Funding Asymmetries in Electoral Competition: How important is a level playing field?

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Abstract

I investigate the idea that campaign spending limits may help to "level the playing field" in electoral competition between parties who have unequal access to campaign funds. The model assumes that the supporters of one party are on average wealthier than those who support a competing party. Contributions are used to finance advertisements that truthfully reveal information about the quality of candidates. Voters update their beliefs rationally based on information revealed during the campaign. Rational beliefs are shown to "compensate" for funding asymmetries in equilibrium. As a result, asymmetries in access to funds do not bias the electoral outcome from an ex ante perspective. A limit on campaign expenditures does not affect the relative chances of the two parties, while leading to unintended negative consequences. I conclude that the "level playing field" argument in support of expenditure limitations is inconsistent with the key assumptions of the analysis and offer some suggestions for future research.

1 Introduction

This paper investigates the policy implications of asymmetries in the access of political parties to campaign contributions. Such asymmetries might be expected to bestow an advantage on the party that has better access to funds. As a result, one might expect that electoral outcomes will be skewed in favor of that party. If this is the case, moderate voters and those supporting the financially weaker party may benefit from a policy that imposes limits on campaign contributions. Those supporting the financially stronger party would be worse off under such a policy. This intuition suggests that a policy aiming to "level the playing field" by imposing limits on contributions will redistribute welfare from a partisan minority to a majority of voters.

In the following analysis, I examine this intuition using a simple model of political competition with campaign contributions and rational voters. The analysis assumes that candidates use contributions to provide truthful information to voters, and that voters update their beliefs rationally based on that information. Contributions are assumed to come from partisan interest groups. Asymmetry in access to funds is modeled as an asymmetry in the membership size of these groups[1].

Specifically, there are two political parties that represent different ideologies. Each party puts forth a candidate who shares its ideology. While it is always possible to find such a candidate, parties may not always be able to find one who is "qualified." A qualified candidate is one who has a verifiable record of previous achievements, such as having held political office or having completed a distinguished career in the military. While voters can infer a candidate's ideological position from her party label, they do not know prior to the campaign whether or not she is qualified. Candidates can use an advertising technology to inform voters of their qualifications. Ideologically moderate swing voters who do not have a strong preference for either party may base their voting decisions on this information. This motivates partisan interest groups to contribute funds to qualified candidates, who spend them on advertisements.

The analysis in this paper shows that the intuition outlined in the opening paragraph is not consistent with the above set of assumptions. In particular, the main result of the paper is that an asymmetry in access to funds does not influence the outcome of the election from an ex ante perspective. Intuitively, the reason is as follows. If a party has good access to funds, voters come to expect that they will see advertisements informing them whenever the party has found a qualified candidate. As a consequence, voters who receive no information about the financially stronger party's candidate will conclude that she is unlikely to be qualified. In contrast, voters are willing to give the weaker party's candidate the "benefit of the doubt". This implies that access to contributions is actually a disadvantage when a party's candidate is unqualified. This disadvantage counterbalances the advantage enjoyed by the financially stronger party when their candidate is qualified. The consequence is that the ex ante probability that a party wins is actually independent of its ability to raise contributions.

A corollary to this result is that the introduction of contribution limits would actually harm moderate voters and those supporting the financially weaker party. This is because such limits will have no effect on the prior probability that either party wins, while lowering the probability that a qualified leader is elected. Surprisingly, the only group that may benefit from such a policy in this context are the members of the financially stronger interest group. The reason for this is that they save some of the money they would otherwise have spent on contributions. It turns out, in fact, that the members of the financially stronger group always prefer at least a marginal limit on contributions.

Thus, the conclusions suggested by the formal analysis of this model are different from what might have been expected intuitively. This tension between the formal results and prior intuition suggests several avenues for future research, including empirical testing and alternative theoretical formulations. This is discussed further in

[1] An alternative interpretation of the size parameter is that the members of one interest group are wealthier, so that they enjoy a lower marginal utility of wealth.

the conclusion. The rest of the paper is organized as follows. Section 2 presents a brief review of related literature. Section 3 presents the formal model and the equilibrium concept. Section 4 discusses properties of the equilibrium in the absence of contribution limits and presents the main result of the paper. Section 5 deals with the effects of a campaign finance policy that imposes limits. Section 6 concludes. Proofs are contained in the appendix.

2 Related Literature

Formal models of elections with campaign contributions can be categorized according to two aspects. The first distinction concerns assumptions about the motivation of campaign contributors. The second concerns assumptions about the effects of campaign spending on voter behavior and election outcomes. The literature reveals that conclusions about the desirability of campaign finance policies differ depending on the particular combination of assumptions employed.

"Influence" vs. "electoral" contribution motives There are essentially two reasons why an interest groups may choose to contribute money to a political candidate. First, the candidate may, in exchange for the contribution, shift her policy position, or promise to perform special favors in the event that she should win the election. This has been referred to as the "influence" or "service" motive to contribute (see Ashworth 2005, Coate 2004, Grossman and Helpman 1996, Prat 2000). Alternatively, an interest group may simply contribute to a candidate in order to improve her chances of winning the election, without expecting the candidate to shift her position or provide favors in return. This has been referred to as the "electoral" or "position induced" motive to contribute (Baron 1994, Austen-Smith 1987, Coate 2001).[2]

The main concern in the previous literature has been that both service and position induced contributions may influence the behavior of politicians. If contributions can improve a candidate's chances of winning, they may induce her to shift her policy stance, or to promise favors to special interests. Thus, both types of models provide an explanation for special-interest legislation and the divergence of policy platforms from the median voter's ideal point. In addition, they point to a potential inefficiency, and the possibility of a welfare-enhancing role for campaign finance policy.

"Impressionable" vs. "rational" voters The second aspect with respect to which existing models differ is in their assumptions about how and why campaign spending affects the outcome of an election. One group of models simply assumes

[2] There is a considerable debate as to which of these motives dominates in reality, although they are by no means mutually exclusive. In fact, most models that allow for service-induced contributions give rise to both kinds of incentives. Prat (2002b) and Grossman and Helpman (1996) argue that the electoral incentive disappears only when the number of interest groups is large, so that each interest group takes the probability with which candidates are elected as given.

that money can be used to directly influence the choices of "impressionable" voters, without explicitly modeling these choices (Grossman and Helpman 1996, Baron 1994). These models have been criticized for a lack of micro-foundation (see Prat 2000). In addition, the lack of specified individual utility functions for some citizens precludes welfare analysis. The main conclusion to be drawn from this literature is therefore positive. If candidates seek to maximize their chances of being elected, they will perceive a trade-off between catering to the median informed voter, in order to gain informed votes, and catering to special interests, in order to obtain money that can be used to "buy" uninformed votes. As a consequence, equilibrium policy positions will diverge significantly from the median informed voter's ideal point when the fraction of "impressionable" voters is large.

Another set of models treats all voters as rational agents, some or all of whom are uncertain about candidate characteristics or policy positions. Campaign spending can influence the behavior of imperfectly informed voters by reducing this uncertainty in one of two ways. One strand of literature argues that campaign spending can be indirectly informative (Prat 2000). This literature tells a classic signalling story. Although candidates cannot send credible messages to voters, they can engage in the costly activity of "burning" money to prove that informed interest groups consider them competent. Therefore voters rationally infer competence from the size of a candidate's "war chest". Another strand of the "rational voter" literature makes the somewhat simpler assumption that money can be used to send costly but directly informative messages to voters (Ashworth 2005, Coate 2001 and 2004). The idea is that campaign messages convey verifiable information, and that candidates cannot lie for reasons outside of the model.

Policy Implications The literature on informative advertising identifies both positive and negative aspects of campaign contributions. On the one hand, the need to attract contributions may induce politicians to move their policy platforms away from the median voter's ideal point, or to offer favors to special interest groups. On the other hand, contributions may help to provide information about the candidates to voters. This trade-off implies that policy recommendations are not straightforward, and it turns out that the models lead to different policy conclusions. Two types of policies are typically considered. The first is a simple contribution cap or ban, the second is a combination of spending caps and public financing through matching funds.

In signalling models, the benefit of campaign spending arises because it induces a separating equilibrium, allowing voters to distinguish between qualified and unqualified candidates. On the other hand, voters incur a cost that results from the policy distortions or "favors" necessary to illicit contributions from interest groups. The median voter will benefit from a contribution ban if she prefers a pooling equilibrium with no policy distortions to a separating equilibrium with distortions. This will be true if the policy distortions in the separating equilibrium are large enough to outweigh the benefit of distinguishing between candidate types (see Prat 2000). While a ban may be beneficial in such models, public financing cannot improve the situation because it would remove the informational value of campaign spending and induce a pooling equilibrium.

In models with directly informative advertising, the policy conclusions depend significantly on whether contributions are service or position induced. In the case of position induced contributions, Coate (2003b) argues that banning contributions may increase the likelihood that parties select ideologically extreme candidates. The argument assumes that contributions are used to inform voters of a candidate's ideological position. This gives moderate candidates an electoral advantage over extremist opponents. Parties therefore have an incentive to select moderate candidates. Put simply, "contributions help to produce more informed choices" and "the existence of contributions provides parties with an incentive to select candidates with characteristics that voters want" (ibid.). In the absence of such incentives, parties are more likely to select candidates preferred by the median party member, rather than the median voter. Therefore, contribution limits actually redistribute welfare from moderate voters to party members, whose preferences diverge from the median.

In contrast, the independent work of Coate (2004) and Ashworth (2005) provides an argument in favor of regulation, based on the assumption that contributions are service-induced. The intuition underlying the argument is as follows. Contributions are used to inform voters that a candidate is "qualified", a characteristic that all voters prefer. However, because contributions are service-induced, rational voters may not have a strong preference for a qualified candidate in equilibrium, because they are aware that such a candidate must promise favors in order to obtain the funds necessary to advertise her qualifications. In the words of Coate (2004), voters are "rationally cynical". Under appropriate conditions, this implies that advertisements are ineffective in equilibrium, so that qualified candidates have no electoral advantage. Banning contributions will therefore have no effect on the quality of the selected leader, while reducing the amount of special interest favors. This implies a Pareto improvement. Even under conditions where a ban is not Pareto improving, Coate shows that an appropriate limit combined with a publicly financed matching grant can always create a Pareto improvement. This is because public money is not tainted by promised favors, so that voters are no longer (as) "cynical", which increases the effectiveness of advertising.

Summary As this brief review shows, the previous literature has primarily been concerned with the effect that campaign contributions may have on the behavior of politicians. In particular, it has shown how politicians may shift their policy positions or offer favors to special interests in order to attract contributions. Thus, the importance of contributions in electoral competition can explain why politicians may take positions or support programs different from those preferred by the median voter. The literature on informative advertising identifies a tradeoff between this (presumably) negative aspect of contributions and their positive effects in terms of information that is made available to voters. This tradeoff makes welfare analysis complicated, and it has turned out that policy conclusions depend on assumptions made about the informational impact of contributions and the motivation of contributors. None the less, the literature has contributed to the debate about campaign finance reform by clarifying and formally elaborating different arguments, without providing a definite policy conclusion.

This paper The question I ask in this paper is slightly different from what has previously been addressed. Rather than focusing on the effect that contributions may have on the behavior of politicians, I focus on the effect that they may have on the outcome of the election. Keeping policy positions fixed, I investigate how an asymmetry in the ability of two competing parties to raise contributions may affect their relative electoral chances. Such an asymmetry may arise, for instance, because the supporters of the two parties differ in average wealth. The purpose of the analysis is to investigate formally the intuition that a contribution limit may help to "level the playing field" and therefore benefit the supporters of the financially weaker party. To my knowledge, this paper is the first to analyze this question formally. If the previous literature is any indication, the conclusions may depend on the particular set of assumptions employed. This will be discussed further in the conclusion. In terms of the taxonomy developed above, the model I analyze assumes that contributions are position-induced, voters are rational, and contributions are used to fund directly informative advertising.

3 Model

3.1 General Setup

There are three types of voters in the model - left partisans, right partisans, and swing voters. The fractions of the population belonging to each group are given by v_L, v_R, and v_S, respectively. These groups differ in their ideology, which is measured on a scale from 0 to 1. Left and right partisans have ideal points 0 and 1, respectively. Swing voters have ideologies that are uniformly distributed on the interval $[\mu - \tau, \mu + \tau]$. The ideology of the median swing voter is ex ante uncertain. Specifically, μ is the realization of a random variable uniformly distributed on $\left[\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right]$, where $\varepsilon < \frac{1}{2} - \tau$.

In addition, there are two political parties, L and R, who put forth a candidate characterized by an ideology and a level of qualification, denoted q_K for $K \in \{L, R\}$. Party L's candidate always has ideology 0, party R's candidate always has ideology 1. Party K's candidate is "qualified" (denoted $q_K = 1$) with probability σ_K, and "unqualified" (denoted $q_K = 0$) with probability $(1 - \sigma_K)$.

A citizen with ideology i enjoys a payoff from having a representative of ideology i' and qualification q that is given by $u(i, i', q) = \delta q - \beta |i - i'|$. The parameters δ and β represent the benefit of having a qualified representative, and the cost of having a representative with a different ideology, respectively. I assume that partisans always vote for their own party's candidate, which implies that $\delta < \beta$. I also assume that $v_L < \frac{1}{2}$ and $v_R < \frac{1}{2}$, so that swing voters are decisive.

Finally, each candidate is supported by one of two interest groups, composed of a subset of the partisan voters. The size (mass) of the interest group K's membership is given by $\gamma_K \leq v_K$. These groups collect contributions from their members to pay for campaign "advertisements"[3]. As noted above, an important assumption in the model is that advertising is truthful, in the sense that it reliably reveals the true qualification of the advertised candidate[4]. The advertising technology is defined in terms of a function $\lambda(C)$ that gives the fraction of swing voters that learn the true qualification of a candidate as a function of campaign expenditures. That is, when interest group K spends C_K on the campaign, a fraction $\lambda(C_K)$ of swing voters will come to know q_K[5]. $\lambda(C_K)$ is a strictly concave function. That is, each additional dollar spent on advertising will cause a smaller and smaller number of additional voters to become informed.

When making their spending decision, interest groups are assumed to know only the type of their own, and not that of the opposing party's candidate. They choose contribution levels to maximize the utility of their members, whose preferences are homogeneous. Swing voters are assumed to vote sincerely based on their ideological preference and rational beliefs about candidate qualifications, given any advertisements they may have seen[6].

Some technical points I define $\varkappa = \frac{v_R - v_L}{v_S}$ as a measure of the ex ante "right-bias" of the population. This parameter is allowed to be nonzero in order to show that the main results of the analysis are unaffected by such an asymmetry. The reader may prefer to assume that $v_R = v_L$, so that $\varkappa = 0$. This simplifies many of the equations without taking away any insight.

In addition, I will make the following assumptions, which make the model more tractable.

Assumption (1): $\tau > \frac{\delta}{2\beta} + \varepsilon$; $\varepsilon > \frac{\delta}{2\beta} + \tau \cdot |\varkappa|$

Assumption (2): $|\lambda''(C)| \geq \frac{\lambda'(C)^2}{1-\lambda(C)}$ for all $C > 0$.

Intuitively, the first assumption says that swing voters are sufficiently heterogenous, and that the preference of the median voter is sufficiently uncertain, so that it is impossible to forecast any election with certainty. It is introduced in order to

[3] More generally, "advertisements" in this model can be interpreted to representent a broader set of activities aimed at increasing the public exposure of a candidate. In reality, such activities include, but are not limited to, television advertisements. The only thing that matters for the present analysis is that such activities provide information about the candidate to voters, and that parties require money to undertake them.

[4] Note that this assumption is not as naive as it may first appear. Intuitively, it says that advertisements cannot be used to "fool" voters. If voters are rational, they will be able to distinguish between credible signals and mere "cheap talk". The assumption says that qualified candidates are able to send credible signals of their qualification.

[5] Technical Note: The voters reached are assumed to be uniformally distributed accross all locations in the swing interval. In addition, there is no correlation between the probabilities of seeing both parties' ads.

[6] That is, voters are not permitted to rationally abstain or vote for a less preferred candidate in this model (see Feddersen and Pesendorfer 1996).

ensure that election probabilities are continuously differentiable, and does not effect the substantive conclusions of the paper. The second assumption puts a lower bound on the concavity of the advertising technology. It allows me to restrict attention to the case where the equilibrium of the model is unique[7]. Both assumptions will be referred to when they become important for the analysis.

3.2 Equilibrium

An equilibrium is a set of contribution strategies for the interest groups, a voting strategy for swing voters, and a set of beliefs for voters, such that (i) interest group strategies are optimal given voter beliefs and given knowledge of their own candidate's type, (ii) voters vote for their most preferred candidate, given beliefs about the probabilities that they are qualified, and (iii) voter beliefs about candidate qualification are rational (i.e. derived from Bayes rule) given the strategies employed by the interest groups.

The model is solved by backward induction, beginning with the behavior of swing voters in the election stage. Analysis of the swing voter's problem will allow me to derive a probability of election function that describes the relationship between contribution levels and election probabilities, given candidate types and voter beliefs. This is then used to derive the interest group's optimal contribution strategies given candidate types. Finally, contribution strategies are used to tie down equilibrium beliefs using Bayes rule.

Behavior of swing voters Suppose that realized candidate types are given by q_L and q_R, and that the interest groups spend C_L and C_R on their campaigns. A swing voter can be in one of four information states, depending on what, if any, advertisements she may have seen. I denote an information state by a pair $s = (I_L, I_R)$, where $I_K = 1$ if the voter has seen an ad for candidate K, and $I_K = 0$ otherwise. The set of information states is $S = \{(1,1),(1,0),(0,1),(0,0)\}$. The spending levels C_L and C_R determine how many swing voters are in each information state. Specifically, a fraction $\lambda(C_L) \cdot \lambda(C_R)$ is in information state $(1,1)$, a fraction $\lambda(C_L) \cdot (1 - \lambda(C_R))$ is in information state $(1,0)$, etc.

Voters in different information states have different beliefs concerning the likelihood that the candidates are qualified. I denote these subjective probabilities by $\rho_L(s)$ and $\rho_R(s)$. Since advertising is truthful, $\rho_K(s) = q_K$ whenever $I_K = 1$. That is, if a voter has seen an advertisement for candidate K, she knows with certainty whether K is qualified. Further, since candidate types are drawn independently and advertising decisions are independent of the opposing candidate's type, $\rho_L(0,0) = \rho_L(0,1)$ and $\rho_R(0,0) = \rho_R(1,0)$. That is, a voter's belief about candidate L must be independent of whether or not she has seen an advertisement for candidate R, and vice versa.

[7] Note that this assumption is satisfied by the functional form assumed in Coate (2003), $\lambda(C) = \frac{C}{C+\alpha}$, for any $\alpha > 0$. See the appendix for a derivation of this condition as well as additional examples.

It follows that equilibrium beliefs can be fully summarized by a pair $(\widehat{q}_L, \widehat{q}_R)$, where $\widehat{q}_K$ is a swing voter's belief about the probability that candidate K is qualified, given that he has not seen her ad. ($\widehat{q}_K$ will be determined by Bayes rule in equilibrium.)

Given the information they have, swing voters are assumed to vote sincerely for the candidate that offers them the highest expected utility if elected. The expected utility of a voter with ideology i in information state s if party K wins the election is $E\left[u(i, i_K, q_K)|s\right] = \delta \cdot \rho_L(s) - \beta \cdot |i - i_K|$. A voter in information state s with ideology i will vote for party L if $E\left[u(i, 0, q_L)|s\right] > E\left[u(i, 1, q_R)|s\right]$, that is if $i < i^*(s)$, where

$$i^*(s) = \frac{1}{2} + \frac{\delta}{2\beta} \cdot \left[\rho_L(s) - \rho_R(s)\right].$$

The probability of election function The function $i^*(s)$ identifies the voter who is just indifferent between the two candidates, given beliefs about the probabilities that they are qualified. This marginal voter is of great importance, as she is in all models of voting. However, it is important to keep in mind that there are several such cut-points in the model discussed here. Specifically, the cut-point depends on the information state. Therefore, the fraction of voters that votes for party L will differ across information states. It follows from the uniform distribution of swing voters that the fraction of those in information state s that votes for L is given by $\alpha(s) = \frac{1}{2} + \frac{i^*(s) - \mu}{2\tau}$[8].

The fraction of all swing voters that votes for party L is then given by the weighted sum of these fractions across information states, $\overline{\alpha}(C_L, C_R) = \lambda\left(C_L\right) \cdot \lambda\left(C_R\right) \cdot \alpha(1,1) + \lambda\left(C_L\right) \cdot (1 - \lambda\left(C_R\right)) \cdot \alpha(1,0) + (1 - \lambda\left(C_L\right)) \cdot \lambda\left(C_R\right) \cdot \alpha(0,1) + (1 - \lambda\left(C_L\right)) \cdot (1 - \lambda\left(C_R\right)) \cdot \alpha(0,0)$. Candidate L will win the election if more than half of all voters vote for L, that is if $v_L + v_S \cdot \overline{\alpha}(C_L, C_R) > \frac{1}{2}$, i.e. if $\overline{\alpha}(C_L, C_R) > \frac{1}{v_S}\left[\frac{1}{2} - v_L\right]$. This will occur if $\mu < \mu^* - \tau \cdot \varkappa$, where $\mu^* = \lambda\left(C_L\right) \cdot \lambda\left(C_R\right) \cdot i^*(1,1) + \lambda\left(C_L\right) \cdot (1 - \lambda\left(C_R\right)) \cdot i^*(1,0) + (1 - \lambda\left(C_L\right)) \cdot \lambda\left(C_R\right) \cdot i^*(0,1) + (1 - \lambda\left(C_L\right)) \cdot (1 - \lambda\left(C_R\right)) \cdot i^*(0,0)$ is the weighted average of the different cutoff points, and $\varkappa = \frac{v_R - v_L}{v_S}$ measures how right-biased the distribution of partisan voters is. It then follows from the distribution of μ that the probability that L will win the election, given candidate types and spending levels, is given by

$$\pi_L(C_L, C_R, q_L, q_R) = \frac{1}{2} - \frac{\tau \cdot \varkappa}{2\varepsilon} + \eta \cdot \left[\overline{\rho}(C_L, q_L, \widehat{q}_L) - \overline{\rho}(C_R, q_R, , \widehat{q}_R)\right], \tag{1}$$

where $\eta = \frac{\delta}{4\varepsilon\beta}$, and

$$\overline{\rho}(C_K, q_K, \widehat{q}_K) = \lambda\left(C_K\right) \cdot q_K + (1 - \lambda\left(C_K\right)) \cdot \widehat{q}_K.$$

$\overline{\rho}(C_K, q_K, \widehat{q}_K)$ can be interpreted as candidate K's average "reputation", given her true type and advertising expenditures[9]. Thus, a candidate's probability of winning

[8] Assumption (1) guarentees that $\alpha(s) \in (0,1)$ for all $s \in S$. This assumption is introduced only to make sure that the probability of election function has a tractable, continuously differentiable form. It does not affect the substantive conclusions of the paper.

[9] Assumption (1) guarantees that L's probability of winning lies strictly between zero and one. Again, it is introduced only to ensure a tractable, continuously differentiable form.

depends on the difference between the two candidates' "reputations" among swing voters.

Interest group contributions A strategy for interest group K is a contribution schedule $C_K(q_K)$. For $q_K \in \{0,1\}$, $C_K^*(q_K)$ is the contribution level that maximizes the expected utility of its representative member. This choice is conditioned only on the type of the interest group's own candidate, as the opposing candidate's type is assumed to be uncertain[10]. Without loss of generality, consider the problem of the left interest group. Assuming that contribution costs are shared equally among the interest group's members, $C_L^*(q_L)$ solves

$$\max_{C_L} E\left[\pi_L(C_L, C_R^*, q_L, q_R) \cdot u(0,0,q_L) + (1-\pi_L(C_L, C_R^*, q_L, q_R)) \cdot u(0,1,q_R) - \frac{C_L}{\gamma_L}\right]$$

The optimal strategy $C_L^*(q_L)$ satisfies, for $q_L \in \{0,1\}$,

$$E\left[\frac{\partial \pi_L(C_L^*(q_L), C_R^*(q_R), q_L, q_R)}{\partial C_L} \cdot (u(0,0,q_L) - u(0,1,q_R))\right] \leq \frac{1}{\gamma_L}(= if C_L^* > 0)$$

Note that, according to equation (1), $\frac{\partial \pi_K(C_L, C_R, q_L, q_R)}{\partial C_K} = \eta \cdot \frac{\partial \overline{p}(C_K, q_K)}{\partial C_K}$. That is, spending affects L's probability of winning by changing her "reputation". Also note that its impact on her reputation is independent of q_R. It is given by $\frac{\partial \overline{p}(C_L, q_L)}{\partial C_L} = \lambda'(C_L) \cdot [q_L - \widehat{q}_L]$. When candidate L is unqualified, so that $q_L = 0$, $\frac{\partial \overline{p}(C_L, q_L)}{\partial C_L} < 0$. This reflects the simple fact that truthful advertising can only harm candidate L's reputation and reduce her chance of being elected when she is unqualified. When $q_L = 1$, $\frac{\partial \overline{p}(C_L, q_L)}{\partial C_L} > 0$, reflecting that a qualified candidate will improve her chance of being elected through advertising.

It follows that only qualified candidates receive contributions and advertise in equilibrium. Interest group K's strategy can therefore be summarized by a single number, C_K^*, which specifies the contribution made when $q_K = 1$. This, in turn, implies that equilibrium beliefs about unadvertised candidates are given by

$$\widehat{q}_K = \frac{\sigma_K \cdot (1 - \lambda(C_K^*))}{1 - \sigma_K \cdot \lambda(C_K^*)}$$

What remains to be determined is C_L^*, the contribution given to a qualified candidate. Given rational beliefs, the equilibrium effect of campaign spending on L's probability of being elected when she is qualified is given by

$$\frac{\partial \pi_L(C_L, C_R^*(q_R), 1, q_R)}{\partial C_L} = \eta \cdot \lambda'(C_L) \cdot (1 - \widehat{q}_L)\,.$$

[10] This assumption makes the analysis significantly less cumbersome and easier to understand. If interest groups can see the other candidate's type, their incentive to advertise will depend on that type's realization. Therefore advertising would implicitly reveal information about both candidates' types. While this may be more "realistic", it would significantly complicate the analysis, and seems unlikely to add insight.

Before proceeding, it is helpful to look at this equation carefully in order to understand the effect of spending in this model. When interest group L spends an additional dollar on advertising, an additional $\lambda'(C_L)$ voters will become informed of L's qualification. These voters will change their beliefs about candidate L's probability of qualification from $\widehat{q}_L$ to 1. Therefore candidate L's "reputation" goes up by $\lambda'(C_L) \cdot (1 - \widehat{q}_L)$. The parameter η translates this increment in reputation into an increment in L's probability of winning[11]. When $q_L = 1$, the group's expected benefit from electing the left candidate is $\beta + (1 - \sigma_R) \cdot \delta$. Thus, the equilibrium contribution level C_L^* satisfies

$$\eta \cdot \lambda'(C_L^*) \cdot (1 - \widehat{q}_L) \cdot (\beta + (1 - \sigma_R) \cdot \delta) \leq \frac{1}{\gamma_L} \quad (= \text{ if } C_L^* > 0).$$

3.3 Existence and uniqueness of the equilibrium

The analysis above has shown that an equilibrium can be summarized by four numbers, $(C_L^*, C_R^*, \widehat{q}_L, \widehat{q}_R)$. C_K^* specifies the contribution made to candidate K when she is qualified, and $\widehat{q}_K$ is the equilibrium belief of an uninformed voter concerning the probability that candidate K is qualified. I have shown that $(C_L^*, C_R^*, \widehat{q}_L, \widehat{q}_R)$ constitute an equilibrium if and only if, for $K = L, R$,

$$\widehat{q}_K = \frac{\sigma_K \cdot (1 - \lambda(C_K^*))}{1 - \sigma_K \cdot \lambda(C_K^*)} \tag{2}$$

and

$$\eta \cdot \lambda'(C_K^*) \cdot (1 - \widehat{q}_K) \cdot (\beta + (1 - \sigma_{-K}) \cdot \delta) \leq \frac{1}{\gamma_K} \quad (= \text{ if } C_K^* > 0). \tag{3}$$

The separability of the equilibrium conditions for the two parties significantly simplifies the analysis of equilibrium. In particular, it allows me to begin by analyzing the problem of one interest groups in isolation. I therefore drop the subscript K when this does not create confusion. A rigorous proof of the following proposition is relegated to the appendix.

Proposition 1 *There exists a unique equilibrium* $(C_L^*, C_R^*, \widehat{q}_L, \widehat{q}_R)$.

Proof. *See appendix.* ■

The following section provides an intuitive graphical representation of the equilibrium[12].

[11] Technical note: $\xi_L^* \equiv \eta \cdot (1 - \widehat{q}_L) = \eta \cdot \frac{1-\sigma_L}{1-\sigma_L \cdot \lambda(C_L^*)}$ can be interpreted as the "effectiveness of advertising" consistent with rational beliefs on the part of voters. This parameter is used to simplify some parts of the formal analysis, but will not be used in the text. See Coate (2003), in which it plays a more central role. In that model, the effectiveness of advertising is a function of the level of "favors" that must be offered in order to finance it. Here, it depends only on the reputation of unadvertised candidates.

[12] For the remainder of the analysis, I will assume that $C_K^* > 0$ for $K = L, R$. That is, I am assuming that interest groups are large enough, so that both parties do, in fact, have some access to campaign funds. In this case, the second equilibrium condition will hold with equality for both parties.

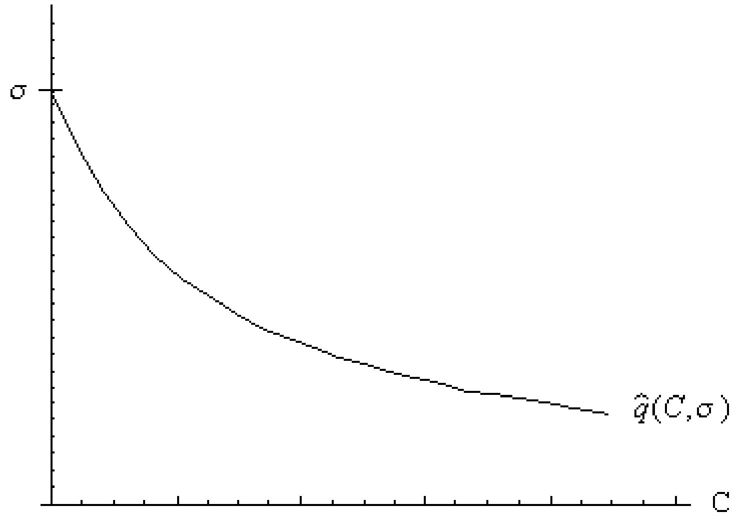

Figure 1: Equilibrium beliefs about unadvertised candidates as a function of contributions

Beliefs as a function of spending Equation (1) implicitly defines a function, $\widehat{q}\,(C,\sigma) = \frac{\sigma\cdot(1-\lambda(C))}{1-\sigma\cdot\lambda(C)}$, which describes how a voter's belief about an unadvertised candidate depends on the level of contributions her party provides to qualified candidates in equilibrium. Rational beliefs imply that $\widehat{q}\,(C,\sigma)$ is decreasing in the equilibrium level of spending (see figure 1). The reason for this is as follows. The more a party spends on qualified candidates in equilibrium, the more likely it is that a voter will see their candidate's ad when she is qualified. Consequently, the more a party spends on campaigns in equilibrium, the less likely it is that their candidate is qualified, given that a voter has not seen her ad.

Spending as a function of beliefs Equation (2) implicitly defines a function $C(\widehat{q},\gamma)$ that describes the contribution that the interest group will make to a qualified candidate, given its size and given beliefs on the part of voters concerning the probability that an unadvertised candidate is qualified. This function is decreasing in $\widehat{q}$. The reason for this is clear. If voters believe that an unadvertised candidate is very unlikely to be qualified, a qualified candidate will significantly improve her reputation if she advertises. In contrast, if voters think very highly even of unadvertised candidates, there is little reason to advertise (see figure 2).

Equilibrium beliefs and spending A convenient way to visualize an equilibrium of the model is by plotting the two equations in $C - \widehat{q}$ space (see figure 3). The intersection of the two curves represents the point at which (1) beliefs are consistent with the contributions made to qualified candidates, and (2) the interest group is reacting optimally to beliefs. Proposition 1 says that the intersection of the two

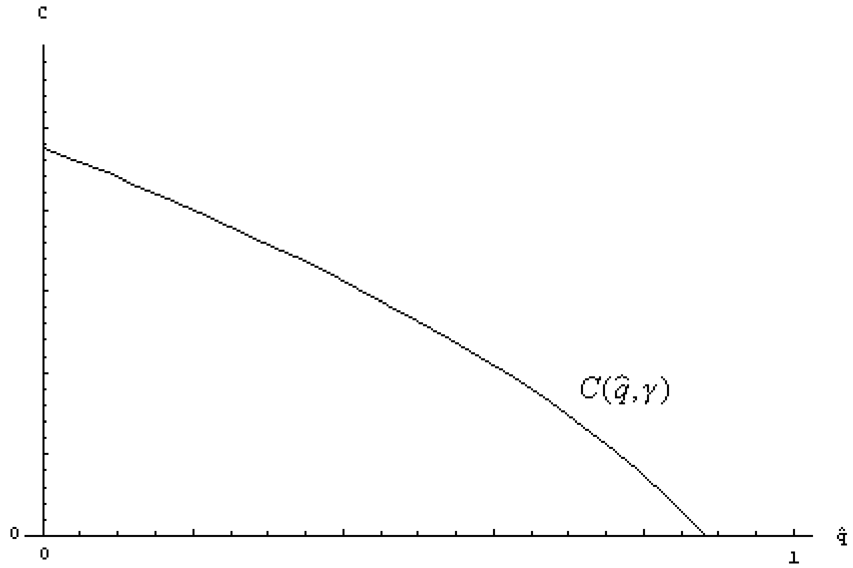

Figure 2: The equilibrium contribution schedule as a function of beliefs about unadvertised candidates.

curves exists and is unique.

3.4 Analysis of Equilibrium

In this section, I want to investigate how access to funds (formally, the parameters γ_L and γ_R) affects three key properties of the equilibrium. These properties are important in evaluating the effect of a campaign finance policy in this model. First, what is the probability that a particular party will win the election? Second, how responsive is the election to the preference of the median swing voter? Third, what is the probability that a qualified candidate will be elected?

The partisan effects of funding asymmetries The result discussed in this section shows that greater access to funds has both positive and negative effects in this model. In order to isolate these effects, the following proposition assumes that both parties have an equal probability of obtaining a qualified candidate, and that the distribution of voter ideologies is symmetric. Further, it assumes that party R has greater access to funds.

Proposition 2 *Suppose that $\sigma_L = \sigma_R = \sigma$, $\varkappa = 0$, and $\gamma_R > \gamma_L$. Then*

1. *Candidate R receives more contributions than L when she is qualified, and*

2. *The conditional probability that candidate R wins when she is qualified is larger than the conditional probability that candidate L wins when she is qualified. However,*

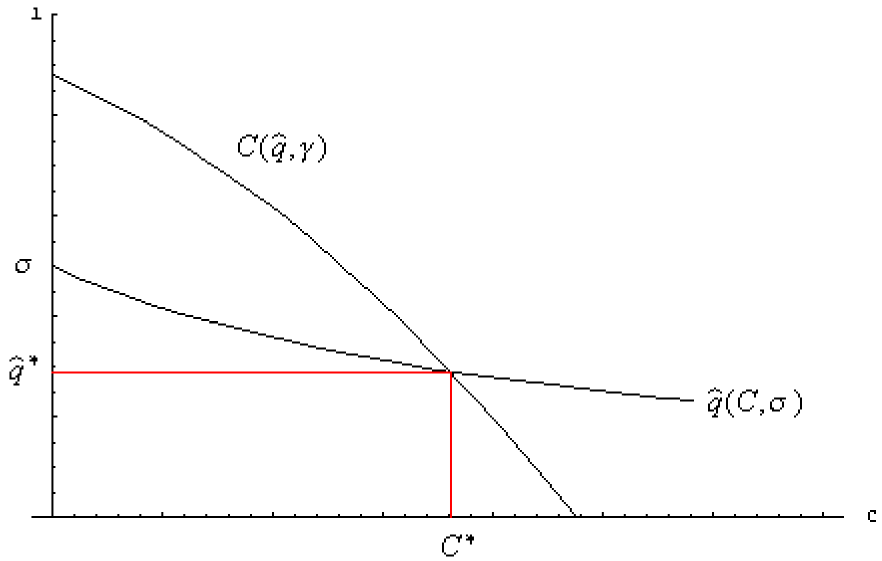

Figure 3: Equilibrium of beliefs and spending

3. *Conditional on not being informed of a candidate's qualifications, voters believe that candidate R is less likely to be qualified than candidate L, and*

4. *The conditional probability that candidate R wins when she is unqualified is smaller than the conditional probability that candidate L wins when she is unqualified.*

Proof. *See appendix.* ■

Parts (1) and (2) of this proposition establish that the party that has better access to contributions (party R in this case) enjoys an electoral advantage whenever its candidate is qualified. Roughly speaking, this is because candidate R can spend more money on advertisements. Therefore, a greater number of undecided voters will learn that she is qualified and switch their votes to her. The consequence of this is that a qualified candidate of party R is more likely to win an election than is a qualified candidate of party L.

In contrast, parts (3) and (4) of the proposition establish that party R actually has a disadvantage when its candidate is unqualified. The reason for this is that voters will receive no information about unqualified candidates. In that instance, they are less likely to give candidate R the "benefit of the doubt". That is, rational beliefs imply that voters factor in the candidates' different signalling abilities and impose a kind of "handicap" on the stronger party's candidate.

To understand the mechanism that underlies this result intuitively, imagine the situation of a voter who has received no information about either candidate. Such an individual will reason something like this: "Candidate R has many wealthy supporters. If she was qualified, she would have been able to advertise aggressively, so that I would probably have found out about it. Candidate L has fewer wealthy supporters. Even if she was qualified, she may not have been able to get the word out, so that it is pretty likely that I would never have found out about it. Therefore, L is more likely to be qualified." The consequence of this is that an unqualified candidate R is less likely to win an election than is an unqualified candidate L.

Proposition (2) isolates the different effects of a funding asymmetry conditional on candidate qualifications. The main result of the paper summarizes the total impact of such asymmetries from an ex ante perspective. It can be stated as follows.

Proposition 3 *The ex ante probability that party L's candidate will win the election is independent of interest group sizes. Specifically, it is given by*

$$\pi_L = \left(\frac{1}{2} - \frac{\tau}{2\varepsilon} \cdot \varkappa\right) + \eta \cdot (\sigma_L - \sigma_R)$$

Proof. *See appendix.* ■

Contrary to the initial intuition that motivated the analysis, proposition (3) establishes that the positive and negative effects identified in proposition (2) cancel out, so that funding asymmetries will not skew electoral outcomes from an ex ante perspective. In the absence of other asymmetries, both parties have an equal chance of winning the election, irrespective of any differences in their access to funds. Any ex ante advantage for either party that does exist (because either $\varkappa \neq 0$ or $\sigma_L \neq \sigma_R$) would be unaffected by a change in funding levels.

Access to contributions and the ideological responsiveness of the election Recall that the ideology of the median swing voter, μ, is ex ante uncertain. From the point of view of swing voters, one of the main functions of the election is to select a leader who is ideologically preferred by the median swing voter. From an ex ante perspective, a useful measure of the "ideological responsiveness" of the election is the expected distance between μ and the ideology of the winner.

Proposition 4 *Increased access to campaign funds (formally an increase in γ_K) for either party implies an increase in the expected ideological distance between the median voter and the winner of the election.*

Proof. *See appendix.* ■

Thus, the more access the parties have to campaign funds, the less responsive the election is to the median swing voter's ideology. At first glance, this result seems to resemble the policy divergence that occurs in some of the models discussed in section 2. However, the logic here is different. Recall that the parties' platforms are exogenously fixed, and assumed to be distinct from the median ideology. Thus, campaign contributions are not causing platforms to move in this context. Rather, they are influencing voters choices, making it less likely that they will choose the candidate who is ideologically closer to them.

The reason for this is as follows. Voters evaluate candidates on two dimensions, ideology and qualification. Better access to funds induces parties to provide information about candidate qualifications to more voters. (In addition, it also increases the amount of implicit information revealed by the absence of such information.) The more information a voter has about the candidates' qualifications (either explicit or implicit), the more likely it is that he will switch his vote from a candidate whom he prefers based on ideology alone to one that he prefers based on the likelihood that she is qualified. Therefore, the more information is made available, the more likely it is that the ideologically less preferred candidate will be elected.

Access to contributions and the probability that the winner of the election is qualified The other major function of the election in this model is to select a qualified leader. Therefore the final question of interest is, with what probability is the winner of the election qualified, and how is this probability related to the parties' access to funds? The corresponding result is no surprise.

Proposition 5 *Increased access to campaign funds (formally an increase in γ_K) for either party implies an increase in the probability that the winner of the election is qualified.*

Proof. *See appendix.* ■

The reasoning behind this result is straightforward. The only point to make is that there are two reinforcing mechanisms at work here. First, the more information voters have about qualification, the more likely it is that they will correctly identify a qualified candidate. Second, the more information voters have about qualification, the more likely it is that they will decide based on this information rather than the candidate's ideology.

Summary To summarize, the results presented in this section suggest that restricting access to funds by limiting contributions will (a) not effect the ex ante probability that either party wins, (b) increase the "ideological responsiveness" of the election to the preference of the median swing voter, and (c) decrease the probability that the winner of the election will be qualified. These effects are investigated in more detail in the next section.

3.5 Contribution Limits

This section discusses the effects of a campaign finance policy that imposes limits on contributions. The analysis has already shown that such a policy cannot effect the ex ante probability that either party is elected. It follows that the welfare effects of such a policy depend on the combination of its impacts on what I have called the ideological responsiveness of the election and the probability that the winner is qualified.

Equilibrium under a limit Suppose a policy is introduced that sets a limit $\overline{C}$ on spending. The effect of such a limit on the equilibrium is straightforward. Formally, contribution levels and beliefs $\left(\widetilde{C}_L, \widetilde{C}_R, \widetilde{q}_L, \widetilde{q}_R\right)$ constitute an equilibrium under a limit $\overline{C}$ if and only if for $K = L, R$,

$$0 \leq \widetilde{C}_K \leq \overline{C}$$

$$\widetilde{q}_K = \frac{\sigma_K \cdot (1 - \lambda(\widetilde{C}_K))}{1 - \sigma_K \cdot \lambda(\widetilde{C}_K)}$$

and

$$\eta \cdot \lambda'(\widetilde{C}_K) \cdot (1 - \widetilde{q}_K) \cdot (\beta + (1 - \sigma_{-K}) \cdot \delta) \begin{cases} \leq \frac{1}{\gamma_K} & if \quad \widetilde{C}_K = 0 \\ = \frac{1}{\gamma_K} & if \quad \widetilde{C}_K \in \left(0, \overline{C}\right) \\ \geq \frac{1}{\gamma_K} & if \quad \widetilde{C}_K = \overline{C} \end{cases}$$

We can define the equilibrium contribution function under a limit $\overline{C}$ using our previously derived function $C^*\left(\gamma_K, \sigma_K, \sigma_{-K}\right)$, as follows.

$$\widetilde{C}\left(\gamma_K, \sigma_K, \sigma_{-K}, \overline{C}\right) = \begin{cases} C^*\left(\gamma_K, \sigma_K, \sigma_{-K}\right) & if \qquad C^*\left(\gamma_K, \sigma_K, \sigma_{-K}\right) \leq \overline{C} \\ \overline{C} & otherwise \end{cases}$$

That is, the new contribution function is simply truncated at $\overline{C}$.

Evaluating the effect of a limit Suppose that the laissez-faire equilibrium involves contribution levels (C_L^*, C_R^*). Suppose also that party R receives more contributions when its candidate is qualified, i.e. that $C_L^* < C_R^*$. In this section, I ask how the welfare of the different agents in the model is effected by the introduction of a contribution limit. In order to understand the effects, it is best to focus on the case where the limit is binding for interest group R, but not for interest group L, i.e. $C_L^* < \overline{C} < C_R^*$. The intuition that motivated this analysis suggested that such a policy might have redistributive effects, benefitting left partisans (and possibly swing voters) at the expense of right partisans. As the following proposition shows, this intuition is incorrect within the model employed here.

Proposition 6 *Suppose that $C_R^* > C_L^*$ in the laissez-faire equilibrium. Then (1) the imposition of a limit $C_L^* < \overline{C} < C_R^*$ will reduce the expected utility of all citizens who are not members of interest group R. Further, (2) there exists a limit $\overline{C}^* < C_R^*$ such that the expected utility of the right interest group's members is higher under that limit than in the laissez-faire equilibrium.*

Proof. *See appendix.* ■

To understand the result intuitively, note first that the limit affects equilibrium spending and electoral outcomes in essentially the same way as a reduction in the size of the right interest group would. It then follows from Proposition 2 that it will have no effect on the ex ante probability that either party will win the election. According to the other propositions, it will (a) reduce the expected ideological distance between the winner of the election and the median swing voter, and (b) reduce the probability that the winner of the election will be qualified. In addition to these effects, the policy will (c) reduce the per person contribution made by the right interest group members from $\frac{C_R^*}{\gamma_R}$ to $\frac{\overline{C}}{\gamma_R}$.

Consider first the welfare of agents who are not members of interest group R. Since partisan voters who are not members of interest group R don't care about (a) or (c), it follows immediately from (b) that they are worse off under the limit than in the laissez-faire equilibrium. Although effect (a) is actually a good thing from the point of view of swing voters, it is not surprising that effect (b) dominates, so that they are also worse off under the limit. Intuitively, the expected ideological distance from the median is lower under the limit only because more voting decisions are based only on the ideological positions of the candidates, with no knowledge of whether or not they are qualified. This changes the outcome of the election only because some uninformed voters are voting for the candidate closer to them in ideology even though they would have preferred the other candidate, had they known that she was qualified. It follows that these voters are making the "wrong" choice, and must be worse off under the limit.

Finally, consider the members of the right interest group. These agents are effected in two ways. First, they suffer a loss due to the reduced probability that the winner will be qualified. Second, they gain because they spend less on contributions. To understand why the second effect can dominate the first, note that from an ex ante perspective, the information revealed by the group's advertising activities is like a public good. It affects only the probability that a qualified leader is elected, not the probability that the group's preferred candidate will win. However, the group will decide how much information to provide after they have learned that their candidate is qualified. At that point in time, the advertising activity has an additional private component. Conditional on being qualified, the contributions will raise the probability that their candidate will win. That is, equilibrium contributions are motivated by ideological (private) as well as qualification (private and public) considerations, and the sum contributed will reflect both of these. Before the realization of candidate types, the private component is absent, so that the members of the interest group will feel that they are contributing "too much" from an ex ante perspective. Therefore, they would prefer to commit to some lower level of contributions. A contribution limit would provide them with a device to do this. From an ex ante perspective, these agents will benefit if the contribution limit is set at (or close to) the level which they would like to commit to from an ex ante perspective. I show formally in the appendix that this level is strictly lower than the equilibrium contribution.

3.6 Extension: Endogenous probability of qualification

An important theoretical objection to the model presented here is that the ability of parties to obtain qualified candidates is treated as an exogenous parameter. It can be argued that qualified candidates may be more likely to emerge if they are more likely to win the election. Since the advantages of funding are enjoyed by qualified candidates in the model analyzed here, this implies that the financially stronger party may be more likely to obtain a qualified candidate in equilibrium, giving it an electoral advantage from an ex ante perspective. As a consequence, electoral outcomes can be said to be biased in favor of that party. In this case, an argument can be made that public financing can help to eliminate this bias by increasing the incentives for the weaker partie's qualified candidates to run. Analyzing the welfare effects of such a policy presents additional challenges that go beyond the scope of the present analysis. Here, I merely want to establish the positive result that an advantage in access to funds will imply that a party is more likely to obtain a qualified candidate.

The simplest way to endogenize the probability that a party finds a qualified candidate is to assume that each party has one qualified and one unqualified "potential candidate". While the unqualified candidate is always willing to run, the qualified candidate faces a random opportunity cost of running and will do so only if he benefit exceeds this cost. For simplicity, assume that candidates are purely office motivated, so that they receive an ego rent equal to $r > 0$ if and only if they win the election. Also assume that the candidate's opportunity cost is distributed uniformly on $[0, \bar{c}]$ where $\bar{c} \geq r$. Then, given a probability of winning π_Q^K, candidate K will run if and only if $c \leq \pi_Q^K \cdot r$. This occurs with probability $\frac{r}{\bar{c}} \cdot \pi_{QK}$. For simplicity, assume $\frac{r}{\bar{c}} = 1$. Then, letting $\pi_Q^K(\sigma_K, \sigma_{-K}, \gamma_K)$ denote the equilibrium probability that party K's qualified candidate wins when $(\sigma_K, \sigma_{-K}, \gamma_K)$ are exogenously given, an equilibrium with endogenous candidate qualities is given by (σ_L^*, σ_R^*) such that

$$\begin{aligned} \sigma_L^* &= s(\sigma_R^*, \gamma_L) \\ \sigma_R^* &= s(\sigma_L^*, \gamma_L) \end{aligned}$$

where $s(\sigma_{-K}, \gamma_K)$ is implicitly defined by

$$s(\sigma_{-K}, \gamma_K) = \pi_Q^K(s(\sigma_{-K}, \gamma_K), \sigma_{-K}, \gamma_K)$$

Then, the following result is proved in the appendix.

Proposition 7 *Suppose that $\varkappa = 0$, and $\gamma_R > \gamma_L$. Then, $\sigma_R^* > \sigma_L^*$ and the ex ante probability that party R's candidate wins is larger than the ex ante probability that party L's candidate wins.*

Proof. See appendix ■

As indicated above, this result raises new questions concerning policy. Specifically, it appears possible that voters may benefit from a combination of contribution limits and public financing in order to increase the incentives for qualified candidates for party L to run. This would reduce the partisan advantage that candidate R enjoys in the laissez-faire equilibrium. At this point, the welfare analysis of such a policy presents a number of technical challenges and is therefore left for future research.

4 Conclusion

There are two arguments commonly raised in support of campaign finance regulation. One is that such policies may reduce incentives for politicians to serve special interests through favors or shifts in their policy positions. Another is that regulation can help to "level the playing field" in the competition between parties and candidates representing the interests of citizens who differ in their ability to provide financial support. As outlined in section 2, the formal literature on campaign finance policy has primarily examined the first argument. One of the central lessons that has emerged from this variable literature has been that both positive and normative conclusions depend strongly on assumptions made about the role of campaign advertisements, the rationality of voters, and the motivation of contributors.

To my knowledge, this paper is the first to investigate the second argument in a formal context. The main conclusions of the analysis are theoretically simple, yet somewhat surprising intuitively. They suggest that, if advertising is informative, and if voters rationally update their beliefs taking into account the ability of candidates to advertise, an advantage in access to funds has both positive and negative effects. In the model presented here, these effects precisely cancel out, so that electoral outcomes are not systematically skewed in favor of the financially stronger party.

The analysis lends some support to Justice Antonin Scalia's dissent to the Supreme Court's decision on Dec. 10th 2003, upholding the Bipartisan Campaign Reform Act. Justice Scalia argues that "the premise of the First Amendment is that the American people are neither sheep nor fools, and hence fully capable of considering both the substance of the speech presented to them and its proximate and ultimate source. (...) Given the premises of democracy, there is no such thing as too much speech."

While the existence of countervailing advantages and disadvantages of financial access is intuitively appealing, the conclusion that the stronger party has no ex ante advantage stands in contrast to casual empiricism, and may be quite sensitive to the particular theoretical specification. This suggests a need for empirical testing and experimentation with alternative theoretical formulations.

Even if voters are "neither sheep nor fools", it may appear questionable whether they can and do engage in the fairly sophisticated Bayesian reasoning that underlies the main results of the paper. It would be interesting to test empirically whether voters take a party's ability to raise campaign funds into account when they evaluate candidates about whom they have obtained little information. Another way to test the model would be to collect evidence about the relative chances of "qualified" and "unqualified" candidates belonging to different parties. Of course, this would require a precise definition of qualification. According to the theory presented here, such an investigation should show that a party's access to funds has positive effects for qualified, and negative effects for unqualified candidates. If such effects should exist, it would also be interesting to quantify their relative magnitudes in order to evaluate the model's rather stark result that they are exactly offsetting.

An important theoretical objection to the model is that the ability of parties to obtain qualified candidates is treated as an exogenous parameter. It can be argued that qualified candidates may be more likely to emerge if they are more likely to win the election. Since the advantages of funding are enjoyed by qualified candidates in the model analyzed here, this implies that the financially stronger party will be more likely to obtain a qualified candidate in equilibrium, giving it an electoral advantage from an ex ante perspective. Future research should aim to develop a tractable model that endogenizes the probability of obtaining a qualified candidate.

5 Appendix

The formal analysis that follows uses an alternative (but equivalent) definition of the equilibrium, which is technically more convenient. Define $\xi(\widehat{q}) = \eta \cdot (1 - \widehat{q})$. Note that for a qualified candidate, $\frac{\partial \pi_K(.)}{\partial C_K} = \lambda'(C_K) \cdot \xi(\widehat{q})$. Thus, $\xi(\widehat{q})$ can be thought of as measuring the "effectiveness of advertising" as a function of beliefs. An equilibrium can then be summarized by four numbers, $(C_L^*, C_R^*, \xi_L^*, \xi_R^*)$ such that for $K = L, R$,

$$\xi_K^* = \eta \cdot \frac{1 - \sigma_K}{1 - \sigma_K \cdot \lambda(C_K^*)} \tag{4}$$

and

$$\xi_K^* \cdot \lambda'(C_K^*) \cdot (\beta + (1 - \sigma_{-K}) \cdot \delta) \leq \frac{1}{\gamma_K} \quad (= \text{ if } C_K^* > 0). \tag{5}$$

Equation 3 implicitly defines a function, $\xi(C, \sigma) = \eta \cdot \frac{1-\sigma}{1-\sigma\cdot\lambda(C)}$, which describes how the "effectiveness of advertising" depends on the level of spending and the ex ante probability of qualification. Note that this function is increasing in C. That is, the more an interest group spends on advertising, the greater is the effect of an advertisement on the behavior of a swing voters who sees it.

To understand why this is so, recall that a voter who sees an advertisement will change his beliefs from $\widehat{q}_K$ to 1. The size of this increment, and therefore the effect of the ad, is decreasing in $\widehat{q}_K$. But $\widehat{q}_K$ itself is decreasing in the equilibrium level of spending (see figure 1). Taken together, this implies that the effectiveness of advertising is increasing in the equilibrium level of spending

Now consider the function $\Phi(C, \sigma) = \xi(C, \sigma) \cdot \lambda'(C)$. This function describes how the effectiveness of an additional dollar spent on advertising depends on the equilibrium level of spending. Assumption (2) implies that $\Phi(C, \sigma)$ is monotone decreasing in C. It then follows that the interest groups' first order conditions have unique solutions.

Proof of Proposition 1

Define $\Phi(C, \sigma) = \xi(C, \sigma) \cdot \lambda'(C)$. Then $(C_L^*, C_R^*, \xi(C_L^*, \sigma_L), \xi(C_R^*, \sigma_R))$ constitute an equilibrium if and only if

$$\Phi(C_K^*, \sigma_K) \leq \frac{1}{\gamma_K \cdot (\beta + (1 - \sigma_{-K}) \cdot \delta)} \quad (= \text{ if } C_K^* > 0) \text{ for } K = L, R$$

The RHS of this condition is constant in C. The derivative of the RHS is

$$\frac{\partial \Phi\left(C,\sigma\right)}{\partial C}=\eta\cdot(1-\sigma_K)\cdot\Big(\sigma_K\cdot\left[\lambda'(C)\right]^2+(1-\sigma_K\cdot\lambda(C))\cdot\lambda''(C)\Big).$$

Since $\sigma_K\in(0,1)$, $\sigma_K\cdot\left[\lambda'(C)\right]^2+(1-\sigma_K\cdot\lambda(C))\cdot\lambda''(C)<\left[\lambda'(C)\right]^2+(1-\lambda(C))\cdot\lambda''(C)<0$ by assumption 2. Therefore there exists a unique C_K^* which satisfies the condition. Q.E.D..

Examples of advertising technologies As examples, it is straightforward to check that Assumption (2) is satisfied for $\lambda(C)=\frac{C}{C+\alpha}$, for all $\alpha>0$, as well as for $\lambda(C)=1-\exp\left[-\alpha\cdot C\right]$, for all $\alpha>0$. I have not been able to come up with an example that satisfies the conditions $\lambda\left(0\right)=0$, $\lambda'(C)>0$ and $\lambda''(C)<0$ for all C, and $\lim_{C\to\infty}\lambda\left(C\right)=1$, is continuously differentiable, but fails to satisfy Assumption (2).

Proof of Proposition 2

Define $C^*(\gamma_K,\ \sigma_K,\ \sigma_{-K})$ to be the unique equilibrium contribution level given γ_K, σ_K, and σ_{-K}. Also define the following functions.

$$\begin{aligned}\Omega(\gamma_K,\sigma_{-K}) &= \frac{1}{\gamma_K\cdot(\beta+(1-\sigma_{-K})\cdot\delta)}\\ \Phi(C,\sigma_K) &= \xi\left(C,\sigma_K\right)\cdot\lambda'(C)\end{aligned}$$

Recall that the unique equilibrium contribution level satisfies the FOC

$$\Phi(C^*(\gamma_K,\sigma_K,\sigma_{-K}),\sigma_K)\leq\Omega(\gamma_K,\sigma_{-K})\quad(=\text{if } C^*(\gamma_K,\sigma_K,\sigma_{-K})>0)$$

This implies that $C^*(\gamma_K,\sigma_K,\sigma_{-K})>0$ iff

$$\gamma_K>\left[\eta\cdot(1-\sigma_K)\cdot\lambda'(0)\cdot(\beta+(1-\sigma_{-K})\cdot\delta)\right]^{-1}$$

By assumption, both γ_K exceed this threshold. Therefore contributions are strictly positive, and $C^*(\gamma_K,\sigma_K,\sigma_{-K})$ is implicitly defined by

$$\Phi(C^*(\gamma_K,\sigma_K,\sigma_{-K}),\sigma_K)=\Omega(\gamma_K,\sigma_{-K})\tag{6}$$

Claims 1 and 3 Differentiating both sides of Equation (5) with respect to γ_K yields

$$\frac{\partial C^*(\gamma_K,\sigma_K,\sigma_{-K})}{\partial\gamma_K}=\frac{\partial\Omega(\gamma_K,\sigma_{-K})}{\partial\gamma_K}\cdot\left(\frac{\partial\Phi(C^*(\gamma_K,\sigma_K,\sigma_{-K}),\sigma_K)}{\partial C}\right)^{-1}>0.$$

Therefore $\gamma_R>\gamma_L$ implies $C^*(\gamma_R,\sigma,\sigma)>C^*(\gamma_L,\sigma,\sigma)$. As a function of C, equilibrium beliefs are $\widehat{q}(C)=\frac{\sigma\cdot(1-\lambda(C))}{1-\sigma\cdot\lambda(C)}$. Since $\lambda'(C)>0$, we have $\widehat{q}'(C)<0$. Therefore $\widehat{q}(C^*(\gamma_R,\sigma,\sigma))<\widehat{q}(C^*(\gamma_L,\sigma,\sigma))$.

Claims 2 and 4 Recall that the probability that L wins the election, given candidate types and spending levels, is

$$\pi_L(C_L, C_R, q_L, q_R) = \frac{1}{2} - \frac{\tau \cdot \varkappa}{2\varepsilon} + \eta \cdot [\overline{\rho}(C_L, q_L, \widehat{q}_L) - \overline{\rho}(C_R, q_R, \widehat{q}_R)].$$

Suppose that candidate L is unqualified and candidate R is qualified. Then, L's reputation is given by $\overline{\rho}(0, 0, \widehat{q}_L) = \widehat{q}_L$, and R's reputation is given by $\overline{\rho}(C_R^*, 1, \widehat{q}_R) = \lambda(C_R^*) + (1 - \lambda(C_R^*)) \cdot \widehat{q}_R$. Therefore the probability that L will win the election is given by $\frac{1}{2} - \frac{\tau \cdot \varkappa}{2\varepsilon} + \eta\,[\widehat{q}_L - \lambda(C_R^*) - (1 - \lambda(C_R^*))\,\widehat{q}_R] = \frac{1}{2} - \frac{\tau \cdot \varkappa}{2\varepsilon} + [(1 - \lambda(C_R))\xi_R - \xi_L]$. Denote this probability by π_L^{UQ}. Similarly, denote the probability that L wins when both candidates are qualified by $\pi_L^{QQ} = \left(\frac{1}{2} - \frac{\tau}{2\varepsilon} \cdot \varkappa\right) + [(1 - \lambda(C_R^*)) \cdot \xi_R^* - (1 - \lambda(C_L^*)) \cdot \xi_L^*]$. Then, the probability that candidate R will win the election, given that she is qualified, is $\pi_R^Q = 1 - \left[\sigma_L \cdot \pi_L^{UQ} + (1 - \sigma_L) \cdot \pi_L^{QQ}\right] = \left(\frac{1}{2} + \frac{\tau}{2\varepsilon} \cdot \varkappa\right) + \eta(1 - \sigma_L) - (1 - \lambda(C_R^*)) \cdot \xi_R^*$. Similarly, the probability that candidate L will win, given that she is qualified, is given by $\pi_L^Q = \left(\frac{1}{2} - \frac{\tau}{2\varepsilon} \cdot \varkappa\right) + \eta(1 - \sigma_R) - (1 - \lambda(C_R^*)) \cdot \xi_L^*$. As a function of γ_K, $\frac{\partial \pi_K^Q}{\partial \gamma_K} = \frac{\partial C^*(\gamma_K, \sigma_K, \sigma_{-K})}{\partial \gamma_K} \cdot \left[\lambda'(C_K^*) \cdot \xi\left(C_K^*, \sigma_K\right) - (1 - \lambda(C_K^*)) \cdot \frac{\partial \xi\left(C_K^*, \sigma_K\right)}{\partial C}\right]$. Notice that $\frac{\partial \xi(C, \sigma)}{\partial C} = \eta \cdot \frac{\sigma \cdot (1 - \sigma) \cdot \lambda'(C)}{(1 - \sigma \cdot \lambda(C))^2} = \sigma \cdot \lambda'(C) \cdot \xi\,(C, \sigma)$. Thus

$$\frac{\partial \pi_K^Q}{\partial \gamma_K} = \frac{\partial C^*(\gamma_K, \sigma_K, \sigma_{-K})}{\partial \gamma_K} \cdot \lambda'(C_K^*) \cdot \xi\left(C_K^*, \sigma_K\right) \cdot [1 - \sigma_K \cdot (1 - \lambda(C_K^*))] > 0.$$

Similar calculations show that the conditional probability of winning when L is unqualified is given by $\pi_L^U = \left(\frac{1}{2} - \frac{\tau}{2\varepsilon} \cdot \varkappa\right) + \eta \cdot (1 - \sigma_R) - \xi_L$, and $\pi_R^U = \left(\frac{1}{2} + \frac{\tau}{2\varepsilon} \cdot \varkappa\right) + \eta \cdot (1 - \sigma_L) - \xi_R$. Therefore

$$\frac{\partial \pi_K^U}{\partial \gamma_K} = -\frac{\partial C^*(\gamma_K, \sigma_K, \sigma_{-K})}{\partial \gamma_K} \cdot \frac{\partial \xi\left(C_K^*, \sigma_K\right)}{\partial C} < 0.$$

Q.E.D.

Proof of Proposition 3

Referring back to the probabilities calculated in the previous proof, we can see that the unconditional ex ante probability that L wins is

$$\begin{aligned}
\pi^L &= \sigma_L \cdot \pi_Q^L + (1 - \sigma_L) \cdot \pi_U^L \\
&= \left(\frac{1}{2} - \frac{\tau}{2\varepsilon} \cdot \varkappa\right) + \eta \cdot (1 - \sigma_R) - \xi_L \cdot [\sigma_L \cdot (1 - \lambda(C_L)) + (1 - \sigma_L)] \\
&= \left(\frac{1}{2} - \frac{\tau}{2\varepsilon} \cdot \varkappa\right) + \eta \cdot (1 - \sigma_R) - \xi_L \cdot [1 - \sigma_L \cdot \lambda(C_L)] \\
&= \left(\frac{1}{2} - \frac{\tau}{2\varepsilon} \cdot \varkappa\right) + \eta \cdot (1 - \sigma_R) - \eta \cdot (1 - \sigma_L)
\end{aligned}$$

Q.E.D.

Proof of Proposition 4

Recall that L wins the election if $\mu < \mu^* - \tau \cdot \varkappa$. Consider the case where $(q_L, q_R) = (0, 0)$. This case occurs with probability $(1 - \sigma_L)(1 - \sigma_R)$. Neither candidate is qualified. Thus, $\mu^* = \frac{1}{2} + \frac{\delta}{2\beta}(\widehat{q}_L - \widehat{q}_R)$. So the left candidate will win if $\mu < \left(\frac{1}{2} - \tau \cdot \varkappa\right) + \frac{\delta}{2\beta}(\widehat{q}_L - \widehat{q}_R)$. If so, then $|\mu - z| = \mu$, otherwise $|\mu - z| = 1 - \mu$. Denote $\left(\frac{1}{2} - \tau \cdot \varkappa\right) + \frac{\delta}{2\beta}(\widehat{q}_L - \widehat{q}_R) \equiv h$. Then $E(|\mu - z| \,|\, (q_L, q_R) = (0, 0)) = \int_{\frac{1}{2}-\varepsilon}^{h} \mu \, dF(\mu) + \int_{h}^{\frac{1}{2}+\varepsilon} (1 - \mu) \, dF(\mu)$. After a bit of work, this simplifies to $E(|\mu - z| \,|\, (q_L, q_R) = (0, 0)) = \frac{1-\varepsilon}{2} + 2\varepsilon \cdot \left((\xi_R - \xi_L) - \frac{\tau\varkappa}{2\varepsilon}\right)^2$. Similar calculations for the other cases show that
$E(|\mu - z| \,|\, (q_L, q_R) = (1, 0)) = \frac{1-\varepsilon}{2} + 2\varepsilon \cdot \left((\xi_R - (1 - \lambda(C_L)) \cdot \xi_L) - \frac{\tau\varkappa}{2\varepsilon}\right)^2$,
$E(|\mu - z| \,|\, (q_L, q_R) = (0, 1)) = \frac{1-\varepsilon}{2} + 2\varepsilon \cdot \left(((1 - \lambda(C_R)) \cdot \xi_R - \xi_L) - \frac{\tau\varkappa}{2\varepsilon}\right)^2$, and
$E(|\mu - z| \,|\, (q_L, q_R) = (1, 1)) = \frac{1-\varepsilon}{2} + 2\varepsilon \left(((1 - \lambda(C_R))\xi_R - (1 - \lambda(C_L))\xi_L) - \frac{\tau\varkappa}{2\varepsilon}\right)^2$.
Then $E[|\mu - z|] = \frac{1-\varepsilon}{2} + \left(\frac{\tau\varkappa}{2\varepsilon}\right)^2 + \frac{\tau\varkappa}{\varepsilon} \cdot \eta \cdot (\sigma_R - \sigma_L)$
$+ 2\varepsilon \cdot \sum_{K=L,R} \{[1 - \sigma_K \cdot \lambda(C_K) \cdot (2 - \lambda(C_K))] \cdot \xi_K^2 - [1 - \sigma_K \cdot \lambda(C_K)][1 - \sigma_{-K} \cdot \lambda(C_{-K})] \cdot \xi_K \cdot \xi_{-K}\}$. Then a bit of algebra shows that

$$\frac{\partial E[|\mu - z|]}{\partial C_K} = \frac{\beta}{\delta} \cdot \sigma_K \cdot \lambda(C_K) \cdot \xi(C_K, \sigma_K)^3 \cdot \lambda'(C_K) > 0.$$

Therefore,

$$\frac{\partial E[|\mu - z|]}{\partial \gamma_K} = \frac{\partial C^*(\gamma_K, \sigma_K, \sigma_{-K})}{\partial \gamma_K} \cdot \frac{\partial E[|\mu - z|]}{\partial C_K} > 0$$

Q.E.D.

Proof of Proposition 5

The probability that the winner of the election is qualified is given by $\pi^Q = \sigma_L \cdot \sigma_R + \sigma_L \cdot (1 - \sigma_R) \cdot \pi_{QU}^L + \sigma_R \cdot (1 - \sigma_L) \cdot \left(1 - \pi_{UQ}^L\right)$. Simple algebra shows that
$\pi^Q(C_L, C_R) = \frac{\sigma_L + \sigma_R}{2} + \sum_K \sigma_K \cdot (1 - \sigma_{-K}) \cdot \left[\xi\left(C_{-K}^*, \sigma_{-K}\right) - (1 - \lambda(C_K^*)) \cdot \xi(C_K^*, \sigma_K)\right] + (\sigma_R - \sigma_L) \cdot \frac{\tau}{2\varepsilon} \cdot \varkappa$. Then $\frac{\partial \pi^Q}{\partial \gamma_K}$ simplifies to

$$\frac{\partial \pi^Q}{\partial \gamma_K} = \frac{\partial C^*(\gamma_K, \sigma_K, \sigma_{-K})}{\partial \gamma_K} \cdot \left[\frac{\sigma_K}{\eta} \cdot \xi(C_K, \sigma_K)^2 \cdot \lambda'(C_K)\right] > 0$$

Q.E.D.

Proof of Proposition 6

Note first that equilibrium contributions under a limit $\overline{C} \in (C_L^*, C_R^*)$ are given by $\left(C_L^*, \overline{C}\right)$. Now consider a marginal reduction of $\overline{C}$. The reader can refer back to the argument in the proof of Proposition 3 to verify that this will not effect the probability that either party will win the election. The proofs of Propositions 4 and 5 show that (a) the effect on the expected ideological distance from the median swing voter is given by $-\frac{\partial E[|\mu - z|]}{\partial C_R} < 0$, and that (b) the effect on the probability that the winner is qualified is given by $-\frac{\partial \pi^Q(C_L^*, \overline{C})}{\partial C_R} < 0$. In addition to these effects, (c) interest group R's per capita contribution level will drop by $\frac{1}{\gamma_R}$.

It's immediately clear that partisan voters who are not members of interest group R will be effected only by (b), so they must be worse off under a limit. To see that swing voters are also worse off, we need to consider the sum of effects (a) and (b). The expected utility of the median swing voter in the laissez-faire equilibrium is

$$\overline{U}(C_L^*, C_R^*) = \pi^Q(C_L^*, C_R^*) \cdot \delta - \beta \cdot E\left[|\mu - z|\,|C_L^*, C_R^*\right].$$

Thus, the change in her utility from lowering the limit is

$$\begin{aligned} -\frac{\partial \overline{U}(C_L^*, \overline{C})}{\partial C_R} &= -\delta \cdot \frac{\partial \pi^Q(C_L^*, \overline{C})}{\partial C_R} + \beta \cdot \frac{\partial E\left[|\mu - z|\,|C_L^*, \overline{C}\right]}{\partial C_R} \\ &= -\frac{\delta}{\eta} \cdot \sigma_R \cdot \xi\left(\overline{C}, \sigma_R\right)^2 \cdot \lambda'(\overline{C}) \cdot \left(1 - \frac{(1-\sigma_R) \cdot \lambda(\overline{C})}{1 - \sigma_R \cdot \lambda(\overline{C})}\right) < 0. \end{aligned}$$

To see part (2), recall that the laissez-faire level of spending $C_R^* > 0$ was chosen by the interest group to satisfy it's FOC,

$$\lambda'(C_R^*) \cdot \xi\left(C_R^*, \sigma_R\right) \cdot \left(\beta + (1 - \sigma_L) \cdot \delta\right) = \frac{1}{\gamma_R}$$

Now, suppose that the group is permitted to commit to some contribution level $\widehat{C}_R$ ex ante. Since spending has no ex ante effect on π_R, then $\widehat{C}_R$ would be chosen to satisfy the following condition.

$$\frac{\partial \pi^Q\left(C_L^*, \widehat{C}_R\right)}{\partial C_R} \cdot \delta \leq \frac{\sigma_R}{\gamma_R} \qquad (= \text{ if } \widehat{C}_R > 0)$$

Where

$$\frac{\partial \pi^Q(C_L^*, \widehat{C}_R)}{\partial C_R} = \lambda'(\widehat{C}_R) \cdot \xi\left(\widehat{C}_R, \sigma_R\right) \cdot \left(\frac{\sigma_R \cdot (1-\sigma_R)}{1 - \sigma_R \cdot \lambda(\widehat{C}_R)}\right)$$

Suppose that $\widehat{C}_R > 0$. Then, combining the two FOC conditions, we have

$$\lambda'(C_R^*) \cdot \xi\left(C_R^*, \sigma_R\right) \cdot \left(\beta + (1 - \sigma_L) \cdot \delta\right) = \frac{\partial \pi^Q\left(\widehat{C}_R\right)}{\partial C_R} \cdot \frac{\delta}{\sigma_R}$$

$$\frac{\lambda'(C_R^*) \cdot \xi\left(C_R^*, \sigma_R\right)}{\lambda'(\widehat{C}_R) \cdot \xi\left(\widehat{C}_R, \sigma_R\right)} = \frac{(1 - \sigma_R)}{1 - \sigma_R \cdot \lambda(\widehat{C}_R)} \cdot \frac{\delta}{\left(\beta + (1 - \sigma_L) \cdot \delta\right)}$$

Recall that $\delta < \beta$ by assumption. Therefore it's clear that

$$(1-\sigma_R)\cdot\delta < \left(1-\sigma_R\cdot\lambda(\widehat{C}_R)\right)\cdot(\beta+(1-\sigma_L)\cdot\delta)$$

Therefore

$$\frac{\lambda'(C_R^*)\cdot\xi\left(C_R^*,\sigma_R\right)}{\lambda'(\widehat{C}_R)\cdot\xi\left(\widehat{C}_R,\sigma_R\right)} < 1$$

By assumption 2, $\lambda'(C)\cdot\xi\left(C,\sigma\right)$ is decreasing in C. It follows that $\widehat{C}_R < C_R^*$. This establishes part (2) of the proposition. Q.E.D.

Proof of Proposition 7

In order to endogenize the probability that candidate K is qualified, we assume that each party's qualified candidate runs iff

$$c \geq \pi_{QK}\left(\sigma_K,\sigma_{-K},\gamma_K\right)\cdot r$$

where $c \sim F(c) = \frac{1}{\bar{c}}$, and $\pi_{QK}\left(\sigma_K,\sigma_{-K},\gamma_K\right)$ is the equilibrium probability of winning when $(\sigma_K,\sigma_{-K},\gamma_K)$ are exogenously given for the model analyzed above.

Then, an equilibrium with endogenous candidate quality is a pair (σ_L^*,σ_R^*) such that

$$\begin{aligned}\sigma_L^* &= s(\sigma_R^*,\gamma_L)\\ \sigma_R^* &= s(\sigma_L^*,\gamma_R)\end{aligned}$$

where $s(\sigma_{-K},\gamma_K)$ is implicitly defined by

$$s(\sigma_{-K},\gamma_K) = \frac{r}{\bar{c}}\cdot\pi_{QK}\left(s(\sigma_{-K},\gamma_K),\sigma_{-K},\gamma_K\right)$$

For simplicity, assume $\frac{r}{\bar{c}} = 1$. Then

$$\begin{aligned}\frac{\partial\sigma_K^*}{\partial\gamma_K} &= \frac{\partial s(\sigma_{-K}^*,\gamma_K)}{\partial\sigma_{-K}}\cdot\frac{\partial\sigma_{-K}^*}{\partial\gamma_K}+\frac{\partial s(\sigma_{-K}^*,\gamma_K)}{\partial\gamma_K}\\ \frac{\partial\sigma_{-K}^*}{\partial\gamma_K} &= \frac{\partial s(\sigma_K^*,\gamma_{-K})}{\partial\sigma_K}\cdot\frac{\partial\sigma_K^*}{\partial\gamma_K}\end{aligned}$$

$$\frac{\partial\sigma_K^*}{\partial\gamma_K} = \frac{\frac{\partial s(\sigma_{-K}^*,\gamma_K)}{\partial\gamma_K}}{1-\frac{\partial s(\sigma_{-K}^*,\gamma_K)}{\partial\sigma_{-K}}\cdot\frac{\partial s(\sigma_K^*,\gamma_{-K})}{\partial\sigma_K}}$$

We know that $\frac{\partial s(\sigma_{-K}^*,\gamma_K)}{\partial\gamma_K} > 0$. (Proof: For all (σ_K,σ_{-K}), we know from the previous analysis that $\pi_{QK}\left(\sigma_K,\sigma_{-K},\gamma_K\right)$ is increasing in γ_K.) Hence $\frac{\partial\sigma_K^*}{\partial\gamma_K} > 0$ iff $1-\frac{\partial s(\sigma_{-K}^*,\gamma_K)}{\partial\sigma_{-K}}\cdot\frac{\partial s(\sigma_K^*,\gamma_{-K})}{\partial\sigma_K} > 0$.

Note that

$$\frac{\partial s(\sigma_{-K},\gamma_K)}{\partial \sigma_{-K}} = \frac{\frac{\partial \pi_{QK}(s(\sigma_{-K},\gamma_K),\sigma_{-K},\gamma_K)}{\partial \sigma_{-K}}}{1-\frac{\partial \pi_{QK}(s(\sigma_{-K},\gamma_K),\sigma_{-K},\gamma_K)}{\partial \sigma_K}}$$

Note that

$$\pi_{QK}\left(\sigma_K,\sigma_{-K},\gamma_K\right) = \frac{1}{2} + \eta\cdot\left(1-\sigma_{-K}-\left(1-\lambda_K^X\left(\sigma_K,\sigma_{-K},\gamma_K\right)\right)\cdot\xi_K^X\left(\sigma_K,\sigma_{-K},\gamma_K\right)\right)$$

where $\lambda_K^X\left(\sigma_K,\sigma_{-K},\gamma_K\right)$ and $\xi_K^X\left(\sigma_K,\sigma_{-K},\gamma_K\right)$ denote the equilibrium outcomes when $\left(\sigma_K,\sigma_{-K},\gamma_K\right)$ are exogenously given.

Then

$$\begin{aligned}
\frac{\partial \pi_{QK}}{\partial \sigma_K} &= \eta\cdot\left[\frac{\partial \lambda_K^X}{\partial \sigma_K}\cdot\xi_K^X-\left(1-\lambda_K^X\right)\cdot\frac{\partial \xi_K^X}{\partial \sigma_K}\right]\\
\frac{\partial \pi_{QK}}{\partial \sigma_{-K}} &= \eta\cdot\left[-1+\frac{\partial \lambda_K^X}{\partial \sigma_{-K}}\cdot\xi_K^X-\left(1-\lambda_K^X\right)\cdot\frac{\partial \xi_K^X}{\partial \sigma_{-K}}\right]
\end{aligned}$$

Where

$$\begin{aligned}
\xi_K^X\left(\sigma_K,\sigma_{-K},\gamma_K\right) &= \frac{1-\sigma_K}{1-\sigma_K\cdot\lambda_K^X\left(\sigma_K,\sigma_{-K},\gamma_K\right)}\\
\frac{\partial \xi_K^X}{\partial \sigma_K} &= \frac{\lambda_K^X-1+\sigma_K\cdot\left(1-\sigma_K\right)\frac{\partial \lambda_K^X}{\partial \sigma_K}}{\left(1-\sigma_K\cdot\lambda_K^X\right)^2}\\
&= \frac{\partial \lambda_K^X}{\partial \sigma_K}\cdot\xi_K^X\cdot\frac{\sigma_K}{1-\sigma_K\cdot\lambda_K^X}-\frac{1-\lambda_K^X}{\left(1-\sigma_K\cdot\lambda_K^X\right)^2}\\
\frac{\partial \xi_K^X}{\partial \sigma_{-K}} &= \frac{\sigma_K\cdot\left(1-\sigma_K\right)\frac{\partial \lambda_K^X}{\partial \sigma_{-K}}}{\left(1-\sigma_K\cdot\lambda_K^X\right)^2} = \frac{\sigma_K}{1-\sigma_K\cdot\lambda_K^X}\cdot\xi_K^X\cdot\frac{\partial \lambda_K^X}{\partial \sigma_{-K}}
\end{aligned}$$

Therefore

$$\begin{aligned}
\frac{\partial \pi_{QK}}{\partial \sigma_K} &= \eta\cdot\left[\frac{\partial \lambda_K^X}{\partial \sigma_K}\cdot\xi_K^X\cdot\left(1-\frac{\left(1-\lambda_K^X\right)\cdot\sigma_K}{1-\sigma_K\cdot\lambda_K^X}\right)+\frac{\left(1-\lambda_K^X\right)^2}{\left(1-\sigma_K\cdot\lambda_K^X\right)^2}\right]\\
&= \eta\cdot\left[\frac{\partial \lambda_K^X}{\partial \sigma_K}\cdot\left(\xi_K^X\right)^2+\frac{\left(1-\lambda_K^X\right)^2}{\left(1-\sigma_K\cdot\lambda_K^X\right)^2}\right]\\
\frac{\partial \pi_{QK}}{\partial \sigma_{-K}} &= \eta\cdot\left[-1+\frac{\partial \lambda_K^X}{\partial \sigma_{-K}}\cdot\left(\xi_K^X\right)^2\right]
\end{aligned}$$

Finally, $\lambda_K^X\left(\sigma_K,\sigma_{-K},\gamma_K\right)$ is implicitly defined by

$$\eta\cdot\gamma_K\cdot\frac{1-\sigma_K}{1-\sigma_K\cdot\lambda_K^X}\cdot\left[\beta+\left(1-\sigma_{-K}\right)\delta\right] = C'\left(\lambda_K^X\right)$$

where C is a convex cost function. Let $\phi(\lambda_K,\sigma_K) = C'(\lambda_K)\cdot\left(\frac{1-\sigma_K}{1-\sigma_K\cdot\lambda_K}\right)^{-1}$ and $g(\sigma_{-K},\gamma_K) = \eta\cdot\gamma_K\cdot[\beta+(1-\sigma_{-K})\delta]$. Then, λ_K^X satisfies

$$\phi\left(\lambda_K^X(\sigma_K,\sigma_{-K},\gamma_K),\sigma_K\right) = g(\sigma_{-K},\gamma_K)$$

and so

$$\begin{aligned}
\frac{\partial\lambda_K^X}{\partial\sigma_K} &= -\frac{\frac{\partial\phi}{\partial\sigma_K}}{\frac{\partial\phi}{\partial\lambda_K}} \\
\frac{\partial\lambda_K^X}{\partial\sigma_{-K}} &= \frac{\frac{\partial g}{\partial\sigma_{-K}}}{\frac{\partial\phi}{\partial\lambda_K}}
\end{aligned}$$

Now, using

$$\begin{aligned}
\xi(\sigma,\lambda) &= \frac{1-\sigma}{1-\sigma\cdot\lambda} \\
\frac{\partial\xi(\sigma,\lambda)}{\partial\sigma} &= -\frac{1-\lambda}{(1-\sigma\lambda)^2} < 0 \\
\frac{\partial\xi(\sigma,\lambda)}{\partial\lambda} &= \frac{\sigma(1-\sigma)}{(1-\sigma\lambda)^2} = \frac{\sigma}{1-\sigma\lambda}\cdot\xi(\sigma,\lambda)
\end{aligned}$$

we have that

$$\begin{aligned}
\frac{\partial\phi}{\partial\sigma_K} &= \frac{-C'\left(\lambda_K^X\right)\cdot\frac{\partial\xi\left(\sigma_K,\lambda_K^X\right)}{\partial\sigma}}{\xi\left(\sigma_K,\lambda_K^X\right)^2} = \frac{1-\lambda_K^X}{(1-\sigma_K)^2}\cdot C'\left(\lambda_K^X\right) > 0 \\
\frac{\partial\phi}{\partial\lambda_K} &= \frac{C''\left(\lambda_K^X\right)}{\xi\left(\sigma_K,\lambda_K^X\right)} - \frac{C'\left(\lambda_K^X\right)\cdot\frac{\partial\xi\left(\sigma_K,\lambda_K^X\right)}{\partial\lambda}}{\xi\left(\sigma_K,\lambda_K^X\right)^2} \\
&= \left(\frac{1}{1-\sigma_K}\right)\cdot\left[C''\left(\lambda_K^X\right)\cdot\left(1-\sigma_K\lambda_K^X\right) - C'\left(\lambda_K^X\right)\cdot\sigma_K\right] > 0 \\
\frac{\partial g}{\partial\sigma_{-K}} &= -\eta\cdot\gamma_K\cdot\delta
\end{aligned}$$

Then

$$\begin{aligned}
\frac{\partial\lambda_K^X}{\partial\sigma_K} &= -\frac{\frac{\partial\phi}{\partial\sigma_K}}{\frac{\partial\phi}{\partial\lambda_K}} = -\frac{1-\lambda_K^X}{1-\sigma_K}\cdot\frac{C'\left(\lambda_K^X\right)}{C''\left(\lambda_K^X\right)\cdot\left(1-\sigma_K\lambda_K^X\right) - C'\left(\lambda_K^X\right)\cdot\sigma_K} < 0 \\
\frac{\partial\lambda_K^X}{\partial\sigma_{-K}} &= \frac{\frac{\partial g}{\partial\sigma_{-K}}}{\frac{\partial\phi}{\partial\lambda_K}} = \frac{-(1-\sigma_K)\cdot\eta\cdot\gamma_K\cdot\delta}{C''\left(\lambda_K^X\right)\cdot\left(1-\sigma_K\lambda_K^X\right) - C'\left(\lambda_K^X\right)\cdot\sigma_K} < 0
\end{aligned}$$

(Recall that by assumption, $(1-\lambda)\cdot C''(\lambda) > C'(\lambda)$ for all λ.)

Claim 1 *Suppose* $(1-\lambda)\cdot C''(\lambda) > C'(\lambda)$ *for all* λ *and* β *is large enough. Then* $\frac{\partial s(\sigma_{-K},\gamma_K)}{\partial \sigma_{-K}} \in (-1,0)$ *for all* (σ_{-K},γ_K), *and so*

$$\frac{\partial \sigma_K^*}{\partial \gamma_K} = \frac{\frac{\partial s(\sigma_{-K}^*,\gamma_K)}{\partial \gamma_K}}{1-\frac{\partial s(\sigma_{-K}^*,\gamma_K)}{\partial \sigma_{-K}}\cdot\frac{\partial s(\sigma_K^*,\gamma_{-K})}{\partial \sigma_K}} > 0$$

and

$$\frac{\partial \sigma_K^*}{\partial \gamma_{-K}} = \frac{\partial s(\sigma_{-K}^*,\gamma_K)}{\partial \sigma_{-K}}\cdot\frac{\partial \sigma_{-K}^*}{\partial \gamma_K} < 0$$

Proof.

$$\begin{aligned}\frac{\partial s(\sigma_{-K},\gamma_K)}{\partial \sigma_{-K}} &= \frac{\frac{\partial \pi_{QK}(s(\sigma_{-K},\gamma_K),\sigma_{-K},\gamma_K)}{\partial \sigma_{-K}}}{1-\frac{\partial \pi_{QK}(s(\sigma_{-K},\gamma_K),\sigma_{-K},\gamma_K)}{\partial \sigma_K}} \\ &= \frac{\eta\cdot\left[-1+\frac{\partial \lambda_K^X}{\partial \sigma_{-K}}\cdot\left(\xi_K^X\right)^2\right]}{1-\eta\cdot\left[\frac{\partial \lambda_K^X}{\partial \sigma_K}\cdot\left(\xi_K^X\right)^2+\frac{\left(1-\lambda_K^X\right)^2}{\left(1-\sigma_K\cdot\lambda_K^X\right)^2}\right]} < 0\end{aligned}$$

To see that this is negative, note that $\frac{\partial \lambda_K^X}{\partial \sigma_{-K}} < 0$, so the numerator is negative. Further, note that the denominator is positive since $\eta\cdot\left[\frac{\partial \lambda_K^X}{\partial \sigma_K}\cdot\left(\xi_K^X\right)^2+\frac{\left(1-\lambda_K^X\right)^2}{\left(1-\sigma_K\cdot\lambda_K^X\right)^2}\right] < \frac{1}{2}$. Thus, $\frac{\partial s(\sigma_{-K},\gamma_K)}{\partial \sigma_{-K}} < 0$. Finally, to see that $\frac{\partial s(\sigma_{-K},\gamma_K)}{\partial \sigma_{-K}} > -1$, note that we need to show

$$\eta\cdot\left[1-\frac{\partial \lambda_K^X}{\partial \sigma_{-K}}\cdot\left(\xi_K^X\right)^2\right] < 1-\eta\cdot\left[\frac{\partial \lambda_K^X}{\partial \sigma_K}\cdot\left(\xi_K^X\right)^2+\frac{\left(1-\lambda_K^X\right)^2}{\left(1-\sigma_K\cdot\lambda_K^X\right)^2}\right]$$

or

$$\frac{1-\eta}{\eta} > \frac{\left(1-\lambda_K^X\right)^2}{\left(1-\sigma_K\cdot\lambda_K^X\right)^2}+\left(\frac{\partial \lambda_K^X}{\partial \sigma_K}-\frac{\partial \lambda_K^X}{\partial \sigma_{-K}}\right)\cdot\left(\xi_K^X\right)^2$$

Note that the LHS is greater than 1. In addition,

$$\left(\frac{\partial \lambda_K^X}{\partial \sigma_K}-\frac{\partial \lambda_K^X}{\partial \sigma_{-K}}\right) = \frac{-\frac{1-\lambda_K^X}{1-\sigma_K}\cdot C'\left(\lambda_K^X\right)+(1-\sigma_K)\cdot\eta\cdot\gamma_K\cdot\delta}{C''\left(\lambda_K^X\right)\cdot\left(1-\sigma_K\lambda_K^X\right)-C'\left(\lambda_K^X\right)\cdot\sigma_K}$$

where

$$C'\left(\lambda_K^X\right) = \eta\cdot\gamma_K\cdot\frac{1-\sigma_K}{1-\sigma_K\cdot\lambda_K^X}\cdot\left[\beta+(1-\sigma_{-K})\,\delta\right]$$

so

$$\left(\frac{\partial \lambda_K^X}{\partial \sigma_K}-\frac{\partial \lambda_K^X}{\partial \sigma_{-K}}\right) = -\frac{(1-\sigma_K)\cdot\eta\cdot\gamma_K}{C''\left(\lambda_K^X\right)\cdot\left(1-\sigma_K\lambda_K^X\right)-C'\left(\lambda_K^X\right)\cdot\sigma_K}\cdot\left(\frac{\left(1-\lambda_K^X\right)\cdot\left[\beta+(1-\sigma_{-K})\cdot\delta\right]}{(1-\sigma_K)\cdot\left(1-\sigma_K\cdot\lambda_K^X\right)}-\delta\right) < 0$$

For β large enough, this is negative. Intuitively, this says that σ_K has a stronger negative impact on equilibrium spending than σ_{-K}. That is, the level of spending in the exogenous quality model is determined mainly by the effectiveness of advertising and not by the probability that the opponent is qualified. Then I've established that $\frac{\partial s(\sigma_{-K},\gamma_K)}{\partial \sigma_{-K}} \in (0,1)$. ■

This establishes that, when $\gamma_R > \gamma_R$, $\sigma_R^* > \sigma_L^*$. Finally, note that the ex ante probability that candidate R wins is given by $\pi_L = \frac{1}{2} + \eta \cdot (\sigma_R^* - \sigma_L^*) > \frac{1}{2}$. Q.E.D.

Example To understand the result, consider the following example. If $\gamma_L = \gamma_R = 0$, we have

$$\pi_{QK}(\sigma_K, \sigma_{-K}, 0) = \frac{1}{2} + \eta \cdot (\sigma_K - \sigma_{-K})$$

then

$$s(\sigma_{-K}, 0) = \frac{1}{2(1-\eta)} - \frac{\eta}{1-\eta} \cdot \sigma_{-K}$$

and the equilibrium satisfies

$$\begin{aligned} \sigma_L^* &= \frac{1}{2(1-\eta)} - \frac{\eta}{1-\eta} \cdot \sigma_R^* \\ \sigma_R^* &= \frac{1}{2(1-\eta)} - \frac{\eta}{1-\eta} \cdot \sigma_L^* \end{aligned}$$

and the unique equilibrium is $\sigma_L^* = \sigma_R^* = \frac{1}{2}$. Note that the slope of $s(\sigma_{-K}, 0)$ is less than 1 in absolute value. Then, suppose γ_R is raised while γ_L remains the same. Now, Mr. R's probability of winning given any pair (σ_L, σ_R) is $\pi_{QR}(\sigma_R, \sigma_L, \gamma_R) = \frac{1}{2} + \eta \cdot (1 - \sigma_L - (1-\lambda_R^X) \cdot \xi_R^X)$ where $(1-\lambda_R^X) \cdot \xi_R^X < (1-\sigma_R)$. Hence $\pi_{QR}(\sigma_R, \sigma_L, \gamma_R) > \frac{1}{2} + \eta \cdot (\sigma_R - \sigma_L) = \pi_{QR}(\sigma_R, \sigma_L, 0)$ for all (σ_R, σ_L). Then, $s(\sigma_L, \gamma_R)$ shifts up while $s(\sigma_R, 0)$ remains the same. Denote the increment in $s(\sigma_L, \gamma_R)$ by $\Delta(\sigma_L, \gamma_L) > 0$. It's actual magnitude is of no concern. Then, the new equilibrium solves

$$\begin{aligned} \sigma_L^* &= \frac{1}{2(1-\eta)} - \frac{\eta}{1-\eta} \cdot \sigma_R^* \\ \sigma_R^* &= \frac{1}{2(1-\eta)} - \frac{\eta}{1-\eta} \cdot \sigma_L^* + \Delta(\sigma_L^*) \end{aligned}$$

so that we must have $\sigma_R^* > \frac{1}{2} > \sigma_L^*$. When both interest groups are larger than zero, the expressions are more difficult to deal with but the result extends provided the assumptions cited are satisfied. Note that those assumptions did not play a role in the example, however.

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