The Reality and Masquerade behind the Bargaining Game of Welfare Policy-Making and Delivery [a]

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Abstract. Current analysis addresses an apparently critical issue for circulation of wealth in the society. Three actors play a game with the welfare-related burden of taxation. The first player, in the role of Negotiator No.1, stands up for citizens' legal and moral rights to social services. The second player, in the role of Negotiator No.2, proceeds from the needs of citizens for the provision and delivery of public goods. Quite the opposite, the Player, hereinafter called No.3, gives private consumption a preference over social services and public goods, i.e., the citizens-taxpayers hope for a reduction in income tax schedules to be deposited in common tax-pool: a shared account of negotiators No.1 and No.2. In fact, backed by a threat to reject the negotiators agreement, the player No.3 through political manoeuvring try to fulfil expectations of voters-citizens about lower taxes, e.g. a *"citizens committee for welfare activity,"* or, in short – the welfare committee must approve a motion against high taxes by unanimous vote. The government assesses and controls the circulation of wealth by poverty line parameter. Through the simulation, we present an evidence for the claim that 50% median income is close enough to be considered as a realistic choice of poverty line within the scope of terms given for Nash bargaining solution and conditions for unanimous vote in the committee.

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1. Introduction

Welfare policy in a democratic State always faces rivalling interests. The paper tries to shed lights on the question how a consensus is reached among different interest groups and whether this consensus satisfies a criterion of reasonable taxation. These questions are set up by a political bargaining with two players, called negotiators, and having different arguments about poverty proposals. The third player controls the game and has an objective to find the least burden of income tax in the State under investigation. First two players in the role of social and public agencies redistribute the wealth what is the classic problem of incomes redistribution addressed by a sweet-cake-cutting model. The cutting scheme explicitly introduces a parameter for the bargaining power of negotiators. The size of the cake depends on poverty proposals agreed with all parties involved including the third player – the taxpayers. We account for the size-effects of the cake by posing a risk for the negotiations to breakdown if the poverty proposal of the first player (social agencies) is too generous.

To clear up any confusion, we call to readers' attention that the risk of breakdown in our scheme originates from political manoeuvring of taxpayers. Furthermore, the public agencies may bear guilt of breakdown if they do reject the social agencies proposal. The fact is that negotiators may come into the region of too generous proposals where both of them, if continuing the bargaining, cannot find any other options at all as *"one agent's welfare necessitates a sacrifice"* for opponents advantage. *"Thus, one would expect the agents to negotiate (or haggle) over these candidates – a process that may in turn expend resources and delay agreement,"* cit. from W. Samuleson [1985: 322]. Therefore, it might be questioned whether the taxpayers will ever accept generous proposals.

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To treat the bargaining power properly, the disagreement point in the bargaining scheme has to be extracted endogenously. Unfortunately, a known objection to the Nash scheme [1950] is an exogenous point of disagreement given beforehand. In contrast, at least in some valuable situations, one can eliminate the necessity of an exogenously given disagreement point on condition we later call *"an equity of breakdown."*

Beyond the perception of how to share and negotiate a given amount of wealth, it is also reasonable to believe that income distribution is, perhaps, the only target for control and a source to assess the policy on poverty. With the regard of income distribution, the reader will understand that the Table 1 is not justified on empirical grounds and that current study does not provide the data and empirical support from the field. Even so, an adequate amount of wealth simulated across an income distribution by example, is close enough to be considered as a realistic match with an empirical 50% median income policy on poverty found useful by Tsumori, Saunders and Hughes [2002], besides by Hunter, Kennedy and Biddle [2002].

There is no part of the literature for a political game between welfare state institutions that directly connects the welfare policy of institutions and the Nash bargaining scheme. Such a connection might be interesting since it adds a new spin to both ideas. To discern the root cause of the situation and to find solutions for the problems, we try to move in the right direction by synthesizing three primary concepts (postulates):

Consistency**:** *The long-term prospective of welfare policy is required.*
Rationality**:** *The need is paramount to strike a mark between incompatible activities (demands) for social services and for public goods.*
Self-interest**:** *To bring a motion for a vote is necessary to meet the consumer perception against high taxes and excessive public spending. Whether it is good or bad, and whether it ought to be acknowledged or not, rejected or accepted, this motion must be carried out unanimously by welfare committee.*

Our position refers to the proclaimed postulates that constitute a cascade of three canonical principles embedded into the welfare policy of institutions. These principals are nothing else than rephrased concepts of the classical economic behaviour aiming at no more than to bring on the surface a convenient form assigned to the presentation of the results.

On methodological level, this synthesis is accessible for the observations and could be a subject to computer simulations in evaluating wealth redistribution policies. By itself, our initiative could also be of certain value for unifying theoretical structure of economic analysis of institutions, for passing judgment on social and political organizations, or for systematic inquiry into impacts of governmental decisions and actions.

To make our view clear and leaving apart the debate on what is good or bad and what is right or wrong, we composed the following list of key assumptions.

Assumptions.

1. We rest on the assumption that the government knows the true incomes of social clients, and thus it requires the social administration to bear an appropriate auditing mechanism. The government does not know the true incomes of wealthy citizens.
2. We emphasize in our assumptions that the government controls the circulation of wealth from the rich to the poor by poverty line as a parameter and decides whether to compensate with subsidies the unfair subsistence of the poor and the needy. However, households behavioural pattern remains endogenous: Households demarcate themselves as being “rich or poor.”
3. We assume that whenever the government makes a decision in respect of welfare policy, and such a policy emerges from legal and moral rights of citizens, then it has been in advance given a chance to be implemented in a long-term horizon. In other words, a balance between the debts and credits for social costs is desirable.
4. The government realizes that its welfare policy may change the behaviour of citizens in a way that subsidies eligible for claims may become more attractive for the needy or less attractive for the steady clients or visa versa. We assume that inverse working incentives feedback (h-factor) effect is likely to happen. In other words, a welfare hazard introduces in the behaviour of households what might “destabilize” the debts to the tax-pool.
5. We deal a) exclusively with proportional (flat) tax, and assume b) that the government is able to enforce tax schedules to be equal to the tax obligations, and c) that the total tax-pool accumulated via tax schedules being spent on welfare without savings.
6. The government can estimate: tax schedules, social and public costs despite the incomes of wealthy citizens are hidden or hard to reveal from the observations when the costs are to be credited to and tax schedules accumulated (debited) in common tax-pool.
7. We consider only progressive cost functions depending on the poverty line as a parameter. Thus, we anticipate an excessive public spending caused by the degree or extent of too generous welfare policies.

In the next section, we discuss the relevant trends in welfare Policy-Making and issues surrounded by three postulates just mentioned above. *“The Sweet-Cake-Cutting”* in Section 3 is the part of the game. In Section 4, we move along the cascade of our welfare policy postulates. At the first stage, we disclose the long-term strategy for stabilization of public spending. The strategy meets specific feasibility criteria, which is an essential instrument for the government to assess and control the circulation of wealth. At the second stage, we make an effort to embrace the ambivalence and multifaceted welfare perception of citizens from the angle of a bargaining game between social and public agencies. Our work here associates the policy on poverty with bargaining related to utility anticipations of negotiators at so-called bargaining frontier – the cross boundary into the set of Pareto efficient deals with positive utilities. Find the proof of the efficiency criteria in the appendix. Only at the third stage, the welfare state negotiators, the social and public agencies on both sides of the bargaining table might come to an agreement. The agreement might not necessarily be in the best self-interest of citizens. However, provided the general conditions for the calculus are smooth enough around the local minimum of taxes, we reveal the detail that among solutions a specific one would possibly enable a policy on poverty with the least burden of taxation. We discuss Self-enforcing (endogenous) solution of the general scheme in Section 5. In Section 6, we learn how to simulate the scheme and bring the results to final judgement by example. Section 7 ends the study with a summary.

2. Relevant trends and issues

Prelude. In the review on “*Handbook of New Institutional Economics,*” Ménard and Shirley (eds.) [2005], Richter [2005: 387] pointed at *“that the sociological analysis…and large institutional structures in economic life is still at an early stage…game theory, and computer simulation could help to further develop the new institutional approach…game theory might be a defendable heuristic device of NIE.”* Therefore, we prefer not to examine publications, which do not match the subject of our study in detail. To our knowledge (or lack of that), the closest to the part of the subject is the public finance.

The main interest area of public finance lies in the tax policy and public spending. In particular, it deals with financing the social security payments to provide a minimum (guaranteed) level of welfare for the citizens endowed by weak productivity, assumption No.2. The materialization of the idea behind the guaranteed welfare is also worth considering on the ground of citizens’ legal and moral obligations. Empirical evidence, consistent with legal obligations, can be found in the literature of social policy: *“...Henderson poverty line. The line was initially set (in 1966) equal to the level of the minimum wage plus family benefits for one-earner couple with two children,”* Saunders [1993: 29]. Hypothesis, consistent with moral obligations, can be found in the literature of economic politics: Eichenberger and Oberholzer-Gee [1996], Feld and Frey [2002].

Musgrave [1959] examined two basic approaches to taxation: the "benefit approach," which puts the taxation into efficiency context, and "the ability-to-pay," which puts the taxation into equity context. We intend to augment the reality of welfare policy as we see it with benefit approach by guaranteed amount of wealth for a reasonable tax. Furthermore we think that to keep taxes *fairly levied*, the best tax for wealth injects optimal equity into the tax system according to ability-to-pay principle of "proportional sacrifice."

Consistency. The purchasing, production and delivery of public goods and services give rise to public spending. A portion of expenses, often referred in common parlance as welfare expenses, reimburses households', those who have a misfortune, through subsidies. To be specific, subsidies for households with low incomes and limited assets provide an adequate chance to improve their disposable income. Households eligible for benefits do not have many assets, they are not flexible on the labour market, and their income lies under poverty line driving them into social exclusion. Therefore, those beneficiaries, who decide to claim for benefits, would be better off under the social administration. On the other extreme, because of implemented declines in welfare and services, the administration revokes the benefits by submitting a request to some steady clients who might find themselves worse off and to decide to be flexible once again on the labour market. That is to say, the emphasis on welfare implementation may manifest itself in hidden ambiguity as a result of economic growth, decline or stagnation, demographic shift, pit or migration, political change, change in the scarcity of resources, property rights, in the level of labour force skills and education, etc., by sending "insecurity waves" into social services. Now, while the services improve or decay, the households, whose disposable income is in the proximity of poverty line, may cause a hazard feedback (h-factor) effect on social costs and benefits, which we think may have an impact on the tax compliance of all citizens.

Thus, the first postulate in the Welfare Policy cascade (see above) discloses an obvious paradigm of public finance: The social agencies have to solve in cooperation with other fiscal institutions the long-term horizon problem for stabilization of public spending as it cannot be solved by market mechanism.

According to ability-to-pay, as how to stabilize inconsistency or distortion of tax polices, the known terms of warranty rely on exogenous taxes enforced by the government on the productivity of households. A variant of classic public finance and the like, Berliant and Gouveia [1994], is the concept when a household of a given productivity does not shift his/her labour supply after all the adjustments to tax formula have been implemented: Optimal taxation enforces optimal labour supply. Yet another "treatment of policies" concerning stabilization, closely related to societal inconsistency, entails equity of pre- and post-tax positions of taxpayers. We continue to rely on policy for stabilization to ensure that the inconsistency does not happen, assumptions No.3 and 4.

The view to demarcate between households attracts attention among economists and tax policy makers: Credit tax-scheme analysis contra income-tested program in "the rich and the poor," two-man economy, Kesselman and Garfinkel [1978], Poverty measurements, Sen [1976], Atkinson [1987], Ebert [2002], Hunter et al. [2002]. Horizontal inequalities seem to occupy a place in Stewart's [2000] paper, which reviews the connection linking income distribution to economic growth. Peñalosa and Wen [2004] investigate income redistribution, which operates as a form of social insurance. Tarp et al. [2002: 8] "*The poverty line acts as a threshold with households falling below the poverty line considered poor and those above poverty line considered nonpoor*."

***Rationality*.** We argue that it will be difficult to explain how the government, to benefit all of the society, assesses and controls the circulation of wealth through the social and public agencies unless we end up in long-term horizon on policy stabilization for public spending. Nevertheless, provided the policy is feasible, by entering the bargaining stage of the cascade, we demonstrate that the government might end up being capable of keeping together incompatible demands for social services and for public goods.

"These flimsy structures, however, are used by individuals to allocate resource flows to participants according to rules that have been devised in tough constitutional and collective-choice bargaining situations over time," Ostrom [2005: 823]. Therefore, by Kalai's [1977a] asymmetric variant of Nash [1950] bargaining solution, we reveal the division of taxpayers' obligations between social agencies, pursuing their own causes on one side, and public agencies on the other. Nonetheless, it might appear that we also be committed to reasoned belief to interpret the Nash solution by virtue of negotiators' strategic interaction. Well, not quite.

Yes, it is realistic to imagine that our actors of social and public agencies play the *"bargaining drama"* of alternating offers between themselves and simultaneously with citizens until all parties reach an agreement. Even though, we do not tell by which bargaining procedure the negotiators actually do arrive at an outcome, but demonstrate the calculus of Nash solution. The procedure of bargaining, whether it is under complete or incomplete information, is secondary for us. Nonetheless, whether the utilities are interpersonally comparable, or the procedure is under the hypothesis of collective rationality or not, makes a difference later. The literature dealing with these serious matters of bargaining occupy a wide range of publications, Alvin E. Roth [1985].

Self-interest. Finally, only at the third stage the well-being of citizens has been taken into consideration. Only at this stage, the government will move to meet the momentum of citizens by focusing on reasonable demands for welfare provision and delivery, to improve the perception and behaviour of consumers against high taxes. Otherwise, destabilized tax schedules or disagreeing social and public agencies at previous stages could block the further turn of the government in making favourable policy for citizens. Therefore, we have to accept that *"…as a result of the intervention of a third party, who exploits the mutual gains…if we perturb a bargaining game of alternating offers by introducing a small exogenous probability of breakdown then…the outcome is close to Nash solution of the appropriately defined bargaining problem,"* Osborn and Rubinstein [1990: 71-76]. To put it other way, the threat, in the terms of a disagreement to implement the breakdown, is an ultimate argument in the debate to achieve the goal of taxpayers (citizens).

Finally, we must emphasize that we do not analyse any voting system or scheme by which voters-citizens express their argument as taxpayers. Rather than, a voting tool of collecting data about who would approve or oppose as to what proposal is best, we design a debating and voting platform for the government policy on poverty, Table 1. As noticed by Roberts [1977: 329], *"The point is not whether choices in the public domain are made through a voting mechanism but whether choice procedures mirror some voting mechanism,"* and as such, we adhere to all voting guidelines where each tax proposal comes with or includes its own tax, c.f. Mueller [2003: 67]. Worth to know that without a goal of minimizing taxes, the policies, what the government will be dedicated to overturn, will not be necessarily approved by unanimous vote, Observation 3, c.f. Buchanan [1967: 4.1.4] reasoning of *"Public choices through professional organizations."*

3. The cake model

Someone thinking about standards how humans make decisions may ask: What a cut it is to divide a piece of cake fairly between two persons: HE – soft negotiator, not very keen on sweets, SHE – tough negotiator, likes sweets. Axiomatic bargaining theory solves the Sweet-Cake-Cutting problem by product maximization of players' gains above the disagreement point $d = (d_1, d_2)$, the asymmetric variant, Kalai [1977a]:

$$\arg\max_{0 \le x+y \le 1} f(x,y,\alpha) = (u(x) - d_1)^{\alpha} \cdot (g(y) - d_2)^{1-\alpha} .$$

Although the answer may probably be obvious to many game theory purists a question often asked is: What is x, y, α, $u(x)$ and $g(y)$? What is the point (d_1, d_2) and how to use the $arg\,max$ procedure? The answer may be:

x is HIS cutting the cake, and α – HIS bargaining power, $0 \le x \le 1$, $0 \le \alpha \le 1$.
$u(x)$ is HIS desire, for example $u(x) \equiv x$, of HIS x cutting the cake,
$1-\alpha$ is HER bargaining power,
$g(y)$ is HER desire of HER y cutting the cake, for example $g(y) \equiv \sqrt{y}$, $0 \le y \le 1$.

In widely accepted vocabulary, we call $s = (u(x), g(y))$ the utility pair. The disagreement point $d = (d_1, d_2)$ is what HE and SHE collect if they disagree how to cut the cake. The sweet cake disagreement point $d = (d_1, d_2) = (0,0)$; disagreeing players collect nothing for clear reason. Further, we indicate that expectations from the cake are more valuable for HER pointing at HER desire $g(½) = \sqrt{½} = 0.707$, which is greater than HIS desire $u(½) = 0.5$.

Now, considering the $arg\,max$ procedure of $f(x,y,\alpha)$ one may ask a new question: "What standards HE, the sweet cake negotiator, will make HIS decision on to obtain the half of the cake?" That is to ask, for example, what standards facilitate HIS negotiating power α to obtain the half of the cake if SHE may only accept or reject the proposal? A technically minded person can shed lights on the solution. First replace $u(x)$ by x, put $y = 1-x$, replace $g(y)$ by $\sqrt{1-x}$ and take the derivative of the result $f(x,1-x,\alpha)$ with respect to the variable x by evaluating $f'_x(x,1-x,\alpha)$. Then, replace $x = ½$, and finally solve the equation $f'_x(½,½,\alpha) = 0$ for α; the equation $f'_x(½,½,\alpha) = 0$ resolves for $\alpha = 1/3$.

In general, one might feel some comfort in following passage. “*Even in the face of the fact that SHE is twice as tough negotiator,* [1] *to count on the half of the cake is a realistic attitude towards HIS position of negotiations. Surely, rather sooner than later, since HE revealed that SHE likes sweets, HE would have HER to agree to a concession.*” This is an example of what ought to be the standard behaviour of a negotiator.

4. The tax-pool model

Consistency. Tax-pool stabilization. When trying to meet incompatible demands for the provision and delivery of social services and public goods, a subject turns out to divide the pool of taxes. Is there a resemblance with the Sweet-Cake-Cutting problem? To follow the same pattern of “cutting the cake-pool of taxes” seems, more or less, reasonable. Indeed, the subsidies on one side, and the public safety, environment protection, education and health services, national defence, roads and highway systems, etc., as public goods on the other side, all together demand to redistribute the wealth. Nevertheless, a difficulty could make the difference because the size of the pool might influence the fair bargain. Note: When HE and SHE tried to cut the cake, its size remained fixed! Is it true that the size of the pool if varying too fast might confuse the negotiators in making their decisions?

On what standards should the State make its policy more relevant towards this matter? One might suggest first looking at incomes of households to personalize the social, public and consumer needs of people. Thus, the situation looks more like a game between the social agencies negotiating their portion of the tax-pool with public agencies, and the taxpayers refunding the same pool. As mentioned above, this is not enough, because the size of the pool could vary too fast and thereby break into the behaviour of our rational negotiators. Surely, welfare costs could smooth out the balance of the tax-pool payments. Therefore, as long as the government does not stabilize the pool, the fair agreement between social and public agencies is at risk; we say not feasible or inconsistent. What policy might be feasible if the government wishes to be in position to meet the legal and moral rights of the poor?

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[1] Let say, she pays her solicitor twice as much as he does.

Let us first consider a policy on poverty without any warranty of consistency from the government. The policy might be set by the poverty line ξ to decide who is living in poverty, and to transfer the wealth from the rich to the poor: poor people's income σ is to *the left from* the line ξ, $0<\sigma\leq\xi$; rich people with income σ are *to the right*, $\xi<\sigma<\infty$.

Now, according to the assumption No.2, the government is ready to transfer the wealth W from the rich $\sigma>\xi$ to the poor $\sigma\leq\xi$. In considering assumption No.6 and based on its perception of income distribution density $P(\sigma)$, the government estimates the tax schedules in the form of $\tau\cdot W$, the costs of subsidies as B, and public costs as g. The government is ready to finance the social services – to refund the costs B via the tax schedules $\tau\cdot W$. It is ready to keep the balance of payments between the credits B and debts $x\cdot\tau\cdot W$ as a portion of schedules $\tau\cdot W$. Should the government shift the costs B to $B(\xi)$, and $\tau\cdot W$ to $\tau\cdot W(\xi)$, it would open the way to meet its readiness to keep the balance $B(\xi)=x\cdot\tau\cdot W(\xi)$.

In our vocabulary, an implemented or taken effect policy ξ, poverty line ξ, social policy, welfare reform, pact, program, etc. will specify the balance $B(\xi)=x\cdot\tau\cdot W(\xi)$. Despite the balance is valid, it might break in the time slots ahead of the beginning – policy ξ bids fair as being unstable in a long-term horizon. As far as balance of wealth is desirable, only few would question the assumption No.3. However, almost all, perhaps for different reasons, would prefer the balanced way to implement a long-term policy. This was but the first fold of the truth. The second embodies the welfare hazard, assumption No.4, as follows.

In fulfilling the policy ξ on poverty, by implementing rises in disposable income level u, all in need, who claimed for benefits, end up better off. A different outcome in level u would worse off some steady clients should their benefits being revoked. So, the level u *tidal waves* may introduce in the behaviour of those whose claims will be accepted or those who loose the benefits. These waves in turn may feedback to affect the income distribution $P(\sigma)$

and to shift it into undisclosed position $P(\sigma,\xi)$. Accordingly, the policy ξ will shift the balance $B(\xi) = x \cdot \tau \cdot W(\xi)$ into the long-term inconsistency $B_f(\xi) \neq x \cdot \tau \cdot W_f(\xi)$.

An issue that justifies the shift involves quantification. We take into account income scale over functions $P(\sigma,\xi)$. Hence, the scale admits only linear transformations $\sigma' = c \cdot \sigma$, $c > 0$ in establishing a ratio scale for all variables and functions. Actually, this scale satisfies a form of interpersonal comparability of utilities, c.f. Narens and Luce [1983: 249].

One will say that the economy must be immune against swings of poverty in a long-term horizon according to Consistency Postulate. That is to say, the immune policy ξ previously taken effect does not need further adjustments. Likewise, to say that implementing a policy ξ under the "immune" or "navigable environment" is all one as to say that the government, to ensure a proper result, implements the policy only once. For this reason, we assume that inconsistency of wealth is resolved – constraint (3) resolves for ξ, i.e. the feedback inconsistency $B_f(\xi) \neq x \cdot \tau \cdot W_f(\xi)$ has been eliminated: $B(\xi) = B_f(\xi)$, and $W(\xi) = W_f(\xi)$. Briefly, the initial tidal wave replication stabilizes after the implementation of the *push or pull* on the policy ξ.

In this mode, the government is developing its policy ξ that outlines the long-term strategy for stabilization of public spending by which the policy might be brought to conclusion. We therefore conclude that the account of expenses $x \cdot \tau \cdot W(\xi)$ meant for social spending, where x is the division of the tax-pool $\tau \cdot W(\xi)$, must be in balance with social costs $B(\xi)$ not only when the particular policy ξ takes effect but also in the whole spectrum of current and future events: the government policy ξ should enforce the long-term stability.

The balance $B(\xi) = x \cdot \tau \cdot W(\xi)$ is a relationship leading to functional dependency $\tau(\xi, x)$ binding ξ and x variables. The dependency τ suggests, that through formula $\pi(\sigma,\tau) = (1-\tau) \cdot (\sigma - \phi) + \phi$, c.f. Malcomson [1986: 266], the function $\pi(\sigma, \tau(\xi, x))$ is the after-tax income position of taxpayers, where $[\phi, \infty)$ is a tax bracket.

At last, this leads to an obvious question as to by which composition of social costs $B(\xi)$ and tax obligations $\tau \cdot W(\xi)$ these functions emerge as a realistic estimates of responses to the government policy ξ on poverty. Using an empirical data seems to be the best option. Yet, another *technical endeavour* could be to allow easy computations by classical method of function maximization (minimization) with constraints. In short, the welfare composition $[B(\xi), W(\xi)]$ is the origin of wealth in current investigation.

Summary. An outcome $\phi,\xi \Rightarrow p,x,\alpha,\langle u,g,\tau \rangle$ constitutes citizens bargaining shield for wealth circulation that relates to a bundle of variables: policy (or control) parameters ϕ,ξ; the status is set to p,x,α; $\langle u,g,\tau \rangle$ embodies the competing arguments (proposals);

- ϕ – the personal allowance establishing the tax bracket $[\phi,\infty)$, it is an ex-ante, the control parameter, $0 < \phi = const < \xi$;
- ξ – the poverty line, a policy to decide who is living in poverty, the choice or the control parameter as well;
- p – the pool $p = \tau \cdot W$ of tax obligations (public spending) in case of proportional taxes, the assumption No.5, p. a);
- x – the division of the tax-pool p to be deposited in favour of social agencies, $0 \le x \le 1$;
- α – a negotiating power of social agencies, $0 \le \alpha \le 1$;
- u – the guaranteed welfare or disposable income level, demands for social services;
- g – demands for public goods, public costs;
- τ – the marginal tax rate, the wealth-tax.

Three constraints must hold:

– delivery constraint, all taxes are spent on welfare (social + public); welfare delivery equals to liabilities of taxpayers', b) and c) of assumption No.5; this concept of wealth makes sense for proportional, i.e. for flat taxes only, p. a) of assumption No.5.

$$\tau \cdot W(\xi) = B(\xi) + g \qquad (1)$$

– balance of social costs with the portion of the tax-pool (portion of taxpayers' obligations) credited to, and deposited (debited) in social agencies' account, assumption No.3; $B(\xi)$ is the social costs mean value shifted by the government policy ξ,

$$B(\xi) = x \cdot \tau \cdot W(\xi) \qquad (2)$$

– necessary constraint to remove the effect of welfare hazard, assumption No.4, (we distinguish the utility levels $u = \pi(\xi,\tau)$ as an indifference curve $(\xi,\tau) \in \Re \subset \Re^2$ in contrast to $(\sigma,\tau) \in \Re^2$!)

$$u = (1-\tau)\cdot(\xi - \phi) + \phi \qquad (3)$$

- Division x and marginal tax rate τ, due to the constraints (1-3), became functions of variables ξ, g: $x = x(\xi, g)$ and $\tau = \tau(\xi, x(\xi, g))$. This form of dependencies appears later at the stage of *wealth-tax minimization*.
- The explicit formulas, for the total costs B of subsidies and the amount W of the wealth, are generally given by parameterised income distribution $P(\sigma, \theta + h \cdot \xi)$:

$$B(\xi) = \int_0^{\xi} s(\xi, \sigma) \cdot P(\sigma, \theta + h \cdot \xi) \cdot d\sigma, \; s(\xi, \sigma) \text{ is the subsidies function;}$$

$$W(\xi) = \int_0^{\xi} (\sigma + s(\xi, \sigma) - \phi) \cdot P(\sigma, \theta + h \cdot \xi) \cdot d\sigma + \int_{\xi}^{\infty} (\sigma - \phi) \cdot P(\sigma, \theta + h \cdot \xi) \cdot d\sigma,$$

where h-factor reveals the inverse working incentives feedback of social clients.

- The policy ξ is an issue shaping the purchasing of wealth W at the provision side of services and goods. It is an issue of average income $a(\theta + h \cdot \xi)$ maintenance to uphold the restraint $a(\theta + h \cdot \xi) > W(\xi)$ by proper choice of personal allowance parameter $\phi > 0$.
- The baseline $P(\sigma, \theta)$ of the income density function is the initial position, when the circulation of wealth from the rich to the poor just starts, i.e., $P(\sigma, \theta)$ embodies the assumption No.6. The feedback factor $h < 0$ leads to the assumption No.4 hiding the tidal wave $Hz(\xi) = \frac{\partial}{\partial \xi} a(\theta + h \cdot \xi) < 0$ of the welfare hazard triggered by the push $\xi + \Delta\xi$ or pull $\xi - \Delta\xi$ on the policy ξ. Factor $h < 0$ brings $P(\sigma, \theta)$ into undisclosed position $P(\sigma, \theta + h \cdot (\xi \pm \Delta\xi))$.

Following justifies the welfare hazard constraint (3) in more rigorous vocabulary.

Observation 1. *Constraint $u = \pi(\xi, \tau)$ is necessary to remove the welfare hazard effect.*

All proofs are in the appendix.

Corollary. *If $u = \pi(\xi, \tau(\xi, x))$ resolves for ξ, excluding the chance for households σ to misbalance $B(\xi) = x \cdot \tau \cdot W(\xi)$ and, hereby, without additional adjustments ξ', ξ'' to remove the effect of welfare hazard, then the long-term policy ξ is feasible.*

Finally, replacing the expression of the constraint (2) via τ into (3), the welfare hazard constraint $u = \pi(\xi, \tau(\xi, x))$ must resolve for feasible policy ξ in the form:

$$L(\xi, x, u) := (\xi - \phi) \cdot B(\xi) - x \cdot (\xi - u) \cdot W(\xi) = 0. \tag{4}$$

We call (4) the volatility constraint that introduces in rigorous form the consistency postulate. Thus, it holds down too generous policy of the government on poverty.

A notice is appropriate. We already know that social costs $B(\xi)$ and the amount $W(\xi)$ of wealth are the keys to the welfare composition in our two-man economy. In addition, we stress that ± rates $W'(\xi) \le 0$, $W'(\xi) \ge 0$ of changes for amounts $W(\xi)$, are essential for the analysis. However, we use the function $B(\xi)$ only with $B'(\xi) > 0$, assumption No.7.

Rationality. Tax-pool bargaining. Let, in accordance with the second postulate of Welfare Policy, both the rich and the poor pay their taxes τ, and let, by calculating the costs $B(\xi)$ and amounts of wealth $W(\xi)$, the government estimates for the policy ξ. How the social and public agencies are going to bargain on the division of the tax-pool?

We first personalize the following arguments of all parties involved:

Arguments of the Negotiators No. 1, 2 and the Player No.3:

Negotiator No.1	u	–	welfare level, the legal and moral argument (the lowest disposable net income per capita, guaranteed level of poverty to be implemented by the government in compliance with the law;
Negotiator No.2	g	–	public goods per capita, argument that benefits all of the society;
Player No.3	$\tau(\xi)$	–	estimated wealth-tax, the less the better. We consider only proportional taxes, so-called flat taxes – the case, which substantially simplifies the method of function maximization (minimization) with constraint, p. a) of the assumption No.5.

Note: When the size of the pool *varies too fast* [2], a feasible outcome of the game could be at risk unless the volatility constraint $L(\xi, x, u) = 0$, Observation 1.

Before advancing any further, let recollect the phenomenon of bargaining problem. Assume that negotiators bargain over possible division (x, y) of the tax-pool $\tau \cdot W(\xi)$, $x + y \le 1$. Below we follow Nash's [1950] axiomatic approach. An arbitrary set S of utility pairs $s = (s_1, s_2)$, $s_1 = u$, $s_2 = g$ evaluated only for feasible policies ξ (policies immune against welfare volatility) can be the outcome of the game. A disagreement occurs at the utility point $d = (d_1, d_2)$, where both negotiators $i = 1,2$ obtain the lowest welfare they count on. The *bargaining problem* is a pair $\langle S, d \rangle$ and there exists $s \in S$ such that $s_i > d_i$ for $i = 1,2$ and $d \in S$. A *bargaining solution* is a function $f(S, d)$ that assigns a unique element of S to

[2] Be aware that the process $\xi', \xi'', \ldots$ for stabilization of the tax-pool has a limit $\xi = \lim \xi', \xi'', \ldots$, whereby the level $u = u(\xi)$ and public goods $g = g(\xi)$ as functions turn into "stable" limits as well. Proof of this fact in parallel with the Observation 1 is beyond the scope of current investigation.

every bargaining problem $\langle S, d \rangle$. We expect that the negotiators to be committed to the bargaining solution f, which satisfies SIR – the *strong individual rationality* hypothesis: $f(S,d) > 0$ for every bargaining problem $\langle S, d \rangle$. The solution must comply with other Nash axioms: Invariance under linear change of scale of utilities, Independence of irrelevant alternatives, and Pareto efficiency. Following efficiency, negotiators never agree on ξ in favour for ξ' if $s(\xi') = s'$ and $s' > s$, $s = s(\xi)$. It means that pragmatic negotiators object an outcome s when a better (more efficient) outcome s' for both of them is feasible.

As already mentioned, the poverty line or policy ξ on poverty is the control or choice parameter of the government. Throughout the paper, we refer to the policy ξ as to poverty proposal, when the bargaining agreement over the division of the tax-pool is under negotiations. Therefore, to place a proposal ξ to reveal the constraint that mediates consumers perception against high taxes, which would thus enable the least wealth-tax, seems to offer an appealing interpretation to the otherwise puzzling welfare anticipation $s(\xi) = (u, g)$.

Comment is in place to clarify a standpoint of confusion. Classical definition of bargaining problem consists of a compact convex set $S \subset \Re^2$ and the disagreement point $d \in S$. Conventionally, the bargaining occurs on contract curve S_b. In our case, we call this curve a bargaining frontier. A typical frontier $S_b = u(g)$ on Fig. 2 represents proposals ξ, which solve every single $\xi \in [\xi_1, \xi_2]$ for the volatility constraint (4) and in the same way for efficiency constraint (6). In this respect, we read from Osborn and Rubinstein [1982: 24] that: *"The compactness and the convexity of S are important only insofar as they ensure that the Pareto frontier of S is well defined and concave. Rather than starting with the set S, we could have imposed our axioms on a problem defined by a non-increasing concave function* (*and a disagreement point d*)*."*

It looks like our environment for bargaining frontier $S_b = u(g)$ expands beyond the traditional framework of Osborn and Rubinstein. The thing is that negotiators may cross or penetrate through the bargaining frontier as a boundary into prohibitive region – a region of too generous poverty proposals ξ. Would the negotiators, nevertheless, be willing to continue bargaining but without commitment to cross the frontier S_b, then an inefficient agreement on policy ξ, or any $s(\xi) = (u, g)$ related to prohibitive region, is not worth all of this bargain, Observation 6. However, within the normal range of poverty to examine pragmatic interaction of negotiators might be an advantage. In short, passing along the normal range of poverty the bargaining frontier S_b arranges an ultimate barrier if the negotiators drive to it improving their payoffs in cooperation. That was the reason why we called the cross passage by a bargaining frontier.

One, perhaps, may recognize here the vocabulary of Laffer curve but for different domain, similar to: First *"…poverty lines being proposed in the normal range of collective rationality;"* Next, *"…by passing through their last bifurcation"* poverty proposals will be assessed and reviewed in the range of prohibited (individual) rationality. We exploit here *normal*, *prohibited* vocabulary of Canto, Joins and Laffer [1981: 12], as well as we paraphrase from Rapoport [1994: 31]: *"Bifurcation of the concept of rationality into 'individual' and 'collective' rationality, which prescribe different courses of action to a 'rational' actor."*

A closer look at what we later will call *"a breakdown manoeuvring of negotiators"* seems to be instructive. In the normal range any utility anticipation $s(\xi) = (u = d_1, g = d_2)$ of poverty suits well as breakdown or disagreement point. To sharpen the meaning, we will interpret the point d as well as the risk it poses not to comply with realistic standards of social services and public goods. It is not immediately apparent when a disagreement point d is a feasible outcome of the game. Following observation rules out such a doubt.

Observation 2. *To testify that the utility point* $d = (d_1, d_2)$ *becomes a feasible outcome of the bargaining game, it is necessary and sufficient that there exists a policy* δ *on poverty resolving the equation:*

$$(\delta - \phi) \cdot (B(\delta) + d_2) - (\delta - d_1) \cdot W(\delta) = 0 . \qquad (5)$$

Recall that $W(\delta)$ is the amount of wealth circulating in the society, $B(\delta)$ – social costs.

Corollary. *Suppose that there exists a policy* δ *resolving the equation* (5) *for the utility pair* $d = (u_1, g_2) \notin S_b$, *i.e., for the pair composed of the endpoints* (u_1, g_1) *and* (u_2, g_2) *in the bargaining interval* $[\xi_1, \xi_2]$, $g_1 > g_2$, $u_1 < u_2$; *condition* $\delta \notin [\xi_1, \xi_2]$ *is necessarily required. Then, for any utility pair* $s \in S_b$ *we have* $s > d$, *and for the bargaining frontier* S_b, *the pair* $\langle S_b, d \rangle$ *is a well-defined bargaining problem.*

Since the volatility constraint $L(\xi,x,u)=0$ resolves for poverty proposal ξ, see (4), the outcome $\phi,\xi \Rightarrow p,x,\alpha,\langle u,g,\tau\rangle$ do not allow too fast variation of the tax-pool $p=\tau\cdot W$ in size. Even so, a different aspect of the bargaining frontier is the efficiency constraint on utility pairs (u,g). We stress it again that bargaining frontier S_{b} is a boundary at which the motivation of negotiators changes. Negotiators *arrived* at the frontier guided by collective rationality, but they have an option, in view of individual interests and without commitment, to continue bargaining by crossing into prohibitive range of *inappropriately high values* of poverty. One thing perhaps is worth. Outcomes in the prohibited range result de facto in *"status quo situations,"* Kalai [1977b: 1623]. That's why, the frontier S_{b} is *"the last chance"* for society to fulfil its mission with appropriate level of subsidies provided for the poor what is fully coherent with the difference principle of justice, Rawls [1971: 303].

Summary. An outcome $\phi,\xi \Rightarrow p,x,\alpha,\langle u,g,\tau\rangle$ on the bargaining frontier S_{b} must satisfy the efficiency constraint

$$D(\xi,x,u):=\frac{\partial}{\partial\xi}\left[(\xi-\phi)\cdot B(\xi)-x\cdot(\xi-u)\cdot W(\xi)\right]=0. \qquad (6)$$

If someone doubts the solution (6) of the bargaining problem $\langle S_{\mathrm{b}},d\rangle$ constrained by $D(\xi,x,u)=0$, we reproduced the necessary and sufficient conditions for criteria (6) in the appendix, Observation 5.

Self-interest. Wealth-tax minimization. Those who need to make decisions know the minimization/maximization problems under constraints.

The marginal tax, i.e. the wealth-tax minimization, is the consumer (M)-problem:

(M) $\min_{\xi,g}\ \tau\equiv\tau(\xi,x(\xi,g))$,
where the tax parameter is a function of ξ and x, $x=x(\xi,g)$ is the division of the tax-pool (taxpayers obligations);

and the bargaining is (N)-problem

(N) $\max_{u,g}\ f(u,g)=(u-d_1)^{\alpha}\cdot(g-d_2)^{1-\alpha}$ for some unknown α.

Both (M) and (N) are subject to constraints:

$Q(\xi,\tau,g)=0$ – delivery;
$L(\xi,x,u)=0$ – volatility;
$D(\xi,x,u)=0$ – efficiency;

as preconditions for the political game between welfare state institutions.

Particularly, in case of proportional (flat) taxes, constraints Q, L and D, following a succession of collections on sub-expressions and simplifications, imply an analytical solution:

$$g(\xi)=\frac{W(\xi)}{v(\xi)}-B(\xi),\ \text{where}\ v(\xi)=1+(\xi-\phi)\cdot\left(\frac{B'(\xi)}{B(\xi)}-\frac{W'(\xi)}{W(\xi)}\right),$$
$$u(\xi)=\xi-\tau(\xi)\cdot(\xi-\phi),\ \tau(\xi)=\frac{p(\xi)}{W(\xi)},\ p(\xi)=B(\xi)+g(\xi),\ B'(\xi)>0.$$

Hereby, the welfare level $u(\xi)$ and public goods $g(\xi)$ depend only on poverty proposals (policies) ξ of the government. Thus, the minimization of consumer (M) as well as the bargaining problem (N), turn to be without constraints:

(M) $\quad \min_{\xi} \tau(\xi)=\dfrac{B(\xi)+g(\xi)}{W(\xi)},$

(N) $\quad \max_{\xi} f(\xi,\alpha)=(u(\xi)-d_1)^{\alpha}\cdot(g(\xi)-d_2)^{1-\alpha}.$

Observation 3. *Condition* $\lambda=\arg\min_{\xi}\ \tau(\xi)$ *is necessary to put forward a poverty proposal* λ *before the welfare committee by unanimous vote. At the bargaining frontier* S_{b}, *the proposal* λ *outlines a unique outcome* $\phi,\xi\Rightarrow p,x,\alpha,\langle u(\lambda),g(\lambda),\tau(\lambda)\rangle\in S_{\mathrm{b}}$.

At last, we arrived at the idea of resemblance between: the *sweet cake Policy-Making*, when two highly pragmatic persons try to cut a sweet cake, and the *welfare Policy-Making*, when the social and public agencies try to divide the tax-pool. Provided $\tau(\xi)$ is smooth enough, we experienced above that the solution of consumer problem (M) is the root λ of the equation $\tau'(\xi)=0$. We also experienced that, to avoid the breakdown, associated with political manoeuvring of taxpayers, the policy λ is the most desirable for the government. But, how to serve the Sweet-Cake-Cutting as a portion of suitable tax package in a way that impacts the power α of the social agencies to meet the finance of welfare level $u(\lambda)$?

To put a new spin on old idea appears to be clear. Indeed, the standards affecting HIS negotiating power α of the Cake-Cutting can be the standards for the social agencies to affect their negotiating power α. Recall that first, we must take the derivative of $f(\xi,\alpha)$ with respect to ξ evaluating $f'_{\xi}(\xi,\alpha)$, and then we must replace ξ by λ (do not replace $\xi=½$ this time like in the case of sweet cake). At last, the negotiating power α of the social agencies solves the equation $f'_{\xi}(\xi|_{\xi=\lambda},\alpha)=0$ for α.

5. Self-enforcing solution

Let sidestep the analysis by focusing first on some shortcomings. It seems we passed by all the troubles with the disagreement poverty proposal δ. Nevertheless, imagine a situation when, in the search for funds financing initial anticipations (u_1, g_1) and (u_2, g_2) of negotiators, the difference in amounts of wealth and taxes could in the beginning contribute to or amplify misunderstandings of the rules in the political game. Below, we not only take care of how to avoid the difference, we are also working on an extra option how to extract the breakdown point $d = (d_1, d_2) = (u_1, g_2)$ endogenously encoded into the scheme.

The frontier $S_b = u(g)$ looks like a curve $(u(\xi), g(\xi))$, parameterised by proposals ξ within the scope of negotiations $\xi \in [\xi_1 < \xi_2]$, $u_1 = u(\xi_1)$, $g_1 = g(\xi_1)$, $u_2 = u(\xi_2)$, $g_2 = g(\xi_2)$, $u_1 < u_2$, $g_1 > g_2$, corollary to the Observation 2, Fig. 2. Social agencies make a proposal ξ to the agreement $(u(\xi), g(\xi)) \in S_b$ what the public agencies can only accept or reject. The risk of policy ξ breakdown emerges when the committee decides, on behalf of voters, to implement the worst-case scenario (u_1, g_2), or, the public agencies decide against the proposal ξ. As it follows from Observation 2, the worst-case proposal but feasible δ, if the equation (5) can be resolved, accumulates, as well, the endpoints of the interval $[\xi_1, \xi_2]$.

In the following lines of reasoning, we label the situation as an ***equity condition of breakdown*** associated with political manoeuvring of taxpayers. Indeed, we already know that the implementation of utility anticipations, i.e. the demands $s_1 = (u_1, g_1)$ and $s_2 = (u_2, g_2)$ at the ends of interval $[\xi_1, \xi_2]$, would require some amounts of wealth $W(\xi_1)$ and $W(\xi_2)$ for taxes $\tau(\xi_1)$ and $\tau(\xi_2)$. The demands s_1, s_2 of negotiators being the "initial amounts of wealth and taxes" are, in mundane terms, the alleged resources for maintenance of social services and public goods. Therefore, to bring negotiators into just and equal positions prior to negotiations, it is reasonable, if possible, to equalize their legal claims for finance of welfare implementation. Thus, the exercise would be to fix an interval $[\xi_1, \xi_2]$ solving a system of two non-linear equations $W(\xi_1) = W(\xi_2)$ and $\tau(\xi_1) = \tau(\xi_2)$ for two variables ξ_1 and ξ_2, i.e. to find a point (W^*, τ^*) where the bargaining frontier crosses its own contour on the plain with W, τ as *XY*-axis coordinates like on Fig. 3. Not a hard exercise to quantify the cross point, but we need a criterion for existence. We found the point in number of cases.

6. The method

Recommendations. We will now go back from the way we sidestepped the bargaining frontier analysis and continue along the guidelines of the general scheme. Eventually, as far as incomes survey analysis is required, additional efforts to estimate or restore the income distribution $P(\sigma)$ from atomic data will be necessary. There are methods, for example, Feser et al. [1994], to cope with this problem.

We recommend performing the analysis by simulation in steps.

1. Parameterisation of $P(\sigma)$ by θ in the form $P(\sigma,\theta)$. Parameter θ must keep to the primary shape of the distribution $P(\sigma)$, but *crash* it to the right when θ increases and *protrude* to the left when decreasing. In the example, we also use a parameter m to make the distribution more equal/flat when m increases and more unequal/peaked when m decreasing. The equal/unequal parameterisation of the primary $P(\sigma)$ is not mandatory.
2. Here, assumption No.4, the h-factor inserts the adverse working incentives of households. Replace the parameter θ by $\theta + h \cdot \xi$, where ξ represents the policy on poverty and choose $h < 0$. In so doing, the average income $a(\theta + h \cdot \xi)$ as an indicator on the provision side of services and goods of the distribution $P(\sigma,\theta + h \cdot \xi)$ inherits the feedback effect of welfare hazard. To remove the effect the social administration must know the incomes of social clients, assumption No.1. The administration accepts all eligible claims and revokes all not eligible claims. Once the policy ξ takes effect all households with incomes σ below ξ claim and receives the benefits, $\sigma < \xi$. No benefits for $\sigma \geq \xi$ or the benefits being revoked even if households fall under ξ in the past but now are above ξ. The tidal wave replication, hidden in income densities $P(\sigma,\theta)$, as well as the wave $P(\sigma,\theta + h \cdot \xi)$ of adverse working incentive hidden in its undisclosed position, both have characteristic "tail" to the right, which is typical for the societies sharply divided into very rich and very poor people, Fig. 1.
3. Selection of the subsidies function. We emphasize that subsidies eligible for claims are paid out as benefits $s(\xi,\sigma)$ in compliance with the law, assumption No.2. The subsidy compensates the unfair income σ of the poor and is a supplement to the poor to compose the eligible "poverty basket" for food, clothing and shelter, fuel and lights, etc., c.f. what Rawls [1971: 92] calls "primary goods".
4. Given $P(\sigma,\theta + h \cdot \xi)$ evaluate the welfare composition $[B(\xi),W(\xi)]$.
5. Notice, that by the restraint $a(\theta + h \cdot \xi) > W(\xi)$ we already pointed at vital importance of the proper choice of personal allowance ϕ-parameter, $\xi > \phi > 0$. Indeed, for $\phi \approx 0$ we obtain *too sterile* solution, $\tau_{min} \approx 0$. In contrast, condition $\phi \approx \xi$, if the delivery constraint (1) is desirable, could lead to excessive public spending. In the example below $\phi = 2.34$.

6. If possible, get hold of the principle associated with equity of breakdown. Otherwise, we run into the shortcomings of Nash bargaining scheme regarding the disagreement point $d=(d_1,d_2)$. Proper choice of breakdown allows to extracts the interval, or the scope of negotiations $\xi \in [\xi_1,\xi_2]$, internally encoded into distribution $P(\sigma,\theta+h\cdot\xi)$.

Finally, a conjecture observed true in the next example and the like.

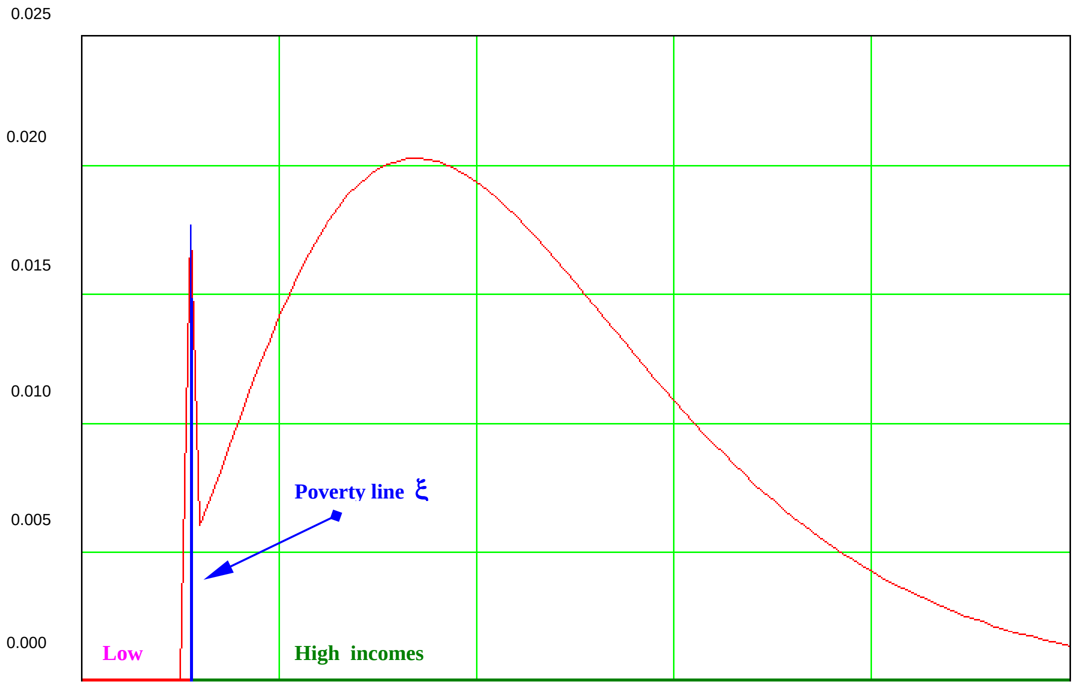
Under condition for equity principle of breakdown manoeuvring, the policy η *on poverty with equal power of negotiators minimizes the wealth amount* $W(\xi)$.

Simulation. Example. We proceed further with a specific simulation for the welfare state positions encapsulating an income distribution density similar to exponential function:

$$P(\sigma,\theta+h\cdot\xi)=\frac{1}{(\theta+h\cdot\xi)\cdot\Gamma(m)}\left(\frac{\sigma}{\theta+h\cdot\xi}\right)^{m-1}\cdot exp\left(-\frac{\sigma}{\theta+h\cdot\xi}\right),$$ where

$m=4.1$, $\theta=11.9$, $\Gamma(m)=6.81$ and $h=-0.09$. Average income of this σ-density equals $a(\theta+h\cdot\xi)=\int_0^\infty \sigma\cdot P(\sigma,\theta+h\cdot\xi)\cdot d\sigma$, $\Gamma(m)$ – extension of $(m-1)!$ to real variables. The subsidy $s(\xi,\sigma)=\xi-\sigma>0$ supplements the income of the poor up to the poverty line. Incorporation of more comprehensive rules of assistance is possible.

Figure 1. Income Distribution Density with characteristic *"bifurcation phenomena"* around the poverty line



Let us verify that a disagreement policy δ might be an outcome of the game in accordance with the equity principle of breakdown manoeuvring. There is no reason why the equation

$(\delta-\phi)\cdot(B(\delta)+d_2)-(\delta-d_1)\cdot W(\delta)=0$, associated to the Observation 2, has a solution in general. However, for the particular example of the income distribution $P(\sigma,\theta+h\cdot\xi)$, see above, the equation may be resolved. Indeed, using utility pairs at the endpoints of the bargaining interval $[5.32, 39.16]$ as $d=(d_1,d_2)=(u_1,g_2)$, $u_1<u_2$, $d_1=u_1$, $d_2=g_2$, $g_1>g_2$, where $(u_1=4.54, g_1=10.9)$ and $(u_2=29.45, g_2=1.29)$, one can check that $W^*=41.32$ and $\tau^*=26.38\%$, while $\delta=4.61\notin[5.32, 39.16]$.

Recall already known incomes – proposals λ, δ (bear in mind that δ is outside the frontier), together with some new poverty proposals k, η and ½μ as follows:

- κ the poverty line, a policy minimizing public spending; minimizing the pool of tax obligations instead of the wealth-tax;
- η the policy on poverty with equal power of negotiators; the social and public agencies are in symmetric positions or equal roles;
- ½μ 50% of the median income; μ such that half of the population having income above μ, and half having income below that amount.
- λ the poverty line or policy minimizing the tax rate, i.e. the best wealth-tax;
- δ the most negative result; a) the policy of breakdown or disagreement in the political game or manoeuvring in welfare committee, or b) when the proposal of social agencies on tax-pool division was rejected by public agencies.

After a quick glance at the table below, we are going to mirror the *"eventualities of the burden of taxation"* by the magnitude and dimension of poverty proposals ought to be debated or implemented.

Table 1. Numerical Experiment behind the Bargaining Game of Welfare Policy-Making and Delivery [3]

Obtained by means of income distribution density, Fig. 1; personal allowance ϕ = 2.34	**Policy, minimizing public spending**	**Policy with equal power of negotiators**	***Policy, minimizing the wealth-tax***	***50% of median income***	**Policy of disagreement, the breakdown**
Policy on poverty	20.05	25.45	*17.36*	*16.76*	4.61
Poverty rate: percentage of households below the poverty line	12.79%	25.65%	*8.1%*	*7.2%*	0.06%
Negotiating power of social agencies	0.28	0.5	*0.2*	*0.19*	Not defined
Welfare level	16.48	20.62	*14.35*	*13.86*	4.54
Public goods	7.07	6.1	*7.43*	*7.5*	1.29
Amount of wealth circulating in society	38.33	37.94	*38.75*	*38.86*	41.47
The wealth-tax, Marginal tax rate	20.16%	20.9%	*20.05%*	*20.06%*	3.1%
Public spending	7.73	7.93	*7.77*	*7.8*	1.29
Average income, Provision indicator	39.98	38.43	*40.71*	*40.87*	43.74
The first day free of tax [4]	12. March	17. March	*11. March*	*11. March*	11. January

[3] Imagine, the column in the table is a proposal laid before the welfare committee the member of which wishes to vote for.
[4] The first day of the year and all succeeding days with 0% tax in contrast to all previous days with 100% tax.

A chance behind the study to masquerade the economic reality might be an answer to the following question: *"Is it none the less true that in the long-term horizon it is going to be difficult to maintain the welfare if the entire tax burden for all citizens is decreasing?"* On the surface, it seems that at some point the fairness and equity may disappear because the rich simply gets richer and the poor gets poorer. The effect of "*a tax relief for the rich*" seems to drop down the welfare. Nevertheless, it remains to be seen what fallout may result from such eventualities. We find out that tax relief actually guarantees a fair level of welfare.

Judgement. It goes without saying that entering the realm of obvious utopia, the table, nonetheless, gives a reason why the social agencies can still make a fair bargain with the public agencies trying to help the poor. Indeed, when engaged in interaction to implement equal policy *25.45* (like HE and SHE engaged the Sweet-Cake-Cutting), the government reviewed the demands for social services and for public goods. The equal power $\alpha = 0.5$ of social agencies negotiators was stronger than *0.2*, see the Table 1. However, the incident with weakened power $\alpha = 0.2$ is yet to be determined and the aim of the agreement can still be reached on policy *17.36* for the tax rate *20.05%* < *20.9%*. Thus, regardless of the reduced liabilities of taxpayers, and even with the weakened bargaining position, the social agencies will be able to find a consensus with the public agencies to maintain a fair level of welfare. Therefore, only the policy $\lambda = 17.36$ on poverty, Fig. 3, has a chance for a vote by the unanimous consent. This is an example of standards as to how the State ought to behave when trying to fulfil its welfare mission.

7. Concluding remarks

It is the moment to pass in review the knowledge what the study brought to us. So far, we shot our loads in two directions. Into the social circle, where households of high and low incomes got split up into two-man economy depending on policy to decide who is living in poverty, to estimate the level of social costs, and thereby to reimburse the poor to treat them fairly. The public circle, where we revised a way of how to estimate the costs of public goods. Together, these estimates become functions of poverty line as a parameter. Therefore, the government policy on poverty, navigated through the tax system, was the choice of poverty line, which influenced the circulation of wealth, and for this reason, had an inverse working effect called welfare hazard feedback or h-factor.

Now we reveal an invisible target for the idea of feasible tax resource formation among households with higher incomes contra lower incomes. We determined the net income to serve as self-interest of households. Without the reader being fully aware of it, in so doing, we gave credit to the tax resource ability to guarantee reasonably high living standards for the poor. Later we used the guaranteed welfare as a welfare indicator. The social agencies took to the role of welfare delivery and were willing to put in all the efforts to increase the indicator. Public agencies argument, trailing a turn of the political wheel, was to meet the demands for public goods. Therefore, to benefit all of the society, the root of our scheme was the welfare formation under governmental institutions pursuing their own causes. Within the limits of the scheme whereas the poverty line increases, it is now a standard exercise to find out that the total resource, which is legally available for taxation, is shrinking in size and declining together with self-interests of households. This finding, a part of welfare formation process, leads to Laffer type $\cap$-peakedness of welfare indicator and plays a major role in establishing a non-trivial bargaining solution on the contract curve called the bargaining frontier. After all, this missing detail becomes merely a pretext to the following.

The *"piece of drama"* to get the reader here, was the bargaining game embedded into the welfare policy of institutions. In search for reasons behind the game, we provided a guide for how the rivalling interests of institutions as the players of the game ought to be analysed. Grounding on the prerogatives of the game story transformation into a normative model, we then embodied from the scheme a welfare masquerade of taxpayers interaction with the interest groups involved. As the result, we could express the consensus of the game status in numbers. The model explicitly introduced a bargaining power of institutions. We looked first due to shortcomings of the Nash scheme at the power parameter realizing that the study cannot provide its suitable calculus. However, we were lucky to finally find the condition, at least true in valuable cases, and called it *"Equity of Breakdown Manoeuvring,"* to fulfil the calculus properly. Last but not least. Despite one experiment in aiming at the simulation of the model does not make a trend, we presented an evidence for the claim that the well-known poverty line defined as 50% of the median income is a poverty proposal minimizing the wealth-tax in terms of the scheme and conditions for the consensus.

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Appendix

Proof of Observation 1. On the contrary, suppose that the government is trying *to implement an improved* welfare level $u' > u$. In other words, let the government attempts to implement $u' > u = \pi(\xi, \tau)$ by making a policy $\xi' > \xi$. At once, for some pragmatic households σ an option becomes visible *to claim for subsidies* to be *better off* because of inequalities $u' \geq \pi(\sigma, \tau) > u$. These highly pragmatic households σ would create *an increase* $B(\xi') > B(\xi)$ in the social costs, i.e. they would shift the balance $B(\xi) = x \cdot \tau \cdot W(\xi)$ onto *deficit* $B(\xi') > x \cdot \tau \cdot W(\xi)$. For the government, to keep the balance, the only option remains to adjust from τ to $\tau(\xi', \xi, x) = \dfrac{B(\xi')}{x \cdot W(\xi)} > \tau$; division x is not under the direct government jurisdiction, otherwise, by keeping the old policy ξ in tact, the government could, but it cannot eliminate the *deficit* through *decrease* in x. If $u' > \pi(\xi', \tau(\xi', \xi, x))$ the government must continue it's adjustment policy of taxes $\tau(\xi'', \xi', x) > \tau(\xi', \xi, x)$, but this time adjusting the welfare policy ξ' trying to eliminate a new *deficit* $B(\xi'') > x \cdot \tau \cdot W(\xi')$ by the sequence $...\xi' < \xi''$ creation. ■

To sum up: trying *an improvement* $u' > u$ means making a sequence of policies $..., \xi' < \xi''$ unless $\xi'' = \xi'$; a policy ξ' previously taken effect does not need further adjustments.

The chain of reasoning with $u' < u$ is similar. Just follow the instructions.

Replace:	*to implement an improved*	by	*to make a decline in*
–	*better off*	–	*worth off*
–	*an improvement*	–	*a worsening*
–	*to claim for subsidies*	–	*that the subsidies have been revoked*
–	*deficit*	–	*surplus*
–	$\geq, >$	–	$\leq, <$
Transpose:	*an increase*	with	*a decrease*

Proof of Observation 2. The statement is correct as soon as we can find a feasible policy δ for implementation of utility pair (d_1, d_2). Really, let us replace the variable g by d_2 in the formula for constraint (1). Then, take out the expression for $\tau = \dfrac{B(\delta) + d_2}{W(\delta)}$ from (1) and substitute it into $(1 - \tau)...$ of constraint (3), where u should be replaced by d_1 in advance. Simplifying we end up with the statement of the observation. ■

Sketch of the proof, Observation 3. The proof is an add-on to the Observation 4. Indeed, looking at the wealth-tax values $\tau > \tau_{min}$, for any outcome $...\langle u, g, \tau \rangle \in S_b$, one may prefer a counter outcome as a motion $...\langle u', g', \tau \rangle$, which outlines $...\langle u' > u, g' < g, \tau \rangle$ or $...\langle u' < u, g' > g, \tau \rangle$. However, since the frontier $S_b = u(g)$ is the curve consisting of top-points of single $\cap$-peaked preferences for the guaranteed welfare u, no one can put forward a motion $u' > u^\circ$ or $g' > g^\circ$ against an outcome $...\langle u^\circ, g^\circ, \tau_{min} \rangle$ at $\tau = \tau_{min}$. We argue that the only way to fulfil the expectations of citizens to carry out the motion $...\langle u^\circ = u(\lambda), g^\circ = g(\lambda), \tau_{min} = \tau(\lambda) \rangle$ is by the unanimous vote. ■

Below we investigate the social services and public goods feasible anticipations $(u,g)\in S$. The bargaining agreement occurs at outcomes $\phi,\xi \Rightarrow p,x,\alpha,\langle u,g,\tau\rangle$ under constraint that the variation of policy ξ does not improve the position of both negotiators at the same time – the point turns up on bargaining frontier $S_b = u(g)$.

In feasible outcomes, the variables of welfare level u, the division x, policy ξ and the tax rate τ depend on each other. The division $x = x^\circ$, if settled as a possible agreement, redirects the level $u = \pi(\xi,\tau(\xi,x^\circ)$ to become a function $u = u(\xi,x^\circ)$. Consequently, the peak point of level u with regard to the government policy looks like:

$$\xi^\circ = \arg\max_\xi u(\xi,x^\circ) \tag{A.1}$$

Observation 4. *Assume that the social agencies do not shift from the division* $x = x^\circ$. *Let the volatility constraint* (4) *solve for two different policies* $\xi_1 < \xi_2$. *Let the welfare level* $u = u(\xi,x^\circ)$ *be a differentiable and strictly convex function of* ξ *within closed interval* $[\xi_1,\xi_2]$, *the derivatives*

$$\frac{\partial}{\partial\xi}u(\xi,x^\circ)\bigg|_{\xi=\xi_1} > 0,\ \frac{\partial}{\partial\xi}u(\xi,x^\circ)\bigg|_{\xi=\xi_2} < 0 \text{ and } \frac{\partial^2}{\partial\xi^2}u(\xi,x^\circ) < 0,$$

then there exists a unique interior policy ξ° *maximizing* u *at* $\frac{\partial}{\partial\xi}u(\xi,x^\circ)\Big|_{\xi=\xi^\circ} = 0$.

Corollary. *The poverty proposal* ξ *higher than* ξ° *can only decline the welfare level* $u(\xi^\circ,x^\circ)$. The proof is elementary. We say that if $\xi \le \xi^\circ$, the poverty proposal ξ for the division x° is in the normal range, if $\xi > \xi^\circ$ the proposal ξ is in the prohibitive range, i.e. the level $u(\xi,x^\circ)$ is single $\cap$-peaked.

The standpoint coming next concerns the necessary and sufficient conditions for feasible policy ξ to occur at the bargaining frontier.

Observation 5. *Assume that the volatility constraint* (4) *is a differentiable of its variables. The welfare level* $u = u(\xi,x^\circ)$ *is differentiable and strictly convex with respect to the policy* ξ *within some closed interval* $[\xi_1,\xi_2]$. *To become a point at which a feasible outcome* $\phi,\xi^\circ \Rightarrow p^\circ,x^\circ,\alpha,\langle u^\circ,g^\circ,\tau^\circ\rangle$ *lies on the bargaining frontier* $S_b = u(g)$ *it is necessary and sufficient that the policy* ξ° *resolves the equation:*

(i) $\frac{\partial}{\partial\xi}L(\xi,x^\circ,u^\circ)\Big|_{\xi=\xi^\circ} = 0$, where $u^\circ = u(\xi^\circ,x^\circ)$ provided that

(ii) $\frac{\partial}{\partial u}L(\xi^\circ,x^\circ,u)\Big|_{u=u^\circ} \ne 0$.

Proof. Necessity. Let a feasible outcome $\phi,\xi^\circ \Rightarrow p^\circ,x^\circ,\alpha,\langle u^\circ,g^\circ,\tau^\circ\rangle$ on the bargaining frontier $S_b = u(g)$ maximizes (A.1) at $u^\circ = u(\xi^\circ,\tau(\xi^\circ,x^\circ))$. Varying policy ξ around ξ° for the outcome $\phi,\xi^\circ \Rightarrow p^\circ,x^\circ,\alpha,\langle u^\circ,g^\circ,\tau^\circ\rangle$ and substituting $u = u(\xi,\tau(\xi,x^\circ))$ into the constraint (4), we obtain an identity $L(\xi,x^\circ,\pi(\xi,\tau(\xi,x^\circ))) \equiv 0$. Within the proximity of ξ°,u° we exhibit for variables ξ,u that

$$\frac{\partial}{\partial\xi}L(\xi,x^\circ,u^\circ)+\frac{\partial}{\partial u}L(\xi^\circ,x^\circ,u)\cdot\frac{\partial}{\partial\xi}\pi(\xi,\tau(\xi,x^\circ))=0\,, \tag{A.2}$$

from which we deduce the necessity statement for $\xi = \xi^\circ$ and $u = u^\circ$.

Sufficiency. Suppose the condition **(ii)** holds. Let (**i**) resolves for ξ° at feasible outcome $\phi,\xi^\circ \Rightarrow p^\circ,x^\circ,\alpha,\langle u^\circ,g^\circ,\tau^\circ\rangle$. Combining (**i**) and (A.2), we conclude that

$$\frac{\partial}{\partial\xi}\pi(\xi,\tau(\xi,x^\circ))\bigg|_{\xi=\xi^\circ}=0\,.$$

The sufficiency statement (A.1) holds since $u = u(\xi,x^\circ)$ is a strictly convex function of ξ . ■

Observation 6. *In any feasible outcome within the prohibited range of poverty proposals* ξ *social agencies can rollback to the bargaining frontier both for their own and for taxpayers' advantage, i.e. the outcomes in the prohibitive range of poverty are inefficient for social agencies upon their coalition with taxpayers.*

Sketch of the proof. Provided that the conditions [5] for the Observation 4 hold, we assume that public agencies propose a shift $g(\xi',x') = g' > g^\circ$ pursuing their own causes by sloping along the indifference curve $u(\xi,x) = u^\circ$ of welfare level u°. The point (ξ',x') necessarily penetrates into the region of prohibited proposals. We already knew what makes the anticipation (u',g') feasible is the increase $\tau' = \tau(\xi',x') > \tau^\circ = \tau(\xi^\circ,x^\circ)$ of taxes that allows $g' > g^\circ$, Observation 1. However, due to some self-financing effects, only in the prohibited range it is possible to *rollback* to $\tau'' = \tau(\xi'',x'') < \tau' = \tau(\xi',x')$, where (ξ'',x'') guarantees the welfare level $u'' > u'$ and (ξ'',x'') occupies some point on the bargaining frontier S_b. Now, because of tax decrease $\tau'' < \tau'$, the opportunities (u'',τ'') of mutual interests may unite the social agencies and voters-citizens against (u',τ') to "*wipe-out*" a motion (u'',τ'') to reject the proposed shift g'. ■

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[5] Actually, within the scope of weaker conditions one can disclose, see Mullat, J.E. [1979] "Stable Coalitions in Monotonic Games," *Automation and Remote Control 40*, 1469-1478, that levels $u(\xi,x^\circ) = \pi(\xi,\tau(\xi,x^\circ))$ are single $\cap$-peaked ξ -functions family of x° parameter, http://www.datalaundering.com/download/monogame.pdf.

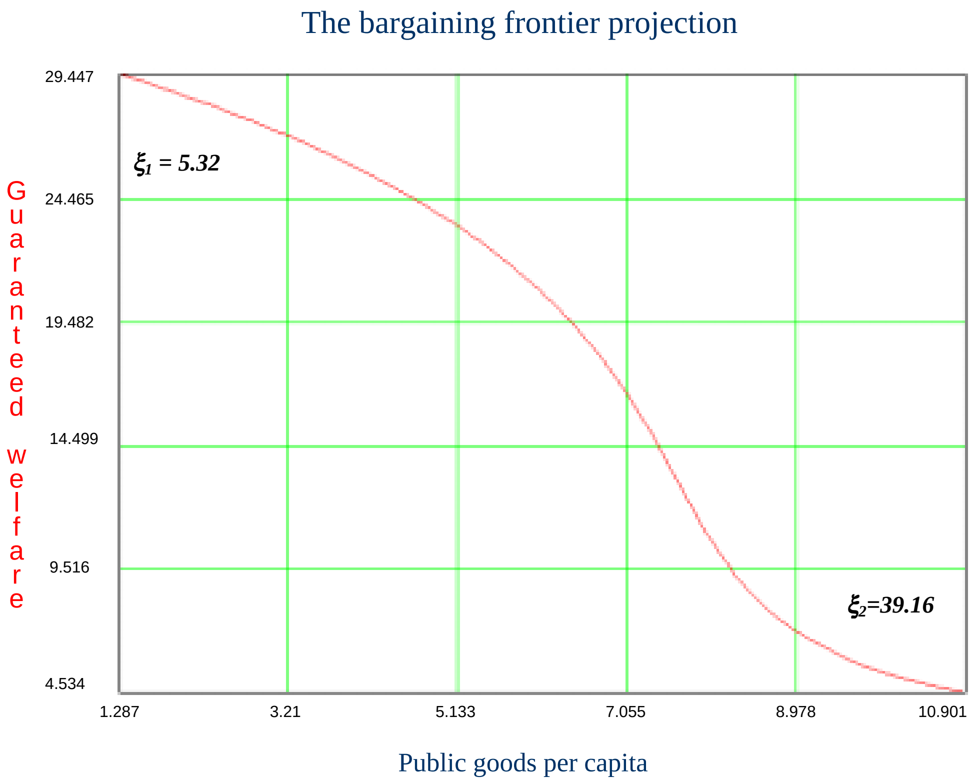

Figure 2. The utilities of the social and the public agencies are depicted on the vertical and the horizontal axes. The graph represents the bargaining frontier $S_b = u(g)$ of guaranteed welfare sloping down from the left-top ξ_1 towards right-bottom ξ_2. It is the projection found by resolving the efficiency constraint (6).

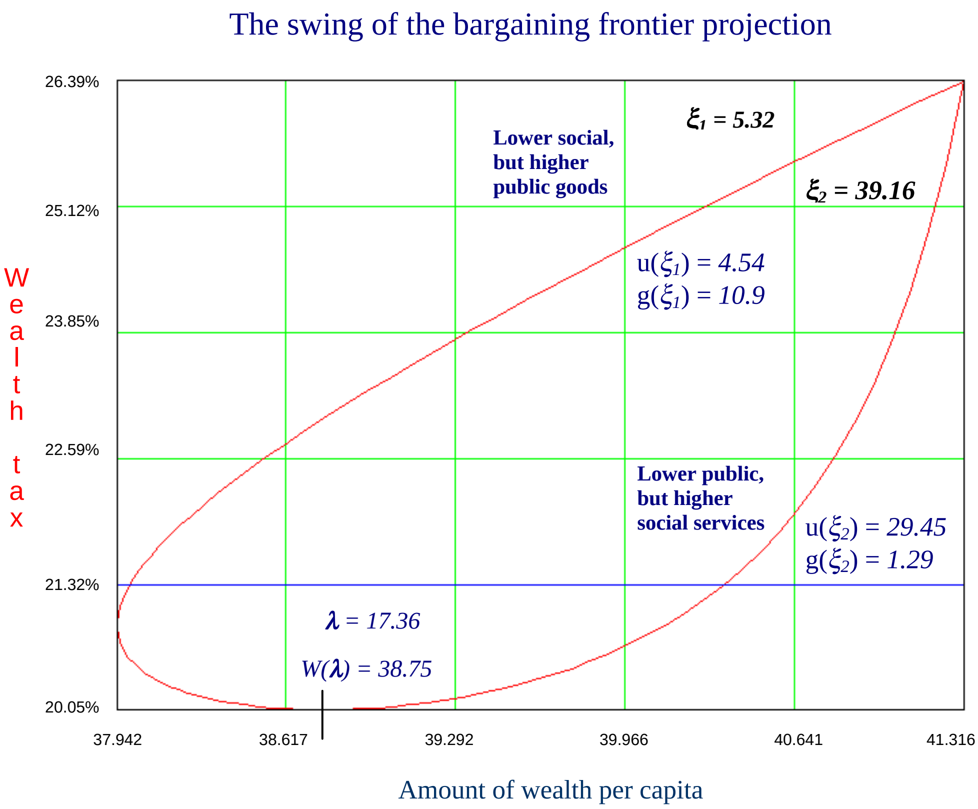

Figure 3. As it follows from the graph, a motion for a vote reg. adequate amount $W=38.75$ of wealth for the least tax $\tau = 20.05\%$ may pass by unanimous consent, the Observation 3. In contrast, the higher tax —— $21.32\% > 20.05\%$ may course a discontent that it might finance both: the higher (lower) social but lower (higher) public goods, Table 1.