The Kindergarten Rule of Sustainable Growth

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Abstract: The relationship between economic growth and the environment is not well understood: we have only limited understanding of the basic science involved and very limited data. Because of these difficulties it is especially important to develop a series of relatively simple theoretical models that generate stark predictions. This paper presents one such model where societies implement "the Kindergarten rule of sustainable growth." Following the Kindergarten rule means implementing zero emission technologies in either finite time or asymptotically. The underlying simplicity of the model allows us to provide new predictions linking the path of environmental quality to pollutant characteristics (stocks vs. flows; toxics vs. irritants) and primitives of the economic system. It also provides a novel Environmental Catch-up Hypothesis.

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1. Introduction

The relationship between economic growth and the environment is, and will likely remain, controversial. Some see the emergence of new pollution problems, the lack of success in dealing with global warming and the still rising population in the developing world as proof positive that humans are a short-sighted and rapacious species. Others however see the glass as half full. They note the tremendous progress made in providing urban sanitation, improvements in air quality in major cities and marvel at continuing improvements in the human condition made possible by technological advance. The first group focuses on the remaining and often serious environmental problems of the day; the second on the long, but sometimes erratic, history of improvements in living standards.

These views are not necessarily inconsistent and growth theory offers us the tools needed to explore the link between environmental problems of today and the likelihood of their improvement tomorrow. But since the relationship between economic growth and the environment is complex and we have such limited data, it is important to develop theoretical models with stark and readily testable predictions. This paper presents one such model where societies implement "the Kindergarten rule of sustainable growth."[1]

We provide conditions under which long run economic growth with rising environmental quality is both feasible and optimal; describe the transition path of environmental quality from short to long run; and link features of the path to both the primitives of pollutants (stocks vs. flows; toxics vs. irritants) and primitives of the economic system (technologies and preference). In doing so we develop a novel

[1] We define sustainable growth as continuous economic growth with non-decreasing environmental quality.

Environmental Catch-up Hypothesis: initially poor countries experience greater environmental degradation than rich countries and a worse environment at all levels of income; despite this, differences in the quality of their environments diminish over time.

The paper differs from others in the literature in two important ways. First, we treat the pollution production and abatement process at a more detailed level than earlier authors. This proves to be absolutely critical. We assume both goods and abatement production functions are linearly homogenous and exhibit diminishing returns; moreover, pollution is a joint and inescapable output of both the production and abatement process. Technological progress is an indirect consequence of production and abatement efforts and our assumptions lead to a specification with constant returns to reproducible factors.[2]

These assumptions rule out situations where sustainable growth is achieved by either accumulating perfectly clean factors of production, or by employing abatement technologies that exhibit increasing returns. They ensure that assumptions we make regarding knowledge spillovers that eliminate diminishing returns to capital accumulation, are also consistently applied to abatement where they eliminate diminishing returns to lowering emissions per unit output. Together they generate sustainable growth when the intensity of abatement is chosen to satisfy what we dub "the Kindergarten rule." This rule is adopted is adopted in finite time or approached asymptotically. In either case, its implementation generates continuing economic growth

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[2] To focus our efforts on the *implications* of ongoing technological progress for the environment and growth, we adopt the very direct link between factor accumulation and technological progress employed by Romer (1986), Lucas (1988) and others.

and rising environmental quality *without* imposing the typical, but restrictive, assumptions ensuring the demand for a clean environment rise rapidly with incomes.

For example, Stokey (1998), Aghion and Howitt (1998), Lopez (1994), etc. all require a rapidly declining marginal utility of consumption to generate rising environmental quality and ongoing growth.[3] This restriction is troublesome for several reasons. As Aghion and Howitt note:

> "Thus it appears that unlimited growth can indeed be sustained, but it not guaranteed by the usual sorts of assumptions that are made in endogenous growth theory. The assumption that the elasticity of marginal utility of consumption be greater than unity seems particularly strong, in as much as it is known to imply odd behavior in the context of various macroeconomics models…" (p. 162.)

This restriction is also troublesome on the empirical front because it implies a large income elasticity of marginal damage and many question whether the demand for a clean environment can be so income elastic.[4] Moreover, a rapidly declining marginal utility of consumption is required in earlier work because, with diminishing returns to abatement, increasingly large investments in abatement are required to hold pollution in check. This implies the share of pollution abatement costs in the value of output approaches one in the limit.[5] This is inconsistent with available evidence.

Our formulation removes the tension between the ubiquitous empirical finding of declining pollution levels with theoretic rationales that require very strong income effects. It does so by introducing a direct role for technological progress in abatement.

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[3] Also see Chapter 3 of Copeland and Taylor (2003) where they present the leading explanations for the EKC showing that many rely on this same restriction.
[4] See our discussion of marginal damage in Section 4.3 where this link is made explicit. For example, McConnell (1997) argues that current empirical estimates from contingent valuation and hedonic studies do not support the very strong income effects needed.
[5] See the discussion in Aghion and Howitt (1998, page 160-161) and our discussion of abatement in Section 5.

This ensures that *all economies* capable of continuing growth will, at some point, experience falling pollution levels and rising environmental quality.

The simplicity of our specification also enables us to develop several predictions that open new avenues for empirical investigation. This is our second contribution. Earlier theoretical work has shown very little interest in developing the empirical implications of their work, apart from generating the inverse U-shaped relationship found in empirical studies of the Environmental Kuznets Curve. This is unfortunate, and in stark contrast to the history of scientific progress in growth theory more generally. Kaldor's stylized facts, the Solow residual, real business cycle theory and recent empirical debates over convergence have all played a major role in the evolution of thought regarding growth. In many cases, theoretical advance opened up new channels of empirical investigation that then led to further refinement of theory.

Until the discovery of the EKC however, theoretical research examining the growth and environment link proceeded with little or no guidance from empirical work.[6] Since then, researchers have developed a set of possible *explanations* for the EKC, but no new hypotheses have been offered. As a consequence the productive give-and-take characteristic of growth theory more generally has been largely absent. In recognition of this fact we develop two types of predictions: predictions holding at or near the balanced growth path; and predictions concerning the overall transition from initial conditions through to balanced growth. We develop our Environmental Catch-up Hypothesis and relate it to the Environmental Kuznets Curve. Throughout we distinguish between

[6] There is of course a large literature on green accounting but this is quite different. See Chapter 11 of Heal (2000) or Dasgupta and Maler (2000).

parameter changes altering the *level* of environmental quality and those affecting its *rate of change*.

For example, we find the *rate* of income growth can be uncorrelated with the *rate* of environmental improvement even though there is a strong relationship between the *levels* of environmental quality and income. Similarly, we find that the income elasticity of marginal damage has no effect on the *rate* of environmental improvement, but does play a role in determining the *level* of environmental quality and the time when countries shift to active abatement. Variation in other pollutant characteristics such as speed of dissipation in the environment, convexity of damages and abatement cost are shown to have both *level* and *rate* of change effects. The level versus rate of change distinction is of course basic to most discussions of growth, but as yet absent from work in this area.

Our formal model is related to many others in the literature. It is similar to the one sector models discussed in Smulders (1993), Smulders and Gradus (1996), Stokey (1998) and Aghion and Howitt (1998) but differs from each of these significantly because of our explicit treatment of abatement. It is also related to work seeking to explain the EKC (Andreoni and Levinson (2001), Jones and Manuelli (2000), Lopez (1994), and John and Pecchenino (1994)) but differs from these both in purpose and in many specifics. Its closest connection may be to the much earlier work of Keeler, Spence, and Zeckhauser (1972) that sought to understand the growth and environment link quite generally and adopted several alternative formulations for abatement and pollution.

The rest of the paper proceeds as follows. In section 2 we detail the model assumptions paying close attention to how individual abatement and production functions aggregate. Section 3 then investigates the balanced growth path of the model and presents our balanced growth predictions. Section 4 examines the two stages of economic growth and develops predictions concerning transition paths and our Environmental Catch-up Hypothesis. Since our detailed treatment of abatement, production and knowledge spillovers is critical to the novelty of our results, in the penultimate section of the paper we relate our specification to those of earlier authors. A short conclusion sums up. All lengthy proofs are relegated to an appendix.

2. The Model

We adopt the conventional infinitely lived representative agent model. There is one aggregate good, labeled Y, which is either consumed or used for investment or abatement purposes. There are two factors of production: labor and capital. We assume zero population growth and hence L(t) = L. In contrast the capital stock accumulates via investment and depreciates at the constant rate δ.

2.1 Tastes

Our representative consumer maximizes lifetime utility given by:

$$W = \int_0^\infty U(C, X)e^{-rt}dt \qquad (1.1)$$

where C indicates consumption, and X is the pollution stock. Utility is increasing and quasi-concave in C and –X and hence X= 0 corresponds to a pristine environment. When we treat X as flow, X = 0 occurs with zero flow of pollution. When we allow pollution to accumulate in the biosphere we assume the (damaging) service flow is proportional to the level of the stock. Taking this factor of proportionality to be one, then X is the damaging service flow from the stock of pollution given by X.

A useful special case of U(C,X) is the constant elasticity formulation where:

$$U(C,X) = \frac{C^{1-e}}{1-e} - \frac{B(L,r)X^g}{g} \quad for\ e \neq 1$$
$$U(C,X) = \ln C - \frac{B(L,r)X^g}{g} \quad for\ e = 1 \tag{1.2}$$

where $g \geq 1$, ε > 0 and B measures the impact of local pollution on a representative individual. We follow Copeland and Taylor (1994) and assume exposure and hence disutility depends positively on population density ρ, and negatively on the physical size of country indexed by population L.[7] For the most part we assume countries are identical in terms of size and density, and suppress the arguments of B in what follows. We consider the impact of changes in these variables in Section 4.6.

2.2 Technologies

Our assumptions on production are standard. Each firm has access to a strictly concave and CRS production function linking labor and capital to output Y. The productivity of labor is augmented by a technology parameter T taken as given by individual agents. Following Romer (1986) and Lucas (1998) we assume the state of technology is proportional to an economy wide measure of activity. In Romer (1986) this aggregate measure is aggregate R&D, in Lucas (1988) it is average human capital levels; in AK specifications it is linked to either the aggregate capital stock or (to eliminate scale effects) average capital per worker. We assume T is proportional to the aggregate capital

[7] This assumes a country is comprised of many identically sized communities, so that an increase in L taking ρ as given implies either an increase in the physical size of the country or the creation of a new community that suffers only from its own pollution. See Copeland and Taylor (1994) for further details.

to labor ratio in the economy, K/L, and by choice of units take the proportionality constant to be one.[8]

2.3 From Individual to Aggregate Production

Although we adopt a social planning perspective, it is instructive to review how firm level magnitudes aggregate to economy wide measures since this makes clear the assumptions made regarding the role of knowledge spillovers. We aggregate across firms to obtain the AK aggregate production function as follows: [9]

$$\begin{aligned} Y_i &= F(K_i, TL_i) \\ Y &= \sum_i F(K_i, TL_i) \\ Y &= TLF(K/TL, 1) \\ Y &= KF(1,1) = AK \end{aligned} \tag{1.3}$$

where the first line gives firm level production; the second line sums across firms; the third uses linear homogeneity and exploits the fact that efficiency requires all firms adopt the same capital intensity. The last line follows from the definition of T.

Summarizing: diminishing returns at the firm level are undone by technological progress linked to aggregate capital intensity leaving the social marginal product of capital constant.

We now employ similar methods to generate the aggregate abatement technology. To start we note pollution is a joint product of output and we take this relationship to be

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[8] As is well known, one-sector models of endogenous growth blur the important distinction between physical capital and knowledge capital and force us to think of "capital" in very broad terms. Extensions of our framework along the lines of Grossman and Helpman (1991) or Aghion and Howitt (1998) seem both possible and worthwhile. These extensions would however add additional state variables making our examination of transition paths difficult.

[9] Implementing our planning solution by way of pollution taxes and subsidies to investment and abatement should be straightforward. We leave this to future work.

proportional.[10] By choice of units we take the factor of proportionality to be one. Pollution emitted is equal to pollution created minus pollution abated. Denoting pollution emitted by P, we have:

$$P = Y^{G} - abatement \tag{1.4}$$

Abatement of pollution takes as inputs the flow of pollution, which is proportional to the gross flow of output Y^G, and abatement inputs denoted by Y_A. The abatement production function is standard: it is strictly concave and CRS. Therefore we can write pollution emitted by the ith firm as:

$$P_i = Y_i^{G} - a(Y_i^{G}, Y_i^{A}) \tag{1.5}$$

note that in this formulation, the abatement activity pollutes since it is counted in Y^G.

Now consider a Romeresque approach where individual abatement efforts provide knowledge spillovers useful to others abating in the economy. To do so we again introduce a technology shift parameter *T*, and assume it raises the marginal product of abatement. To be consistent with our earlier treatment of technological progress in production we assume *T* is proportional to the average abatement intensity in the economy, Y^A/Y^G. Then much as before we have the individual to aggregate abatement technology transformation as:

$$\begin{aligned} &P_i = Y_i^{G} - a(Y_i^{G}T, Y_i^{A}) \\ &P_i = Y_i^{G}[1 - Ta(1, Y_i^{A}/Y_i^{G}T)] \\ &\sum_i P_i = \sum_i Y_i^{G}[1 - Ta(1, Y_i^{A}/Y_i^{G}T)] \\ &P = Y^{G}[1 - Ta(1,1)], a(1,1) > 1 \\ &P = Y^{G}[1 - q\, a(1,1)], q \equiv Y^{A}/Y^{G} \end{aligned} \tag{1.6}$$

[10] Nothing is lost if we assume pollution is produced in proportion to the services of capital inputs. The service flow of capital is proportional to the stock of capital, and the stock of capital is proportional to output.

where the first line introduces the technology parameter T; the second exploits linear homogeneity of the abatement production function; the third aggregates across firms; the fourth recognizes that efficiency requires all firms choose identical abatement intensities, uses the definition of T and notes that for abatement to be productive it must be able to clean up after itself. The fifth line defines the intensity of abatement, $q \equiv Y^A / Y^G$. Since abatement can only reduce the pollution flow we must have $q \leq 1/a(1,1)$.[11]

It is important to note that the aggregate relationship between pollution and abatement given by the last line in (1.6) is consistent with empirical estimates finding rising marginal abatement costs at the firm level. Each individual firm has abatement costs given by foregone output used in abatement, and hence partially differentiating the second line of (1.6) and rearranging we find:

$$\partial Y_i^A / \partial P_i = -1/[\partial a / \partial Y_i^A] < 0,\ \partial Y_i^{A^2} / \partial P_i^2 > 0 \tag{1.7}$$

Marginal abatement costs are rising at the firm level.

Marginal abatement costs at the society level, are however, constant. To see why totally differentiate (1.6) allowing T and individual abatement to both vary. We find:

$$dY_i^A / dP_i = -1/[\partial a / \partial Y_i^A] - \frac{[\partial a / \partial Y_i^G] Y_i^G}{[\partial a / \partial Y_i^A]} \frac{dT}{dP_i} \tag{1.8}$$

T is the average abatement intensity in the economy, which given identical firms, is just the abatement intensity for the typical ith firm. Using $dT / dP_i = [1/Y_i^G][dY_i^A / dP_i]$ and rearranging we obtain:

[11] Adding the possibility of investments in restoration would probably strengthen the case for sustainable growth. Abatement of pollution and restoration are however distinct activities. We imagine that a restoration production function would take as an input the current damage to the environment – our stock variable X – and then apply inputs to restore it. This is quite different from abatement which operates to lower the current flow of pollution by use of variable inputs.

$$dY_i^A / dP_i = -\frac{Y_i^A}{[[\partial a / \partial Y_i^A]Y_i^A + [\partial a / \partial Y_i^G]TY_i^G]}$$
$$= -\frac{Y_i^A}{a(Y_i^A, TY_i^G)} = -\frac{1}{a(1,1)} < 0 \tag{1.9}$$

where the first line follows from rearrangement and the second by CRS in abatement. Note the result given in (1.9) is identical to what we find by differentiating the aggregate relationship between pollution and abatement given in the last line of (1.6).

Summarizing: diminishing returns at the firm level, that lead to rising marginal abatement costs, are undone by technological progress linked to aggregate abatement intensity leaving the social marginal cost of abatement constant.

Putting these pieces together our planner faces the aggregate production relations for output and abatement given by the last lines of (1.3) and (1.6) together with the atemporal resource constraint linking gross output, abatement and net production:

$$Y = Y^G - Y^A \tag{1.10}$$

2.4 Endowments

We treat pollution as a flow that either dissipates instantaneously – such as noise pollution – or a stock that is only eliminated over time by natural regeneration – such as lead emissions or radioactive waste. When X is a stock we have:

$$\dot{X} = AK[1-\theta a] - \eta X \tag{1.11}$$

where $\eta > 0$ represents the speed of natural regeneration, and where for economy of notation we have denoted a(1,1) by a. When X is a flow we have:

$$X = AK[1-\theta a]$$

3. The Kindergarten Rule

We focus first on the possibility of balanced and continual growth, leaving to the next section a discussion of transition paths. Consider the following problem:

$$
\begin{aligned}
&Maximize \int_0^\infty U(C,X)e^{-\rho t}\, dt \\
&s.t.\, K(0)=K_0,\ \ X(0)=X_0,\ \ and\ \ \theta \le 1/a \\
&\dot{K} = AK[1-\theta]-\delta K - C \\
&\dot{X} = AK[1-\theta a]-\eta X
\end{aligned}
\tag{1.12}
$$

where we adopt U(C,X) from (1.2). Recall the fraction of gross output allocated to abatement is θ and since the flow of pollution into the environment cannot be negative this will never exceed 1/a. We can write the Hamilton-Jacobi-Bellman equation as:

$$
\rho W(K,X) = Max\left\{ H = \frac{C^{1-\varepsilon}}{1-\varepsilon} - \frac{BX^{\gamma}}{\gamma} + \lambda_1[AK[1-\theta]-\delta K - C] + \lambda_2[AK[1-\theta a]-\eta X] \right\} \tag{1.13}
$$

where ρW(K,X) is the maximized value of the program for the given initial conditions $\{K_0, X_0\}$, and H is the current value Hamiltonian for our problem. The controls for this problem are consumption, C, and abatement intensity, θ.

Observe the term involving our control variable, θ,

$$
Max\{AK\theta[-\lambda_1 - a\lambda_2]\}\ \ s.t.\ 0 \le \theta \le 1/a
$$

where λ_1 is the positive shadow value of capital and λ_2 is the negative shadow cost of pollution. Since the Hamiltonian is linear in θ, the value of the term $S = [-\lambda_1 - a\lambda_2]$ will largely determine the optimal level of abatement. When $S > 0$, the shadow cost of pollution is high relative to that of capital. In this case abatement is relatively cheap and maximal abatement will be undertaken. Conversely when $S < 0$ the shadow value of capital is high relative to that of pollution. In this case abatement is relatively expensive and zero abatement will occur. Finally, when S=0, the shadow values are equated and active, but not necessarily maximal, abatement will occur. Therefore, the value of S determines when and if the economy switches from a zero-to-active-to-maximal abatement regime. We deal with these possibilities in turn.

Regardless of the value of S, the optimal level of consumption will always satisfy

$$\frac{\partial H}{\partial C} = C^{-e} - l_1 = 0 \tag{1.14}$$

although the shadow value of capital and its dynamic path may differ across regimes.

When S > 0, maximal abatement occurs and the dynamics are given by:

$$\begin{aligned}
&S > 0 \\
&q = q^K, q^K \equiv 1/a \\
&\dot{l}_1 = -g\, l_1,\ \ g \equiv A[1 - q^K] - d - r \\
&\dot{K} = [g + r]K - C(l_1),\ K(0) = K_0,\ \ C(l_1) \equiv l_1^{-1/e} \\
&\dot{l}_2 = l_2[r + h] + BX^{g-1} \\
&\dot{X} = -h X,\ X(0) = X_0
\end{aligned} \tag{1.15}$$

By choosing the intensity of abatement θ = θ^K there are no net emissions of pollution and the environment improves at a rate given by natural regeneration. We dub θ^K "the Kindergarten rule" because when economies adopt the Kindergarten rule pollution is cleaned up when it is created. [12]

Alternatively, S may be exactly zero. In this case we have an interior solution for abatement, with the following dynamics:

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[12] This is one of the most common rules taught in Kindergarten. For a list of common Kindergarten rules see <u>All I Really Need to Know I Learned in Kindergarten: Uncommon Thoughts on Common Things</u> by Robert Fulgham. Fulgham argues that the basic values we learned in grade school such as "clean up your own mess" (in effect our Kindergarten rule) and "play fair" are the bedrock of a meaningful life.

$$
\begin{aligned}
&S = 0 \\
&\theta \in [0, \theta^K] \\
&\dot{\lambda}_1 = -g\,\lambda_1 \\
&\dot{K} = [A[1-\theta] - \delta]K - C(\lambda_1),\ K(0) = K_0,\ \ C(\lambda_1) \equiv \lambda_1^{-1/\varepsilon} \\
&\dot{\lambda}_2 = \lambda_2[\rho + \eta] + BX^{g-1} \\
&\dot{X} = AK\left[1 - \alpha\theta\right] - \eta X,\ X(0) = X_0
\end{aligned}
\tag{1.16}
$$

In this situation pollution is not completely abated, and hence the evolution of environmental quality reflects both the level of active abatement and natural regeneration. And finally, with no abatement at all we must have S < 0 yielding:

$$
\begin{aligned}
&S < 0 \\
&\theta = 0 \\
&\dot{\lambda}_1 = -\lambda_1[A - \delta - \rho] - \lambda_2 A, \\
&\dot{K} = [A - \delta]K - C(\lambda_1),\ K(0) = K_0,\ \ C(\lambda_1) \equiv \lambda_1^{-1/\varepsilon} \\
&\dot{\lambda}_2 = \lambda_2[\rho + \eta] + BX^{g-1} \\
&\dot{X} = AK - \eta X,\ X(0) = X_0
\end{aligned}
\tag{1.17}
$$

Consider growth paths with active abatement. Then from (1.16) and (1.15) we find the shadow value of capital falls over time at a constant exponential rate:

$$
-g \equiv \frac{\dot{\lambda}_1}{\lambda_1} = -\left[A[1-\theta^K] - \rho - \delta\right] < 0, \tag{1.18}
$$

provided the net marginal product of capital, at the Kindergarten rule level of abatement, $A[1-\theta^K]$, can cover both depreciation and impatience. We leave for now a detailed discussion of what this requires and assume it is true: g > 0. Then it is immediate that consumption rises at the constant rate $g_C = g/\varepsilon > 0$.

From the capital accumulation equations in both (1.15) and (1.16) we can now deduce that capital and output must grow at the same rate as consumption if θ is constant over time. To determine whether the intensity of abatement *is* constant over time, consider the accumulation equation for pollution:

$$\dot{X} = AK[1-\theta\alpha]-\eta X \tag{1.19}$$

There are two ways (1.19) can be consistent with balanced growth. The first possibility is that we have a maximal abatement regime where S > 0 holds everywhere along the balanced growth path. In this situation, K grows exponentially over time and θ is set to the Kindergarten rule level. Using (1.15), this balanced growth path must have:

$$\dot{X} = -\eta X \quad and \quad \theta = \theta^{K} \tag{1.20}$$

In this scenario, the environment improves at the rate η over time and abatement is a constant fraction of output $1 > \theta^K > 0$. As time goes to infinity the economy approaches a pristine level of environmental quality. Therefore the balanced growth path exhibits constant growth in consumption, output, capital and environmental quality. Consumption is a constant fraction of output and we have:

$$g_C = g_K = g_Y = \gamma/\varepsilon > 0,\ g_X = -\eta < 0 \tag{1.21}$$

A second possibility is that abatement is active but not maximal. Define the deviation of abatement from the Kindergarten rule as $D(\theta) = (\theta^K-\theta)/\theta^K$. Using this definition rewrite (1.19) to find:

$$\frac{\dot{X}}{X} = \frac{AK[D(\theta)]}{X} - \eta \tag{1.22}$$

It is apparent that if the deviation of abatement from the Kindergarten rule fell exponentially, then it may be possible for X to fall exponentially while K rises. That is, in obvious notation, a possible balanced growth path would have:

$$g_K + g_D = g_X < 0 \tag{1.23}$$

In this situation abatement is at an interior solution at all times and becomes progressively tighter over time approaching the Kindergarten rule asymptotically. The inflow of pollution from production into the environment is always positive but environmental quality improves nevertheless. This intuitive description suggests that the possibility of this outcome must rely on both the pace of economic growth and the ability of the environment to regenerate. This is indeed the case as we now show.

To do so it proves useful to assume S=0 and explore its implications. When S = 0, the shadow values for capital and pollution are related by:

$$l_1 = -a l_2 \tag{1.24}$$

which implies, since *a* is constant, that the rate of change of the shadow cost of pollution must fall over time at the same rate the shadow value of capital falls over time. From (1.18) this rate is g. Now using (1.16) and the fact that the time rates of change and levels of the shadow prices are pinned down by (1.24), we can show that the instantaneous marginal rate of substitution between consumption and pollution must remain constant along the balanced growth path. That is, we have:[13]

$$\frac{1}{a} = \frac{BX^{g-1}C^{e}}{h+g+r} \tag{1.25}$$

[13] Note this is just the equality between marginal abatement costs, 1/a, and the marginal damage from pollution. We show that this implies a unique K for every X at the start of Stage II and call this relationship the Switching Locus. See (1.32).

Therefore, when abatement is active, but not maximal, we must have:

$$\frac{\dot{X}}{X} = \frac{e}{1-g}\frac{\dot{C}}{C} = \frac{g}{1-g} < 0 \tag{1.26}$$

As consumption rises along the balanced growth path the environment improves. The only question remaining is whether the growth rate given by (1.26) is in fact consistent with remaining in the S=0 abatement regime.

To investigate solve the state equation for X from (1.16) to find the transition of θ towards the Kindergarten rule. Manipulating the state equation from (1.16) and exploiting our definition of D(θ), we find:

$$D(q) = \frac{X}{AK}\left[\frac{h(g-1)-g}{(g-1)}\right] \tag{1.27}$$

D(θ) must be non-negative while X and K are always positive. Together these imply that a necessary condition for us to remain in a S=0 regime is simply:

$$h(g\text{-}1)>\mathrm{g} \tag{1.28}$$

This condition reflects two different requirements. The first is simply that γ cannot equal one. If it did then the (instantaneous) marginal disutility of pollution is a constant and λ_2 is a constant as well. To see this manipulate (1.25) and (1.24) to find:

$$l_2(t) = -\frac{BX(t)^{g-1}}{h+g+r} < 0 \tag{1.29}$$

hence when γ is one, the shadow cost of pollution is independent of time. This would also imply, if S=0 from (1.24), that consumption be fixed as well. This is inconsistent with growth of any sort.

Assuming γ not equal to one is necessary for balanced growth with an interior solution for abatement. But a second condition must also hold. Natural regeneration, η, must be sufficiently large relative to the growth rate g. If the rate of regeneration is high and growth rates quite low, then the optimal plan is to use nature's regenerative abilities to partially offset the costs of abating because the shadow value of foregone output is high in slow growth situations. Conversely, if regeneration is low and the growth rate g relatively high, then no amount of abatement short of the Kindergarten rule will hold pollution to acceptable levels.

Finally we need to rule out a zero abatement balanced growth path. To do so we assume balanced growth occurs and use this to generate a contradiction. Note the last equation of (1.17) shows balanced growth in X requires equal growth in both capital and pollution. Since positive growth in the economy requires $g_K > 0$, it also implies:

$$g_X \equiv \dot{X}/X = \frac{AK}{X} - h, \Rightarrow g_X = g_K > 0 \tag{1.30}$$

But $g_X > 0$ implies a monotonically rising shadow cost of pollution that moves us towards an active abatement regime. To complete the argument use S < 0 to find:

$$\dot{l}_1 = -[g + x(t)]l_1 < 0, x(t) \equiv A[l_1 - l_2 a]/a > 0 \tag{1.31}$$

where g > 0 is required for growth. The shadow value of capital is falling while the shadow cost of pollution is rising and S < 0 is contradicted in finite time. Summarizing:

<u>Proposition 1</u>. Assume g > 0, then sustainable economic growth with an ever improving environment is possible and optimal.

i) If $\eta(\gamma-1) > g$, then the intensity of abatement approaches the Kindergarten rule level of abatement, θ^K, asymptotically.

ii) If $\eta(\gamma-1) < g$, then $\theta = \theta^K$ everywhere along the balanced growth path.

iii) Balanced growth with zero abatement is not optimal.

Proof: See Appendix.

Proposition 1 suggests a natural corollary for the case of flow pollutants. If pollution has only a flow cost it is "as if" the environment is regenerating itself infinitely fast. This intuition suggests that as we let η get large, our results in the stock pollutant case should replicate those for a flow. This intuition is, in fact, correct.

Proposition 2. Assume $g > 0$ and X is a flow pollutant, then sustainable economic growth with an ever improving environment is possible and optimal.

iv) If $(\gamma-1)>0$, the intensity of abatement approaches the Kindergarten rule level of abatement, θ^K, asymptotically.

v) If $(\gamma-1)=0$, $\theta = \theta^K$ everywhere along the balanced growth path.

Proof: See Appendix.

Propositions 1 and 2 are important in showing how the Kindergarten rule generates sustainable growth. The requirement for sustainable growth, $g > 0$, is very similar to that in standard endogenous growth models. The assumption $g > 0$ requires the marginal product of capital, adjusted for the ongoing costs of abatement, be sufficiently high. A necessary condition is that $A[1-\theta^K]$ be positive, but this is guaranteed as long as abatement is a productive activity. Recall that abatement, like all other economic activities, pollutes. One unit of abatement creates one unit of pollution, but cleans up $a > 1$ units of pollution. It is only this surplus between costs and benefits, $1-1/a = 1- \theta^K > 0$ that makes abatement useful at all!

Given abatement is productive, we still require the adjusted marginal product of capital, $A[1-\theta^K]$, to offset both impatience and depreciation. If abatement is not very productive, then $1/a$ will be close to one and growth cannot occur. If capital is not very productive or if the level of impatience and depreciation are high then ongoing economic growth cannot occur. These are however very standard requirements for growth under any circumstances; therefore our addition of the further requirement that abatement be productive seems both innocuous and natural in our setting.[14]

4. Empirical Implications

The model we have presented relies heavily on the assumed role of technological progress in staving off diminishing returns to both capital formation and abatement. It is impossible to know apriori whether technological progress can indeed be so successful and hence it is important to distinguish between two types of predictions before proceeding. The first class of predictions are those regarding behavior at or near the balanced growth path. This set has received almost no attention in the empirical literature on the environment and growth, although balanced growth path predictions and their testing are at the core of empirical research in growth theory proper (see the review by Durlauf and Quah (1999)). The second set of predictions concern the transition from inactive to active abatement and these are related to the empirical work on the Environmental Kuznets Curve (See Grossman and Krueger (1993,1995) and the review by Barbier (2000)).

[14] In Keeler, Spence and Zeckhauser (1972) a similar condition describes their Golden Age capital stock. In their model with no endogenous growth the Golden Age capital stock is defined by (in our notation) the equality $f'(K)[1-1/a]-\delta = \rho$; simulations of the model assume a to be 12 (see p.22). Chimeli and Braden (2002) assume a similar condition. Both studies assume abatement or clean up is a linear function of effort thereby ignoring the reality of diminishing returns and the necessity of ongoing technological progress.

4.1 Balanced Growth Path Predictions

Using our previous results it is straightforward to show that near the balanced growth path we must have: convergence in the quality of the environment across all countries sharing parameter values but differing in initial conditions; the share of pollution abatement costs in output approaching a positive constant less than one; overall emissions rates falling and environmental quality rising; and emissions per unit output falling as production processes adopt methods that approach zero emission technologies.

An especially sharp prediction is that the rate of change in per capita income is uncorrelated with the rate of change in environmental quality when pollutants are long-lived. They are negatively correlated when they are short-lived.[15] To see why let η be small. In this case, the fast growth scenario arises, and the environment improves at rate η. If we now vary g, but remain in the fast growth scenario, this has no effect on the pace of environmental improvement. Note however that countries with higher g's produce greater income gains over any period of time.

This result is counter-intuitive since, as we will show subsequently, higher levels of income are always associated with higher levels of environmental quality along the balanced growth path. Nevertheless the logic is simple: in the fast growth scenario, the constraint on environmental improvement is not income, but rather technology. Since we have assumed that net inflows of pollution cannot be negative – and hence no restoration activities can occur - improvement can at most occur at rate η. This result highlights the fact that income growth may not be the constraining factor in some circumstances. The

[15] The reader should note that in most cases the ratio of η/g is critical in determining these results, and not the level of either η or g.

rate of environmental improvement can be limited by other factors – such as the availability of better technologies - so that the rate of income growth can be uncorrelated with the rate of environmental improvement even though income levels and environmental quality are positively related. [16]

Conversely, when η is large, the slow growth scenario arises and environmental improvement occurs at rate $g/(\gamma-1)$. In this case, faster growth implies faster environmental improvement. This faster growth could occur from a higher marginal product of capital or abatement, or increased patience. This result is surprising since we would normally associate fast growth with large increases in pollution unless there was a very strong demand for a clean environment (which we have not assumed). Faster growth is linked with faster environmental improvement in this case because income growth *is* the limiting factor. When g rises the shadow value of capital falls very quickly and this is, of course, the opportunity cost of abatement.

Whether these predictions are borne out by empirical work is as yet unknown but an examination of US data shows the model's strongest predictions – those regarding falling emissions and improving environmental quality - are not grossly at odds with available U.S. data. For example, the U.S. EPA reports that over the 1971 to 2001 period, total emissions of the 6 criteria air pollutants (Nitrogen Dioxide, Ozone, Sulfur Dioxide, Particulate matter, Carbon Monoxide, and Lead)[17] decreased 25%. Over this

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[16] A suggestive plot of growth rates versus the rate of improvement in an environmental index can be found in Figure 2.3, p. 27 of the World Bank's 2003 Development Report. The index aggregates across measures of deforestation, water pollution and CO2 emissions. The plot shows no relationship between the two rates of change. In contrast, Carson et al. (1997) finds a negative and significant relationship between the rate of state income growth and the rate of decline in emissions of air toxics.

[17] All US figures are taken from the EPA's Latest Findings on National Air Quality, 2001 Status and Trends, available at http://www.epa.gov/air/aqtrnd01/ published September 2002. Corresponding measures of environmental quality (ground level concentrations of these same pollutants) have also been improving.

same period, gross domestic product rose 161%. Looking at the individual pollutants only nitrogen oxide emission rose over the 1971 period and that by only 15%. In contrast emissions of carbon monoxide fell 19%, volatile organic compounds 38%, sulfur dioxide 44%, particulates 76% and lead 98%. Moreover, abatement costs as a fraction of output are both small and exhibit no strong trend over time as our analysis would suggest. For example, if we consider the size of pollution abatement costs specifically directed to air pollutants and scale this by real US output, the ratio is incredibly small – approximately one half of one percent of GDP - and has remained so for over twenty years (See Vogan (1996)). Moreover, the share of pollution abatement costs in manufacturing value-added has remained a relatively small 1-2%. Changes in the composition of U.S. output surely play some role in these figures, but it is difficult to eliminate the inference we draw: abatement efforts have become much more productive over time.

This partial snapshot of the US environmental record is not meant to imply that environmental quality is always and everywhere rising in the US or elsewhere.[18] Nor should it be as our framework suggests that some environmental problems will be addressed while others are left ignored. Looking across pollutants we would expect this variability whenever pollutants differ in their abatement costs, their rates of dissipation in the environment and their marginal damage.[19] Active regulation of the six criteria

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[18] One important omission from the above is any discussion of air toxics such as benzene (in gasoline), perchloroethylene (used by dry cleaners), and methyl chloride (a common solvent). These are chemicals are believed to cause severe health effects such as cancer, damage to the immune system, etc. At present the EPA does not maintain an extensive national monitoring system for air toxics, and only limited information is available. Figures from the US National Toxics Inventory show emissions nationwide have fallen 24% from the baseline (1990-1993) to 1996. Emissions of 33 toxics, believed to be the most deleterious to human health, have fallen 31%. See U.S. EPA, Toxic Air Pollutants, available at www.epa.gov/airtrends/toxic.html.

[19] It will become clear as we proceed that if we treat the damage from different pollutants separately in utility and assume abatement is separable in pollutants, then for each pollutant the planner solves a problem

pollutants would not imply abatement of all pollutants everywhere. Our point is simply that the historic trends of these most regulated pollutants are roughly consistent with our model's balanced growth path predictions.

4.2 The Environmental Catch-up Hypothesis

We have so far focused on balanced growth paths and these always exhibited active abatement. It is however natural to ask whether abatement is always undertaken even off the balanced growth path. The analysis suggests quite strongly that it is not.

To start we present several transition paths in Figure 1. One of these paths is that of a Poor country having small initial capital K^P but a pristine environment. The other is the path of a Rich country starting again with a pristine environment but with a much larger initial capital K^R. Each economy starts with a pristine environment in stage I and grows. During this stage the environment deteriorates, X rises, and the capital stock grows until the trajectory hits the Switching Locus labeled SL. Once the economy hits the Switching Locus it begins stage II. The exact position and shape of the locus depend on whether parameters satisfy the fast growth or slow growth scenario. For the most part we will proceed under the assumption that economic growth is fast relative to environmental regeneration; that is (1.28) fails strongly and we have $\eta(\gamma-1)< g$. This implies $\theta(t) = \theta^K$ everywhere along the balanced growth path (Figure 1 implicitly assumes this result). For illustrative purposes we will sometimes discuss the parallel flow case (where we can think of η approaching infinity but (1.28) failing because $\gamma = 1$).

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identical to the one we discuss here. This would generate a theory of optimal sequencing of abatement across pollutants with different characteristics.

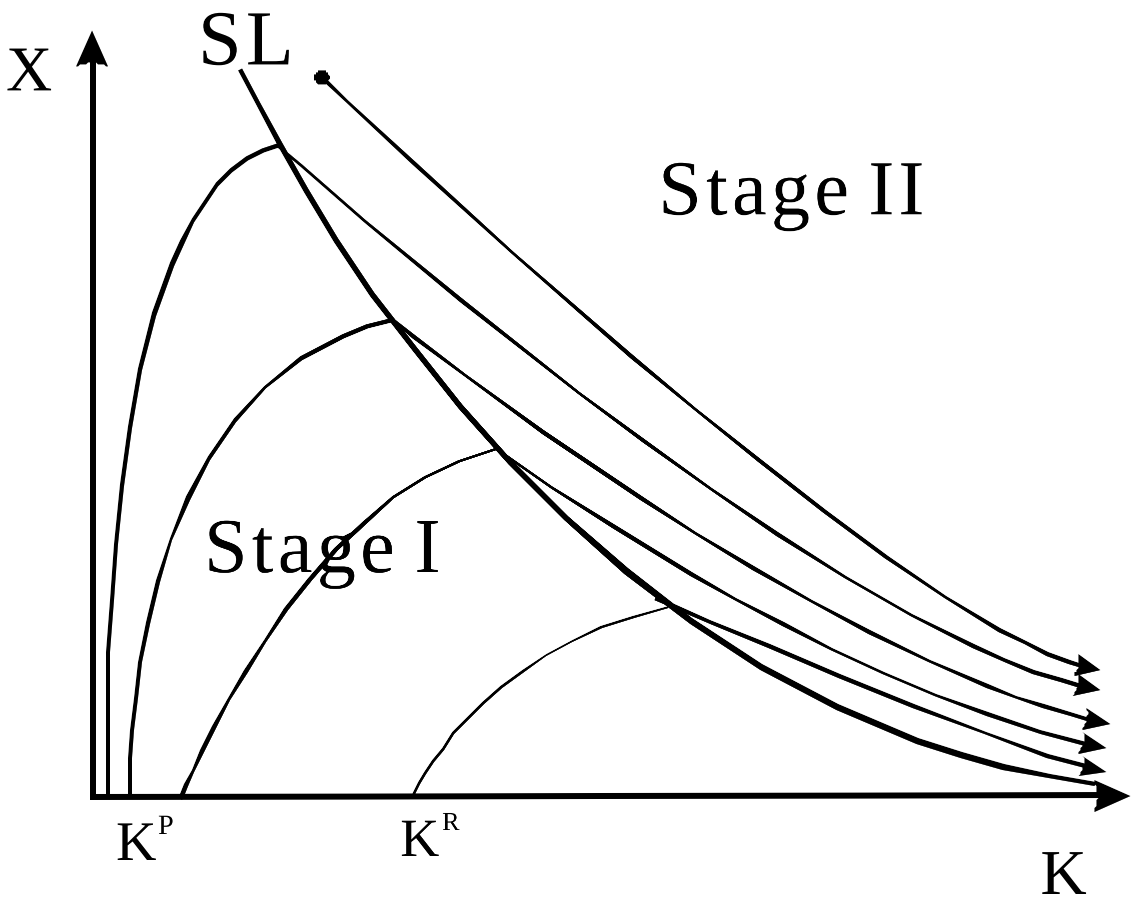

Figure 1
Transition Paths

It is apparent from the figure that the Poor country experiences the greatest environmental degradation at its peak, and at *any* given capital stock, (i.e. income level) the initially Poor country has worse environmental quality than the Rich. Moreover, since both Rich and Poor economies start with pristine environments, the qualities of their environments at first diverge and then converge. This is our Environmental Catch-up Hypothesis.

Divergence occurs because the opportunity cost of abatement (and consumption) is much higher in capital poor countries. A high shadow price of capital leads to less consumption, more investment and rapid industrialization in the Poor country. Nature's ability to regenerate is overwhelmed. The quality of the environment falls precipitously. In capital rich countries the opportunity cost of capital is lower: consumption is greater and investment less. Industrialization is less rapid and natural regeneration has time to work. The peak level of environmental degradation in the Rich country is therefore much smaller.

But once we enter Stage II abatement is undertaken and since abatement is an investment in improving the environment, it is only undertaken when the rate of return on this investment equals (or exceeds) the rate of return on capital. Since economies are identical, except for initial conditions, rates of return are the same across all countries in Stage II. Equalized rates of return require equal percentage reductions in the pollution stock. Therefore absolute differences in environmental quality present at the beginning of Stage II disappear over time.

To verify these assertions we start with Stage II.

4.3 Stage II

Stage II starts at the moment the economy's shadow prices for capital and pollution satisfy (1.24) and abatement begins. Denote this transition time t=t* and the associated capital and pollution stocks at the moment of transition K* and X*. Recall the shadow values in (1.24) are partial derivatives of the value function W(K,X) evaluated at K*, X*. Where W(K*,X*) is the maximized value of the program starting from capital and pollution stocks {K*, X*} over the period t = t* until $t = \infty$. This implies the set of {K*, X*} at which a transition to active abatement occurs must satisfy:

$$-\frac{\partial W(K^*, X^*)/\partial K}{\partial W(K^*, X^*)/\partial X} = \frac{l_1(K^*, X^*)}{-l_2(K^*, X^*)} = a \tag{1.32}$$

We refer to the set of {K*,X*} satisfying (1.32) as the Switching Locus, since any trajectory of the economy switches from inactive to active abatement when it crosses it.[20] The Switching Locus represents the set of points where the equality of marginal abatement costs, MAC, and marginal damage, MD(K,X), yields an interior solution for abatement.

To go further we need to solve for the shadow values as functions of capital and pollution stocks and then impose (1.24) to find the Switching Locus. We have denoted the switching time as t*, and hence at t* the S > 0 set of dynamics come into play. Using (1.15) solve the differential equation for the shadow value of capital. Eliminate consumption and then solve for the capital stock. This yields:

[20] Trajectories in the fast growth case cross the Switching Locus; trajectories in the slow growth case enter a similar Switching Locus and then follow it thereafter. We discuss the slow growth case in section 4.5.

$$\lambda_1(t) = \lambda_1(t^*)\exp[-gt]$$
$$K(t) = \exp[[g+\rho]t]\left[K^* - \lambda_1(t^*)^{-1/\varepsilon}\int_{t^*}^{t}\exp[-[g(1-1/\varepsilon)+\rho]s]ds\right] \tag{1.33}$$

where both K* and $\lambda_1(t^*)$ are unknowns. We can eliminate one of these unknowns by using the transversality condition, TVC, $\lim_{t\to\infty}[\exp[-\rho t]\lambda_1(t)K(t)] = 0$. Subbing (1.33) into the TVC we find it requires:

$$\lim_{t\to\infty}\left\{\left[K^* - \lambda_1(t^*)^{-1/\varepsilon}\int_{t^*}^{t}\exp[-[g(1-1/\varepsilon)+\rho]s]ds\right]\right\} = 0, \tag{1.34}$$

Integrating and solving (1.34) gives us the solution for $\lambda_1(t^*)$ as:

$$\lambda_1(t^*) = [hK^*]^{-\varepsilon},\ h \equiv g(1-1/\varepsilon)+\rho > 0 \tag{1.35}$$

and using (1.35) in (1.33) solves for $\lambda_1(t)$. It is important to note the shadow value of capital is only a function of K* and not {K*,X*}. This is because, in the fast-growth-slow-regeneration case, the economy's shadow value of capital falls so rapidly that abatement efforts are always maximal. Since abatement is maximal, the {K, λ_1} part of the problem separates and solves a simple AK model with power utility. Not surprisingly it is now possible to show that the condition h > 0 is equivalent to the standard sufficient condition for existence of an optimum path for an AK model with power utility.[21]

With the $\lambda_1(t^*)$ in hand we now turn to solve for the shadow cost of pollution, $\lambda_2(t^*)$. Assuming we remain in S > 0 forever, we can solve the state and co-state equations in this case to yield:

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[21]Denote the growth rate in an AK model with power utility by g*, then in terms of our parameters we have $g^* = g/\varepsilon$ and the standard condition is $\rho+(\varepsilon-1)g^*>0$. This is equivalent to h>0. See Aghion and Howitt (1998, Equation (5.3)).

$$X(t) = X^* \exp[-h(t-t^*)],$$
$$l_2(t) = \exp[[r+h](t-t^*)][[l_2(t^*) + BX^{*g-1}[1-\exp[-c_1(t-t^*)]/c_1] \quad (1.36)$$
$$c_1 \equiv r + gh$$

where $\lambda_2(t^*)$ and X^* are unknowns. We can again use a transversality condition to eliminate one unknown. Using $\lim_{t\to\infty}[\exp[-rt]\, l_2(t)\, X(t)] = 0$ and manipulating yields:

$$l_2(t) = l_2(t^*)\exp[[1-g]h(t-t^*)], \quad l_2(t^*) = -BX^{*g-1}/[r+gh] \quad (1.37)$$

The shadow cost of pollution falls at rate $\eta(\gamma-1)$, which in the fast growth case, is less than g, the rate the shadow value of capital falls. This implies we have abatement at its maximum – the Kindergarten rule level – because MAC < MD(K(t), X(t)) for t > t*.

We can now employ (1.35) and (1.37) in (1.24) and set t = t* to find a closed form expression for the Switching Locus. Manipulating to obtain familiar terms we find:

$$MAC = \frac{1}{a} = \frac{B}{[r+gh]} X^{*g-1}[hK^*]^e = MD(K^*, X^*) \quad (1.38)$$

which is a downward sloping and convex relationship between pollution and capital. The left hand side of (1.38) represents marginal abatement costs. The right hand side is marginal damage evaluated at {K*,X*}. Marginal damage is increasing in the pollution stock provided γ exceeds one, and since the flow of national (and per-capita) income Y is always proportional to K, it is apparent that the income elasticity of marginal damage with respect to flow income is given by $\varepsilon > 0$. Large values of ε correspond to the strong income effects referred to earlier.

The discussion thus far is incomplete because it has implicitly assumed two key points yet to be proven. The first is simply that a country starting in a situation below the Switching Locus – such as the natural initial conditions $K_0 > 0$ and $X_0 = 0$ – will indeed reach the Switching Locus and make the transition to Stage II. Proposition 1 indicates a balanced growth path is not possible with $S < 0$ but this does not imply every country must cross the Switching Locus in finite time. The second is that the locus does indeed represent an irrevocable change so that once we cross over to Stage II we remain there forever. These points are possible to establish and hence we record:

Proposition 3. Assume $g > 0$ and $g > \eta(\gamma-1)$, then:

i) every economy starting in Stage I must enter Stage II in finite time

ii) every economy in Stage II remains in Stage II.

iii) The locus of $\{K, X\}$ at which Stage I ends is given by (1.38)

Proof: See appendix

It is possible to extend this result to flow pollutants. To remain in the relatively fast growth case with $\eta(\gamma\text{-}1) < g$ we now assume $\gamma=1$ and find:

Proposition 4 Assume $g > 0$ and $\gamma = 1$, then:

iv) every economy starting in Stage I must enter Stage II in finite time

v) every economy in Stage II remains in Stage II.

vi) The Switching Locus is vertical at a finite $K^* > 0$.

Proof: See appendix

4.4 Slow Growth and Fast Regeneration

The previous analysis was undertaken under the fast-growth-slow-regeneration assumptions because this case illustrates the various forces at work very clearly. Since many of the same conclusions hold when growth is relatively slow we only provide a sketch here of some differences, leaving a complete analysis to the interested reader.

We can again solve for a Switching Locus which divides Stage I from Stage II. If we let W(K,X) represent the state valuation function the new Switching Locus is again defined by (1.32). The interpretation given to the Switching Locus remains the same. The most important difference is that once a trajectory of the system hits this new Switching Locus it remains within it forever. This implies the economy's choice of abatement remains interior; i.e. the trajectory follows along the Switching Locus maintaining MAC = MD(K,X) throughout. As time progresses the intensity of abatement approaches the Kindergarten rule in the limit.

Our transition path predictions remain virtually unchanged except that the model now predicts an especially strong form of convergence. All transition paths remain on the Switching Locus once active abatement begins; therefore policy active countries share the same path for environmental quality and income levels in Stage II.

Solving for the Switching Locus involves similar steps and is left to the appendix. The Switching Locus again defines a unique X* that is declining in K*. If the economy is below this locus then abatement is inactive and K rises at a rapid rate. X increases rapidly until the Switching Locus is reached in finite time. If the initial (K,X) is above the locus, maximal abatement is undertaken but this drives down the shadow cost of pollution very quickly and we again hit the locus, this time, from above. Once on the locus, countries remain trapped within it thereafter.

4.5 The ECH and the EKC

Proposition 3 and 4 prove that all economies follow the stage I – stage II life cycle. We have assumed $g > 0$ and the standard sufficient condition required in models of endogenous growth given here as $h > 0$ in (1.35). We have met the Aghion and Howitt test of generating sustainable growth while adopting the usual sort of assumptions made in endogenous growth models.

The model also presents an interesting new prediction, the ECH: countries differentiated only by initial capital exhibit initial divergence in environmental quality followed by eventual convergence.[22] Moreover, as Figure 1 makes clear countries make the transition to active abatement at different income *and* peak pollution levels. This of course throws into question empirical methods seeking to estimate a unique income-pollution path. More constructively it suggests that an important feature of the data may well be a large variance in environmental quality at relatively low-income levels with little variance at high incomes. Empirical work by Carson et al. (1997) relating air toxics to U.S. state income levels is supportive of this conjecture:

> "Without exception, the high-income states have low per-capita emissions while emissions in the lower-income states are highly variable. We believe that this may be the most interesting feature of the data to explore in future work. It suggests that it may be difficult to predict emission levels for countries just starting to enter the phase, where per capita emissions are decreasing with income", p. 447-8.

In some cases however, (initially) Rich and Poor will make the transition at the same income level but still exhibit our Catch-up Hypothesis. Let γ approach one. Then the slope of the Switching Locus approaches infinity and all countries attain their peak pollution levels at the same K*. From (1.38), at γ equal to one, we have:

$$K^* = \frac{1}{[g(1-1/e)+r]}\left[\frac{[r+h]}{aB}\right]^{1/e} \tag{1.39}$$

[22] It is possible to show X is rising throughout Stage I and this ensures points along the Switching Locus do indeed represent peaks in pollution levels.

Even with a common turning point differences in environmental quality remain. Moreover, these are not simple level differences because countries initially diverge and then converge after crossing K* as shown in Figure 2.

To eliminate our catch-up hypothesis we must assume regeneration is infinitely fast: X is a flow. The Switching Locus is now vertical at K* and given by:[23]

$$K^* = \frac{1}{[g(1-1/e)+r]}\left[\frac{1}{aB}\right]^{1/e} \qquad (1.40)$$

Since pollution is proportional to production before K*, and policies are identical after K*: initial conditions no longer matter.

This tells us that when pollution is strictly a flow, all countries share the same income pollution path. We have generated an EKC, and empirical methods used to estimate a unique income-pollution path are appropriate. But when pollution does not dissipate instantaneously, initial conditions matter. We have the Environmental Catch-up Hypothesis, and empirical methods must now account for the persistent role of initial conditions.[24] It is easy to see the two hypotheses are mutually exclusive and exhaustive.

We now ask whether the process of catch-up begins at low or high-income levels. One important consideration is the income elasticity of marginal damage. To illustrate its role consider the fast growth regime and let the gross marginal product of capital, A, rise.

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[23] The turning point in the flow case is related to that of the stock in a transparent manner. Recall from (1.37) that the shadow cost of pollution is equal to the perpetuity value of the constant marginal disutility of – B (when γ=1). The perpetuity value arises since any unit of pollution has a constant instantaneous cost of –B but this cost is discounted by time preference, ρ, and eliminated by natural regeneration at rate η from the present time until infinity. Therefore, the perpetuity value of any unit of pollution is simply $-B/[\rho+\eta]$. Once pollution is a flow, it is eliminated immediately from the environment and has only a current flow cost of –B. Not surprisingly then (1.40) only differs from (1.39) by the absence of the perpetuity term.

[24] This result may explain why empirical research investigating the EKC has been far more successful with air pollutants like SO_2, than with water pollutants or other long lasting stocks (see the review by Barbier (2000)).

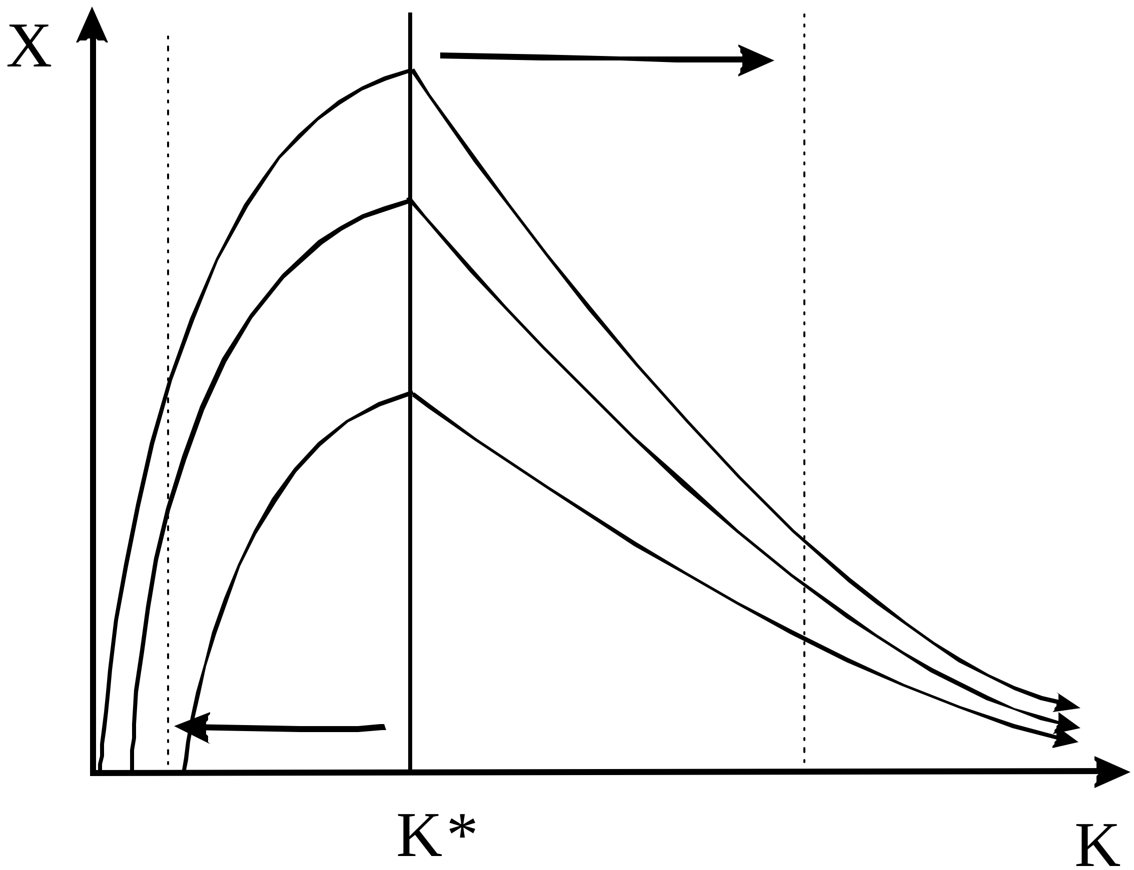

Figure 2
Common Turning Point

This necessarily raises g and if $\varepsilon > 1$, the Switching Locus shifts in. Abatement is hastened. The same result holds with constant marginal damage, (1.39), and with a pure flow pollutant (1.40). In terms of Figure 2, peak pollution levels shift left. When $\varepsilon < 1$, abatement is delayed and peak pollution levels shift right.[25] A similar set of results holds for increases in the productivity of abatement although there is an additional conflicting force. From (1.39) it is clear that an increase in abatement productivity, holding g constant, necessarily lowers K* and hastens abatement. This is direct impact of higher productivity on marginal abatement costs. But to this we must add the income response created by the resulting change in g. When the income elasticity of marginal damage is greater than one both forces work to hasten abatement. When it is less than one, the result appears ambiguous. Therefore, in contrast to earlier work we find that while the income elasticity of marginal damage has an important role to play in determining *the income level* at which abatement occurs and the resulting *pollution level*, it plays virtually no role in determining if the environment will improve nor its rate of improvement.

4.6 Pollution Characteristics

The ECH focuses on cross-country comparisons in pollution levels, but says little about how predictions vary with pollutant characteristics. There is however good data in the U.S. and elsewhere that could be fruitfully employed to test within country but across pollutant predictions. We have already examined the role of income effects and the productivity of abatement, therefore we focus on the remaining pollutant characteristics: permanence in the environment, marginal disutility, and convexity of marginal damages.

[25] Our use of the terms delayed or hastened does not refer to calendar time, but rather to whether actions occur at a higher or lower income level.

Consider regeneration and start from a position where $\eta = 0$ (radioactive waste). Then it is immediate from (1.38) that the Switching Locus shifts outwards as we raise η. The response is to delay action and allow the environment to deteriorate further. Delay is obvious when the marginal disutility of pollution is constant, because (1.39) shows K* necessarily rises with η. Once we raise η sufficiently the economy enters the fast regeneration regime and here we find abatement delayed in another manner – it is introduced slowly by the now gradual implementation of the Kindergarten rule. Faster regeneration then implies that countries either begin abatement at higher income levels or allow their environments to deteriorate more before taking action. Surprisingly, fast regeneration will be associated with lower and not higher environmental quality - at least over some periods of time or ranges of income.[26]

A change in regeneration rates also affects the pace of abatement. If a pollutant has a long life in the environment, then once abatement begins it is clear that natural regeneration can play only a small role. Consequently the optimal plan calls for an initial period of inaction before starting a very aggressive abatement regime – the Kindergarten rule. Putting these two features together we find that very long-lived pollutants should be addressed early with their complete elimination compressed in time. It is optimal to delay action on short-lived pollutants and adopt only a gradual program of abatement. This description of optimal behavior is of course consonant with the historical record in

[26] An especially colorful example of delay in abatement caused by rapid natural regeneration is that of the City of Victoria in British Columbia. Every day, the Victoria Capital Regional District (CRD) dumps approximately 100,000 cubic meters of sewage into the Juan de Fuca Strait. Scientific studies have long argued that since the sewage is pumped through long outfalls into cold, deep, fast moving water there is no need for treatment. The CRD has always used these studies to delay building a treatment facility. Current plans are for secondary treatment to begin in 2020, but until then over 40 square kilometers of shoreline remains closed to shell fishing. Background information can be found at the Sierra Legal Defense fund site http://www.sierralegal.org/m_archive/1998-9/bk99_02_04.htm

several instances where long-lived chemical discharges and gas emissions were eliminated very quickly by legislation, whereas short-lived criteria pollutants have seen active regulation but not elimination over the last 30 years.

Pollutants also differ in their toxicity. The marginal disutility of toxics could exceed those classified as irritants, and damages from toxics may rise more steeply with exposure. The first feature of toxics implies their abatement should come early. This is clear from (1.38)-(1.40) where increases in B hasten abatement. Note that since exposure is rising with population density and falling with country size it is now also apparent that abatement should come early when population is dense and late when pollutants can be spread over large geographic areas. Surprisingly very convex marginal damages call for the gradual and not aggressive elimination of pollution. The logic is that any reduction in the concentration of toxics has a large impact on marginal damage. Therefore, only by lowering emissions slowly can we match a steeply declining value of marginal reductions with a falling opportunity cost of abatement. Therefore, although toxics may have large absolute negative impacts on welfare, this argues for their early, but not necessarily aggressive, abatement.

5. Alternative Approaches to Abatement

Despite several articles examining one-sector models it remained unclear whether sustained growth with non-decreasing environmental quality was possible in an AK framework. Early work in a one-sector framework by Smulders (1993) and Smulders and Gradus (1996) demonstrated how continuing economic growth and constant environmental quality are compatible. In constrast, Stokey (1998) demonstrated how continuing growth and constant environmental quality are not possible within a one-sector model. The difference lies in their assumptions on abatement. To verify this assertion start with (1.5), ignore knowledge spillovers, and work forward using now familiar steps to find:

$$P_i = Y_i^G f(1-q), f(q) \equiv [1-a(1,1-q)] \tag{1.41}$$

Stokey employs the specific function form for φ given by (1-θ)$^{\beta}$ for β > 1, and this implies a CRS abatement production function given by:

$$a(Y_i^G, Y_i^A) = Y_i^G[1-(1-Y_i^A / Y_i^G)^b] \tag{1.42}$$

Using (1.41) it is now easy to show abatement is subject to diminishing returns:

$$\partial P_i / \partial Y_i^A = -b(1-q)^{b-1} < 0, \partial P_i^2 / \partial Y_i^{A2} > 0$$

which implies marginal abatement costs are rising at the aggregate level. Setting θ = 0 we find the first unit of abatement lowers pollution by the amount β > 1, somewhat similar to our formulation where a(1,1) > 1. If we now combine (1.41) with (1.3), recall net output is (1-θ) times gross output, and introduce the variable z = 1-θ, we find the specification employed in Stokey (1998).

$$Y = AKz, P = AKz^b \tag{1.43}$$

Therefore, Stokey's (1998) result that growth is not possible follows from matching an AK aggregate production function with strictly neoclassical assumptions on abatement adopted from Copeland and Taylor (1994).

Comparing our approach to Smulders is more difficult because abatement is not specifically modeled and he considers a variety of formulations. By specializing his framework to the AK paradigm we find:

$$Y = aK, \ P = [K/A]^g \tag{1.44}$$

The first element is just a standard AK production function. The second relates what Smulders refers to as net or emitted pollution to the capital stock and abatement. If we employ (1.44) and solve for emissions per unit of gross output we find:

$$P / Y = [K / A]^{g} / a K \tag{1.45}$$

If the economy allocates a fixed fraction of its output to abatement, K/A is constant, and emissions per unit of gross output *fall* with the size of the economy. This reflects a strong degree of increasing returns. Moreover, the reader may note from (1.44) that pollution emitted goes to infinity as abatement goes to zero, which is inconsistent with pollution being a joint product of output. Therefore, Smulders and Gradus (1993, 1996) match AK aggregate production with assumptions on abatement ensuring increasing returns; and, in contrast with our specification, assume pollution is not a joint product of output.

6. Conclusion

The relationship between economic growth and the environment is not well understood: we have only limited understanding of the basic science involved – be it physical or economic – and we have very limited data. Because of these difficulties it is especially important to develop a series of relatively simple theoretical models to generate stark predictions testable with currently available data. To do so we developed a model relying on technological progress to drive long run growth and showed how growth is sustainable whenever the planner implements the Kindergarten rule – clean up your messes as they are created. This rule may be implemented in finite time or approached asymptotically.

We provided predictions concerning the path of environmental quality for pollutants differentiated by their toxicity, their permanence in the environment, and their immediate disutility. We have shown that changes in abatement productivity, the convexity of marginal damages, and regeneration rates all affect both the level and rate of change in environmental quality. Conversely changes in disutility and changes in the income elasticity of marginal damage have only level effects. These predictions can be evaluated by exploiting available data for the U.S. and other developed countries. We also introduced a novel Environmental Catch-up Hypothesis predicting eventual convergence in environmental quality across both Rich and Poor countries. Together these predictions widen the scope for empirical research considerably while offering a theoretically based alternative to the presently dominant EKC methodology.

To generate these results much simplification was required. Some of our strongest assumptions produced features that some may find undesirable; nevertheless the model's simplicity is a virtue since it has laid bare previously unknown features of the growth and environment relationship. And by paying special attention to abatement we demonstrate how sustainable growth need not rely on strong income effects, increasing returns to abatement, or the assumed existence of perfectly clean factors of production.

7. Appendix

Proof of Proposition 1

Start with part ii) and assume $\eta(\gamma\text{-}1) < g$ then given our analysis in the text all that remains to prove is that the first order condition dynamics of S > 0 and $\theta = \theta^K$ remain in the S > 0 set forever. To start write out the state/co-state equations for S > 0 to find:

$$\begin{aligned} &\dot{l}_1 = -l_1 g, \quad g \equiv A(1-q^k) - d - r \\ &\dot{K} = [g+r]K - C(l_1), \; K(0) = K_0, \; C(l_1) \equiv l_1^{-1/e} \\ &\dot{C}/C = g/e, \; C(t) = C(0)\exp[(g/e)t] \\ &\dot{X} = -hX, \; X(0) = X_0 \quad \dot{l}_2 = l_2[r+h] + BX^{g-1} \end{aligned} \tag{A.1}$$

In Section 4 of the text we solve these to obtain the switching locus and the shadow values. In doing so we implicitly assumed:

$$-a\,l_2(t) > l_1(t), \; for \; \forall t > t^S \tag{A.2}$$

where t^S is the switching time. Solving (A.1) for the shadow value of capital we have:

$$l_1(t) = l_1(0)\exp\{-gt\}, \; l_1(0) = [hK(0)]^{-e} \tag{A.3}$$

where the initial value can be found by using the transversality condition (see section 4 in the text). Similarly solve for the shadow cost of pollution to find:

$$l_2(t) = \frac{-BX(0)^{g-1}\exp\{(1-g)ht\}}{r+gh} \tag{A.4}$$

where again we have employed a transversality condition to pin down the initial value of the shadow price in terms of X(0). Hence using (A.3) and (A.4) we find that (A.2) requires:

$$\frac{BX(0)^{(g-1)}}{[r+gh][hK(0)]^{-e}} > \exp\{[-g-[1-g]h]t\} \tag{A.5}$$

for all t > t^S. By definition (A.5) is an equality when t = t^s. Using the definition of t^s and multiplying both sides of (A.5), we obtain:

$$1 > \exp\{-[g-(g-1)h][t-t^s]\} \tag{A.6}$$

which is true for t > t^s if and only if

$$h(g-1) < g \tag{A.7}$$

This is of course what we have assumed and this parameter restriction defines the fast growth-slow regeneration scenario. Note that when (A.7) fails – so we are in the fast regeneration-slow growth situation – and initial conditions put us above the switching locus so that S > 0, then (A.6) also implies the S > 0 dynamics drive the system to the switching locus and S=0 in finite time. This ends the proof to part ii).

Consider part i). The text does not show the economy approaches a balanced growth path with capital, output and consumption growing. To do so solve (1.19) for θ and substitute into the capital accumulation equation to obtain:

$$\dot{K} = [g+r]K + [X(0)/a]\exp\{((g/(1-g))t\} - C(0)\exp\{(g/e)t\} \tag{A.8}$$

where use has been made of the solution for λ_1(t) and the solutions for C(t) and X(t) given by:

$$\begin{aligned} C(t) &= C(0)\exp\{(g/e)t\} \\ X(t) &= X(0)\exp\{(-g/(g-1)t\} \end{aligned} \tag{A.9}$$

Now integrate (A.8) to find:

$$\begin{aligned} &K(t) = -\Pi_1 X(0)\exp\{((g/(1-g))t\} + \Pi_2 C(0)\exp\{(g/e)t\} + \Pi_3\exp\{(g+r)t\} \\ &\Pi_1 \equiv \left[\frac{(g-1)}{g\,g - r(1-g)}\right] > 0, \Pi_2 \equiv \left[\frac{e}{re - g(1-e)}\right] > 0 \end{aligned} \tag{A.10}$$

Multiply (A.10) by exp{-ρt}λ_1(t) and invoke the transversality condition, that:

$$\lim_{t\to\infty} l_1(t)K(t)\exp\{-r\,t\} = 0 \qquad \text{(A.11)}$$

This in turn implies

$$0 = \lim_{t\to\infty}\begin{bmatrix} -\Pi_1 X(0)\, l_1(0)\exp\{(gg/(1-g) - r)t\} \\ +\Pi_2 C(0)\, l_1(0)\exp\{(g(1-e)/e - r)t\} + \Pi_3 \exp\{gt\} \end{bmatrix} \qquad \text{(A.12)}$$

The limit of the term multiplying X(0) goes to zero as its exponent is negative. The limit of the term multiplying C(0) goes to zero as long as: g(1-ε)/ε -ρ < 0 which is a standard condition for the transversality condition to be met in AK models. Together they imply Π_3 must be zero in (A.10). We can rewrite this condition in a useful way to find:

$$K(t) = \exp\{(g/e)t\}\left[\Pi_1 X(0)\exp\{-e/(g-1)t\} + \Pi_2 C(0)\right] \qquad \text{(A.13)}$$

First note the term in square brackets approaches Π_2C(0) as time goes to infinity. This implies the growth rate of capital approaches that of consumption, g_C, in the limit. As the growth rate of capital and consumption converge, the abatement intensity must be approaching the Kindergarten rule. Therefore, for a given rate of consumption growth, g_C, the speed of convergence of θ(t) to the Kindergarten rule level is greater the larger is ε and the larger is γ. This makes a great deal of sense. In situations where ε is large, the marginal utility of consumption falls very rapidly and hence the sacrifice of marginal units of consumption for greater abatement is relatively painless. Similarly, when γ is large the marginal disutility of pollution is quite high and hence it is important to reduce it quickly. This is achieved by approaching the Kindergarten rule rapidly. Finally we must ensure the TVC for X(t) holds as well along this balanced growth path. Solving for λ_2(t) we find:

$$l_2(t) = \exp\{(r+h)t\}\left[l_2(0) + \int_0^t BX(s)^{g-1}\exp\{-(r+h)s\}ds \right] \qquad \text{(A.14)}$$

The TVC requires the limit of X(t)λ_2(t)exp{-ρt} go to zero as t goes to infinity. The solution for X(s) is given in (A.9). Substituting this into (A.14) and integrating shows the TVC requires:

$$\lim_{t\to\infty} X(0)\exp\{(h-g/(g-1))t\}\left[l_2(0) + \frac{BX(0)^{g-1}}{g+r+h}\left[1-\exp\{-(g+h+r)t\right] \right] = 0 \quad \text{(A.15)}$$

The first term multiplying the brackets goes to plus infinity since the exponent is positive in the fast regeneration-slow growth scenario we are investigating. Therefore, we are going to need the initial value of pollution to be given by:

$$l_2(0) = -\frac{BX(0)^{g-1}}{g+r+h} \qquad \text{(A.16)}$$

Finally we need to prove that a balanced growth path with no abatement is not possible. To do so, recall from the text that this requires a growth rate of pollution equal to that of capital. By solving for the shadow value of pollution we obtain:

$$l_2(t) = \exp\{(r+h)t\}\left[l_2(0) + BX(0)^{(g-1)}\int_0^t \exp\{[g_K(g-1)-(r+h)]s\}ds \right] \qquad \text{(A.17)}$$

and invoking the transversality condition yields:

$$\lim_{t\to\infty}\left[\begin{array}{l} X(0)\exp\{(h+g_K)t\}\left[l_2(0) - \dfrac{BX(0)^{g-1}}{g_K(g-1)-r-h} \right] \\ +X(0)\exp\{(g_K g - r)t\}\left[\dfrac{BX(0)^{g-1}}{g_K(g-1)-r-h} \right] \end{array} \right] = 0 \qquad \text{(A.18)}$$

Hence a necessary condition for the TVC to be met is $g_K\gamma<\rho$ as this sends the second term in brackets to zero. An additional condition is that $\lambda_2(0)$ take on a specific value to make the first term in the brackets zero. This in turn, implies, since this shadow value must be negative, that $g_K(\gamma-1)<\rho+\eta$. Using this information we can now write out the solution for the shadow value completely as:

$$l_2(t) = \exp\{g_K(g-1)t\}\left[\frac{BX(0)^{g-1}}{g_K(g-1)-(r+h)}\right] < 0 \tag{A.19}$$

and now note that it rises with t. Therefore, if S < 0 at some t, it will be contradicted in finite time since the shadow values move in opposite directions – exponentially.

This completes the proof of Proposition 1.

Proof of Proposition 2

The problem in the flow case is given by:

$$\begin{aligned} &Maximize \int_0^\infty \left[\frac{C^{1-e}}{1-e} - \frac{BX^g}{g}\right] e^{-rt}\, dt \\ &s.t.\ K(0) = K_0,\ \ and\ \ q \le 1/a \\ &\dot{K} = AK(1-q) - dK - C \\ &X = AK[1-qa] \end{aligned} \tag{A.20}$$

Forming the Hamiltonian we obtain:

$$rW(K) = \underset{\{C,q\}}{Max}\left\{H = \frac{C^{1-e}}{1-e} - \frac{B[AK[1-qa]]^g}{g} + l[AK[1-q] - C - dK]\right\} \tag{A.21}$$

with first order conditions,

$$\frac{\partial H}{\partial C} = C^{-e} - l = 0 \tag{A.22}$$

$$\underset{q}{Max}\left[-\frac{B[AK[1-qa]]^{g}}{g}+l\,AK[1-q],\ s.t.\,q\le 1/a\right] \qquad \text{(A.23)}$$

$$\dot{l}-r\,l=-\frac{\partial H}{\partial K}=-\left[\,l\,[A[1-q]-d]-BX^{g-1}A[1-qa]\right] \qquad \text{(A.24)}$$

Define the sets, S_-, S_0, S_+ as follows:

$$\begin{aligned} S_- &= \{(K,l)\,|\,G'(q=0)=aB(AK)^{g-1}-l<0 \\ S_0 &= \{(K,l)\,|\,G'(q=0)=aB(AK)^{g-1}-l=0 \\ S_+ &= \{(K,l)\,|\,G'(q=0)=aB(AK)^{g-1}-l>0 \end{aligned} \qquad \text{(A.25)}$$

Then note that when S_- is relevant, no abatement would occur at all. When S_0 is relevant abatement is just worthwhile. And when S_+ is relevant some abatement is necessary.

We start with the simple $\gamma = 1$ case. Assume $\{K,\lambda\}$ are in S_+. Then, under this parameter restriction, we obtain from the above that:

$$if\ G'(q=0)=aB-l>0,\,then\,q=q^{k} \qquad \text{(A.26)}$$

since marginal damage from pollution is a constant, abatement is either nothing, or it is set to the Kindergarten rule. When $\theta = \theta^K$ we have:

$$\begin{aligned} &l(t)=l(0)\exp\{-gt\} \\ &K(t)=\exp[(g+r)t][K(0)-l(0)^{-1/e}\int_0^t \exp\{-[g(1-1/e)+r]s\}ds] \end{aligned} \qquad \text{(A.27)}$$

Using the TVC we find $\lambda(0)=[hK(0)]^{-\varepsilon}$ where $h=g(1-1/\varepsilon)+\rho > 0$ is again the standard sufficient condition for the TVC to hold. To show that ongoing growth is possible in this situation, note that $\lambda(t)$ is a strictly decreasing function of K(0). Therefore choosing K(0) large enough we obtain must be in S_+. Once in S_+, the dynamics in (A.27) indicate that λ falls. Hence the dynamics are forward invariant. Balanced growth in consumption, capital and output occur at rates:

$$g_Y = g_K = g_C = \gamma/\varepsilon = [A[1-\theta^k] - (\delta + \rho)]/\varepsilon > 0 \qquad \text{(A.28)}$$

Suppose instead that K(0) puts us in S_-. Then θ = 0 from (A.25), and (A.27) holds if we replace $[1-\theta^k]$ with 1. Since the solution for λ shows it must fall exponentially over time approaching zero, there exists a finite time t at which S_+ is entered at which the dynamics shift to the case previously discussed. This ends the γ=1 proof.

Let γ not equal one, and assume we start in S_+. Then from (A.25) we have

$$\alpha B(AK[1-\theta\alpha])^{\gamma-1} - \lambda = 0 \qquad \text{(A.29)}$$

and $\theta=\theta^K$ is never optimal if λ is positive. Using G'(θ) = 0, we can now write the state-co-state equations as:

$$\begin{aligned} &\dot{\lambda} = -\gamma\lambda, \\ &\dot{K} = (\gamma+\rho)K + D\lambda^{1/(\gamma-1)} - C(\lambda), D \equiv (1/\alpha)[1/\alpha B]^{1/(\gamma-1)} \end{aligned} \qquad \text{(A.30)}$$

Using the TVC we can now obtain a relationship between the initial shadow value of capital and the initial capital stock. Manipulation of the TVC yields an implicit relationship between these two variables:

$$\begin{aligned} &K(0) + D\lambda(0)^{1/(\gamma-1)}/\alpha_1 = \lambda(0)^{-1/\varepsilon}/\alpha_2, \\ &-\alpha_1 \equiv -(\gamma+\rho) - \gamma/(\gamma-1), \alpha_2 \equiv \eta \end{aligned} \qquad \text{(A.31)}$$

Straightforward differentiation shows that λ(0) is a decreasing function of K(0). Therefore, for K(0) large we must be in S_+. Once in S_+, the dynamics given above shows λ falls exponentially over time leaving the solution in S_+. Note from (A.30) that consumption grows at rate $g_C=g/\varepsilon > 0$; and from the solution for λ(t) that as t goes to infinity, θ goes to θ^K as required. The growth rates of capital and output approach those of the γ equal to one case. Finally, suppose our initial conditions put us in S_-. Then it is again straightforward to show λ falls exponentially over time until some finite time t, such that S_+ is entered. This completes the proof for Proposition 2.

Proof of Proposition 3

Part ii) has already been proven in Proposition 1. Part iii) is proven in the text. We prove part i) by contradiction. Assume it is never optimal to leave Phase I. This requires we stay in S < 0 for all t, or that:

$$0 \le -a\,l_2(t) < l_1(t) \tag{A.32}$$

The difficulty in proving this is false arises from the fact that when S < 0 we cannot solve for either shadow value independently. Accordingly we adopt an indirect approach by bounding λ_1(t) from above and bounding λ_2(t) from below and then showing these bounds violate (A.32) generating the contradiction. Note since S < 0 is assumed throughout we must have an upper bound on λ_1(t) given by:

$$l_1(t) < l_1(0)\exp\{-gt\} \tag{A.33}$$

To bound λ_2(t) we start with its solution which is given by:

$$l_2(t) = \exp\{(r+h)t\}\left[l_2(0) + \int_0^t BX(s)^{g-1}\exp\{-(r+h)s\}\,ds \right] \tag{A.34}$$

Use the TVC to solve for the constant and find:

$$l_2(0) = -\int_0^\infty BX(s)^{g-1}\exp\{-(r+h)s\}\,ds \tag{A.35}$$

And hence the solution becomes:

$$l_2(t) = \exp\{(r+h)t\}\left[-\int_t^\infty BX(s)^{g-1}\exp\{-(r+h)s\}\,ds \right] \tag{A.36}$$

The solution for X(t) involves λ_1(t) so we bound λ_2(t) without solving for X(t) by noting that when S < 0:

$$\begin{aligned} \dot{X} &= AK - hX \geq -hX, \\ &\Rightarrow X(t) \geq X(0)\exp\{-ht\} \end{aligned} \tag{A.37}$$

Now substitute this solution into (A.36) since this bounds λ_2(t) from below. That is, we now have constructed the fictitious $\lambda_2(t)^*$ where:

$$\begin{aligned} -l_2(t) &\geq -l_2(t)^* = \exp\{(r+h)t\}\left[\int_t^\infty BX(0)^{g-1}\exp\{-(r+hg)s\}ds\right] \\ -l_2(t)^* &= \frac{BX(0)^{g-1}}{hg+r}\exp\{-h(g-1)t\} \end{aligned} \tag{A.38}$$

Now employing the inequality in (A.32), substituting using (A.33) and (A.38), and manipulating slightly we see that staying in S < 0 requires that for all t

$$\frac{aBX(0)^{g-1}}{hg+r}\exp\{g - h(g-1)t\} < l_1(0) \tag{A.39}$$

which is false for t sufficiently large since g > $\eta(\gamma-1)$.

Proof of Proposition 4

The proof of Proposition 2 demonstrates that λ(t) is monotonically falling during Stage II. This is shown by (A.27). Moreover when γ=1 we can find the unique capital stock at which Stage II begins. Note when Stage II begins we must have from (A.25) and γ=1, that λ = aB. By definition then, K* must satisfy λ=aB = $[hK^*]^{-\varepsilon}$ Therefore, once K* is reached the Stage II dynamics will keep it in stage II since λ(t) falls monotonically. Since K* is uniquely determined, the Switching Locus is vertical. To show that every economy must enter Stage II in finite time locate the dK/dt = 0 locus during Stage I. This locus must have $\lambda_1(t) = [(A-\delta)K]^{-\varepsilon}$ where we have denoted the shadow value in Stage I by the subscript 1. In stage II the dK/dt = 0 locus is given by $\lambda_2(t) = [(Az^k-\delta)K]^{-\varepsilon}$ where 2 stands for Stage II. Construct a phase diagram in $\{\lambda,K\}$space and note while both loci are necessarily downward sloping the Stage II dK/dt = 0 locus lies everywhere above that of the Stage I dK/dt = 0 locus. Also draw in the horizontal line λ=aB. K* is defined by the intersection of this horizontal line and the Stage II dK/dt = 0 locus. Notice for any K(0) < K*, a $\lambda(0)$ can be found such that Stage I dynamics carries the point (K(0), $\lambda(0)$) in finite time T to the point (K*, λ*). This is true because the steady state value of λ assuming we stay in Stage I, is less than aB when g > 0 which we have assumed. For t > T the Stage II dynamics apply as discussed above. To prove that this constructed candidate solution in (K,λ) space, that joins the two candidate Stage I and Stage II dynamics, is optimal, apply the sufficiency theory of Arrow and Kurz (1970, pp. 43-49). First the candidate solution satisfies the state/co-state equations by construction. The TVC is met in Stage II by construction of the Stage II solution of the state/costate equations. It is easy to see that the projection of the graph of Stage I in (K,λ) space covers the set (0,K*) on the K-axis as can be seen by the relative location of the switching locus aB = λ and dλ/dt = 0 in Stage I and by the construction of Stage I by running the Stage I state/costate dynamics backwards in time from the point (K*, λ*=aB). Thus we have a well-defined candidate solution for all positive initial K(0). Using the location of the locus aB = λ it is easy to see that $\theta = 0$ in Stage I and $\theta = \theta^K$ in Stage II. Thus the control θ actually optimizes H. The control C optimizes H. Hence we are done by sufficiency theory.

Switching Locus in the Slow Regeneration Case

We have from (A.16) one relationship between the shadow value of pollution and the pollution level when we move to Stage II. Along the Switching locus we also have from (1.32) in the text that:

$$dX/dt + hX = AK[1 - qa] = (g_X + h)X^* \exp\{g_X t\} \tag{A.40}$$

And the accumulation equation for capital gives us:

$$\begin{aligned} dK/dt &= AK[1-q] - C(l_1) \\ &= [g+r]K + \left[\frac{(g_X + h)X^*}{a}\right]\exp\{g_X t\} - l_1{}^{*-1/e}\exp\{(g/e)t\} \end{aligned} \tag{A.41}$$

Integrate (A.41) and use the transversality condition to obtain:

$$\left[\frac{l_1{}^{*-1/e}}{g(1-1/e)+r}\right] = K^* + \frac{(g_X + h)}{(g+r-g_X)(X^*/a)} \tag{A.42}$$

Along the Switching Locus we have $\lambda_1 = -a\lambda_2$ and using this and (A.16) we obtain:

$$\left[\frac{aBX^{*g-1}}{g+r+h}\right]^{-1/e} / \left[g(1-1/e)+r\right] = K^* + \left[\frac{g_X + h}{(g+r-g_X)(X^*/a)}\right] \tag{A.43}$$

Now recall $g_X = g/(1-\gamma) < 0$. Since $\eta(\gamma-1) > g$ together they imply $g_X + \eta > 0$ and the right hand side of (A.43) is an increasing function of X*. The left hand side is a declining function of X* since h > 0. Therefore we obtain a unique X* > 0 for each K*>0. Rearranging (A.43) yields the more friendly form:

$$\frac{1}{a} = \left[\frac{BX^{*g-1}}{g+r+h}\right]\left[hK^* + \frac{h(g_X + h)}{(g+r-g_X)(X^*/a)}\right]^e \tag{A.44}$$

The optimal solution may now be sketched. If (K,X) lies below the SL we put $\theta = 0$ and the FOC is increasing in K at rapid rate. X increases rapidly until SL is reached in finite time. If (K,X) lies above SL we put $\theta = 1/a$. The FOC values of K and C grow slowly.

Since $\eta(\gamma-1)>g$, the shadow value $a\lambda_2$ is falling faster than the shadow value of capital on the FOC solution above SL. Therefore SL will be hit in finite time and which θ is adjusted to keep the FOC dynamics in SL.

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