fact that workers belonging to high-income groups often have more expensive education, we assume that the cost of creating human capital stock $a_{t,i}e^{\beta}_{t-1,i}$ equals $a_{t,i}e_{t-1,i}$. The number of workers belonging to the ability class (t, i) is $n_{t,i}$. We normalize the measure of ability so that the average ability is unity in each period, implying that $\sum_i n_{t,i}a_{t,i} = N_t$. Replacing e_{t-1} by $e_{t-1,i}$ and the stock of human capital e^{β}_{t-1} by $a_{t,i}e^{\beta}_{t-1,i}$ in the individual maximization problem (9) and solving the first-order conditions for different ability classes i yields identical investments in human capital $\widehat{e}_{t-1,i}$. Through adopting this normalization, this translates into an aggregate stock of human capital in the economy and factor prices both of which are identical to the case without heterogeneity.

What remains to be analyzed are the welfare effects of a social security reform for different ability classes. Subsequently, we denote the share of domestically-owned land held in period t by the elderly members of the ability class $(t-1, i)$ by $\psi_{t-1,i}$. Corresponding to Section II, we can now derive the condition under which social security reform benefits the members of this ability class:

$$\frac{n_{t-1,i}a_{t-1,i}}{N_{t-1}}\frac{\Delta B_t(\tau^s,\tau^w,\omega,\alpha_H,\alpha_K,\beta,g,n,r)}{Y_t}+\psi_{t-1,i}\theta\frac{\Delta V_t(\tau^s,\tau^w,\omega,\alpha_H,\alpha_K,\beta,g,n,r)}{Y_t}>0, \quad (17)$$

in which $\frac{n_{t-1,i}a_{t-1,i}}{N_{t-1}}$ denotes the share of social security benefits belonging to ability class $(t-1,i)$. If $\frac{n_{t-1,i}a_{t-1,i}}{N_{t-1}}=\psi_{t-1,i}$, the results from Section II can be generalized to an economy with a heterogeneous population. In this case, land ownership is distributed in the same way as wage income and, importantly, social security benefits.

Therefore, what is crucial in analyzing the feasibility of a Pareto-improvement is $\min_i\left(\frac{\psi_{t-1,i}}{n_{t-1,i}a_{t-1,i}/N_{t-1}}\theta\right)$, which is the ratio of the share of land owned by the members of ability class $(t-1,i)$ to its share of wage income in period $t-1$, which is again equal to its share of social security benefits in period t. Let us denote the minimum value of these ratios in period t by λ_{t-1}. If all land is owned domestically and asset ownership is perfectly correlated with wage income, then $\lambda_{t-1}=1$. If land ownership were to be distributed either less or more equally than wage income and entitlement to social security benefits, then this disparity would reduce scope for a Pareto-improvement in the absence of capital gains taxation above the normal rate of return on fixed asset holdings. For instance, starting at the parameter baseline, the critical value for λ_{t-1} becomes ≈ 0.96.[22] Disparity is not allowed to be too large in order to achieve a Pareto-improvement for each ability class. If λ_{t-1} drops below the threshold level, there is demand for capital gains taxation in order to redistribute the aggregate increments in land value (exceeding the reduction in the social security budget) intragenerationally to achieve Pareto-improvement for all households.

Appendix A2: Proof of Propositions 1 and 2

[22] Increased popularity of defined contribution plans, as documented by Poterba et al. (2001), has probably increased the value of λ among those retirees participating.

Proof of Proposition 1:

The change in benefits relative to GDP is $\frac{\Delta B_t(\cdot)}{Y_t} = -\omega\tau^s\alpha_H$ and the change in land valuation relative to GDP is

$$\frac{\Delta V_t(\cdot)}{Y_t} = (1-\alpha_H-\alpha_K)\frac{z}{1-z}\left[\left(\frac{(1-\tau^s(1-\omega)(1-z)-\tau^w)}{(1-\tau^s(1-z)-\tau^w)}\right)^{\gamma\beta\alpha_H} - 1\right]. \tag{18}$$

Differentiating with respect to ω evaluated at $\omega = 0$ gives

$$\frac{\partial}{\partial\omega}\frac{\Delta B_t(\cdot)}{Y_t}\bigg|_{\omega=0} = -\tau^s\alpha_H \tag{19}$$

and

$$\begin{aligned}\frac{\partial}{\partial\omega}\frac{\Delta V_t(\cdot)}{Y_t}\bigg|_{\omega=0} &= (1-\alpha_H-\alpha_K)\frac{z}{1-z}\gamma\beta\alpha_H \\ &\quad \left(\frac{(1-\tau^s(1-\omega)(1-z)-\tau^w)}{1-\tau^s(1-z)-\tau^w}\right)^{\gamma\beta\alpha_H-1} \\ &\quad \frac{\tau^s(1-z)}{1-\tau^s(1-z)-\tau^w} \\ &= (1-\alpha_H-\alpha_K)\frac{z}{1-z}\gamma\beta\alpha_H\frac{\tau^s(1-z)}{1-\tau^s(1-z)-\tau^w},\end{aligned} \tag{20}$$

where $z := \frac{1+q}{1+r}$. The last equality follows from $\left(\frac{(1-\tau^s(1-\omega)(1-z)-\tau^w)}{(1-\tau^s(1-z)-\tau^w)}\right)^{\gamma\beta\alpha_{H-1}} = 1$ at $\omega = 0$. Since $\frac{\Delta B_t(\cdot)}{Y_t}\Big|_{\omega=0} = \frac{\Delta V_t(\cdot)}{Y_t}\Big|_{\omega=0} = 0$, a necessary and sufficient condition for a Pareto-improvement is

$$\frac{\partial}{\partial\omega}\frac{\Delta B_t(\cdot)}{Y_t}\bigg|_{\omega=0} + \frac{\partial}{\partial\omega}\frac{\Delta V_t(\cdot)}{Y_t}\bigg|_{\omega=0} > 0. \tag{21}$$

Inserting Eqs. (19) and (20) into condition (21), using $\gamma := \frac{1}{1-\alpha_K-\beta\alpha_H}$, and factoring out $\tau^s\alpha_H$

yields

$$\tau^s \alpha_H \left[-1 + \beta \frac{1-\alpha_H-\alpha_K}{1-\alpha_K-\beta\alpha_H} \frac{z}{1-z} \frac{1-z}{1-\tau^s(1-z)-\tau^w} \right] > 0$$

$$\Leftrightarrow \beta \frac{1-\alpha_H-\alpha_K}{1-\alpha_K-\beta\alpha_H} \frac{z}{1-\tau^s(1-z)-\tau^w} > 1. \tag{22}$$

Note, the term in (22) is strictly increasing in β and z. Furthermore, observe that

$$\lim_{z\to 1} \frac{z}{1-\tau^s(1-z)-\tau^w} = \frac{1}{1-\tau^w} > 1$$

and

$$\lim_{\beta\to 1} \beta \frac{1-\alpha_H-\alpha_K}{1-\alpha_K-\beta\alpha_H} = 1.$$

Thus,

$$\lim_{z\to 1, \beta\to 1} \beta \frac{1-\alpha_H-\alpha_K}{1-\alpha_K-\beta\alpha_H} \frac{z}{1-\tau^s(1-z)-\tau^w} > 1. \tag{23}$$

Consequently, there is always a value of z and a sufficiently high value of $\beta^*(z)$ such that

$$\beta^*(z) \frac{1-\alpha_H-\alpha_K}{1-\alpha_K-\beta^*(z)\alpha_H} \frac{z}{1-\tau^s(1-z)-\tau^w} = 1. \tag{24}$$

Denote the set of z-values which induce $\beta^*(z) \in (0,1)$ by Γ which is non-empty by the limes (23). Given by Eq. (22), for all values $z \in \Gamma$ and $\beta \in (\beta^*(z), 1)$ the social security reform is Pareto-improving.

Proof of Proposition 2:

(i) Since $\beta\frac{1-\alpha_H-\alpha_K}{1-\alpha_K-\beta\alpha_H}$ is strictly increasing in β and $\frac{z}{1-\tau^s(1-z)-\tau^w}$ is strictly increasing in z, $\beta^*(z)$, defined by Eq. (24), is downward-sloping.

(ii) $\frac{z}{1-\tau^s(1-z)-\tau^w}$ is strictly increasing in z. Furthermore, $\beta\frac{1-\alpha_H-\alpha_K}{1-\alpha_K-\beta\alpha_H}$ is strictly decreasing in α_K and strictly increasing in α_H. Now, choose the infimum of Γ denoted $z_{\inf}$. Using Eq. (24), $z_{\inf}$ rises (falls) in response to a higher value of α_K (α_H). Now, keep z, $z \in \Gamma$, constant. The corresponding level of $\beta^*(z)$ is increasing in α_K and decreasing in α_H and τ^w. The latter results from the fact that $\frac{z}{1-\tau^s(1-z)-\tau^w}$ is strictly increasing in τ^w.

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