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Labor Mobility and Income Tax Competition

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Abstract

This paper provides a model of nonlinear income taxation in a context of international mobility. We consider two identical countries, in which each government chooses non-cooperatively redistributive taxes.

It is shown that when skilled workers can move at low cost, the income taxation does not involve distortions. When the cost to move becomes high for skilled workers, taxation policy is less redistributive but qualitatively similar to the taxation policy in autarky. Moreover, the mobility of the unskilled workers does not affect the income taxation when both countries have Rawlsian objectives.

Keywords: Fiscal Competition, Labor Mobility, Optimal Taxation, Mechanism Design.
JEL Classification: H21, H23, H77.

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Résumé

On se place dans un cadre traditionnel de taxation optimale du revenu. La population est constituée d'individus (qui sont à la fois contribuables et travailleurs) qui prennent des décisions quant à leur consommation et leur travail. Les contribuables peuvent changer de pays et profiter éventuellement d'une fiscalité plus avantageuse à l'étranger.

Le premier résultat est que si les deux gouvernements ont des objectifs rawlsiens (ce qui revient ici à maximiser l'utilité de l'individu le moins productif) les taxations d'équilibre sont indépendantes de la faculté des individus non productifs à changer de pays.

Le second résultat est que la taxation optimale, si le coût de déplacement des individus les plus productifs est assez faible, n'entraine pas de distortion dans l'offre de travail: les individus travaillent comme s'il n'y avait aucune taxation.

Enfin, si les gouvernments ne sont pas rawlsien, s'ils prennent en compte aussi le bien-être des individus les plus productifs, alors les possibilités de déplacement des individus les moins productifs vont aussi influencer les taxations d'équilibre.

Keywords: Concurrence fiscale, Mobilité internationale, Taxation optimale.
JEL Classification: H21, H23, H77.

1 Introduction

Common wisdom suggests that mobility across countries leads to a “race-to-the-bottom”: generous countries will see their low-skilled population increase and in the same time they their more skilled workers immigrate to escape hight tax rate. It will make redistribution and social programs more difficult. Thus, any economic integration would cause reduction in social programs, or less progressive tax schedules.Then one would conclude that free migration constrains the shape of possible redistribution schemes for each government.

This effect of international competition when factor are mobile has been studied in the literature[1], especially for capital income taxation. The primary objective of this paper is to provide a model of income tax competition between two countries when workers are mobile.

Most papers on income tax competition focus on linear income taxes [2]. We depart from these papers by considering nonlinear income taxes. This makes the analysis more complex and general results difficult to obtain. But, as shown by Diamond (1998), the distribution of population between more or less skilled workers affects the shape of the optimal income taxes. Therefore the possibility of migration may have different impacts on marginal income rates set by the government, which cannot be seen if we consider only linear income tax.

Optimal nonlinear taxation usually consists in transferring income from the rich to the poor. In this context Bertrand competition between governments leads to a unsatisfactory equilibrium: trying to attract skilled worker in order to increase tax revenue, each government has incentives to reduce the tax rate on high wages. If there is no restriction on mobility, the “laissez-faire” is the only outcome of the competition game. Using Swiss data Kirchgässner and Pommerehne (1996) show that fiscal competition has empirically an impact. Their results strongly suggest that high income earners choose their place of residence depending on the amount of income tax they have to pay. But they also show that even if fiscal competition matters, there is no “race to the bottom”, geographically close regions exhibiting very different taxation policies.

In the remainder of this paper, we will consider different restrictions on mobility and their consequences on the equilibrium. Workers will have different costs to move, depending on their preferences.

Hindriks (1999) provides a model close to ours. The author discusses the level of redistribution when workers are imperfectly mobile. But it does not allow for imperfect information between government and workers. Moreover, labor supply is taken as exogenous. On the contrary, we will focus on imperfect information. The closest paper is

[1]For a more detail survey, see Cremer, Fourgeaud, Leite-Monteiro, Marchand, and Pestieau (1996) or Wilson (1999).

[2]See for examples Gordon (1983), Wildasin (1991) and Wildasin (1994).

Hamilton, Lozachmeur, and Pestieau (2002) but it deals only with a particular case: the governments are Rawlsian; the unskilled workers are perfectly mobile while the skilled workers are perfectly immobile. They show that in that case international mobility does not affect redistribution, which is a very intuitive result: a Rawlsian government has no incentive to attract unskilled workers as long as it maximizes their per-person utility by transferring them income, and it has incentives to attract skilled workers who pay taxes, but in their model these skilled workers are immobile. Our paper generalizes this result.

Hamilton and Pestieau (2001) also study nonlinear income tax competition. They consider both Rawlsian, despotic governments and majority voting outcomes under different assumption on the mobility of workers. But their assumptions on mobility are quite restrictive: workers are perfectly mobile or perfectly immobile and both kinds of workers cannot be mobile together. Moreover, they consider small open economies: each country does not anticipate any effect of its own redistribution scheme on international migration. Their governments are not strategic competitors. In the following we consider the opposite assumption. Our governments are strategic players who anticipate that their taxes affect migration[3].

From a technical point of view, this paper is close to Rochet and Stole (2002) which analyses the competition between two principals in a duopoly framework with random participation. In our model the cost of mobility plays the same role as the random participation in limiting the effect of the Bertrand competition.

The paper is organized as follows. Section 2 introduces the basic framework of the study. Section 3 provides the basic properties of the optimal taxation. Section 4 and 5 derive the properties of the competitive outcome under different assumptions on the welfare criterion. Section 6 concludes. All the proof are moved in appendix.

2 Model

For tractability and for ease of comparison we adopt the discrete-skill setting of Stiglitz (1982). We consider an economy with two kinds of agents: the workers (who are also tax-payers) and two governments.

2.1 Workers

The workers are characterized by their identical preferences and some different abilities. The preferences can be formalized by a quasi-linear utility function $U(\cdot,\cdot)$:

$$U(Z,L) = Z - v(L),$$

where $Z = I - T(I)$ is the after-tax income with I being the before-tax income, L the labor supply, $T(\cdot)$ the tax schedule set by the government. v is the disutility of labor

[3]Our model does not generalized Hamilton and Pestieau (2001)'s one. Both assumptions on strategic behavior of the government can be justify.

satisfying: $v(0) = 0$, $v' > 0$, and $v'' > 0$. The ability is denoted by ω_k, the (constant) marginal productivity. We assume that there are two types of workers, skilled workers who have a high productivity and unskilled workers who have a low marginal productivity. Formally: $\omega_k \in \{\omega_1, \omega_2\}$, with $\omega_1 < \omega_2$.

Given that the labor market is competitive, the wages are equal to the marginal productivities of workers, which implies $I = \omega_k L$. The utility can be written as a function of I as:

$$U_k = I - T(I) - v\left(\frac{I}{\omega_k}\right).$$

We denote by p the proportion of skilled workers.

There are two countries denoted by $i = A, B$. Workers can move (once) from their native country to the foreign country. For an worker of ability k born in country A, the cost of changing country is $(1-x)\sigma_k$, with x denoting a preference parameter depending upon the individual, $x \in [0,1]$ and σ_k a preference parameter depending upon the individual productivity. Therefore, the worker with $x = 0$ (resp. $x = 1$) is the least mobile. On the contrary, a worker with $x = 1$ is the most mobile. So x can be interpreted as a personal mobility parameter. The higher σ_k is, the less one worker with productivity ω_k is mobile, σ_k can be also interpreted as mobility parameter.

The couple (x, ω) is private information: it is only known by the worker. We assume that x is uniformly distributed (for either skilled or unskilled workers) on the segment $[0,1]$. Moreover the whole population in each country is equal to 1.

2.2 Governments

The governments [4] are both benevolent. They have the same preferences over the utility space represented by the same welfare function $W(U_1, U_2)$. We will consider two particular cases the Rawlsian case:

$$W(U_1, U_2) = \min(U_1, U_2),$$

and the weighted utilitarian case:

$$W(U_1, U_2) = \alpha U_1 + U_2,$$

where $alpha$ is an exogenous weight such that $\alpha > \frac{1-p}{p}$.

Therefore we restrict our analysis to welfare function which do not depends on the proportion p. If we include p in the welfare function some strange effects may arrive: attracting skilled workers may increase the total welfare even if utilities remain the same. The second welfare function is not exactly the utilitarian social welfare function, it will use to test the robustness of the results obtained in the Rawlsian case.

[4] We take the separation between the two countries as given. Obviously, the merge of the two governments would be welfare improving. For a discussion on principals separation, see Martimort (1999).

The governments choose non-cooperatively their income tax schedule under a budget constraint. The taxation has an unique purpose: redistribution from the "rich" to the "poor" since the governments do not need to finance public goods [5].

2.3 Firms

Our results are derived assuming that firms are competitive, and that the two types of labor are perfect substitutes.

3 Autarky

The results presented in this section have first been obtained by Stiglitz (1982). If we consider a Rawlsian government, the function W does not depend on p, i.e. the distribution of type affects only the budget constraint. On the contrary, if we consider a weighted utilitarian government, the distribution of type affects both the objective function of the government and its budget constraint.

To characterize the optimal tax policy, we rely on a "revelation mechanism". For our purpose, a mechanism consist in a set of specific after-tax and before-tax income. The government maximizes with respect to (I_1, Z_1, I_2, Z_2) the welfare function under two incentive constraints and the budget constraint.

$$\max W\left[Z_1 - v\left(\frac{I_1}{\omega_1}\right), Z_2 - v\left(\frac{I_2}{\omega_2}\right)\right]$$

$$s.t.$$

$$Z_1 - v\left(\frac{I_1}{\omega_1}\right) \geq Z_2 - v\left(\frac{I_2}{\omega_1}\right),$$

$$Z_2 - v\left(\frac{I_2}{\omega_2}\right) \geq Z_1 - v\left(\frac{I_1}{\omega_2}\right),$$

$$(1-p)(I_1 - Z_1) + p(I_2 - Z_2) \geq 0.$$

With some change of variables this program can also be written as

[5] We could introduce an exogenous amount G of public goods financed by the government, it would not change our results.

$$\max W(U_1, U_2)$$

$$s.t.$$

$$U_1 + v\left(\frac{I_1}{\omega_1}\right) - v\left(\frac{I_1}{\omega_2}\right) - U_2 \leq 0, \tag{1}$$

$$U_2 + v\left(\frac{I_2}{\omega_2}\right) - v\left(\frac{I_2}{\omega_1}\right) - U_1 \leq 0, \tag{2}$$

$$(1-p)\left[I_1 - U_1 - v\left(\frac{I_1}{\omega_1}\right)\right] + p\left[I_2 - U_2 - v\left(\frac{I_2}{\omega_2}\right)\right] \geq 0. \tag{3}$$

Now, the government maximizes with respect to (I_1, U_1, I_2, U_2).

We denote by δ_1 and δ_2 the Lagrangian multipliers associated with constraints (1) and (2), and by λ the multiplier associated with the budget constraint (3).

We define I_2^* and I_1^* as the two before-tax incomes that would be optimal without asymmetric information. Formally they are defined by the following equations[6]:

- $v'\left(\frac{I_2^*}{\omega_2}\right) = \omega_2,$
- $v'\left(\frac{I_1^*}{\omega_1}\right) = \omega_1.$

In the same way, we define I_1^{**} and I_2^{**} as the two optimal before-tax incomes when ω_1 and ω_2 are private information.

Proposition 1 (Mirrlees 1971, Stiglitz 1982) *Let us consider a pure Rawlsian government or an utilitarian government with* $\alpha \geq (1-p)/p$. *At the optimum, constraints* (1) *and* (3) *are binding, and the labor supply (or the before-tax income) of the skilled workers is not distorted i.e.*

$$I_2^{**} = I_2^*,$$

while the labor supply of the unskilled individuals is distorted i.e.

$$I_1^{**} < I_1^*.$$

The optimal taxation exhibits an important feature. The labor supply of the less skilled individual is distorted in order to reduce the incentives for the skilled to misreport their type. It shapes the form of the tax function, which can not be convex everywhere. The international mobility affects this property when the cost to move is sufficiently low as it shown in the following sections.

[6]We restrict our attention to interior solutions.

4 Rawlsian governments

First, we consider two Rawlsian governments. Each government chooses its tax function taking the tax function of the other country as given and anticipating correctly the migrations induced by taxes. Equilibrium is a fixed point in which no worker wants to move and no government wants to change its redistribution policy (given the policy of the other country).

We adopt the Rothschild-Stiglitz-Nash concept of equilibrium. This could be controversial since a government does not anticipate that after a deviation from equilibrium the policy of the other government could not be sustainable. The budget constraint depends on the proportions of both kinds of workers: after a migration from one country to the other induced by a change in the fiscal policy of one country, the other country's budget constraint is not balanced anymore.

We adopt this concept of equilibrium for mainly two reasons. First, there is no consensus on an alternative definition of equilibrium. Second, we want to keep the Nash equilibrium concept to be consistent with the usual assumption made in the economic literature.

We solve this game by using the "revelation principle": we assume that there is no restriction in considering that both governments offer truthful mechanisms. In a more general setting where we consider competition between two principals there is some loss of generality in restricting the analysis to this set of mechanisms[7].

In our context, the agent deals either one of the two principals, but he cannot deal with the two simultaneously: a worker works and pays taxes in country A or in the country B, any arbitrage is forbidden. In this case, Martimort and Stole (2002) argue that the revelation principal applies, as long as we consider only pure strategy equilibria.

The government A chooses a tax function which can be summarized by the mechanism

$$(I_{A1}, U_{A1}, I_{A2}, U_{A2}),$$

A worker with preference x ($0 \leq x \leq 1$) and ability ω_k ($k = 1,2$) from B moves if and only if:

$$U_{Ak} - (1-x)\,\sigma_k \geq U_{Bk}.$$

i.e. if

$$x \geq 1 + \frac{U_{Bk} - U_{Ak}}{\sigma_k}.$$

In the same way, a worker with preference x and ability ω_k from A, moves if and only if:

[7] For a more detail survey, see Martimort and Stole (2002) or Peters (2001).

$$x \geq 1 + \frac{U_{Ak} - U_{Bk}}{\sigma_k}.$$

Given the two tax functions, the total number of workers with ability k who live in B is

$$p + p\frac{U_{Bk} - U_{Ak}}{\sigma_k} \equiv p P_k (U_{Ak}, U_{Bk}),$$

where $P_k (U_{Ak}, U_{Bk})$ can be either lower or higher than 1 according to the sign of $U_{Ak} - U_{Bk}$.

The governments do not observe x, they cannot design mechanisms which reveal this information. This property is a consequence of the additivity of the moving cost[8]. In this context, the optimal mechanism is still a set of specific after-tax and before-tax income.

With a Rawlsian objective, the government B maximizes the utility of less productive individuals. Since we can restrict our analysis to truthful mechanisms, we impose that workers reveal their ability, which gives the two usual incentive constraints (1) and (2). The budget constraints becomes:

$$(1-p) P_1 (U_{A1}, U_{B1}) \left[U_{B1} + v\left(\tfrac{I_{B1}}{\omega_1}\right) - I_{B1}\right] + p P_2 (U_{A2}, U_{B2}) \left[U_{B2} + v\left(\tfrac{I_{B2}}{\omega_2}\right) - I_{B2}\right] \leq 0. \tag{4}$$

The program of government B is the same its the program in the autarchy case, except that the number of the skilled and unskilled individuals in country B is now endogenous. Formally:

$$\max U_{B1}$$

$$s.t.$$

$$U_{B2} + v\left(\tfrac{I_{B2}}{\omega_2}\right) - v\left(\tfrac{I_{B2}}{\omega_1}\right) - U_{B1} \leq 0,$$

$$U_{B1} + v\left(\tfrac{I_{B1}}{\omega_1}\right) - v\left(\tfrac{I_{B1}}{\omega_2}\right) - U_{B2} \leq 0,$$

$$(1-p) P_1 (U_{A1}, U_{B1}) \left[U_{B1} + v\left(\tfrac{I_{B1}}{\omega_1}\right) - I_{B1}\right] + p P_2 [U_{A2}, U_{B2}) \left[U_{B2} + v\left(\tfrac{I_{B2}}{\omega_2}\right) - I_{B2}\right] \leq 0.$$

Given this, it is convenient to define two values of σ_2:

$$\widehat{\sigma} = (1-p)\left[I_2^* - v\left(\tfrac{I_2^*}{\omega_2}\right)\right] - (1-p)\left[I_1^* - v\left(\tfrac{I_1^*}{\omega_2}\right)\right],$$

$$\widetilde{\sigma} = (1-p)\left[I_1^* - v\left(\tfrac{I_1^*}{\omega_2}\right)\right] - (1-p)\left[I_2^* - v\left(\tfrac{I_2^*}{\omega_2}\right)\right].$$

[8]If the mobility were not additively it would be possible to use taxation to make reveal x. Using random taxation it would be also possible.

Given the properties of the function v, it is easy to see that $\widehat{\sigma} > 0$ and $\widetilde{\sigma} < 0$.

We restrict the analysis to symmetric equilibria.

Proposition 2 *At equilibrium, the optimal allocations have the following properties.*

- *If* $\sigma_2 \in [0, \hat{\sigma}]$, *none of the incentive constraints is binding: labor supplies are not distorted. Each skilled worker pays a total income tax equal to* σ_2*.*
- *If* $\sigma_2 \in [\hat{\sigma}, +\infty)$, *the incentive constraint* (2) *is binding: the labor supply of the skilled worker is not distorted while the labor supply of the unskilled workers is distorted.*
- *For any given* σ_2*, the tax policy does not depend on* σ_1*.*

The proposition can be summarized by the following figure:

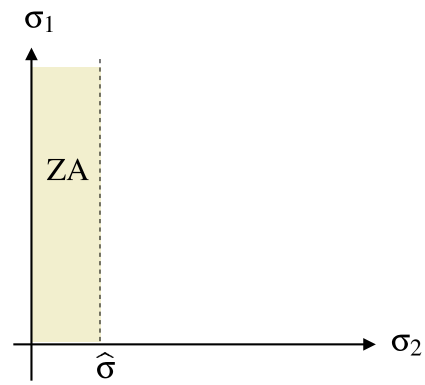

Figure 1

There are two regimes of taxation, depending on the value of σ_2. If it is small, tax policiy does not involve distortion (Area *ZA* in figure 1). On the contrary, if σ_2 is large enough, the tax polices are qualitatively similar to the tax policy in autarchy.

A government has three relevant constraints. First, the government must respect a budget constraint. Second, the government has to leave enough rent to skilled workers in order to dissuade them from misreporting their type which is formalized by the incentive constraint (2). But here the government has also to prevent migration of the skilled workers (which is new with respect to the autarky case).

When σ_2 is small (moving from one country to the other is quite easy) only the third constraint prevails: in order to prevent migration, the government leave enough rent to the skilled workers, so skilled workers have no incentive to misreport their type. When σ_2 becomes higher (greater than $\hat{\sigma}$), it is less difficult to prevent migration, therefore the government leaves less rent to the skilled workers, which may incourage them to misreport their type.

If we make extremes assumptions on mobility, we can deduce from this proposition two interesting corollaries. First, if skilled individual are immobile, e.i. σ_2 goes to infinity, the budget constraint becomes:

$$(1-p)\,P_1\,(U_{A1},U_{B1})\left[U_{B1}+v\left(\frac{I_{B1}}{\omega_1}\right)-I_{B1}\right]+p\left[U_{B2}+v\left(\frac{I_{B2}}{\omega_2}\right)-I_{B2}\right]\leq 0, \quad (5)$$

Corollary 1 *At equilibrium, when σ_2 goes to infinity, the optimal tax policy does not depends on σ_1, and does not differ from the autarky case.*

This corollary is a generalization of Hamilton, Lozachmeur, and Pestieau (2002)'s main result. The intuition of the result is the same. A Rawlsian government has no incentive to attract unskilled workers: to maximize only their utility, they receive some transfer from the government. If a unskilled worker moves from A to B, it means that for a given tax revenue, each unskilled worker receives lower transfer. On the contrary, a Rawlsian government has incentives to attract skilled workers who pay taxes, but here they cannot move.

If unskilled individuals are immobile, e.i. σ_1 goes to infinity, the budget constraint becomes:

$$(1-p)\left[U_{B1}+v\left(\tfrac{I_{B1}}{\omega_1}\right)-I_{B1}\right]+p\,P_2\,(U_{A2},U_{B2})\left[U_{B2}+v\left(\tfrac{I_{B2}}{\omega_2}\right)-I_{B2}\right]\leq 0. \quad (6)$$

and we can establish the following corollary:

Corollary 2 *At equilibrium when σ_1 goes to infinity, the optimal allocations have the following properties:*

- *If $\sigma_2 \in [0,\hat{\sigma}]$, none of the incentive constraints is binding, so labor supplies are not distorted. Each skilled worker pays a total income tax equal to σ_2.*

- *If $\sigma_2 \in [\hat{\sigma},+\infty)$, incentive constraint (2) is binding: the labor supply of skilled workers is not distorted while the labor supply of unskilled individuals is distorted. Each skilled worker pays a total income tax smaller than σ_2.*

As in proposition 2, the tax policies do not depends on σ_1. The governments care only about less skilled workers, so they only try to attract skilled workers.

5 Other welfare criterion

Proposition 2 contrasts with results obtained by Hindriks (1999) when we compare his results on tax-transfer competition with ours. His optimal transfer policy depends on σ_1. This important difference does not come from his assumption on information, but from the objective of the governments. If we consider other welfare functions, we get optimal tax policies which depends on σ_2 and σ_1. The following example shows this.

The country's B government maximizes the following social welfare:

$$\max \alpha U_{B1} + U_{B2}, \tag{7}$$

under constraints (1), (2), and (4).

We will use the following notations

$$S(\sigma_1) = \left\{ s > 0, \left[1 - \frac{1-p}{\alpha p} - \frac{\widehat{\sigma}}{\alpha \sigma_1} \right] s > \widehat{\sigma} \right\},$$

and

$$T_2 = -\frac{\left(1 - \alpha \frac{p}{1-p}\right) \sigma_1}{\left(\frac{p}{1-p} + \alpha \frac{p}{1-p} \frac{\sigma_1}{\sigma_2}\right)}.$$

Proposition 3 *When both governments have objective* (7)*, at equilibrium:*

- *If $\sigma_2 \notin S(\sigma_1)$ none of the incentive constraints is binding: labor supplies are not distorted. Each skilled worker pays a total income tax T_2,*
- *If $\sigma_2 \in S(\sigma_1)$ incentive constraint* (2) *is binding: the labor supply of the skilled worker is not distorted while the labor supply of the unskilled individuals is distorted. Each skilled worker pays a total income tax smaller than T_2.*

Figure 1 becomes:

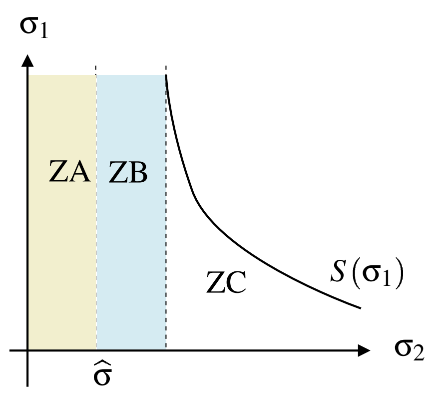

Figure 2

The area to the left of $S(\sigma_1)$ where labor supplies are not distorted can be divided in three subareas: *ZA*, *ZB* and *ZC*.

First the domain of values of σ_2 for which labor supplies are not distorted is enlarged: the area *ZB* is added to *ZA*. The welfare of skilled workers affects social welfare, then it is less costly for a government to reduce the tax on skilled workers to attract them.

Second, σ_1 now matters. In the autarchy case, a government would be better off with less unskilled worker in its country: less unskilled workers means less taxes on skilled workers (or higher transfer to the less skilled workers for a Rawlsian government). Then when σ_1 is small a government has incentives to reduce transfer to the unskilled workers, (which reduce their welfare) to make them move. The area *ZC* is added to *ZA* and *ZB*.

This "strange behavior" is due to the specification of the welfare function. Since it does not depend on p, a government has the same utility whatever the number of "poor" living in its country may be. As long as it cares about the welfare of the skilled workers who are net taxpayers, for a given number of skilled worker, its welfare objective decreases with the number of unskilled workers for a given amount of their transfer, just because it implies less taxes for skilled workers of a given number.

The conclusions from proposition 3 are twofold.

First, it shows that the neutrality of σ_1 holds no longer when we consider more general welfare functions.

Second, it also shows the role of welfare criterion, which can imply counter-intuitive and questionable behavior of governments.

6 Conclusion

In this paper we have shown how mobility affects the possibility of redistribution, and the shape of the taxation policy. The key variable in our analysis is the mobility of skilled workers, who are the victims of redistribution. If they can move, they use the fiscal competition between the two governments to reduce their income tax rate. On the contrary, the ability to move of the unskilled workers does not affect the optimal tax policy of Rawlsian governments [9].

The second important result is that when the cost to move of the skilled worker is small the equilibrium of the game leads to first best allocations, i.e. efficient labor supplies. It does not mean that fiscal competition has no effect on redistribution, which is on the contrary reduced by this competition.

Finally, we have shown, using one particular example, the role of the welfare function. If the government cares about the whole population, it may have some strange behavior, such as to induce the less rich part of the population to migrate to other countries.

[9] The result would be exactly the contrary if we had consider "despotic" governments, i.e. government who maximize the utility of the richest worker.

Even if we use a standard model, some of our assumptions are restrictive.

First, all our results have been derived using two particular welfare functions. Using more general welfare function would give intuition on the robustness (or not) of the mains conclusions of this paper. The role of the mobility played by skilled workers in our model is due to the aim of the government, which is roughly speaking to take from the rich to give to the poor. But the specification of the welfare function play a central role in our model.

We have considered a tractable discrete-type model, but even in a traditional setting discrete and continuous models are not strictly equivalent. In particular, first best allocation are sometimes implementable in a discrete type setting, which is not possible in continuous setting.

The existence of non symmetric equilibria remains an open question. As long as we have symmetric countries, focusing on symmetric equilibria makes sense. But a clear possible extension would be to consider asymmetric countries, and then we would have no reason to focus on symmetric equilibria.

As capital taxation is an important question in economic literature, introducing generalized production function including capital as production factor and capital taxation (as in Huber (1999)) would probably gives interesting results.

Finally, as we have stressed the importance of mobility cost for the skilled workers, it would be interesting to try to get empirical estimations for this cost. This could have interesting consequences for fiscal policy, for example Kirchgässner and Pommerehne (1996) suggest that fiscal competition has a significant effect in Switzerland, but this could be much less important in European Union because of higher costs to move.

These issues are on our current research agenda.

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A Proof of Proposition 1

From the first order conditions, we obtain:

$$\alpha(1-p)+\delta_1-\delta_2-(1-p)\lambda=0, \tag{8}$$

and

$$p-\delta_1+\delta_2-p\lambda=0. \tag{9}$$

One easily check that $\lambda = 0$ is impossible. Let us assume that $\delta_1 \geq 0$ and $\delta_2 = 0$. Then from equations (8) and (9) :

$$\alpha(1-p)\,p - p\,(1-p) = -\delta_1,$$

which contradicts $\delta_1 \geq 0$.

The proof is similar if we consider a Rawlsian government.

B Proof of Proposition 2

The Lagrangian associated with the problem is:

$$U_{B1} - \delta_1\left[U_{B2} + v\left(\tfrac{I_{B2}}{\omega_2}\right) - v\left(\tfrac{I_{B2}}{\omega_1}\right) - U_{B1}\right] - \delta_2\left[U_{B1} + v\left(\tfrac{I_{B1}}{\omega_1}\right) - v\left(\tfrac{I_{B1}}{\omega_2}\right) - U_{B2}\right]$$

$$-\lambda\left\{(1-p)\,P_1(U_{A1},U_{B1})\left[U_{B1} + v\left(\tfrac{I_{B1}}{\omega_1}\right) - I_{B1}\right] + p\,P_2[U_{A2},U_{B2}]\left[U_{B2} + v\left(\tfrac{I_{B2}}{\omega_2}\right) - I_{B2}\right]\right\},$$

Where δ_1, δ_2 and λ are multipliers associated with the constraints. The first order conditions are the following:

$$\begin{cases} 1 - \delta_2 + \delta_1 - (1-p)\,\lambda P_1(U_{A1},U_{B1}) - (1-p)\,\frac{\lambda}{\sigma_1}\left[U_{B1} - I_{B1} + v\left(\frac{I_{B1}}{\omega_1}\right)\right] = 0, \\ -\delta_1 + \delta_2 - p\lambda P_2(U_{A2},U_{B2}) - p\frac{\lambda}{\sigma_2}\left[I_{B2} - v\left(\frac{I_{B2}}{\omega_2}\right) - U_{B2}\right] = 0, \\ \delta_1\left[\frac{1}{\omega_1}v'\left(\frac{I_{B2}}{\omega_1}\right) - \frac{1}{\omega_2}v'\left(\frac{I_{B2}}{\omega_2}\right)\right] + p\lambda P_2(U_{A2},U_{B2})\left[1 - \frac{1}{\omega_2}v'\left(\frac{I_{B2}}{\omega_2}\right)\right] = 0, \\ \delta_2\left[\frac{1}{\omega_2}v'\left(\frac{I_{B1}}{\omega_2}\right) - \frac{1}{\omega_1}v'\left(\frac{I_{B1}}{\omega_1}\right)\right] + \lambda(1-p)\,P_1(U_{A1},U_{B1})\left[1 - \frac{1}{\omega_1}v'\left(\frac{I_{B1}}{\omega_1}\right)\right] = 0. \end{cases}$$

Given a value of U_{B1}, and a value of U_{B2} we try to find conditions on first order condition in order to have a best response.

We consider symmetric equilibria: $U_{A1} = U_{B1} = U_1$, $U_{A2} = U_{B2} = U_2$, $I_{A1} = I_{B1} = I_1$ and $I_{A2} = I_{B2} = I_2$.

Solution $\delta_1 = \delta_2 = 0$

First order conditions become:

$$1 - \lambda\,(1-p) - \frac{\lambda}{\sigma_1}(1-p)\left[U_1 - I_1^* + v\left(\frac{I_1^*}{\omega_1}\right)\right] = 0, \tag{10}$$

$$1 - \frac{1}{\sigma_2}\left[U_2 - I_2^* + v\left(\frac{I_2^*}{\omega_2}\right)\right] = 0, \tag{11}$$

$$I_2 = I_2^*, \tag{12}$$

$$I_1 = I_1^*. \tag{13}$$

It gives us a value for λ :

$$\lambda = \frac{1}{(1-p)\left[1 - \frac{1}{\sigma_1}\left(I_1^* - v\left(\frac{I_1^*}{\omega_1}\right) - U_1\right)\right]} \tag{14}$$

Using equation (11) we define U_2^* as:

$$U_2^* = I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - \sigma_2. \tag{15}$$

Budget constraint implies:

$$U_1 = I_1^* - v\left(\frac{I_1^*}{\omega_1}\right) + \frac{p}{1-p}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2^*\right]. \tag{16}$$

The condition $\lambda \geq 0$ is equivalent to:

$$U_1 \geq I_1^* - v\left(\frac{I_1^*}{\omega_1}\right) - \sigma_1. \tag{17}$$

Using (15) and (16) we obtain a value for $U_B(\omega_1)$:

$$U_1 = I_1^* - v\left(\frac{I_1^*}{\omega_1}\right) + \frac{p}{1-p}\frac{\sigma_2}{2}. \tag{18}$$

This implies that condition (17) is satisfied. Incentive constraints are satisfied if and only:

$$v\left(\frac{I_2^*}{\omega_1}\right) - v\left(\frac{I_2^*}{\omega_2}\right) \geq U_2 - U_1 \geq v\left(\frac{I_1^*}{\omega_1}\right) - v\left(\frac{I_1^*}{\omega_2}\right), \tag{19}$$

or equivalently:

$$\begin{cases} \frac{1}{1-p}\sigma_2 \geq I_2^* - v\left(\frac{I_2^*}{\omega_1}\right) - I_1^* + v\left(\frac{I_1^*}{\omega_1}\right) \\ \\ \frac{1}{1-p}\sigma_2 \leq I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - I_1^* + v\left(\frac{I_1^*}{\omega_2}\right) \end{cases} \tag{20}$$

or using a reduce form:

$$\widetilde{\sigma} \leq \sigma_2 \leq \widehat{\sigma}.$$

As $\widetilde{\sigma} < 0$, the only relevant condition is $\sigma_2 \leq \widehat{\sigma}$.

Conclusion

If $\sigma_2 \leq \widehat{\sigma}$, and if the government A chooses a taxation such that

$$\begin{cases} I_{A2} = I_2^*, \\ I_{A1} = I_1^*, \\ U_{A2} = U_2^* = I_2^* + v\left(\frac{I_2^*}{\omega_2}\right) - \sigma_2, \\ U_{A1} = U_1^* = I_1^* - v\left(\frac{I_1^*}{\omega_1}\right) + \frac{p}{1-p}\sigma_2. \end{cases}$$

The best response for the government B, is to set the taxation

$$\begin{cases} I_{B2} = I_2^*, \\ I_{B1} = I_1^*, \\ U_{B2} = U_2^* = I_2^* + v\left(\frac{I_2^*}{\omega_2}\right) - \sigma_2, \\ U_{B1} = U_1^* = I_1^* - v\left(\frac{I_1^*}{\omega_1}\right) + \frac{p}{1-p}\sigma_2. \end{cases}$$

The income tax pay by the skilled workers is equal to σ_2.

Solution $\delta_1 = 0$, $\delta_2 > 0$

First order conditions become:

$$1 - \delta_2 - \lambda(1-p) + \frac{\lambda}{\sigma_1}(1-p)\left[I_1 - v\left(\frac{I_1}{\omega_1}\right) - U_1\right] = 0, \tag{21}$$

$$\delta_2 - \lambda p + \frac{\lambda}{\sigma_2} p \left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right] = 0, \tag{22}$$

$$\frac{1}{\omega_1} v'\left(\frac{I_1}{\omega_1}\right) - 1 = \frac{\delta_2}{\lambda(1-p)}\left[\frac{1}{\omega_2} v'\left(\frac{I_1}{\omega_2}\right) - \frac{1}{\omega_1} v'\left(\frac{I_1}{\omega_1}\right)\right], \tag{23}$$

$$I_2 = I_2^*. \tag{24}$$

These first order conditions give:

$$\lambda = \frac{\delta_2}{p - \frac{p}{\sigma_2}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right]}, \tag{25}$$

and

$$\delta_2 = \frac{p - \frac{p}{\sigma_2}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right]}{1 - \frac{2p}{\sigma_2}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right] - \frac{(1-p)}{\sigma_1}\left[I_1 - v\left(\frac{I_1}{\omega_1}\right) - U_1\right]}. \tag{26}$$

From (23) and (25), we can conclude that the value of I_1 given U_2 is independent of σ_1 and U_1 :

$$\frac{1 - \frac{1}{\omega_1}v'\left(\frac{I_1}{\omega_1}\right)}{\frac{1}{\omega_2}v'\left(\frac{I_1}{\omega_2}\right) - \frac{1}{\omega_1}v'\left(\frac{I_1}{\omega_1}\right)} = \frac{p - \frac{p}{\sigma_2}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right]}{(1-p)}. \tag{27}$$

The budget constraint can be rewrite as:

$$U_1 = I_1 - v\left(\frac{I_1}{\omega_1}\right) + \frac{p}{1-p}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right]. \tag{28}$$

The incentive constraints become

$$U_2 - (1-p)\left[I_1 - v\left(\frac{I_1}{\omega_1}\right)\right] \leq p\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right)\right] + (1-p)\left[v\left(\frac{I_2^*}{\omega_1}\right) - v\left(\frac{I_2^*}{\omega_2}\right)\right], \tag{29}$$

and

$$U_2 = p\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right)\right] + (1-p)\left[I_1 - v\left(\frac{I_1}{\omega_2}\right)\right]. \tag{30}$$

If we set $U_2 = U_2^*$, then equation (26) implies $\delta_2 = 0$ and $I_1 = I_1^*$. Because $\sigma_2 > \widehat{\sigma}$ the incentive constraint (2) is not valid:

$$U_2^* < p\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right)\right] + (1-p)\left[I_1^* - v\left(\frac{I_1^*}{\omega_2}\right)\right]. \tag{31}$$

If from the point $(U_2 = U_2^*, I_1^*)$, we sufficiently increase U_2 and decrease I_1, will find values for U_2 and I_1 such that:

$$U_2 = p\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right)\right] + (1-p)\left[I_1 - v\left(\frac{I_1}{\omega_2}\right)\right]. \tag{32}$$

Given that $v(.)$ is well behaved, there is only one solution to the preceding equation such that $U_2 > U_2^*$ and $I_1 < I_1^*$.

From first order equations, we have $\delta_2 = \frac{p - \frac{p}{\sigma_2}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right]}{1 - \frac{p}{\sigma_2}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right]}$, which implies $\delta_2 > 0$, and $\delta_2 < 1$, and thus $\lambda > 0$. The condition:

$$U_2 - (1-p)\left[I_1 - v\left(\frac{I_1}{\omega_1}\right)\right] \leq p\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right)\right] + (1-p)\left[v\left(\frac{I_2^*}{\omega_1}\right) - v\left(\frac{I_2^*}{\omega_2}\right)\right], \tag{33}$$

is equivalent the condition $v\left(\frac{I_2^*}{\omega_1}\right) \geq v\left(\frac{I_1}{\omega_1}\right)$, which is satisfied because $I_2^* > I_1^*$ and $>$ $I_1^* \geq I_1$.

By construction budget constraint is satisfied, as all the first order conditions.

Conclusion

If $\sigma_2 > \widehat{\sigma}$ and country A adopt the following policy:

- $I_{A2} = I_2^*$,
- U_{A1} and I_{A1} define as solutions of (27) and (30).
- U_{A2} define by equation (28).

The best response of country B is to adopt the same policy. By construction, I_2, U_1, I_2, and U_2 do not depend on σ_1.

If $P_1(U_{A1}, U_{B1}) = P_2(U_{A2}, U_{B2}) = 1$, equations defining I_2, U_1, I_2, and U_2 are the same as in the autarky case. Thus income taxation is not influenced by the labor mobility.

C Proofs of corollaries 1 and 2

The preceding arguments apply in both cases.

D Proof of Proposition 3

The Lagrangian associated with the problem is:

$$\alpha U_{B1} + U_{B2} - \delta_2\left[U_{B2} + v\left(\frac{I_{B2}}{\omega_2}\right) - v\left(\frac{I_{B2}}{\omega_1}\right) - U_{B1}\right] - \delta_1\left[U_{B1} + v\left(\frac{I_{B1}}{\omega_1}\right) - v\left(\frac{I_{B1}}{\omega_2}\right) - U_{B2}\right]$$

$$-\lambda\left\{(1-p) P_1(U_{A1}, U_{B1})\left[U_{B1} + v\left(\frac{I_{B1}}{\omega_1}\right) - I_{B1}\right] + p P_2(U_{A2}, U_{B2})\left[U_{B2} + v\left(\frac{I_{B2}}{\omega_2}\right) - I_{B2}\right]\right\}.$$

The first order conditions are:

$$\begin{cases} \alpha - \delta_2 + \delta_1 - \lambda(1-p) P_1(U_{A1}, U_{B1}) + x\frac{\lambda}{\sigma_1}(1-p)\left[I_{B1} - v\left(\frac{I_{B1}}{\omega_1}\right) - U_{B1}\right] = 0, \\ 1 - \delta_1 + \delta_2 - 2p\lambda P_2(U_{A2}, U_{B2}) + 2p\frac{\lambda}{\sigma_2}\left[I_{B2} - v\left(\frac{I_{B2}}{\omega_2}\right) - U_{B2}\right] = 0, \\ -\delta_2\left[\frac{1}{\omega_2}v'\left(\frac{I_{B1}}{\omega_2}\right) - \frac{1}{\omega_1}v'\left(\frac{I_{B1}}{\omega_1}\right)\right] + \lambda(1-p) P_1(U_{A1}, U_{B1})\left[1 - \frac{1}{\omega_1}v'\left(\frac{I_{B1}}{\omega_1}\right)\right] = 0. \\ -\delta_1\left[\frac{1}{\omega_1}v'\left(\frac{I_{B2}}{\omega_1}\right) - \frac{1}{\omega_2}v'\left(\frac{I_{B2}}{\omega_2}\right)\right] + \lambda p P_2(U_{A2}, U_{B2})\left[1 - \frac{1}{\omega_2}v'\left(\frac{I_{B2}}{\omega_2}\right)\right] = 0, \end{cases}$$

As in the previous case, we consider symmetric equilibria: $U_{A1} = U_{B1} = U_1$, $U_{A2} = U_{B2} = U_2$, $I_{A1} = I_{B1} = I_1$ and $I_{A2} = I_{B2} = I_2$.

Solution $\delta_1 = \delta_2 = 0$

First order conditions become:

$$\begin{cases} \frac{\alpha}{1-p} - \lambda + \frac{\lambda}{\sigma_1}\left[I_1 - v\left(\frac{I_1}{\omega_1}\right) - U_1\right] = 0, \\ \frac{1}{p} - \lambda + \frac{\lambda}{\sigma_2}\left[I_2 - v\left(\frac{I_2}{\omega_2}\right) - U_{B2}\right] = 0, \\ \lambda p\left[1 - \frac{1}{\omega_2}v'\left(\frac{I_2}{\omega_2}\right)\right] = 0, \\ \lambda(1-p)\left[1 - \frac{1}{\omega_1}v'\left(\frac{I_1}{\omega_1}\right)\right] = 0. \end{cases}$$

These first order conditions gives two values for λ :

$$\lambda = \frac{\frac{\alpha}{1-p}}{1 - \frac{1}{\sigma_1}\left[I_1^* - v\left(\frac{I_1^*}{\omega_1}\right) - U_1\right]}, \tag{34}$$

and

$$\lambda = \frac{\frac{1}{p}}{1 - \frac{1}{\sigma_2}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right]}. \tag{35}$$

Solving these two equations give a relation between U_1 and U_2 :

$$1 - \frac{1}{\sigma_1}\left[I_1^* - v\left(\frac{I_1^*}{\omega_1}\right) - U_1\right] = \alpha\frac{p}{1-p}\left\{1 - \frac{1}{\sigma_2}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right]\right\} \tag{36}$$

From the budget constraint (4) we have a other relation between U_1 and U_2 :

$$U_1 = I_1^* - v\left(\frac{I_1^*}{\omega_1}\right) + \frac{p}{1-p}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right]. \tag{37}$$

Solving the system gives a value for U_2:

$$U_2 = I_2^* + \frac{\left(1 - \alpha\frac{p}{1-p}\right)\sigma_1}{2\left(\frac{p}{1-p} + \alpha\frac{p}{1-p}\frac{\sigma_1}{\sigma_2}\right)} - v\left(\frac{I_2^*}{\omega_2}\right), \tag{38}$$

or equivalently a value for T_2:

$$U_2 = I_2^* + \frac{\left(1-\alpha\frac{p}{1-p}\right)\sigma_1}{2\left(\frac{p}{1-p}+\alpha\frac{p}{1-p}\frac{\sigma_1}{\sigma_2}\right)} - v\left(\frac{I_2^*}{\omega_2}\right). \tag{39}$$

The incentive constraints become:

$$I_1^* - v\left(\frac{I_1^*}{\omega_1}\right) - \left[I_2^* - v\left(\frac{I_2^*}{\omega_1}\right)\right] \geq \left[I_1^* - U_1 - v\left(\frac{I_1^*}{\omega_1}\right)\right] - \left[I_2^* - U_2 - v\left(\frac{I_2^*}{\omega_2}\right)\right], \tag{40}$$

and

$$\left[I_1^* - U_1 - v\left(\frac{I_1^*}{\omega_1}\right)\right] - \left[I_2^* - U_2 - v\left(\frac{I_2^*}{\omega_2}\right)\right] \geq I_1^* - I_2^* + v\left(\frac{I_2^*}{\omega_2}\right) - v\left(\frac{I_1^*}{\omega_2}\right). \tag{41}$$

Using the budget constraint:

$$-\left[I_1^* - U_1 - v\left(\frac{I_1^*}{\omega_1}\right)\right] = \frac{p}{1-p}\left[I_2^* - U_2 - v\left(\frac{I_2^*}{\omega_2}\right)\right], \tag{42}$$

the optimal value U_2 check these two conditions if and only if:

$$\sigma_2 \leq \frac{\widehat{\sigma}}{1 - \frac{1-p}{\alpha p} - \frac{\widehat{\sigma}}{\alpha\sigma_1}}. \tag{43}$$

Conclusion

If $\sigma_2 \leq \widehat{\sigma}(\sigma_1)$ and if the country A adopt the following policy:

- $I_{A2} = I_2^*$,
- $I_{A1} = I_1^*$,
- U_{A2} and U_{A1} define as solutions of (36) and (37).

The best response of country B is to adopt the same policy. It implies that skilled workers pay an income tax T_2, T_2 define as indicated in the proof.

Solution $\delta_1 = 0$, $\delta_2 > 0$

$$\begin{cases} \frac{\alpha}{1-p} - \frac{\delta_2}{1-p} - \lambda + \frac{\lambda}{\sigma_1}\left[I_1 - v\left(\frac{I_1}{\omega_1}\right) - U_1\right] = 0, \\ \frac{1}{p} + \frac{\delta_2}{p} - \lambda + \frac{\lambda}{\sigma_2}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right] = 0, \\ \frac{1}{\omega_1}v'\left(\frac{I_1}{\omega_1}\right) = 1 - \frac{\delta_2}{\lambda(1-p)}\left[\frac{1}{\omega_2}v'\left(\frac{I_1}{\omega_2}\right) - \frac{1}{\omega_1}v'\left(\frac{I_1}{\omega_1}\right)\right], \\ \frac{1}{\omega_2}v'\left(\frac{I_2^*}{\omega_2}\right) = 1. \end{cases}$$

First order conditions give two different values for λ

$$\lambda = \frac{1+\delta_2}{p - \frac{p}{\sigma_2}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right]}, \tag{44}$$

and

$$\lambda = \frac{\alpha - \delta_2}{(1-p) - \frac{1-p}{\sigma_1}\left[I_1 - v\left(\frac{I_1}{\omega_1}\right) - U_1\right]}, \tag{45}$$

which give a relation between U_1 and U_2:

$$\left[I_1 - v\left(\frac{I_1}{\omega_1}\right) - U_1\right] = \frac{\sigma_1}{2} - \frac{(2\alpha-\delta_2)\,p}{(2+\delta_2)\,(1-p)}\frac{\sigma_1}{2}\left(1 - \frac{2}{\sigma_2}\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right]\right). \tag{46}$$

Using the budget constraint it follows:

$$\left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right) - U_2\right] = -\frac{\left(1 - \frac{\alpha-\delta_2}{1+\delta_2}\frac{p}{1-p}\right)\sigma_1}{\frac{p}{1-p} + \frac{\alpha-\delta_2}{1+\delta_2}\frac{p}{1-p}\frac{\sigma_1}{\sigma_2}}, \tag{47}$$

Then, the binding incentive constraint gives a relation between I_1 and δ_2

$$I_1 - v\left(\frac{I_1}{\omega_2}\right) - \left[I_2^* - v\left(\frac{I_2^*}{\omega_2}\right)\right] = \frac{1}{1-p}\frac{\left(1 - \frac{\alpha-\delta_2}{1+\delta_2}\frac{p}{1-p}\right)\sigma_1}{\frac{p}{1-p} + \frac{\alpha-\delta_2}{1+\delta_2}\frac{p}{1-p}\frac{\sigma_1}{\sigma_2}}. \tag{48}$$

From the first order condition we have an other relation between I_1 and δ_2 :

$$\frac{\frac{1}{\omega_1}v'\left(\frac{I_1}{\omega_1}\right) - 1}{\frac{1}{\omega_1}v'\left(\frac{I_1}{\omega_1}\right) - \frac{1}{\omega_2}v'\left(\frac{I_1}{\omega_2}\right)} = \frac{\delta_2}{\lambda(1-p)}. \tag{49}$$

Using equations (44) and (47) one can write λ as a function of δ_2. We have two equations, and then possible implicit solutions for I_1 and δ_2.

We derive interesting properties of $\hat{\delta}_2(I_1)$, the set of the solutions of equation (48). If we set $\delta_2 = 0$, given our assumptions on α, the right-hand-side of (48) is negative and nonzero. Then we have two solution for I_1, denoted $I_1^{\bullet}$ and $I_1^{\bullet\bullet}$, $I_1^{\bullet} < I_1^{\bullet\bullet}$. In the same way, if we set $I_1 = I_2^*$, we find a unique value for δ_2, denoted $\bar{\delta}_2$ and define by:

$$\bar{\delta}_2\,(\alpha+1)\,p - 1.$$

Since $\alpha \geq (1-p)/p$, $\bar{\delta}_2 \geq 0$. It also possible to show that:

$$\forall\,\delta_2 \leq \bar{\delta}_2, \quad \frac{p}{1-p} + \frac{\alpha-\delta_2}{1+\delta_2}\frac{p}{1-p}\frac{\sigma_1}{\sigma_2} > 0.$$

From that, we can deduce that between $I_1^\bullet$ and $I_1^{\bullet\bullet}$, the set $\hat{\delta}_2(I_1)$ is well define, and $\hat{\delta}_2(I_1)$ is unique and positive. As both side of equation (48) are continuous function, the function $\hat{\delta}_2(I_1)$ is continuous between $I_1^\bullet$ and $I_1^{\bullet\bullet}$.

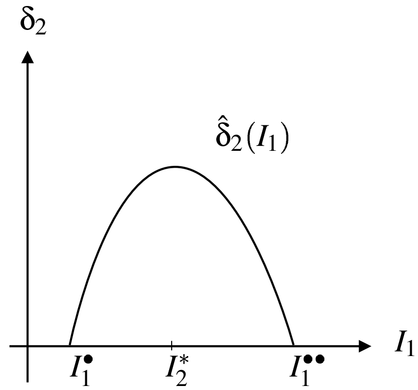

Figure 3

From equation (49), we can deduce an another relation between I_1 and δ_2, denoted $\tilde{\delta}_2(I_1)$.

From (44) one can remark that λ is strictly positive for any value of δ_2. Thus, if $I_1 = I_1^*$, then $\delta_2 = 0$. Because $\sigma_2 \in S(\sigma_1)$, we have $I_1^* > I_1^\bullet$.

Moreover, the right-hand-side of equation (49) goes to finite number when δ_2 goes to infinity. The left-hand-side of equation (49) is increasing with I_1 and goes to 1 when I_1 goes to infinity.

Let us assume that $\delta_2 = \bar{\delta}_2$. Then

$$\frac{\bar{\delta}_2}{\lambda(1-p)} = \frac{(1+\alpha)\,p-1}{(1+\alpha)\,p}\frac{p}{1-p} \geq 0.$$

If

$$\frac{(1+\alpha)\,p-1}{(1+\alpha)\,p}\frac{p}{1-p} < 1,$$

it exists a $\bar{I}_1 > I_1^*$ such that:

$$\frac{\frac{1}{\omega_1}v'\left(\frac{\bar{I}_1}{\omega_1}\right)-1}{\frac{1}{\omega_1}v'\left(\frac{\bar{I}_1}{\omega_1}\right)-\frac{1}{\omega_2}v'\left(\frac{\bar{I}_1}{\omega_2}\right)} = \frac{(1+\alpha)\,p-1}{(1+\alpha)\,p}\frac{p}{1-p}.$$

As both side of equation (49) are continuous function, for $I_1^* \leq I_1 \leq \bar{I}_1$, the relation $\tilde{\delta}_2(I_1)$ is well define and continuous. We can deduce from this, that $\tilde{\delta}_2(I_1)$ must cross $\hat{\delta}_2(I_1)$.

(see figure D).

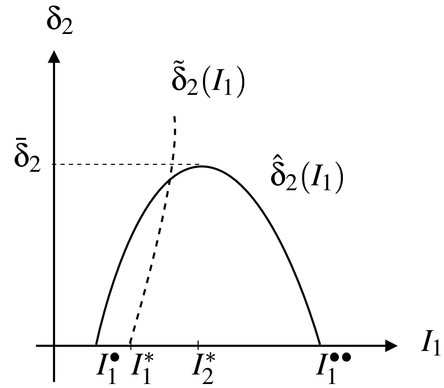

Figure 4

If

$$\frac{(1+\alpha)\,p-1}{(1+\alpha)\,p}\frac{p}{1-p} \geq 1,$$

it exists $0 > \delta_2 > \bar{\delta}_2$ such that for some $I_1 > I_1^{\bullet\bullet}$, as both side of equation (49) are continuous fonction, for $I_1 \geq I_1^*$, the relation $\tilde{\delta}_2(I_1)$ is well define and continuous. We can deduce from this, that $\tilde{\delta}_2(I_1)$ must cross $\hat{\delta}_2(I_1)$. (see figure D).

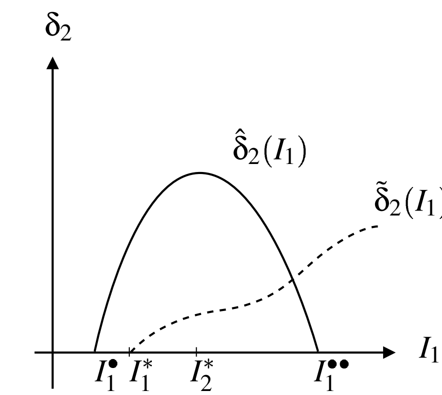

Figure 5

Equations (49) and (49) have a common solution (δ_2, I_1). By contruction, $\delta_2 >\geq 0$ and $I_1 \geq I_1^{\bullet}$.

Conclusion

If $\sigma_2 \in S(\sigma_1)$, and if the country A adopt the following describe in the proof, The best response of country B is to adopt the same policy.