A PARTIAL EQUILIBRIUM MODEL OF CONFLICT WITH INTERDEPENDENT INSTRUMENTS, RETURNS TO SCALE AND INTEGRATIVE GRANTS

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DRAFT VERSION
JUNE 2005

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Abstract

This paper considers a partial equilibrium model of conflict where two asymmetric, rational and risk-neutral opponents conflict in order to redistribute future income in their favour. In order to do that, they can choose between one single instrument, say violence, and two instruments, namely violence and negotiating. In the first section, the paper explores the incentives and conditions leading to the optimal endogenous choice for both instruments. In the second part, the basic model is extended in order to consider the impact of different degrees of the aggregate returns to scale attached the stake of the conflict. The results suggest that an asymmetry in the degree of the opponents' aggregate returns to scale modifies the incentives to fight and negotiate in favour of negotiating. Moreover, in the last section the case for providing an integrative grant is analysed. The existence of an integrative grant provided by the agent with the higher evaluation of the stake in the conflict partly seems to pave the way for the conflict resolution enlarging the Potential Settlement Region under some conditions.

Keywords: Conflict, returns to scale, contest success function, integrative grant, settlement region.

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* Università Cattolica del Sacro Cuore di Milano. Paper prepared for the European Jan Tinbergen Peace Science Conference, June 26- 29 2005, Amsterdam. I must warmly thank Damiano Palano and Andrea Locatelli for their comments and enduring support. The Derive mathematical program was used for calculations and plots.

INTRODUCTION

The growing economic theory of conflict rests to a large extent on the assumption that agents involved in conflict interactions have to choose an optimal level of efforts or resources devoted to the unproductive activity of conflicts[1].

In many situations, however, agents have a choice as to what kind of strategy they will use. In fact, it is a great mistake to think of many conflicts as an exclusively violent activity. Beyond violence, applied to send actual or potential threats, agents are used to apply some other instruments to end successfully the contest. In international relations, for instance, the exploitation of potential or actual violence is often interlinked with the diplomatic efforts.

Of course, any of the instruments used is interdependent with the others. Agents still want to gain economic payoffs. Therefore, the core assumption of this paper is that once involved in conflict interactions, agents face the option of choosing also a second instrument trying to improve the outcome of the conflict. Thus, rational agents can choose whether to use only one instrument or two instruments. Their own choice is rational, that is, they will choose upon the consideration of the attainable payoffs.

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[1] A significant element in economic theory of conflict is that investing resources in conflict is necessarily immiserizing. This is central to theory of conflict as well as to theory of rent-seeking and contests. In particular, conflicts, contests and rent-seeking can be considered directly unproductive activities in the spirit of the definition provided by Bhagwati (1982).

The remainder of the paper is organised as follows: In the first part the basic hypothesis and definitions are presented. In the second part a basic model allowing for the second instrument is presented. In the third part, the foregoing model is extended in order to take into account the existence of aggregate returns to scale attached to the stake of the conflict. In the last section the case for providing an integrative grant is analysed.

I. The Basic Model

There are two risk-neutral agents indexed by $i = 1,2$. They conflict over a positive stake denoted by $X_i \in (0, \infty), i = 1,2$. The agents have different evaluations of the stake in the conflict. Then, in this two-agents model it is possible to write that $X_i \neq X_j, \forall i \neq j, i, j = 1,2$.

The stake of the conflict can be interpreted in different ways. It might be for example a contested natural resource, a territory or a homogenous input. Possible reasons for one agent to attach a greater value to the stake of the conflict can include the following among others: (i) one agent has better information, (ii) one agent has a greater access to markets. (iii) there had been mistakes in evaluation; (iv) there is a great relevance of this input within the aggregate production structure of the economy.

Let $\delta \in (0,1)$ denote the level of asymmetry between the stakes of the two agents. In particular, assuming that $X_1 > X_2$, it is possible to write that $X_2 = \delta X_1$. There is common knowledge about such hypotheses.

Given the assumption of risk-neutrality, agents see the outcome of the non-cooperative conflict game as deterministic. Then, the conflict is supposed to give each party control over a positive fraction of the contested stake in order to maximize its utility.

Agents can use two different instruments, say violent appropriation and negotiation respectively. Let $z_i, i = 1,2$ denote the amount of violent efforts and $h_i, i = 1,2$ denote the negotiation efforts.

The core assumption is that agents behave rationally and choose the elements of their strategy sets in order to maximize their own expected payoff. It is assumed that violence and appropriation constitute the first option for both agents. Hence, agents will choose to use a second instrument if and only if the attainable payoffs will be greater than in the standard 'one-instrument' contest mechanism. The two instruments complement each other. Namely, the outcome of the conflict depends upon the mixed effect of violence and negotiation.

Let π_i^o, π_i^t for $i = 1,2$ denote the payoff achievable under the standard contest and the payoff achievable using the two instruments respectively. In particular, in their general form the payoff functions can be written as $\pi_i^o = \pi_i^o(z_i, z_j, X_i, X_j)$ and $\pi_i^t = \pi_i^t\left(z_i, z_j, h_i, h_j, X_i, X_j\right)$ respectively. Then, each agent will choose to use the second instrument if and only if $\pi_i^t > \pi_i^o, i = 1,2$. In the remainder of the paper, I will define it as **negotiating condition**. Whenever it is satisfied it would be possible to say that a willingness to negotiate emerges. To summarise formally:

$$\begin{cases} z_i > 0 \quad h_i > 0 \quad \Leftrightarrow \quad \pi_i^T > \pi_i^o \quad i = 1,2 \\ z_i > 0 \quad h_i = 0 \quad \Leftrightarrow \quad \pi_i^T < \pi_i^o \quad i = 1,2 \end{cases} \qquad \text{(c.1)}$$

In simpler words, agents will choose also the second instrument if and only if it reinforces the effect of the first instrument. That is, agents will devote resources to negotiations if and only if this behaviour appears to provide a higher payoff.

Whenever both agents choose to use the second instrument, that is to devote resources to negotiating, a settlement region can be identified. Then, another limiting hypothesis is that a settlement region is feasible if and only if both agents choose to negotiate. To summarise, formally in the limiting two-agents scenario a **potential settlement region** (hereafter PSR for brevity) does exist if and only if:

$$\begin{cases} \pi_1^T > \pi_1^o \\ \pi_2^T > \pi_2^o \end{cases} \qquad \text{(c.2)}$$

Since according to (c.2) the payoff with two instruments must exceed the attainable payoff with only one instrument for both agents, this also means that there exists a positive value $\gamma_i = \pi_i^T - \pi_i^o, i = 1,2$. Then, the PSR can be defined as the set of all the positive values for γ, namely $PSR \equiv \{(\gamma_i, \gamma_j) \in \Re : \gamma_i > 0, \gamma_i > 0, i = 1,2, i \neq j\}$. In this PSR some agreement can be reached.

Consider first the standard setting with one instrument. A partial equilibrium model of conflict is not technically distinguishable from standard rent-seeking model. The cornerstone of this class of models is the Contest Success

Function[2] (hereafter CSF for brevity). The payoff function is given by:

$$\pi_i^o = \frac{z_i}{z_i + z_j} X_i - z_i \tag{1}$$

Each agent maximizes its own payoff with respect to its own level of violent efforts. This yields to the first order conditions:

$$\frac{\partial \pi_i^o}{\partial z_i} = \frac{X_i z_j}{\left(z_i + z_j\right)^2} - 1 = 0, i = 1,2, i \neq j \tag{2}$$

Solving the first order conditions, the equilibrium (denoted by stars superscripted) choices of violent efforts are given by:

$$z_i^{*O} = \frac{X_i^2 X_j}{\left(X_i + X_j\right)^2}, i = 1,2; i \neq j \tag{3}$$

In equilibrium the payoffs for agent i are given by:

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[2] The Contest Success Function is a mathematical relation that links the outcome of a contest and the efforts of the players. It is actually a founding pillar of many models. Selective seminal contributions are by Tullock (1980), O'Keeffe et al. (1984) and Rosen (1986). Dixit (1987) develops a general framework for contests using the general properties of logit functions. Hirshleifer (1989) focuses on a different form for the CSF: the ratio form and logit form. See then Skaperdas (1996) and Clark and Riis (1998) for a basic axiomatization.

$$\pi_i^{*O} = \frac{X_i^3}{\left(X_i + X_j\right)^2}, i = 1,2; i \neq j \tag{4}$$

Given $X_1 > X_2$ by assumption, then payoff for agent 1 is greater than payoff for agent 2, $\pi_1^o > \pi_2^o$.

II. The second instrument

Consider now the option of a second instrument. That is, parties can commit themselves to the use of a second instrument in order to affect the result of the contest. The basic model presented hereafter follows Epstein and Hefeker (2003). The ordinary Contest Success Function is modified in order to allow for a second instrument. The two instruments are assumed to be complementary to each other. Then, the use of the second instrument would strengthen the effect of the first instrument. The payoff function for each agent becomes:

$$\pi_i^T = \frac{z_i(h_i + 1)}{z_1(h_1 + 1) + z_2(h_2 + 1)} X_i - z_i - h_i \tag{5}$$

And follows the conditions below:

$$\begin{cases} \frac{\partial \pi_i^T}{\partial z_i} > 0 & \frac{\partial^2 \pi_i^T}{\partial z_i} < 0 \\ \frac{\partial \pi_i^T}{\partial z_j} < 0 & \frac{\partial^2 \pi_i^T}{\partial z_j} > 0 \\ \frac{\partial \pi_i^T}{\partial h_i} > 0 & \frac{\partial^2 \pi_i^T}{\partial h_i} < 0 \\ \frac{\partial \pi_i^T}{\partial h_j} < 0 & \frac{\partial^2 \pi_i^T}{\partial h_j} > 0 \end{cases} \tag{5.1}$$

Also in this case, I assume a Nash-Cournot behaviour for both agents. Therefore, each party maximizes its own payoff. The first order conditions for maximization are:

$$\frac{\partial \pi_i}{\partial z_i} = 0 \quad \frac{\partial \pi_i}{\partial h_i} = 0 \tag{6}$$

Solving the four first order conditions for both agents yields the equilibrium level both for violent appropriation and negotiation efforts:

$$\begin{aligned} z_1^{*T} &= \frac{(X_1^3 X_2^2)}{\left(X_1^2 + X_2^2\right)^2} \qquad h_1^* = \frac{X_1^3 X_2^2}{\left(X_1^2 + X_2^2\right)^2} - 1 \\ z_2^{*T} &= \frac{\left(X_1^2 X_2^3\right)}{\left(X_1^2 + X_2^2\right)^2} \qquad h_2^* = \frac{X_1^2 X_2^3}{\left(X_1^2 + X_2^2\right)^2} - 1 \end{aligned} \tag{7}$$

Using also $X_1 = \delta X_2$, the payoff for both agents are expressed in terms of X_1 and given by:

$$\pi_1^T = \frac{\delta^4 + \delta^2 (2 - X_1) + X_1 + 1}{\left(\delta^2 + 1\right)^2} \tag{8.1}$$

$$\pi_2^T = \frac{X_1 \delta^3 \left(\delta^2 - 1\right) + \delta^4 + 2\delta^2 + 1}{\left(\delta^2 + 1\right)^2} \tag{8.2}$$

Given $\delta \in (0,1)$, it would be simple to verify that $\pi_1^T > \pi_2^T$. That is, the agent with a higher evaluation of the stake of the conflict is able to achieve a higher payoff.

Through a comparison of equations (4) and (8) and using $X_2 = \delta X_1$ it would be possible to verify whether both agents choose to use the second instrument. The negotiating condition for agent 1 and agent 2 are respectively denoted by (9.1) and (9.2).

$$\frac{\delta^4 + \delta^2(2 - X_1) + X_1 + 1}{(\delta^2 + 1)^2} > \frac{X_1}{(\delta + 1)^2} \qquad (9.1)$$

$$\frac{X_1\delta^3(\delta^2 - 1) + \delta^4 + 2\delta^2 + 1}{(\delta^2 + 1)^2} > \frac{X_1\delta^3}{(\delta + 1)^2} \qquad (9.2)$$

The plot below (figure 1) captures the different willingness to negotiate. The area denoted by B contains all the points which satisfy the **negotiating condition** for agent 2, namely the agent with the lower evaluation of the stake of the conflict. In a parallel way, the area denoted by A contains all the points which satisfy the negotiating condition for agent 1, namely the agent with a higher evaluation of the stake of the conflict.

Note first that as the difference in evaluation decreases (namely as δ approaches the unity) the willingness to negotiate decreases for both agents. More precisely, as the difference in evaluation decreases the stake of the conflict needs to be small in order to allow for negotiating.

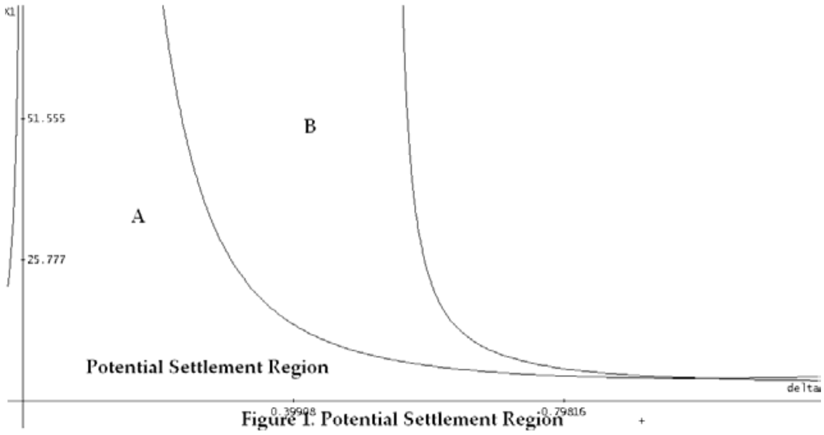

Figure 1. Potential Settlement Region

It is also clear that all the points in A are also contained in area B. This would mean that an asymmetric evaluation lead to different willingness to negotiate. In particular, the agent with a lower evaluation retains a higher willingness to negotiate. Third, according to (c.1), the area denoted by A constitutes a PSR. In fact, since all the points in A are also contained in B, the area A satisfies the condition denoted by (c.2).
Moreover, the plot clearly shows that as the value of the stake of the conflict rises the PSR shrinks. At relatively small values for the stake of the conflict parties will negotiate whatever the level of asymmetry in evaluations of the stake.

III. Returns to Scale and Incentives to conflict

In this section I extend the basic model expounded in the foregoing paragraph. In particular, I try to analyse the relationship between the incentives to conflict and to negotiate and the existence of returns to scale for the stake of the conflict. The role of returns to scale is highly relevant, but hard to pin down. The economic literature on conflict commonly disregards the importance of returns to scale. Such kind of literature focuses mainly on the technology of conflict leading to different non-cooperative results. [3]

Now consider that the stake of the conflict can be interpreted as an input for a productive process. Given the assumption of risk neutrality, agents interpret the outcome of the conflict as deterministic. Therefore, appropriating a deterministic positive fraction of a contestable input can be also interpreted as the choice of the production level. This also would imply that agents look at the stake of the conflict taking into account the technical properties of the productive process. Agents attach to the value of the stake the technical properties of the production function. This also implies that while appropriating a positive fraction of a contestable input each agent does take into account its own production function as well as that of its opponent. That is, it would be reasonable to think that both incentives to conflict and negotiate change when the existence of returns to scale are considered.

Then, imagine that production functions for both agents can be represented by simple exponential functions denoted by:

$$\begin{cases} Y_1 = X_1^{\alpha} \\ Y_2 = X_2^{\beta} \end{cases} \qquad (10)$$

[3] To my knowledge, a first attempt is in Hirshleifer (1995) that deals with the existence of returns to scale through attaching a productivity parameter to a contestable income. However, it does not affect the result of the conflict.

Where α, β now denote the degrees of returns to scale for agent 1 and 2 respectively. This simple form can be interpreted as an intensive production function. Let also $\alpha \neq \beta$, namely agents have different degrees of returns to scale. Note that the degrees of returns to scale are supposed to be independent with respect the degree of asymmetry in evaluation of the stake. This implies, for instance, that even if agent 2 has a lower evaluation of the stake but at the same time it can exhibit increasing returns to scale in its productive process. Intuitively this can be explained considering asymmetric information as the main source of the difference in evaluation. Then, equations (10) could be considered equivalent to the stake of the conflict for both agents.
Given the existence of returns to scale, hereafter the negotiating condition (c.1) and the condition for the existence of a PSR (c.2) will depend upon the relationship of the different degree of productivity, namely the different degree of aggregate returns to scale.

With a single instrument the payoff function of each agent becomes respectively:

$$\begin{cases} \pi_1^o = \dfrac{z_1}{z_1 + z_2} X_1^\alpha - z_1 \\ \pi_2^o = \dfrac{z_2}{z_1 + z_2} X_2^\beta - z_2 \end{cases} \qquad (11)$$

In such a view, equation (1) can be considered a special case of (11) exhibiting constant returns to scale for both agents $(\alpha = \beta = 1)$. Then, as in the foregoing section agents choose simultaneously and maximize their own payoffs by means of violent appropriation. The equilibrium choice of violent efforts are given by:

$$\begin{cases} z_1^{*o} = \dfrac{X_1^{2\alpha} X_2^{\beta}}{\left(X_1^{\alpha} + X_2^{\beta}\right)^2} \\ z_2^{*o} = \dfrac{X_1^{\alpha} X_2^{2\beta}}{\left(X_1^{\alpha} + X_2^{\beta}\right)^2} \end{cases} \tag{12}$$

Plugging (12) into (11) the equilibrium level of payoffs can be derived and are given by:

$$\begin{cases} \pi_1^{*o} = \dfrac{X_1^{3\alpha}}{\left(X_1^{\alpha} + X_2^{\beta}\right)^2} \\ \pi_2^{*o} = \dfrac{X_2^{3\beta}}{\left(X_1^{\alpha} + X_2^{\beta}\right)^2} \end{cases} \tag{13}$$

note that equations (13) reduce to (8.1) and (8.2) for $\alpha = \beta = 1$. However, according to (13) it would also be possible that the agent with a lower evaluation of the stake can get a higher payoff than the opponent. This result will depend upon the different degrees in aggregate returns to scale. In such a case, an asymmetry of productivity determines the outcome of conflict.

Now consider the option for negotiation. The payoff functions when the second instrument is considered are:

$$\begin{cases} \pi_1^{T} = \dfrac{z_1(h_1+1)}{z_1(h_1+1) + z_2(h_2+1)} X_1^{\alpha} - z_1 - h_1 \\ \pi_2^{T} = \dfrac{z_2(h_2+1)}{z_1(h_1+1) + z_2(h_2+1)} X_2^{\beta} - z_2 - h_2 \end{cases} \tag{14}$$

Following an ordinary maximization process the optimal level both of violent and negotiating efforts are given by:

$$\begin{cases} z_1^T = \dfrac{X_1^{3\alpha} X_2^{2\beta}}{\left(X_1^{2\alpha} + X_2^{2\beta}\right)^2} & h_1^T = \dfrac{X_1^{3\alpha} X_2^{2\beta}}{\left(X_1^{2\alpha} + X_2^{2\beta}\right)^2} - 1 \\ z_2^T = \dfrac{X_1^{2\alpha} X_2^{3\beta}}{\left(X_1^{2\alpha} + X_2^{2\beta}\right)^2} & h_2^T = \dfrac{X_1^{2\alpha} X_2^{3\beta}}{\left(X_1^{2\alpha} + X_2^{2\beta}\right)^2} - 1 \end{cases} \tag{15}$$

Note that if $\alpha = \beta = 1$ equations (15) reduce to (7). The equilibrium level of payoffs are given by:

$$\begin{cases} \pi_1^{*T} = \dfrac{X_1^{3\alpha}\left(X_1^{2\alpha} - X_2^{2\beta}\right) + \left(X_1^{2\alpha} + X_2^{2\beta}\right)^2}{\left(X_1^{2\alpha} + X_2^{2\beta}\right)^2} \\ \pi_2^{*t} = \dfrac{X_1^{4\alpha} + X_1^{2\alpha} X_2^{2\beta}\left(2 - X_2^{\beta}\right) + X_2^{4\beta}\left(X_2^{\beta} + 1\right)}{\left(X_1^{2\alpha} + X_2^{2\beta}\right)^2} \end{cases} \tag{16}$$

As defined before, each agent will choose the use of the second instrument if and only if $\pi_i^t > \pi_i^o, i = 1,2$. Whenever such negotiating condition is satisfied simultaneously for both agents it would possible to define a PSR. Recalling equations (13) and (16) and using $X_2 = \delta X_1$ it is possible to write the negotiating condition for agent 1 and agent 2 respectively:

$$\frac{X_1^{3\alpha}\left(X_1^{2\alpha} - (\delta X_1)^{2\beta}\right) + \left(X_1^{2\alpha} + (\delta X_1)^{2\beta}\right)^2}{\left(X_1^{2\alpha} + (\delta X_1)^{2\beta}\right)^2} > \frac{X_1^{3\alpha}}{\left(X_1^{\alpha} + (\delta X_1)^{\beta}\right)^2}$$

(17.1)

$$\frac{X_1^{4\alpha} + X_1^{2\alpha}(\delta X_1)^{2\beta}\left(2-(\delta X_1)^{\beta}\right)+(\delta X_1)^{4\beta}\left((\delta X_1)^{\beta}+1\right)}{\left(X_1^{2\alpha}+(\delta X_1)^{2\beta}\right)^2} > \frac{(\delta X_1)^{3\beta}}{\left(X_1^{\alpha}+(\delta X_1)^{\beta}\right)^2}$$

(17.2)

Given the analytical complexity, hereafter, I will attach some arbitrary values to the stake of the conflict as well as to the parameter denoting the asymmetry in evaluation. In such a way, I intend to focus upon the relationship between the different degrees of returns to scale. Looked at from the point of view of the propositions herein developed any settlement region will take shape under different combinations of returns to scale for both agents.

Within the framework developed up to this point, building a PSR using the different degrees of returns to scale gives the opportunity to evaluate four different possible cases: (i) both agents exhibit decreasing returns to scale; (ii) both agents exhibit increasing returns to scale, (ii) the agent with a higher evaluation of the stake of the conflict does exhibit increasing returns to scale whilst the agent with the lower evaluation of the stake does exhibit decreasing returns to scale. (iv) The agent with the higher evaluation of the stake does exhibit decreasing returns to scale whilst the agent with a lower evaluation of the stake does exhibit decreasing returns to scale.

Then, first consider the case of $X_1 = 100$ and $\delta = 0.5$. That is, agent 2 has an evaluation of the stake of the conflict which is half worth with respect to agent 1. On the horizontal axis I put the degree of returns to scale for agent 1 (namely α) whereas on the vertical axis I put the degree or returns to scale for agent (namely β). For sake of clarity I also draw the solid lines denoting $\alpha = 1$ and

$\beta = 1$ respectively. Using (17.1) and (17.2) as strict equalities it is possible to plot the curves depicted denoting respectively the negotiating condition for agent 1 and agent 2. In particular, the left-hand curve (***a1a1***) indicates the negotiating condition for agent 1, whereas the curve ***a2a2*** denotes the negotiating condition for agent 2. All the points below these curves satisfy inequalities (17.1) and (17.2) respectively.

For sake of clarity, let me use some capital letters to indicate different areas in the plot. Inequality (17.1) for agent 1 is satisfied for all the points below the curve ***a1a1*** contained in the areas C1, C, B, D3, A and A1. At the same time inequality (17.2) is satisfied for all the points below the curve ***a2a2*** contained in the areas D1,C1,C, B, A and A1. Note that the area denoted by D2 does not satisfy neither the first nor the second inequality.

The PSR is therefore given by A+B+C+C1+A1. The point of interest is that a PSR does exist at different returns to scale. In area C, for instance, agent 1 does exhibit decreasing returns to scale whilst agent 2 does exhibit increasing returns to scale. In area B both opponents exhibit decreasing returns to scale. In the area given by the sum C1+A1 both parties exhibit increasing returns to scale.

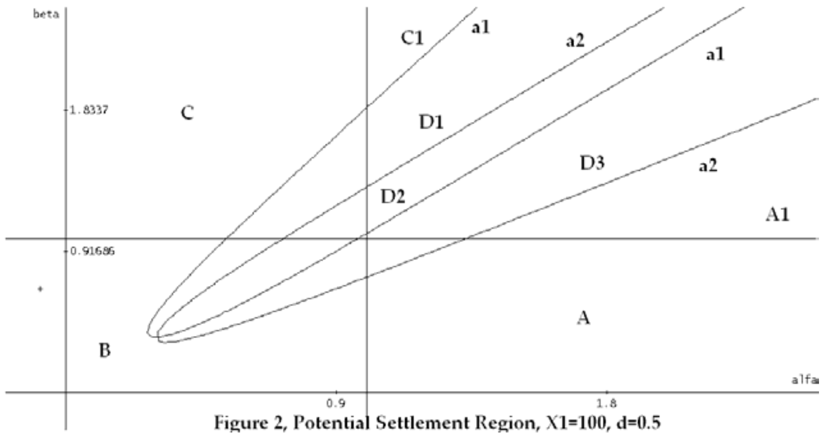

Figure 2, Potential Settlement Region, X1=100, d=0.5

Albeit based on arbitrary values, the plot throws light on the particular relationship between the different degree of productivities. In area C, when agent 2 exhibits increasing returns to scale and agent 1 displays decreasing returns, there is a large room for negotiation. Note that such a room for negotiation does exist if the degree for agent 2 is not too close to unity or when the degree for agent 2 is sufficiently high (see the upper-left quadrangle). Area A also shows a large room for negotiating. In such a case the agent with the higher evaluation of the stake of the conflict also exhibits increasing returns to scale.

The latter considerations lead to the preliminary result:

Result 1 *An asymmetry in degree of the aggregate returns to scale of the opponents modifies the incentives to conflict and to negotiate in favour of negotiating.*

This result is confirmed even with different numerical values. Figure 3 and figure 4 below plot the curves and the PSR with $\delta = 0.9$ and $\delta = 0.25$ respectively. Different values for the degree in asymmetry in evaluations of the stake of the conflict does not affect significantly the foregoing result. More precisely, a different degree of asymmetry does not change the relationship between the asymmetry in returns to scale and the willingness to negotiate. The higher such asymmetry, the greater the willingness to negotiate of both parties. The point of interest in this case is that the degree of asymmetry in evaluation also affects significantly the size and the shape of the PSR.

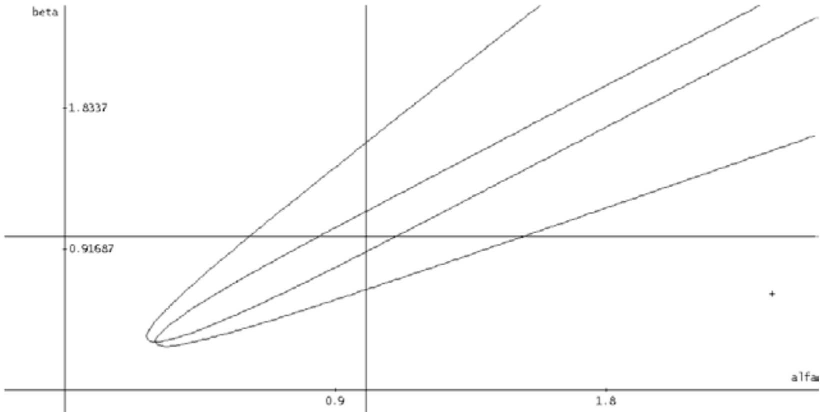

Figure 3, Potential Settlement Region, X1=100, d=0.9

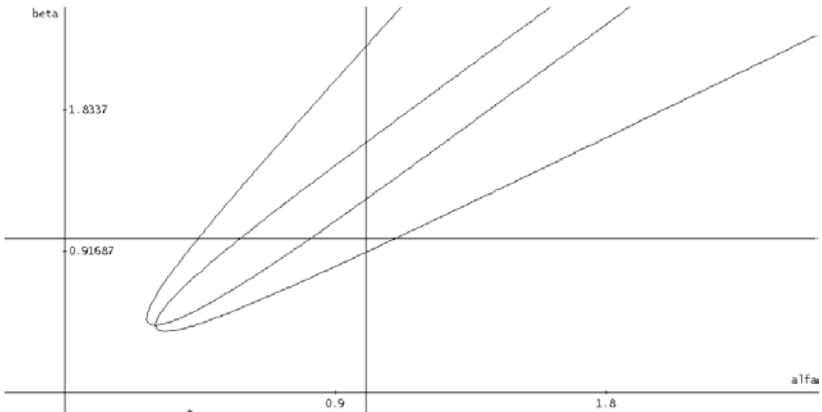

Figure 4, Potential Settlement Region, X1=100, d=0.25

IV. The impact of an Integrative Grant.

In conflict interactions there are often specific measures applied in order to strengthen the impact of efforts devoted to the resolution of conflict. In fact, negotiating does not imply necessarily a resolution. The existence of a PSR does not lead to an agreement according a deterministic cooperative solution. Of course, the larger is the PSR, the higher will be the probability of a peaceful settlement. That is, parties can have an interest in enlarging the PSR. The larger the PSR, the higher is the probability to reach an agreement. Hereafter, I intend to analyse the impact on PSR of an **integrative grant**. The concept of integrative grant I apply does partially fit the idea of integrative as developed and expounded by Boulding (1973). An integrative relationship is characterised by the existence of grants. According to this approach, a grant is supposed to be an unilateral transfer from an individual, a group or a social unit to another. Whenever it takes place, the agent does not receive anything in return.

Now consider that within the framework of this work the agent with a higher evaluation of the stake, say agent 1, decides to grant positively the opponent. This is supposed to modify the incentives to negotiation of both agents. In such a view, it might happen that agent 1 is willing to make some concessions to the opponent. This behaviour is intended to enlarge the range of negotiation, namely the PSR. Moreover, through such integrative approach, each agent can signal to the contender its willingness to settle. This is supposed to modify the incentives to negotiation for both agents. Thus, through a positive transfer, agent 1 commits itself to influence the opponent's behaviour. This influence is not provided by means of coercion but of an integrative approach. Essential to understand the impact of an integrative grant is the awareness that agents do not give up their rational and maximizing behaviour. They are still utility-maximizers and behave simultaneously à la Nash-Cournot.

There might be several interpretations about this integrative grant. In international relations, for instance, it might be a positive monetary aid flowing from one country to another. In a conflict over a territorial dispute it could be a voluntary concession of a fraction of the contested land to the other party. In addition such kind of grant can take the shape of a process of information sharing which is able to make able agent 2 to better evaluate the stake of the conflict.

Then, suppose that such an integrative grant is worth a fraction of the stake of the conflict. In particular, let $\sigma \in (0,1-\delta)$ denote the fraction of the stake of the conflict agent 1 chooses to grant agent 2. That is, the maximum grant agent can provide is given by the difference between the degree of asymmetry in evaluation and the unity (namely, the full evaluation). The payoff functions become for agent 1 and agent 2 respectively:

$$\pi_1^T = \frac{z_1(h_1+1)}{z_1(h_1+1)+z_2(h_2+1)} X_1^{\alpha} - z_1 - h_1 - \sigma X_1 \qquad (18)$$

$$\pi_2^T = \sigma X_1 + \frac{z_2(h_2+1)}{z_1(h_1+1)+z_2(h_2+1)} X_2^{\beta} - z_2 - h_2 \qquad (19)$$

The optimal level for both violent and negotiating efforts are exactly as in (14), whilst the equilibrium payoffs are given by:

$$\pi_1^{T*} = \frac{X_1^{5\alpha} - X_1^{3\alpha} X_2^{2\beta} + (1-\sigma X_1)(X_1^{2\alpha} + X_2^{2\beta})^2}{(X_1^{2\alpha} + X_2^{2\beta})^2}$$

(20.1)

$$\pi_2^{T*} = \frac{X_1^{4\alpha}(1+\sigma X_1) + X_1^{2\alpha} X_2^{2\beta}\left[2(1+\sigma X_1) - X_2^{\beta}\right] + X_2^{4\beta}\left(X_2^{\beta} + \sigma X_1 + 1\right)}{(X_1^{2\alpha} + X_2^{2\beta})^2}$$

(20.2)

Then, for sake of simplicity consider now the case of $\sigma = (1-\delta)/2$. That is, the agent with a higher evaluation of the stake of the conflict gives up a fraction worth the half of the asymmetry in evaluations.

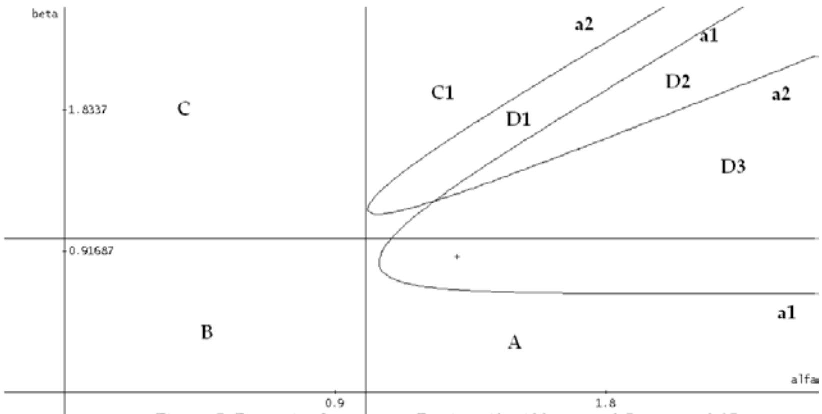

Figure 5, Potential Settlement Region, X1=100, delta=0.5, sigma=0.25

In figure 5 it is clear that the existence of the integrative grant changes the shape of the PSR. In particular, given the arbitrary values imposed, the PSR moves right-ward and no longer falls in the diminishing returns area. Recall the example of a contested fishery. Suppose it does exhibit diminishing return for both rival parties. The agent with the higher evaluation can voluntary grant the opponent with a fixed quota of fishing rights trying to influence the opponent's behaviour towards negotiation.

Preliminary Result 2: *when the agent with a higher evaluation of the stake of the conflict provides the opponent with a positive integrative grant, the PSR enlarges, that is both parties have a higher willingness to settle. In particular, the effect of the integrative grant emerges when (i) they both exhibit decreasing returns to scale; (ii) the grantee exhibits increasing returns to scale whilst the grantor exhibits decreasing returns to scale.*

The point of interest in such a case is two-fold. First, an integrative grant enlarges the PSR. Secondly, although it appears with the negative sign in the payoff function of agent 1, the final payoff of the agent is higher than the pure violence option.

Returning to the basic hypothesis of this paper, this would also mean that the positive output of negotiating reinforces the effect of the first instrument, say violence. However, the result is still ambiguous when agent 1 exhibits increasing returns to scale. In particular when both agents exhibit increasing returns of scale, the result is different with respect to the case plot in figure 2. In figure 5, in fact, it appears clear that there is no settlement region when the degree of returns to scale of agent 1 is relatively high. Moreover, note that differently from areas B and C, agents cannot exhibit an equal degree of aggregate returns. Area C1, which is contained in the PSR, is characterised by both conditions. In area C1 agents differ in degree of increasing returns to scale and at the same time the agent with the lower evaluation has a higher degree. By contrast, when agent 1 exhibits increasing returns to scale at a higher degree than agent 2 there is no possible settlement. In particular, consider that the area A1 which was in figure 2 totally disappear.

The shrinkage of PSR is confirmed if I attach different values to the parameters. Consider the case of $\delta = 0.9$ and $\sigma = 0.05$. Figure 6 plots the PSR under these arbitrary values. In such a case, the difference in evaluation of the contested stake is relatively small.

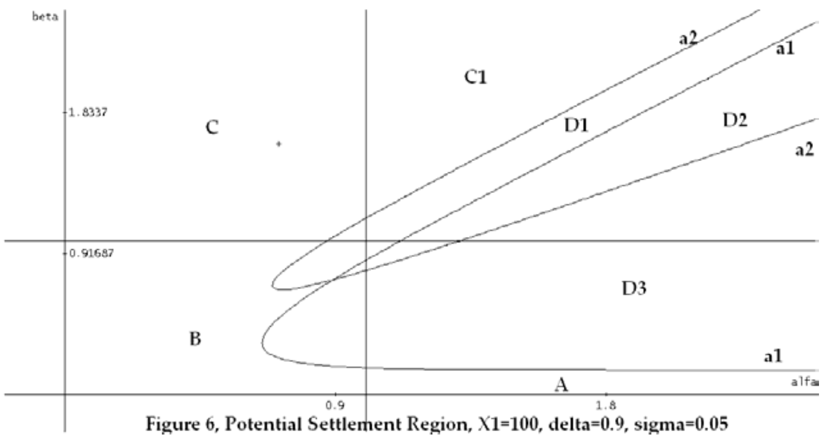

Figure 6, Potential Settlement Region, X1=100, delta=0.9, sigma=0.05

The foregoing notes lead to the following proposition:

Preliminary Result 3: *when the agent with a higher evaluation of the stake of the conflict exhibits increasing returns to scale, the PSR is larger when the other agent also exhibits increasing returns to scale at a higher degree. This kind of impact is more evident when the difference in evaluation of the stake of the conflict is relatively small.*

CONCLUSION

Albeit preliminary, the analysis already appears to be interesting. In some sense, this appears to be a worst-case scenario. The first option for both agents is violence, whilst negotiation appears to be only a complementary instrument. Another point of interest is the existence of returns to scale. Although modelled in a very simple form, the existence of returns to scale strongly affects the size and the shape of a PSR. The first result suggests that an asymmetry in degree of the aggregate returns to scale of the opponents modifies the incentives to conflict and to negotiate in favour of negotiating. By contrast, a similarity in productivity does not allow for negotiation.

It is noteworthy that there is no destruction parameter as commonly used in the economic literature on conflict. In such a case, the opportunity cost of conflict partly disappears and the strategy set is clearly biased towards conflict.

Eventually, the existence of an integrative grant provided by the agent with a higher evaluation of the stake of the conflict partly seems to pave the way for the resolution of the conflict. However, this still leads to ambiguous results. In particular, the effect of the integrative grant emerges when (i) they both exhibit decreasing returns to scale; (ii) the grantee exhibits increasing returns to scale whilst the grantor exhibits decreasing returns to scale. When the agent with a higher evaluation of the stake of the conflict exhibits increasing returns to scale, the PSR is larger when the other agent also exhibits increasing returns to scale at a higher degree. This kind of impact is more evident when the difference in evaluation of the stake of the conflict is relatively small. Otherwise, there is no room for negotiating.

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Appendix

First Order Conditions

II. The Second Instrument

$$\frac{\partial \pi_i}{\partial z_i} = \frac{X_i z_j (h_i + 1)(h_j + 1)}{(h_i z_i + h_j z_j + z_i + z_j)^2} - 1 = 0$$

$$\frac{\partial \pi_i}{\partial h_i} = \frac{X_i z_i z_j (h_j + 1)}{(h_i z_i + h_j z_j + z_i + z_j)} - 1 = 0$$

$i = 1,2, i \neq j$

III. Returns to Scale and Incentives to Conflict

$$\frac{\partial \pi_1}{\partial z_1} = \frac{z_2^2 X_1^\alpha}{(z_1 + z_2)^2} - 1 = 0$$

$$\frac{\partial \pi_2}{\partial z_2} = \frac{z_1^2 X_2^\beta}{(z_1 + z_2)^2} - 1 = 0$$

$$\frac{\partial \pi_1}{\partial z_1} = \frac{z_2 X_1^\alpha (h_1 + 1)(h_2 + 1)}{(h_1 z_1 + h_2 z_2 + z_1 + z_2)^2} - 1 = 0$$

$$\frac{\partial \pi_1}{\partial h_1} = \frac{z_1 z_2 X_1^\alpha (h_2 + 1)}{(h_1 z_1 + h_2 z_2 + z_1 + z_2)^2} - 1 = 0$$

$$\frac{\partial \pi_2}{\partial z_2} = \frac{z_1 X_2^\beta (h_1 + 1)(h_2 + 1)}{(h_1 z_1 + h_2 z_2 + z_1 + z_2)^2} - 1 = 0$$

$$\frac{\partial \pi_2}{\partial h_2} = \frac{z_1 z_2 X_2^\beta (h_1 + 1)}{(h_1 z_1 + h_2 z_2 + z_1 + z_2)^2} - 1 = 0$$