Is Increased Public Schooling Really a Policy for Equality?*

- The Role of Within-the-Family Education –

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Abstract

This is a theoretical study of human-capital accumulation, where parental, as well as public investments are essential. Policy influence rich and poor parents differently when they make educational decisions. Rich parents allocate resources efficiently between physical bequests and educational investments, while poor parents only afford investments in children's human capital. Therefore, educational equality between rich and poor children is not necessarily promoted by further investments in public schooling. Moreover, I show that educational investments in low-skilled parents may have substantial spill-over effects on their children. Tax policy may also be used to influence human-capital accumulation, and I show that tax policy may have unexpected effects on the educational gap between rich and poor children.

Key words: Human capital, Public Education, Bequest, Altruism
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1 Introduction

During the last decades a lot of research has been done, in order to get a grip on the accumulation process of human capital. In many theoretical models the accumulation of human capital of the young generation is solely determined by parental choice. Becker, Murphy & Tamura (1990), Becker (1991), Caballé (1995), and Rangazas (1996) are only a few examples where private parental investment is modelled as the only way to acquire human capital. In many countries, this is, of course far from true. In most OECD countries we actually observe well developed public schooling systems, where everyone has access to free schooling. This is also reflected in some recent theoretical works, which include public, as well as private investments in children's education, see *e.g*. Becker & Tomes (1986), Kaganovich & Zilcha (1999) and Glomm & Kaganovich (1999). These models assume that *both* private and public investments are important to children's accumulation of human capital, an assumption that also finds support in the empirical literature.[1] Haveman & Wolfe (1995) review and compare some empirical evidence, and conclude that in the U.S. parental investments are almost twice the public investments. This is perhaps not surprising in a country, which largely is characterised by private solutions and a small public sector. However, also in Sweden with its large public sector Klevmarken & Stafford (1999) observe a similar pattern. For Swedish children less than seven years old, total private investments are almost twice the public investments. For older children parental investments are smaller, but still exceed those undertaken by the government. Hence, it is an important task to figure out what determines these parental investments, which we can call *within-the-family education*, and how they interact with public education. Heckman (2000) argues that learning starts early in life before formal education begins, and that policies directed to families therefore may increase children's human capital more than would further schooling investments. Just like Heckman I think that parental efforts are enormously important, and I explicitly model them jointly with public investments to see what consequences this might have for over all human-capital accumulation.

The present paper is a theoretical attempt to investigate the importance of within-the-family education, and how it responds to economic incentives. Once we have understood that, we have a better chance to promote human-capital accumulation by economic policy. In my

[1]Quite a few empirical studies have been done, trying to analyse the effects of public, and/or parental investments on children's achievements, see *e.g*. Heckman (2000), Meghir & Palme (1999), Klevmarken & Stafford (1999), Lindahl (2001), Hallberg & Klevmarken (2000) for some recent studies using Swedish and U.S. data.

model parental and public investments interact, and the former is highly dependent on the economic situation of the family. I analyse how different categories of parents respond to different economic policy, which results in conclusions about what policies enhance, and what policies mitigate educational inequality between rich and poor.

I believe my model has the capability of shedding new light on a number of policy issues. Sjögren (1998) shows that Swedes' occupational, and thus educational, choice often follows that of their parents. It has also been noticed that children of well-educated parents get better results already at secondary school,[2] and that few children of blue-collar workers go to university. The reasons for this educational gap may be many, *e.g.* the parents' general attitudes towards schooling. The social and cultural environment at home also influence children's possibilities at school, factors which may well be closely connected with the educational level of the parents. However, in this paper I show that also the pure economic incentives to engage in children's education are stronger for richer and more well-educated parents. This makes it possible to reduce the educational gap in society by altering economic incentives, which affect rich and poor parents in different ways. My results show that it is not at all certain that increased public education is the best way to help poor children catch up with the rich ones, which is a widespread assumption, see *e.g.* Thurow (1972). On the contrary, I show that increased investments in public education may even *widen* the educational gap. Moreover, I show that due to spill-over effects on children, investments in the human capital of the least educated *parents* would unambiguously promote welfare and equality among children. It is a perhaps surprising result that increased adult education has a larger effect on equality than education of the young, but this is due to the fact that more educated parents are more prone to invest in their children's human capital.

In principle, within-the-family education can be either a substitute or a complement to public education, and it may consist of both time and goods. It could *e.g.* be efforts to teach children manners, to help them do their homework, to buy a computer, or to send them on a language course abroad. Also health and other physical aspects are important to the human capital of an individual, so nutritious food and medicine could also be considered human-capital investments, although it is the educational aspect of human capital, which is stressed throughout this paper. Maybe it is most likely that public and private human-capital investments are comple-

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[2] The Swedish National Agency for Education (Skolverket) (1999).

ments. The main reason for assuming this complementarity is that in many Western countries, elementary and secondary schooling are mandatory and publicly provided free of charge, and higher education is free, or, at least heavily subsidised. Then private investments are likely to take the more informal form of providing children with complementary skills, and of helping them do as well as possible at the public school, rather than to substitute public for private education. However, it could even be possible that private and public investments are complements in some families and substitutes in others, as suggested by Kim (2001). In the analysis I allow for the possibilities that private and public education may be either substitutes or complements.

In the literature on human-capital accumulation authors have stressed various motives to invest in children's human capital. Cremer, Kessler & Pestieau (1992), Balestrino (1997), Lagerlöf (1997) assume an exchange motive, so a larger investment is "paid back" in terms of a larger contribution from the adult child to the retired parent.[3] If the child is altruistic towards the parent (Lagerlöf (1997)), or the parent can "bribe" the child to care for him in his later years (Cremer *et al*. (1992)), or if there are social norms forcing the child to support him (Balestrino (1997)), even a completely selfish parent will find it worthwhile to invest in the child's human capital, because this will be a substitute for savings. Many authors instead assume that parents are purely altruistic towards their children, and therefore undertake human-capital investments, see *e.g*. Becker (1991), Becker & Murphy (1988), Becker *et al*. (1990), Caballé (1995) and Rangazas (1996). Others assume that parents are not concerned with their children's utility, but rather with their "quality", represented by *e.g*. their level of education. Becker & Tomes (1976), Kaganovich & Zilcha (1999), Glomm & Kaganovich (1999), and Kim (2001) are some papers assuming this more paternalistic kind of altruism. When there is two-sided altruism between parent and child, like in Rangazas (1991) and Chakrabarti, Lord, & Rangazas (1993), the parent has strategic, as well as altruistic reasons to invest in the child's human capital.

In this paper, I stick to the strand of research, which assumes that parents are purely altruistic towards their children. Parents want to give their children as good consumption opportunities as possible, without noticing that children might be altruistic in return. I assume that parents make transfers to their children through within-the-family education and possibly by leaving a

[3] The exchange motive appears particularly relevant in countries without well-established social security systems

bequest of physical capital. In the model parents are characterised by their propensity to leave physical bequests in addition to investments in within-the-family education. If there are diminishing returns to within-the-family education and constant returns to physical-capital investment, rich parents will choose an efficient level of human-capital investment where marginal returns are equal between human and physical capital investments. Parents who do not have that much assets, or earnings possibilities will only invest in human capital, because the returns to within-the-family education exceed those of bequeathing physical capital. Parents who only invest in their children's human capital will therefore invest an inefficiently small amount. This inefficiency stems from the fact that parents cannot force their children to share their future income with them. This implies that children of poor parents get a lower level of human capital than do children of rich parents, in spite of an equal level of public education for everyone. The paper shows that by improving the economic situation for poor parents by means of fiscal policy this inequality could be decreased.

Another way to mitigate the underinvestment caused by the inability to force children to share the returns of education is to introduce a pay-as-you-go (PAYGO) pension system, where children are actually forced to pay their parents a certain amount. This has been proposed by several authors, see *e.g*. Drazen (1978) and Becker (1991), as a means of reducing inefficiency in the economy. Also in the model in the present paper efficiency could be promoted by the introduction of a PAYGO system, and with a sufficiently large lump-sum transfer from young to old, the educational gap between rich and poor totally vanishes.

The rest of the paper is organised as follows. Section 2 presents the model, and the characteristics of utility maximisation. Section 3 analyses the optimal behaviour of poor parents, who can only afford to transfer resources to their children through within-the-family education, and of rich parents who also transfer physical capital. The analysis is made with means of comparative statics, that explores the different economic incentives for the two groups of parents. In section 4 I analyse and discuss some policy implications, with emphasis on public education and taxation, to see what can be done to decrease inequality in society, both in terms of education and in terms of consumption possibilities. In section 5 I discuss the introduction of a PAYGO system, and study under what conditions such a policy would be Pareto efficient. Section 6 concludes.

2 The model

Let us consider a family, consisting of a parent and a child, where the former is altruistic towards the latter, insofar that he includes the child's utility from consumption in his own utility function. Each generation is assumed to live for three periods, as young, as middle aged, and as retired. When the parent is middle aged, his child is young, and when he is retired, the child has turned middle aged. When young, agents make no economic decisions, so we can concentrate on the periods of adulthood. The utility of the parent is then represented by

$$U_p = u(c_1) + u(c_2) + \gamma U_k(c_k), \tag{1}$$

with subscripts p and k for parent and kid, respectively. U_k is the child's utility of consumption when adult, c_k, and the parent's total utility, U_p is additively separable between consumption in the two periods, c_1 and c_2, and the child's utility. This additive separability implies that consumption is a normal good, with respect to own consumption in both periods, as well as to the child's consumption. Both u and U_k are strictly concave, twice continuously differentiable, and fulfil the Inada conditions. For simplicity the time preference factor is set to unity. Furthermore, for simplicity but without any loss of generality, I disregard any intertemporal decisions made by the child. In effect, I regard the child as being an adult for one period only. The degree of altruism from parent to child is represented by $\gamma > 0$, and the higher the value of γ, the more weight is given to the child.

During their active period of adulthood, agents work, and get paid in proportion to their level of human capital. The parent's human capital h_p is given, while the child acquires human capital through the public schooling system, h_0, and the parent's own investments, $\hat{x}$, according to the production function

$$h_k = f(h_0, \hat{x}), \tag{2}$$

which is strictly concave in $\hat{x}$. Furthermore, the partial derivatives, f_i, are positive $(i = h_0, \hat{x})$, and the Inada conditions are fulfilled for both inputs. The private investments reflect the specific time share, x, actually devoted to educate the child.[4] I assume that the parent's time investment is more productive, the higher is his level of human capital h_p, and that it is the ef-

[4] Allowing also for goods investments in within-the-family education does not alter the results of the paper, it just complicates the analysis.

fective time, $\hat{x} = xh_p$ that matters in the production function.[5] The variable h_p may moreover reflect the social and cultural environment the child has at home, which is also important for the child's achievements at school. Skolverket (1999) even concludes in a report that the most important variable for achievements at school is the educational level of the parents. I assume that the level of h_0 is equal for all children in the economy, so that a higher level of public schooling increases human capital for everyone *ceteris paribus*. It is not obvious whether h_0 and $\hat{x}$ should be regarded as substitutes or as complements in human-capital formation.[6] Some of the results in the paper are sensitive to assumptions about substitutability or complementarity, and the issue will be further discussed below.

The budget constraints for the parent are

$$c_1 = w_p h_p (1-x) - s \tag{3}$$

$$c_2 = s(1-\tau_s) - b \tag{4}$$

In the first period (disregarding childhood) the parent has a human capital endowment, h_p. He is also endowed with one unit of time, of which a fraction x is used to educate the child, and (1-x) is supplied in the labour market. He recieves an exogenous net-of-tax wage, w_p,[7] per unit of supplied human capital $h_p(1-x)$. Income is used for consumption, c_1, and saving, s. In the second period the parent uses his savings net of wealth taxation, $(1-\tau_s)$, for own consumption, c_2, and possibly a bequest, b, to the child. For convenience, the interest rate is set at zero.

When it comes to the child, we abstract from savings, as well as from endogenous labour supply, and concentrate on consumption possibilities in the second, active period. The child's consumption equals his income, which consists of labour income proportional to the achieved level of human capital, h_k, and possibly a bequest from the parent. This bequest is taxable at the rate τ_b. Hence, it follows that

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[5] One might argue that h_p should be included separately, reflecting the biological inheritance of abilities and intelligence, which is unintendedly transferred from parent to child, like in Becker (1991), Becker & Tomes (1976, 1979, 1986), or Giannini (1999). However, it appears likely that also inherited abilities are more developed, the more time the parent spends with the child to share his knowledge. The model of this paper is more in line with Heckman (2000) who suggests that abilities are developed and can be influenced by policy.

[6] There is no consensus about this in the existing literature. Glomm & Ravikumar (1992) and Kaganovich & Zilcha (1999) assume that public and private investments are complements, whereas Becker & Tomes (1986) instead are inclined to the assumption that they are substitutes. Kim (2001) suggests that for highly educated parents they are complements, whereas they are substitutes for poor parents.

[7] Hence, a constant marginal tax rate on labour income is implicitly assumed.

$$c_k = w_k h_k + b(1-\tau_b), \tag{5}$$

where h_k is defined in (2), and w_k is the child's net-of-tax wage rate.

Inserting (3), (4), and (5) into (1) we obtain the parent's maximisation problem

$$\max_{\{x,b,s\}} U_p = u\big(w_p h_p (1-x) - s\big) + u\big(s(1-\tau_s) - b\big) + \gamma U_k \big(w_k h_k + b(1-\tau_b)\big) \tag{6a}$$

$$\text{s.t. } 0 \le x \le 1, \quad b \ge 0. \tag{6b}$$

In principle, s can be either positive or negative. However, a positive s is the only way for the parent to get positive period-two consumption, so the Inada conditions of the utility function assure that $s > 0$. There are still the three inequality constraints in (6b) to consider, but we may in fact disregard the two regarding x. Because $f_{\hat{x}} \to \infty$ as $x \to 0$ (*i.e.* the Inada condition), the constraint $x \ge 0$, will never bind; thus there will always be some time invested in educating the child. Neither will $x \le 1$ bind; because of the Inada condition of the utility function, $\partial u / \partial c \to \infty$ as $c \to 0$. The non-negativity constraint on bequests, $b \ge 0$, comes from the legal restriction implying that the parent cannot leave legally binding debt to the child, and as there is no altruism from the child to the parent, the child will not voluntarily give any assets to the parent. Hence, if the parent would actually want to receive something from the child, all he can do is to abstain from bequeathing.

The first-order conditions with respect to x, b, and s which are necessary and sufficient for a utility maximum then become

$$\frac{\partial U_p}{\partial x} = -w_p u_1 + \gamma U_k' \, w_k f_{\hat{x}} = 0, \tag{7}$$

$$\frac{\partial U_p}{\partial b} = -u_2 + \gamma(1-\tau_b) U_k' \le 0, \tag{8a}$$

$$b \ge 0, \ \frac{\partial U_p}{\partial b} b = 0 \tag{8b}$$

$$\frac{\partial U_p}{\partial s} = -u_1 + (1-\tau_s)\, u_2 = 0 \tag{9}$$

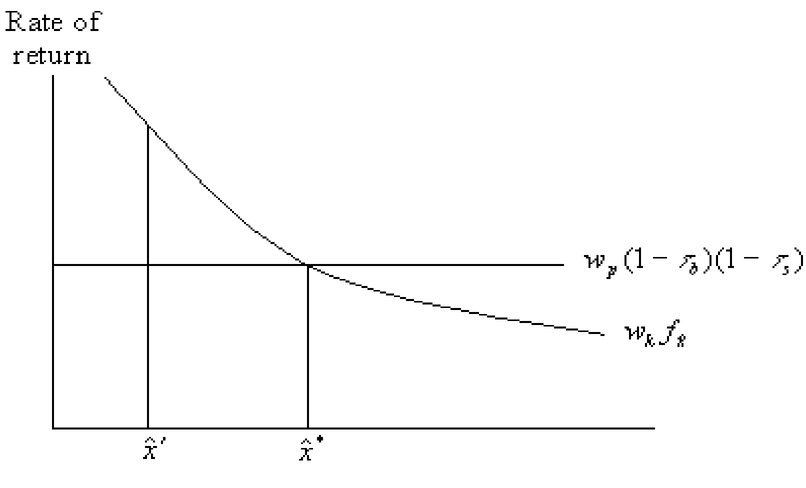
where u_i, i=1, 2 denotes the marginal utility of parental consumption in the i:th period, and (8b) is the complementary slackness condition.

If $b > 0$, the condition (8a) holds with equality. Since the marginal rate of return on within-the-family education is decreasing, while the return to saving and bequeathing is constant, the parent will only invest in human capital to the point where these marginal returns are the same; see point $\hat{x}^*$ in Fig. 1. Additional transfers to the child will be in terms of physical bequests. Combining (7)-(9) then gives the following arbitrage-like condition:

$$w_p(1-\tau_b)(1-\tau_s) = w_k f_{\hat{x}}, \qquad (10)$$

which says that at the optimum, the marginal return from investing one hour in the child's human capital should equal the constant rate of return to working that hour, and then save and bequeath the received wage, see $\hat{x}^*$ in Fig. 1.[8] Because the left-hand side of (10) is identical for every parent in the economy, so must the right-hand side be. If we assume the same production function of human capital for everyone, it is clear from (10) and $\hat{x}^*$ in Fig. 1 that all children who also receive physical bequests will have the same level of human capital.

Fig. 1 Efficient and inefficient human-capital investments from the parent's point of view.



[8] From an individual point of view, $\hat{x}^*$ is the efficient time investment, but due to the tax wedges, $\hat{x}^*$ is not likely to be optimal in terms of Pareto efficiency.

If the inequality in (8a) is strict, we have a corner solution with zero bequests, *i.e.* $b = 0$, and the parent only invests in the child's human capital. This situation occurs when the parent's marginal utility of consumption as retired exceeds the child's marginal utility of the extra consumption enabled by the bequest, weighted by the degree of altruism, *i.e.* when $u_2 > \gamma(1-\tau_b)U'_k$. Together, (7), (8a), and (9) give the following intuitive condition for the zero-bequest solution:

$$w_p(1-\tau_b)(1-\tau_s) < w_k f_{\hat{x}}\,, \tag{11}$$

which says that the marginal return from human-capital investment exceeds the constant rate of return from bequeathing, at the point where $b = 0$. In terms of Fig. 1 we may think of the zero-bequest solution as occurring at a point such as $\hat{x}'$. For a given individual, with a specific value of h_p, the share x' is less than x^*, which would be the efficient amount of time invested. Hence the bequest constrained parent invests too little in within-the-family education, a result we recognise from Becker & Tomes (1986) who also find that only those who bequeath physical assets make efficient human-capital investments. The reason for the under-investment is that negative bequests are ruled out. Parents would invest more if they could share the return from education with the child, but because they cannot force their children to pay them back, and since we assume that children are not altruistic toward their parents (at least, parents do not count on it), bequest constrained parents will instead invest too little.[9] The model thus leaves us with two possible solutions, one interior with physical bequests $(b > 0)$ as well as within-the family education, and a corner one, where the bequest constraint is binding $(b = 0)$.

3 Comparative statics

3.1 Rich or poor?

What factors determine whether parents end up in the corner or in the interior solution? From equation (11), it follows that people end up in the corner solution when the return to within-the-family education is very high, or when taxes on saving and bequeathing are high. Hence, one could imagine a situation where no parent in the economy finds it worthwhile to leave a

[9] Rangazas (1991) and Chakrabarti *et al*. (1993) show that when there is two-sided altruism and the parent is aware that larger human-capital investments imply larger voluntary contributions from the child, the parent will, in fact, make efficient investments. In the present paper it is thus the child's lack of altruism that generates inefficiency, which has the same effect as the liquidity constraint that generates inefficiency in *e.g*. Becker (1991).

bequest, but that they all end up in the corner solution. However, since we actually observe a non-negligible part of the population leaving positive bequests, it seems more plausible to assume that only some subgroup of the population is constrained, in the sense that they do not have enough resources to leave a physical bequest. With the same wage and tax rates and the same access to public schooling, it is straightforward to show that the parents in the interior solution are the ones with a large human-capital endowment, or the rich ones, while those in the corner solution are the poor parents, who have less human capital and therefore earn less money in the labour market.[10] A proof of this result is shown in Appendix A.

3.2 The behaviour of the poor

Let us start by analysing the effects of different public policies on poor parents; *i.e.* parents with such a low level of human capital that they cannot afford to leave a physical bequest $(b = 0)$. Table 1 shows the qualitative effects that different exogenous variables have on time in within-the-family education, x, on the child's total human capital, h_k, and on the child's consumption, $c_k = w_k h_k$. All the tedious derivations are found in Appendix B, together with some additional discussion. Note also that all changes are marginal in the sense that the parent is assumed to remain in the corner solution.

Table 1. Comparative statics for the corner solution

	dx	dh_k	dc_k
dh_0	?	?	Sign[dh_k]
dh_p	?	+	+
dw_p	?	Sign[dx]	Sign[dx]
dw_k	?	Sign[dx]	+
$d\tau_s$	?	Sign[dx]	Sign[dx]

Let me briefly discuss some of these results, proceeding down the rows of Table 1, trying to clarify the many question-marks. For the full analysis, see Appendix B.

[10] If parents also had endowments of physical capital, a larger physical endowment would also increase the prob-

A means that naturally comes to mind when we think of raising the level of human capital is, first of all, publicly provided education, h_0. But, if we also consider within-the-family education, the child's over-all level of human capital, h_k, does not necessarily increase, just because more resources go to public schooling. The parent namely alters x as a consequence of an increase in h_0.

$$\frac{\partial x}{\partial h_0} = \frac{\gamma h_p w_k^2 U_k''\left(u_{11} + u_{22}(1-\tau_s)^2\right)\left(\sigma_k h_k f_{h_0 \hat{x}} - f_{h_0} f_{\hat{x}}\right)}{|A_C|}, \tag{12}$$

where $\sigma_k = -U_k' / U_k'' w_k h_k$ is the child's intertemporal elasticity of substitution, and $|A_C| > 0$ is the system determinant of the corner-solution equation system, presented in Appendix B.

Some inspection reveals that the sign of $\frac{\partial x}{\partial h_0}$ is ambiguous without further assumptions about the human-capital production function. Especially the sign of the cross-derivative $f_{h_0 \hat{x}}$ is crucial; if the two inputs are substitutes, *i.e.* $f_{h_0 \hat{x}} \leq 0$, the parent unambiguously decreases his educational investments, when public investments increase.[11] This is because income and substitution effects go in the same directions in this case. According to the income effect, x tends to decrease because the child becomes relatively richer with increased h_0, and the parent then wants to increase his own material well-being as well. The substitution effect says that the marginal productivity of x decreases when h_0 increases, which also tends to decrease x. If, on the other hand, h_0 and $\hat{x}$ amplify each other, so that $f_{h_0 \hat{x}} > 0$, we cannot tell whether the parent increases or decreases time spent educating the child, because income and substitution effects now work in opposite directions. The income effect is the same as above, but now the parent tends to increase within-the-family education due to the substitution effect, because marginal productivity increases with h_0.

However, it is the total effect on the child's human capital that is relevant, including the direct effect that public education actually has. The total effect on h_k is achieved by differentiating (2) with respect to h_0:

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ability of being in the interior solution.

[11] Kim (2001) finds that mothers with low education significantly decrease their time investment when h_0 increases.

$$\frac{\partial h_k}{\partial h_0} = f_{\hat{x}} h_p \frac{\partial x}{\partial h_0} + f_{h_0}, \tag{13}$$

which implies

$$\frac{\partial h_k}{\partial h_0} = \frac{\overbrace{f_{h_0} w_p^2 h_p^2 u_{11} u_{22} (1-\tau_s)^2}^{+} + \overbrace{\gamma w_k h_p^2 U_k' \left(u_{11} u_{22} (1-\tau_s)^2\right)}^{-} \overbrace{\left(f_{h_0} f_{\hat{x}\hat{x}} - f_{\hat{x}} f_{h_0\hat{x}}\right)}^{?}}{|A_C|}. \tag{14}$$

If $f_{h_0\hat{x}} \geq 0$ the total effect is positive, that is, even if the parent might decrease his own investment, he will not decrease it in a one-to-one proportion, but the child's level of human capital will unambiguously increase from increased public education. If, on the other hand $f_{h_0\hat{x}} < 0$, it is not possible (without further assumptions) to tell whether the child's total human capital increases or decreases. The parent unambiguously decreases *x*, and the total effect depends on which effect is the strongest, and the most important to total human-capital accumulation, public, or within-the-family education. We can thus draw the somewhat awkward conclusion, that increased public investments in schooling may actually imply a *lower* level of human capital and consumption for children with bequest constrained parents.

When parental human capital, h_p increases, the parent's productivity both in the labour market and in within-the-family education increases. Because of this we cannot tell whether the effect on *x* is positive or negative. The total effect on human capital is, however, positive, irrespective of the effect on time investment, because time itself in within-the-family education becomes more productive:

$$\frac{\partial h_k}{\partial h_p} = f_{\hat{x}} \left(h_p \frac{\partial x}{\partial h_p} + x \right) = \frac{f_{\hat{x}} u_{11} u_{22} (1-\tau_s)^2 w_p^2 h_p^2}{|A_C|} > 0. \tag{15}$$

If the parent gets an increased net-wage rate, w_p, the income and substitution effects go in opposite directions. On the one hand, the parent gets more resources, and tends to increase total within-the-family education, so as to transfer some resources to the child (the income effect). On the other hand, it becomes more profitable to work in the market, which tends to decrease *x* (the substitution effect). If the income effect dominates, *x* increases with an increase in w_p, and *vice versa*. If the parent has a constant intertemporal elasticity of substitution less than unity, which according to many empirical studies appears to be the most realistic case, an increased parental wage rate makes the parent increase his educational investments, so that the child receives a higher human-capital level as well as higher consumption.

An increase in the child's net-wage rate w_k makes the child richer, so due to the income effect, the parent wants to reduce within-the-family education. However, the return to education also becomes higher, which instead tends to increase x. It is now the elasticity of substitution of the *child's* utility function that determines the direction of the total effect. When this elasticity exceeds unity, the substitution effect dominates, and the parent increases within-the-family education. If the elasticity of substitution instead is less than unity, a higher return to educational investments actually leads to a *decreased* human-capital level. However, because w_k itself increases, so does the child's consumption, irrespective of the effect on within-the-family education.

If the wealth tax τ_s increases, within-the-family education tends to decrease due to the income effect. The substitution effect goes in the opposite direction, because the parent would like to increase contributions to the child, as well as period-one consumption, in order to avoid the increased taxation in the second period. The intertemporal elasticity of substitution of the parent's utility function determines which effect dominates. When it is less than unity, x decreases with τ_s. With a higher propensity to substitute intertemporally, the substitution effect dominates, and within-the-family education instead increases with a higher wealth tax.

3.3 The behaviour of the rich

For a rich parent who leaves a physical bequest to the child in addition to human-capital investments, the results from the comparative statics are much more clear-cut than those for the poor parent. The reason is that there is now only a substitution effect connected with within-the-family education; the income effect only operates via the bequest, b. This is because the return to working, saving and bequeathing, $w_p(1-\tau_b)(1-\tau_s)$, is fixed, while the marginal return to within-the-family education is a decreasing function of x. Table 2 shows the qualitative effects that different exogenous variables have on x, b, h_k, and on the child's consumption in the interior solution, $c_k = w_k h_k + b(1-\tau_b)$. Appendix C presents the detailed derivation of the results in Table 2.

Table 2. Comparative statics for the interior solution

	dx	Db	dh_k	dc_k
dh_0	?	?	?	+
dh_p	-	+	0	+
dw_p	-	+	-	+
dw_k	+	-	+	+
$d\tau_s$	+	-	+	-
$d\tau_b$	+	?	+	-

Let me again outline the intuition underlying some of the results. If the level of publicly provided education, h_0 increases, also the relative return to x is affected, which alters within-the-family education and the bequest:

$$\frac{\partial x}{\partial h_0} = -\frac{f_{h_0\hat{x}}}{h_p f_{\hat{x}\hat{x}}} \tag{16}$$

$$\frac{\partial b}{\partial h_0} = -\frac{\left(u_{11} + u_{22}(1-\tau_s)^2\right)U_k''(1-\tau_b)w_k \gamma f_{h_0}}{\Omega} + \frac{w_p(1-\tau_s)f_{h_0\hat{x}}}{f_{\hat{x}\hat{x}}}, \tag{17}$$

where $\Omega > 0$ is defined in Appendix C.

If $f_{h_0\hat{x}} > 0$, increased public education increases the marginal productivity of $\hat{x}$, which induces the parent to spend more time educating the child. Instead the parent decreases b, both because of the income and of the substitution effect.[12] If $f_{h_0\hat{x}} < 0$ the parent will instead decrease x, and we cannot tell whether the bequest increases or decreases, because income and substitution effects go in opposite directions. Like in the case of the poor parent, if $f_{h_0\hat{x}} < 0$ we may actually end up in a situation where the child's total level of human capital decreases when public education increases. However, the rich parent would compensate such a loss by

[12] We assume that the increase in h_0 is marginal, in the sense that the parent will keep on leaving a positive bequest, but if the increase in h_0 goes on, bequests will decrease, and eventually cease, and the parent will end up in the corner solution.

an increased bequest, so that the rich child is always left with a higher consumption after increased h_0, irrespective of the sign of $f_{h_0\hat{x}}$. Thus we have that

$$\frac{\partial c_k}{\partial h_0} = w_k\left(f_{\hat{x}} h_p \frac{\partial x}{\partial h_0} + f_{h_0}\right) + (1-\tau_b)\frac{\partial b}{\partial h_0} = \frac{f_{h_0} w_k u_{11} u_{22}}{\Omega} > 0\,. \tag{18}$$

If parental human capital, h_p, increases, the parent's productivity will rise, in paid work, as well as in the efforts to educate the child. Because it is the effective time that counts, the parent will reduce time spent on within-the-family education, and instead work more, just enough to leave the effective time unchanged:

$$\frac{\partial x}{\partial h_p} = -\frac{x}{h_p} \Rightarrow \frac{\partial \hat{x}}{\partial h_p} = x + h_p \frac{\partial x}{\partial h_p} = x - h_p \frac{x}{h_p} = 0\,. \tag{19}$$

This solution, however, hinges on the assumption that it is effective time, $\hat{x} = xh_p$, rather than x and h_p separately that enters the human-capital production function.[13] With a higher h_p the parent works more, and gets richer, while the child receives the same level of human capital as before. Due to the positive income effect, the parent then increases the bequest, leaving the child better off than before.

If the parent's net wage, w_p increases, for instance because of a tax cut, it is more profitable to work an extra hour in the labour market, and hence reduce x. Instead, the parent increases b, both because he substitutes away from within-the-family education, and because he has become richer, and wants to transfer some of the extra resources to the child. If the child's net wage w_k increases, the result is reversed. The bequest is reduced, since the child becomes richer. However, the return to education increases, so a larger proportion of the transfer is made through x. The child is left with a larger consumption after either type of wage increase.

An increased wealth tax, τ_s, makes it less profitable to save, and thereby to bequeath; hence the parent substitutes away from b, towards x. An increased bequest tax, τ_b has the same effect on within-the-family education; it becomes relatively more profitable to educate the child than

[13] If x and h_p would be separate variables of the human-capital production function, there would be no clear effects on x from an increase in h_p. The wage rate and the within-the-family production function would then determine if the parent would choose to spend more time at work, or more time educating the child.

to leave a bequest. When it comes to the bequest the parent wants to reduce b due to the substitution effect, but on the other hand he wants to compensate the child for the decreased income, created by heavier taxation. Hence, we cannot tell whether b increases or decreases with τ_b. However, due to the direct effect of τ_b, c_k unambiguously decreases after an increase in the bequest tax.

4 Policy for equality?

Tables 1 and 2 show that rich (*i.e.* those at interior solutions) and poor (*i.e.* those at corner solutions) parents respond in different ways to various policies, and that the difference primarily depends on the rich parent's ability to shift between intergenerational transfers in the form of physical and human capital investments. This means that there is only a substitution effect connected with within-the-family education for rich parents, and that all income effects will operate entirely via changes in bequests. For poor parents who do not leave physical bequests there are income as well as substitution effects on within-the-family education. Hence, rich and poor parents respond in different ways.

A major aim for governments is increased equality between rich and poor. This may be done in several different ways, but one important aspect is the distribution of human capital. Governments are concerned with getting less well-situated people to acquire more human capital. Then it is of utmost importance to explore what kinds of policies that increase the level of human capital of the poor ones relative to the rich ones, and what policies that diminish the differences in consumption between the two groups.

4.1 Public education

Investments in public education have always been regarded as a powerful tool to give the poor the opportunity to catch up with the rich. However, in a society with already well-developed public schooling, it is not necessarily the case that further investments in public schooling really promote equality between rich and poor pupils. If we also consider within-the-family education, increased public education may actually increase inequality between rich and poor. For the educational gap to decrease the following derivative has to be positive:

$$\frac{\partial \dfrac{h_k^{poor}}{h_k^{rich}}}{\partial h_0} = \frac{\dfrac{\partial h_k^{poor}}{\partial h_0} h_k^{rich} - \dfrac{\partial h_k^{rich}}{\partial h_0} h_k^{poor}}{h_k^{rich\,2}} = \frac{\left[f_{\hat{x}}^{poor} h_p^{poor} \dfrac{\partial x}{\partial h_0}^{poor} + f_{h_0}^{poor} \right] h_k^{rich} - \left[f_{\hat{x}}^{rich} h_p^{rich} \dfrac{\partial x}{\partial h_0}^{rich} + f_{h_0}^{rich} \right] h_k^{poor}}{h_k^{rich\,2}} \tag{20}$$

where I have made use of (13). We know that $h_p^{poor} < h_p^{rich}$, and from equations (10) and (11) we also know that $f_{\hat{x}}^{poor} > f_{\hat{x}}^{rich}$, but we cannot tell the sign of the complete expression without further assumptions about functional forms. There are three different effects to consider. [1] The poor parent has a negative income effect connected with within-the-family education, which tends to decrease his investments, because the child gets richer *c.p.* when h_0 increases. For the rich parent, on the other hand, there is no such income effect to be considered. [2] For both categories the direct effect f_{h_0} is positive, but we cannot tell who benefits the most from it. A simplifying, but not self-evident assumption is that the sign of the cross derivative $f_{h_0\hat{x}}$ is the same for both parents.[14] Then $f_{h_0\hat{x}} > 0$ implies that the direct effect is stronger for the rich, and thus the direct effect tends to widen the educational gap further. If $f_{h_0\hat{x}} < 0$, the direct effect instead decreases the gap, because it is stronger for the poor child who gets less within-the-family education. [3] The last effect is the substitution effect, which has the same sign as the cross derivative, for both rich and poor. However, we cannot tell who has the strongest effect, neither can we determine the sign of the total effect in equation (20). Therefore we cannot say anything about equality in terms of consumption either. It may even be the case that equality increases in one respect, and decreases in another. Hence, we conclude that

> **PROPOSITION 1**: *Increased public education may either increase or decrease inequality in society.*

Special case – logarithmic utility

Logarithmic utility is commonly used in the literature on human capital.[15] Although somewhat restrictive, it is a functional form which is analytically tractable, and which generates closed-form solutions. If we apply it to the present model, and furthermore assume that the

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[14] Kim (2001) suggests that school expenditures and parental time are complementary only for rich parents, and not for the poor ones. In the context of this paper, that would imply increased inequality with certainty when h_0 is increased.

[15] Zhang (1997), Kaganovich & Zilcha (1999) and Behrman *et al.* (1999) are just a few examples.

production function of human capital is of Cobb-Douglas type, we get a model that gives unambiguous results. The analysis is performed in Appendix D, where I show that

> **PROPOSITION 2**: *With logarithmic utility and Cobb-Douglas production function in the human-capital sector, increased public education increases the educational gap between rich and poor, but decreases inequality in terms of consumption.*

PROOF: See Appendix D.

4.2 Education for poor parents

In days of unemployment, labour market training programmes and other types of adult education are often primarily used as kind of labour market policy. There has been intense debate on whether such education really is well invested money. On the basis of U.S. data, Heckman (2000) concludes that public education investments are more profitable for society the earlier in life they are undertaken, and Björklund & Kjellström (2000) conclude that the same holds true in Sweden. Forslund & Krueger (1997) analyse the Swedish labour market programmes, which they do not find to have been very effective in reducing unemployment or in enhancing workers skills. Therefore it may be argued that such programmes are not worthwhile, and that educational resources should be allocated to children, rather than to adults. However, none of the above-mentioned studies explicitly takes into account the spill-over effects on the next generation in terms of within-the-family education. Maybe one should wait some 20 years with the evaluation of labour market training programmes, so one can analyse their total effects, including those on children's human capital. For instance, Behrman *et al*. (1999) show with data from rural India, that the spill-over effects are large enough to justify education of mothers, although they may not work in the market at all, but only help their children to accumulate human capital. The authors remark that one should be cautious, and not generalise the importance of maternal schooling to other countries, but at least their paper points out the fact that parental human capital is not unimportant for the productivity in within-the-family education. It could, hence, be useful to increase parental human capital, h_p if the major aim is to increase levels of human capital and consumption for poor children. In fact,

PROPOSITION 3: *Policies that increase human capital of poor parents will promote equality between rich and poor children, both with respect to human capital and consumption.*

PROOF: When the poor parent's human capital increases, also his child gets a higher human-capital level, because the parent's time becomes more productive, see equation (15). This results in a higher income for the child, and thus higher consumption. Because only poor parents get increased human capital, rich families are unaffected. Hence, equality increases.

4.3 Parental wage rate

If the government wants to increase equality by *e.g.* increasing h_0 or h_p it might choose to finance the reform by increasing the parental wage tax, which is equivalent to decreasing the net wage rate per unit of human capital, w_p. This would, however, not be a very wise thing to do, because decreasing w_p is most likely to increase the educational gap between the rich and the poor. For the rich, a lower w_p unambiguously increases educational investments (see Table 2), whereas for the poor, income and substitution effects go in opposite directions. In fact,

PROPOSITION 4: *If parents have an intertemporal elasticity of substitution less than or equal to unity, a decreased wage rate for both rich and poor parents increases the educational gap between rich and poor children.*

PROOF: According to equation (B9) in Appendix B, the poor parent decreases within-the-family education when w_p decreases, and his intertemporal elasticity of substitution is less than unity. Consequently, the poor child receives a lower level of human capital. If the elasticity of substitution equals unity (logarithmic utility) within-the-family education is unaffected by the change in w_p The rich child, on the other hand gets a higher level of human capital if w_p is lowered, according to (C7) in Appendix C, irrespective of the magnitude of the intertemporal elasticity of substitution. Hence, educational inequality increases.

4.4 Returns to education

Increased returns to education have been pointed out as a way to get children of less educated parents to acquire more human capital. A higher return to education could make up for a potential social obstacle to attend higher education (Sjögren (1998)). In that sense it is presumed that increased w_k would promote equality. However, when parents, rather then children themselves, make investments in within-the-family education, it could well be the case that rich parents take advantage of the increased returns and increase their investments, while poor parents see an opportunity to increase their own well-being, and instead decrease educational investments when w_k increases, contrary to the government's intentions.

> **PROPOSITION 5**: *If children has an intertemporal elasticity of substitution less than or equal to unity the educational gap is enhanced if returns to education increase.*

PROOF: The rich parent always increases within-the-family education when w_k is increased, see equation (C10) in Appendix C. If the poor child has an elasticity of substitution, σ_k, less than one, the poor parent will actually decrease his educational investments, due to the fact that the wage increase makes the child richer, see equation (B10) in Appendix B. If $\sigma_k = 1$, the poor parent will not alter x at all, and because the rich child unambiguously gets more education, the educational gap increases in either case.

4.5 Wealth tax

Increased taxation on wealth and capital income is sometimes advocated as a tool to increase equality in society, because the rich are affected the most. However, if we take human-capital formation into account, the picture might change. On the one hand, rich parents tend to give up some physical investments, and instead increase human-capital investments. On the other hand, this is not likely to be true for a poor, bequest constrained parent, who instead might decrease within-the-family education in consequence of the increased tax, due to counteracting income- and substitution effects. It may thus be shown that:

> **PROPOSITION 6**: *If parents have an intertemporal elasticity of substitution less than or equal to unity, increased capital taxation implies increased educational inequality between rich and poor children.*

PROOF: According to equation (B13) in Appendix B, the poor parent decreases *x*, after an increased τ_s if and only if the intertemporal elasticity of substitution, $\sigma_p < 1$. If $\sigma_p = 1$ the poor parent will not alter his behaviour after the increased tax. For the rich parent, (C13) in Appendix C implies that within-the-family education instead increases, irrespective of σ_p. Hence, educational inequality increases.

4.6 Bequest tax

Rich parents increase within-the-family education when τ_b is increased, in the same way as after an increased wealth tax. However, the bequest tax does not affect the poor, at all. Hence, educational inequality unambiguously increases.

5 A PAYGO Pension System – a Pareto Improvement?

The human-capital investment undertaken by a rich parents who also leaves a physical bequest is efficient from the parent's point of view, in the sense that he invests in within-the-family education up to the point where the personal marginal return to investments in physical and human capital are equal (equation (10), and point $\hat{x}^*$ in Fig. 1). However, the constraint that bequests must be non-negative binds for the poor parent, who therefore will only invest in human capital to a point where the marginal return to within-the-family education exceeds the constant return of saving and bequeathing (equation (11) and point $\hat{x}'$ in Fig.1). If the government has the ambition to generate a more efficient and equal distribution of human capital, it should undertake policies that neutralise the constraint for poor parents, and/or that push more parents into the interior solution. If this can be done without generating further inefficiencies for those already in the interior solution, the policy is actually Pareto improving.

One way could be to introduce a pay-as-you-go (PAYGO) pension system, where the adult child has to pay a lump-sum transfer, *P*, to his parent. Barro (1974) shows that when generations are altruistically linked, such a transfer would have no effect; parents just increase their voluntary transfer to the child. However, Drazen (1978) shows that this Ricardian Equivalence does not necessarily hold when intergenerational transfers may take the form of both physical and human capital. In fact, lump-sum transfers have real effects when physical and human capital investments are not perfect substitutes, and the parent only makes the latter investment. Also Rangazas (1996) finds real effects from lump-sum transfers in an economy where parents are bequest constrained and only transfer human capital to their children.

In the present model such a lump-sum transfer could be regarded as an induced loan from parent to child. The parent invests resources in the young child who, when adult has to pay a transfer to his retired parent. In this case the parent maximises

$$\max_{\{x,b,s\}} U_p = u\left(w_p h_p (1-x) - s\right) + u\left(s(1-\tau_s) - b + P\right) + \gamma U_k\left(w_k h_k + b(1-\tau_b) - P\right). \quad (21)$$

Note that with the (exogenous) transfer, *P*, own savings are not the parent's only period-two income anymore, so *s* may well be negative.

Within-the-family education undertaken by rich parents is not affected by the lump-sum transfer, *P*, because there is no substitution effect connected with it.[16] However, the bequest will increase, because the parent becomes richer and the child becomes poorer. If Ricardian equivalence holds, there will be a one-to-one relation between increased pensions and increased bequests, and the family is unaffected by the lump-sum transfer (see Barro (1974)). However, with a positive bequest tax, τ_b, Ricardian equivalence fails to hold, and both the parent and the child end up with less consumption than before the introduction of the PAYGO system. For the rich families a lump-sum PAYGO system would thus imply no improvements (in presence of a bequest tax they would actually be worse off). Those who actually are expected to benefit from such a system are, however, those in the corner solution.

Because the lump-sum transfer, *P* implies that the poor parent gets resources from the child, he is willing to increase his educational investment. If the enforced redistribution from child to parent is large enough within-the-family education will even increase sufficiently to generate a switch from the corner to the interior solution, where $w_p(1-\tau_b)(1-\tau_s) = w_k f_{\hat{x}}$. Then we get rid of the inefficiency problem, and we also get a completely equal distribution of human capital. The inequality in terms of consumption would, however, remain, although to a decreased extent. In absence of a bequest tax, such a policy would be Pareto improving, because the rich family would be unaffected by the policy, while the former corner-solution family gets a better situation, where both parent and child get higher levels of consumption. Hence,

[16] The results below are derived in Appendix E.

PROPOSITION 7: *A PAYGO system with a large enough lump-sum transfer from child to parent, would eliminate the educational inefficiency of the poor, and all children would get an optimal level of human capital.*

Another way to increase equality by pushing the corner-solution parents into the interior solution could be to increase their human capital, h_p. That would, however, imply an extra cost for the government, and financing that might cause undesirable distortions.

6 Conclusion

This paper has dealt with equality issues concerning the distribution of human capital, and especially the importance of educational investments made by parents. I call these parental investments within-the-family education; albeit informal, they respond to parents' economic situation, and may be altered by political means. A rich parent allocates resources in a, for him efficient way between within-the-family education and bequeathing, which makes him react to fiscal policy in other ways than the less wealthy parent who only invests in within-the-family education. This is the basic mechanism, which opens up for my unconventional policy results.

One often thinks of increasing publicly provided education as a means to increase over-all education, and particularly education of those who do not have rich or well-educated parents. This paper has shown that increased public education of the young ones, may actually *widen* the income and educational gap between rich and poor children. This is because rich parents can afford to take advantage of the increased productivity generated by public investments, while the poor may instead substitute away from own educational investments when public investments increase, to be able to increase their own consumption.

From an equality perspective, increasing human capital of less educated parents, would unambiguously increase equality; poor parents get higher earnings capacity, as do their children who benefit from increased within-the-family education. This contradicts the common view that educational investments are more profitable the earlier in life they are undertaken; see *e.g.* Heckman (2000), and Björklund & Kjellström (2000). However, my result is completely due to the spill-over effects on the next generation. Increasing the wage tax (or decreasing the parental wage rate) could be a way to finance public investments. However, it is likely that such reform would increase inequality between children, which is also the likely outcome of increased wealth taxation.

A lump-sum PAYGO system could mitigate the market failure by forcing children to pay a certain amount to their parents. This would encourage poor parents to increase their human-capital investments. If the lump-sum transfer from child to parent is large enough, the poor child will even get the same level of human capital as the rich child, and also inequality with respect to consumption would be reduced. In the absence of a bequest tax, the rich family would be unaffected by the PAYGO system, and such reform would be Pareto improving, and the inefficiency would completely vanish.

Needless to say, the model has its drawbacks. It is very simple to the structure, and the comparative statics analysis has totally omitted the general equilibrium aspect. This was done deliberately, in order to distinguish the effects from different policy. However, combined effects from different fiscal policy, including public education, would be a desirable task for future research, as would the inclusion of an infinite number of consecutive generations, which would enable us to analyse also long-run effects from different policy. Furthermore, assuming that parents are the ones who make all educational decisions, makes results somewhat different, than if some decisions where instead made by children. It would be interesting to find out how the child's decision would affect parental decisions and overall human capital. Another important question is what the human-capital production function really looks like. The assumption in this paper that everyone receives the same amount of public education is rather strong. Even though public education in many countries is provided for free to everyone, we know that not everyone makes use of it to the same extent (this is especially true when it comes to higher education), and the access to public education may also vary between communities. For instance, Benabou (1996) shows that when public schooling investments are made on a local basis, this in itself may cause inequality, because local governments in areas mostly inhabited by highly educated people tend to invest more in schools than those in areas where inhabitants have lower education. It is also important to analyse the way in which private and public investments really interact in the human-capital production process, as it seems to be crucial to the effects of various fiscal policies.

Hence, there is much research left to be done on this issue, and if we want to increase the educational level in society and remedy the socially unrepresentative intake to universities, it will be necessary to look beyond formal education, and also focus on within-the-family education in one form or the other.

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Appendix A – Rich or poor?

The condition for not leaving a bequest is:

$$\Delta = w_k f_{\hat{x}} - w_p (1-\tau_b)(1-\tau_s) > 0\,, \tag{A1}$$

which is merely a rearrangement of (11). Wage and tax rates are the same for all parents in the economy, so the only variable which differs across parents is their level of human capital, h_p.

If a bequest constrained parent gets a larger human capital, then he will unambiguously approach the interior solution as the value of Δ decreases:

$$\frac{\partial \Delta}{\partial h_p} = w_k f_{\hat{x}\hat{x}} \left(h_p \frac{\partial x}{\partial h_p} + x \right) = \frac{w_k f_{\hat{x}\hat{x}} u_{11} u_{22} (1-\tau_s)^2 w_p h_p^2}{|A_C|} < 0\,. \tag{A2}$$

Hence, the corner-solution parent is a parent who has less human capital than the interior solution parent, and who therefore earns less money; *i.e.* a parent who is poorer than the interior-solution parent.

Appendix B – The corner solution (the poor)

The equation system representing the solution for a poor parent, *i.e.* the corner solution with zero bequests is presented in (B1) on the next page.

The determinant of the matrix on the left-hand side is:

$$|A_C| = w_p^2 h_p^2 u_{11} u_{22} (1-\tau_s)^2 - \gamma w_k^2 h_p^2 U_k'' \left(u_{11} + u_{22}(1-\tau_s)^2\right)\left(\sigma_k h_k f_{\hat{x}\hat{x}} - f_{\hat{x}}^2\right) > 0\,, \tag{B2}$$

where $\sigma_k = -U_k' / w_k h_k U_k''$ is the elasticity of substitution of the child's utility function.

By using Cramer's rule, all partial derivatives connected with the equation system above are determined. All changes are assumed to be marginal, in the sense that the parent will not move to the interior solution.

(B1) The equation system representing the solution for the poor parent

$$\begin{bmatrix} w_p^2 h_p^2 u_{11} + \gamma w_k^2 h_p^2 U''\left(f_{\hat{x}}^2 - \sigma_k h_k f_{\hat{x}\hat{x}}\right) & w_p h_p u_{11} \\ w_p h_p u_{11} & u_{11} + u_{22}(1-\tau_s)^2 \end{bmatrix} * \begin{bmatrix} dx \\ ds \end{bmatrix} =$$

$$= \begin{bmatrix} \gamma w_k^2 h_p U_k''\left(\sigma_k h_k f_{h_0 \hat{x}} - f_{\hat{x}} f_{h_0}\right) & u_{11} w_p^2 h_p (1-x) + \gamma w_k^2 h_p x U_k''\left(\sigma_k h_k f_{\hat{x}\hat{x}} - f_{\hat{x}}^2\right) & h_p u_1 + w_p h_p^2 u_{11}(1-x) & \gamma h_p f_{\hat{x}} w_k h_k U_k''(\sigma_k - 1) & 0 \\ 0 & u_{11} w_p (1-x) & h_p u_{11}(1-x) & 0 & u_2 + u_{22} s(1-\tau_s) \end{bmatrix} * \begin{bmatrix} dh_0 \\ dh_p \\ dw_p \\ dw_k \\ d\tau_s \end{bmatrix}$$

Public education h_0

If the government increases its educational investments, h_0, the parent will respond to it in the following way:

$$\frac{\partial x}{\partial h_0} = \frac{\gamma h_p w_k^2 U_k''\left(u_{11} + u_{22}(1-\tau_s)^2\right)\left(\sigma_k h_k f_{h_0\hat{x}} - f_{h_0} f_{\hat{x}}\right)}{|A_C|}. \quad (12)$$

If public and within-the-family education are substitutes, *i.e.* $f_{h_0\hat{x}} \leq 0$, the parent decreases his educational investments, when public investments increase. This is because income and substitution effects then go in the same direction. The child becomes relatively richer with increased h_0, so the parent tends to decrease x. Furthermore, the marginal productivity of x decreases when h_0 increases if $f_{h_0\hat{x}} \leq 0$, which also tends to decrease x.

If, on the other hand, h_0 and $\hat{x}$ amplify each other, so that $f_{h_0\hat{x}} > 0$ we cannot tell whether the parent increases or decreases time spent educating the child, as income and substitution effects now work in opposite directions, since marginal productivity of x now increases with h_0.

The total effect on the child's human capital, h_k is achieved by implicitly differentiating (2) with respect to h_0:

$$\frac{\partial h_k}{\partial h_0} = f_{\hat{x}} h_p \frac{\partial x}{\partial h_0} + f_{h_0}, \quad (13)$$

which implies

$$\frac{\partial h_k}{\partial h_0} = \frac{\overbrace{f_{h_0} w_p^2 h_p^2 u_{11} u_{22} (1-\tau_s)^2}^{+} + \overbrace{\gamma w_k h_p^2 U_k' \left(u_{11} u_{22} (1-\tau_s)^2\right)}^{-}\overbrace{\left(f_{h_0} f_{\hat{x}\hat{x}} - f_{\hat{x}} f_{h_0\hat{x}}\right)}^{?}}{|A_C|}. \quad (14)$$

If $f_{h_0\hat{x}} \geq 0$ the total effect is positive, so even if the parent might decrease his own investment, he will not decrease it in a one-for-one proportion. If, on the other hand, $f_{h_0\hat{x}} < 0$, we cannot tell whether h_k increases or decreases.

Because the effect on h_k is ambiguous, so is the effect on the child's consumption, c_k, because

$$\frac{\partial c_k}{\partial h_0} = w_k \frac{\partial h_k}{\partial h_0}. \quad (B3)$$

Parental human capital h_p

When h_p increases, there is no clear effect on x. The parent becomes richer, and therefore tends to increase the transfer to the child. However, he also becomes more productive in the labour market, which tends to have the opposite effect. Thus, we have

$$\frac{\partial x}{\partial h_p} = \frac{w_p^2 h_p (1-x) u_{11} u_{22} (1-\tau_s)^2 - \gamma w_k^2 h_p x U_k'' \left(u_{11} + u_{22}(1-\tau_s)^2\right)\left(\sigma_k h_k f_{\hat{x}\hat{x}} - f_{\hat{x}}^2\right)}{|A_C|} = \\ = -\frac{x}{h_p} + \frac{w_p^2 h_p u_{11} u_{22} (1-\tau_s)^2}{|A_C|}. \tag{B4}$$

We cannot tell whether this derivative is positive or negative. The total effect on within-the-family education is, however, positive, irrespective of the effect on time investment, because time itself becomes more productive.

$$\frac{\partial h_k}{\partial h_p} = f_{\hat{x}} \left(h_p \frac{\partial x}{\partial h_p} + x \right) = \frac{f_{\hat{x}} u_{11} u_{22} (1-\tau_s)^2 w_p^2 h_p^2}{|A_C|} > 0, \tag{15}$$

and thus also c_k increases:

$$\frac{\partial c_k}{\partial h_p} = w_k \frac{\partial h_k}{\partial h_p} > 0. \tag{B5}$$

Parental wage rate w_p

When the parental net wage rate increases, the income effect is positive, but it also becomes relatively more profitable to work in the market than to spend time on within-the-family education, so we cannot tell what will actually happen to educational time investment:

$$\frac{\partial x}{\partial w_p} = \frac{h_p u_1 \left(u_{11} + u_{22}(1-\tau_s)^2\right) + w_p h_p^2 (1-x) u_{11} u_{22} (1-\tau_s)^2}{|A_C|}. \tag{B6}$$

The effects on h_k and c_k will go in the same direction as the effect on x:

$$\frac{\partial h_k}{\partial w_p} = f_{\hat{x}} h_p \frac{\partial x}{\partial w_p}, \tag{B7}$$

$$\frac{\partial c_k}{\partial w_p} = w_k \frac{\partial h_k}{\partial w_p}. \tag{B8}$$

If we assume that the parent has a constant elasticity of intertemporal substitution,

$$\sigma_p = -\frac{u_1}{u_{11}\left(w_p h_p (1-x) - s\right)} = -\frac{u_2}{u_{22} s (1-\tau_s)} = -\frac{u_1}{u_{22} s (1-\tau_s)^2},$$

we can rewrite (B6) as

$$\frac{\partial x}{\partial w_p} = \frac{w_p h_p^2 (1-x) u_1 (1-\sigma_p)}{|A_C| \sigma_p^2 s \left(w_p h_p (1-x) - s\right)}. \tag{B9}$$

We then conclude that the following holds:

$$\sigma_p < 1 \Rightarrow \frac{\partial x}{\partial w_p} > 0$$

$$\sigma_p = 1 \Rightarrow \frac{\partial x}{\partial w_p} = 0$$

$$\sigma_p > 1 \Rightarrow \frac{\partial x}{\partial w_p} < 0 .$$

The child's wage rate w_k

If the child's net wage rate increases, the child becomes richer, so the income effect tends to decrease within-the-family education. On the other hand, educational investments become more favourable when the returns increase, so we have two counteracting effects:

$$\frac{\partial x}{\partial w_k} = \frac{\gamma w_k h_k U_k'' h_p f_{\hat{x}} \left(u_{11} + u_{22}(1-\tau_s)^2\right)(\sigma_k - 1)}{|A_C|}, \tag{B10}$$

where $\sigma_k = -U_k' / U_k'' w_k h_k$ is the elasticity of substitution of the child's utility function, which determines the qualitative effect of w_k on x. It follows that

$$\sigma_k < 1 \Rightarrow \frac{\partial x}{\partial w_k} < 0$$

$$\sigma_k = 1 \Rightarrow \frac{\partial x}{\partial w_k} = 0$$

$$\sigma_k > 1 \Rightarrow \frac{\partial x}{\partial w_k} > 0 \ .$$

In the same way, total human capital, h_k also depends on σ_k, because

$$\frac{\partial h_k}{\partial w_k} = f_{\hat{x}} h_p \frac{\partial x}{\partial w_k} \ . \tag{B11}$$

However, because w_k itself increases, the child's income, and thereby consumption will increase, irrespective of the effect on within-the-family education:

$$\frac{\partial c_k}{\partial w_k} = h_k + w_k \frac{\partial h_k}{\partial w_k} = \frac{h_k w_p^2 h_p^2 u_{11} u_{22} (1-\tau_s)^2 + \gamma w_k h_p U_k' \left(u_{11} + u_{22}(1-\tau_s)^2\right)\left(h_k h_p f_{\hat{x}\hat{x}} - f_{\hat{x}}^2\right)}{|A_C|} > 0 \ . \tag{B12}$$

The wealth tax τ_s

If taxation on savings increases, the parent becomes poorer, which tends to decrease within-the-family education. However, it becomes relatively more profitable to invest in within-the-family education. Theses two effects go in opposite directions, and we cannot clearly say which effect dominates the other; *i.e.* we have that

$$\frac{\partial x}{\partial \tau_s} = \frac{w_p h_p u_{11} u_{22} s (1-\tau_s)(\sigma_p - 1)}{|A_C|} \ . \tag{B13}$$

Hence, the size of σ_p determines in what direction within-the-family education changes in case of an increased wealth tax:

$$\sigma_p < 1 \ \Rightarrow \ \frac{\partial x}{\partial \tau_s} < 0$$

$$\sigma_p = 1 \ \Rightarrow \ \frac{\partial x}{\partial \tau_s} = 0$$

$$\sigma_p > 1 \ \Rightarrow \ \frac{\partial x}{\partial \tau_s} > 0 \ .$$

Total human capital and the child's consumption is altered in the same direction as x.

(C1) The equation system representing the solution for the rich parent

$$\begin{bmatrix} w_p^2 h_p^2 u_{11} + \gamma w_k h_p^2 \left(w_k U_k'' f_{\hat{x}}^2 + U_k' f_{\hat{x}\hat{x}} \right) & \gamma w_k h_p U_k''(1-\tau_b) f_{\hat{x}} & w_p h_p u_{11} \\ \gamma w_k h_p U_k''(1-\tau_b) f_{\hat{x}} & u_{22} + \gamma U_k''(1-\tau_b)^2 & -u_{22}(1-\tau_s) \\ w_p h_p u_{11} & -u_{22}(1-\tau_s) & u_{11} + u_{22}(1-\tau_s)^2 \end{bmatrix} * \begin{bmatrix} dx \\ db \\ ds \end{bmatrix} =$$

$$= \begin{bmatrix} -\gamma w_h h_p \left(U_k' f_{h_0 \hat{x}} + w_k U_k'' f_{\hat{x}} f_{h_0} \right) & u_{11} w_p^2 h_p (1-x) - \gamma w_k h_p x \left(U_k' f_{\hat{x}\hat{x}} + w_k U_k'' f_{\hat{x}}^2 \right) & h_p u_1 + w_p h_p^2 u_{11}(1-x) & -\gamma h_p f_{\hat{x}} \left(U_k' + w_k h_k U_k'' \right) & 0 & \gamma w_k U_k'' h_p b f_{\hat{x}} \\ -\gamma w_k U_k'' f_{h_0}(1-\tau_b) & -\gamma w_k U_k'' x f_{\hat{x}}(1-\tau_b) & 0 & -\gamma (1-\tau_b) U_k'' h_k & -u_{22} s & \gamma \left(U_k' + b(1-\tau_b) U_k'' \right) \\ 0 & u_{11} w_p (1-x) & h_p u_{11}(1-x) & 0 & u_2 + u_{22} s (1-\tau_s) & 0 \end{bmatrix} * \begin{bmatrix} dh_0 \\ dh_p \\ dw_p \\ dw_k \\ d\tau_s \\ d\tau_b \end{bmatrix}$$

Appendix C – The interior solution (the rich)

The equation system representing the solution for a rich parent, *i.e.* a parent in the interior solution with a positive bequest is presented in equation (C1) on the previous page. The system determinant of the left-hand side matrix in (C1) is

$$|A_I| = \gamma w_k h_p^2 U'_k f_{\hat{x}\hat{x}} \Omega < 0 \,, \text{ where } \Omega = u_{11}u_{22} + \gamma(1-\tau_b)^2 U''_k \left(u_{11} + u_{22}(1-\tau_s)^2\right) > 0 \,.$$

For a rich parent in the interior solution comparative statics are carried out using Cramer's rule, in the same way as in the corner solution. Except for x, and, b, I also study the effect on the child's total human capital, h_k, and consumption, c_k. Contrary to the corner solution, valid for poor parents, there is now only a substitution effect connected with the parent's educational investment. The income effect only affects the physical bequest.

Public education h_0

If the level of publicly provided education, h_0 increases, also the relative return to x is affected, which alters within-the-family education:

$$\frac{\partial x}{\partial h_0} = -\frac{f_{h_0\hat{x}}}{h_p f_{\hat{x}\hat{x}}} \,. \tag{16}$$

If $f_{h_0\hat{x}} > 0$, increased h_0 increases the marginal productivity of x, which induces the parent to spend more time educating the child. If $f_{h_0\hat{x}} < 0$ the parent will instead decrease x. The total effect on h_k, also including the direct effect caused by h_0 is

$$\frac{\partial h_k}{\partial h_0} = f_{h_0} + f_{\hat{x}} h_p \frac{\partial x}{\partial h_0} = \frac{f_{h_0} f_{\hat{x}\hat{x}} - f_{\hat{x}} f_{h_0\hat{x}}}{f_{\hat{x}\hat{x}}} \,. \tag{C2}$$

If $f_{h_0\hat{x}} \geq 0$ we get an unambiguous effect on human capital; increased h_0 implies increased x, and of course, increased over-all human capital. If, on the other hand, $f_{h_0\hat{x}} < 0$ the total effect on h_k is unclear, because public and private investments go in opposite directions. Also the effect on the bequest is undetermined if $f_{h_0\hat{x}} < 0$:

$$\frac{\partial b}{\partial h_0} = -\frac{\left(u_{11} + u_{22}(1-\tau_s)^2\right)U_k''(1-\tau_b)w_k\gamma f_{h_0}}{\Omega} + \frac{w_p(1-\tau_s)f_{h_0\hat{x}}}{f_{\hat{x}\hat{x}}}. \quad (17)$$

The income- and substitution effects go in opposite directions when $f_{h_0\hat{x}} < 0$, but if $f_{h_0\hat{x}} \geq 0$ it is more favourable to invest in human capital (the substitution effect) at the same time as the child becomes richer with increased h_0 (the income effect). Both these effects tend to decrease the bequest, so in this case we get an unambiguous effect on b.

However, irrespective of the mix of within-the-family education and bequests, the rich child is always left with a higher consumption after an increase in h_0:

$$\frac{\partial c_k}{\partial h_0} = w_k\left(f_{\hat{x}}h_p\frac{\partial x}{\partial h_0} + f_{h_0}\right) + (1-\tau_b)\frac{\partial b}{\partial h_0} = \frac{f_{h_0}w_k u_{11}u_{22}}{\Omega} > 0. \quad (18)$$

Parental human capital h_p

If parental human capital, h_p increases, the parent's productivity will rise, in paid work, as well as in the efforts to educate the child. Because it is the effective time that counts, the parent will reduce time spent on within-the-family education, and instead work more:

$$\frac{\partial x}{\partial h_p} = -\frac{x}{h_p} < 0. \quad \text{(C3)}$$

The effective time in within-the-family education, however, remains unchanged:

$$\frac{\partial h_k}{\partial h_p} = f_{\hat{x}}\frac{\partial \hat{x}}{\partial h_p} = f_{\hat{x}}\left(x + h_p\frac{\partial x}{\partial h_p}\right) = f_{\hat{x}}\left(x - h_p\frac{x}{h_p}\right) = 0. \quad \text{(C4)}$$

This specific solution, however, hinges on the assumption that it is the effective time, $\hat{x} = xh_p$, which is the input in within-the-family education. If x and h_p would have been separate variables there would be no unambiguous effect on x from an increase in h_p. The wage rate and the within-the-family production function would then determine if the parent would choose to spend more time at work, or more time educating the child.

With a higher h_p the parent works more, and gets richer, while the child receives the same level of human capital as before. Due to the positive income effect, the parent then increases the bequest:

$$\frac{\partial b}{\partial h_p} = \frac{w_p u_{11} u_{22} (1-\tau_s)}{\Omega} > 0 \,, \tag{C5}$$

and, naturally also consumption increases,

$$\frac{\partial c_k}{\partial h_p} = (1-\tau_b) \frac{\partial b}{\partial h_p} > 0 \,. \tag{C6}$$

Parental wage rate w_p

The only effect on within-the-family education from an increased parental net wage rate is the negative substitution effect, due to the higher profitability of market work. The positive income effect, that in the corner solution made the effect undetermined, now only impacts on the bequest:

$$\frac{\partial x}{\partial w_p} = \frac{1}{w_p h_p f_{\hat{x}\hat{x}}} < 0 \quad \Rightarrow \quad \frac{\partial h_k}{\partial w_p} < 0 \tag{C7}$$

$$\frac{\partial b}{\partial w_p} = \frac{u_{11} u_{22} (1-\tau_s) h_p (1-x)}{\Omega} - \frac{(1-\tau_s) f_{\hat{x}}}{f_{\hat{x}\hat{x}}} > 0 \,. \tag{C8}$$

The total effect on the child's consumption is, however, positive due to the income effect:

$$\frac{\partial c_k}{\partial w_p} = w_k f_{\hat{x}} h_p \frac{\partial x}{\partial w_p} + (1-\tau_b) \frac{\partial b}{\partial w_p} = \frac{u_{11} u_{22} (1-\tau_s) h_p (1-x)}{\Omega} > 0 \,. \tag{C9}$$

The child's wage rate w_k

Within-the-family education becomes more favourable after an increase in the child's net-wage rate w_k, but as it also makes the child richer, the parent wants to reduce the total transfer.

For the poor parent, the opposite directions of income and substitution effects implied an undetermined effect on within-the-family education. For a rich parent, who also leaves a physical bequest, the reduction will be made entirely on that bequest, and within-the-family education will actually increase, due to the substitution effect:

$$\frac{\partial x}{\partial w_k} = -\frac{f_{\hat{x}}}{w_k h_p f_{\hat{x}\hat{x}}} > 0 \qquad \Rightarrow \quad \frac{\partial h_k}{\partial w_k} > 0 \tag{C10}$$

$$\frac{\partial b}{\partial w_k} = -\frac{\gamma h_k U_k''(1-\tau_b)\left(u_{11} + u_{22}(1-\tau_s)^2\right)}{\Omega} + \frac{(1-\tau_s)w_p f_{\hat{x}}}{w_k f_{\hat{x}\hat{x}}} < 0\,. \tag{C11}$$

The child's consumption, c_k is influenced by the shifts in bequests and education, but also by the wage rate, itself:

$$\frac{\partial c_k}{\partial w_k} = h_k + w_k f_{\hat{x}} h_p \frac{\partial x}{\partial w_k} + (1-\tau_b)\frac{\partial b}{\partial w_k} = \frac{h_k u_{11} u_{22}}{\Omega} > 0\,. \tag{C12}$$

The wealth tax τ_s

If the wealth tax increases, it becomes less profitable to save and leave a bequest. Hence x increases due to the substitution effect:

$$\frac{\partial x}{\partial \tau_s} = -\frac{(1-\tau_b)w_p}{w_k h_p f_{\hat{x}\hat{x}}} > 0\,. \tag{C13}$$

The parent instead decreases b, both due to the income and substitution effects:[17]

$$\frac{\partial b}{\partial \tau_s} = \frac{u_{22}\left(u_1 - u_{11}s\right)}{\Omega} + \frac{w_p f_{\hat{x}}}{f_{\hat{x}\hat{x}}} < 0\,. \tag{C14}$$

Together, the total effect on the child's consumption is negative:

$$\frac{\partial c_k}{\partial \tau_s} = w_k f_{\hat{x}} h_p \frac{\partial x}{\partial \tau_s} + (1-\tau_b)\frac{\partial b}{\partial \tau_s} = \frac{(1-\tau_b)u_{22}\left(u_1 - u_{11}s\right)}{\Omega} < 0\,. \tag{C15}$$

[17] I assume that the parent will remain in the interior solution, also after the marginal increase in τ_s. However, if τ_s becomes large enough, the parent will not find it worthwhile to save and leave a bequest anymore, but we switch to the corner solution.

Tax on bequests τ_b

If the tax that the child has to pay on the bequest, increases, the parent will substitute away from bequeathing, and instead spend more on within-the-family education:

$$\frac{\partial x}{\partial \tau_b} = -\frac{w_p(1-\tau_s)}{h_p f_{\hat{x}\hat{x}}} > 0 . \tag{C16}$$

The effect on the bequest is less straightforward. On the one hand, the parent wants to reduce b due to the substitution effect, but on the other hand he wants to compensate the child for the decreased income, created by heavier taxation. However, there is a tax wedge connected with this compensation, which makes it impossible to completely compensate the loss. We have that:

$$\frac{\partial b}{\partial \tau_b} = \frac{\gamma\left(u_{11} + u_{22}(1-\tau_s)^2\right)\left(U_k' + (1-\tau_b)bU_k''\right)}{\Omega} + \frac{w_p^2(1-\tau_s)^2}{f_{\hat{x}\hat{x}}} . \tag{C17}$$

The sign is undetermined. The curvature of the child's utility function becomes crucial for the outcome of b, but c_k unambiguously decreases:

$$\frac{\partial c_k}{\partial \tau_b} = w_k \frac{\partial h_k}{\partial \tau_b} + (1-\tau_b)\frac{\partial b}{\partial \tau_b} - b = \frac{\gamma(1-\tau_b)U_k'\left(u_{11} + u_{22}(1-\tau_s)^2\right) - bu_{11}u_{22}}{\Omega} < 0 . \tag{C18}$$

Appendix D – Proof of Proposition 2

Assume that the utility function is logarithmic, and that the production function for human capital is of Cobb-Douglas type. Then the parent maximises

$$\max_{\{x,b,s\}} U_p = \ln\left(w_p h_p(1-x) - s\right) + \ln\left(s(1-\tau_s) - b\right) + \gamma \ln\left(w_k h_k + b(1-\tau_b)\right), \tag{D1}$$

where $h_k = h_0^\alpha \hat{x}^\beta$, $\alpha, \beta \in (0,1)$.

In this case the poor parent invests the following amount in the child's human capital:

$$x^{poor} = \frac{\gamma\beta}{2+\gamma\beta}, \tag{D2}$$

which, together with public investments, h_0 result in the following level of human capital:

$$h_k^{poor} = h_0^{\alpha}\left[\frac{\gamma\beta h_p^{poor}}{2+\gamma\beta}\right]^{\beta}. \tag{D3}$$

A rich parent, on the other hand, makes the following investment in within-the-family education:

$$x^{rich} = \frac{h_0^{\frac{\alpha}{1-\beta}}}{h_p^{rich}}\left[\frac{w_k\beta}{w_p(1-\tau_s)(1-\tau_b)}\right]^{\frac{1}{1-\beta}}, \tag{D4}$$

which makes the total level of human capital:

$$h_k^{rich} = h_0^{\frac{\alpha}{1-\beta}}\left[\frac{w_k\beta}{w_p(1-\tau_s)(1-\tau_b)}\right]^{\frac{\beta}{1-\beta}}. \tag{D5}$$

The ratio between the human capital of poor and rich children is

$$\frac{h_k^{poor}}{h_k^{rich}} = h_0^{\frac{\beta}{\beta-1}}\left[\frac{w_k\beta}{w_p(1-\tau_s)(1-\tau_b)}\right]^{\frac{\beta}{\beta-1}}\left[\frac{\gamma h_p^{poor}}{2+\gamma\beta}\right]^{\beta}, \tag{D6}$$

and it changes with public education:

$$\frac{\partial \frac{h_k^{poor}}{h_k^{rich}}}{\partial h_0} = \frac{\beta}{\beta-1}h_0^{\frac{1}{\beta-1}}\left[\frac{w_k\beta}{w_p(1-\tau_s)(1-\tau_b)}\right]^{\frac{\beta}{\beta-1}}\left[\frac{\gamma h_p^{poor}}{2+\gamma\beta}\right]^{\beta} < 0, \tag{D7}$$

implying that the educational gap between rich and poor increases when more resources are spent on public education. However, not only the educational equality is of economic interest; also the relative change in consumption possibilities has to be considered. It is easy to show that in the Cobb-Douglas case, we have that:

$$\frac{c_k^{poor}}{c_k^{rich}} = \frac{w_k h_k^{poor}}{w_k h_k^{rich} + (1-\tau_b)b}, \tag{D8}$$

where $b = \dfrac{\gamma w_p h_p^{rich}(1-x^{rich})(1-\tau_s)(1-\tau_b) - 2w_k h_k^{rich}}{(1-\tau_b)(2+\gamma)}$, and h_k^{poor} and h_k^{rich} are defined above.

Hence, it follows that:

$$\frac{\partial \dfrac{c_k^{poor}}{c_k^{rich}}}{\partial h_0} = \frac{(2+\gamma)\alpha w_k h_k^{poor} w_p h_p^{rich}(1-x^{rich})(1-\tau_s)(1-\tau_b)}{\gamma h_0 \left(w_k h_k^{rich} + w_p h_p^{rich}(1-x^{rich})(1-\tau_s)(1-\tau_b)\right)^2} > 0\,. \tag{D9}$$

Hence, increased public education does not increase equality in terms of human capital, but it increases equality in terms of consumption.

Appendix E – A PAYGO system

If a lump-sum transfer, *P*, is paid by the adult child, and given to the elderly parent, the parent maximises

$$\max_{\{x,b,s\}} U_p = u\left(w_p h_p (1-x) - s\right) + u\left(s(1-\tau_s) - b + P\right) + \gamma U_k\left(w_k h_k + b(1-\tau_b) - P\right). \tag{21}$$

Rich parent

For the rich parent, who leaves a bequest, $b > 0$ the comparative statics with respect to *P* are

$$\frac{\partial x}{\partial P} = 0 \tag{E1}$$

$$\frac{\partial b}{\partial P} = \frac{\left(u_{11} + u_{22}(1-\tau_s)^2\right)\gamma U_k''(1-\tau_b) + u_{11}u_{22}}{\left(u_{11} + u_{22}(1-\tau_s)^2\right)\gamma U_k''(1-\tau_b)^2 + u_{11}u_{22}} \tag{E2}$$

$$\frac{\partial s}{\partial P} = \frac{U_k'' u_{22}(1-\tau_s)(1-\tau_b)\tau_b}{\left(u_{11} + u_{22}(1-\tau_s)^2\right)\gamma U_k''(1-\tau_b)^2 + u_{11}u_{22}} \tag{E3}$$

$$\frac{\partial c_k}{\partial P} = (1-\tau_b)\frac{\partial b}{\partial P} - 1 = \frac{\left(u_{11} + u_{22}(1-\tau_s)^2\right)\gamma U_k''(1-\tau_b)^2 + u_{11}u_{22}(1-\tau_b)}{\left(u_{11} + u_{22}(1-\tau_s)^2\right)\gamma U_k''(1-\tau_b)^2 + u_{11}u_{22}} - 1. \tag{E4}$$

The rich parent's consumption in both periods is affected in the following way:

$$\frac{\partial c_{p1}}{\partial P} = -\frac{\partial s}{\partial P} \tag{E5}$$

$$\frac{\partial c_{p2}}{\partial P} = (1-\tau_s)\frac{\partial s}{\partial P} - \frac{\partial b}{\partial P} + 1 = -\frac{\gamma(1-\tau_b)U_k'' u_{11}\tau_b}{\Omega}. \tag{E6}$$

From (E2) - (E6) it is clear that in absence of the bequest tax, τ_b, Ricardian Equivalence holds, and the lump-sum transfer would make no difference to the rich family. However, if $\tau_b > 0$, there will be a tax wedge, such that

$$\tau_b > 0 \Rightarrow \frac{\partial b}{\partial P} > 1,\ \frac{\partial c_k}{\partial P} < 0,\ \frac{\partial c_p}{\partial P} < 0.$$

Hence, the rich family is worse off, the higher is the transfer *P*, if the bequest tax $\tau_b > 0$.

Poor parent

For the poor parent in the corner solution, the lump-sum transfer will have a positive effect on educational investments:

$$\frac{\partial x}{\partial P} = \frac{\gamma U_k'' w_k h_p f_{\hat{x}}\left(u_{11} + u_{22}(1-\tau_s)^2\right) + u_{22}(1-\tau_s)w_p h_p u_{11}}{|A_C|} > 0, \tag{E7}$$

where $\left|A_C\right| > 0$ is defined in Appendix B.

If P continues to increase, x will eventually reach the point where $w_p(1-\tau_b)(1-\tau_s) = w_k f_{\hat{x}}$, and also the former corner-solution parent will end up in the interior solution. Hence, with a sufficiently high P, everyone in the economy will be in the interior solution with the same amount of human capital, and the corner-solution inefficiency will be gone. If there is no bequest tax, such a policy would be Pareto improving.