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ENVIRONMENTAL DECISION MODELS WITH JOINT OUTPUTS

by

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Frequently undesirable outputs are produced together with desirable. This joint production of bad and good outputs in connection with environmental decision models is the subject of this paper. Recently Haynes, Ratick, Bowen and Cummings-Saxton (1993) addressed the same subject matter. They observed that: "... input resources, including the chemicals of concern, produce as joint products the saleable product and the undesired environmental residuals." (p. 271) However, rather than model the undesirable residuals as output, they view them as inputs in their DEA (= Data Envelopment Analysis) model.

Building on earlier work by Shephard and Färe (1974), we show how the jointness between desirable and undesirable outputs can be incorporated into environmental decision models. We follow Färe, Grosskopf and Pasurka (1986) (1989) and Färe, Grosskopf, Lovell and Pasurka (1989), who also discuss various approaches to modeling pollution and environmental regulation.

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[1]This terminology was introduced by Charnes, Cooper and Rhodes (1978).

This paper expands on the new approach "to monitoring of industrial pollution abatement" (p. 261) introduced by Haynes, Ratick, Bowen and Cummings-Saxton (1993). This generalized model is discussed in Section 1. Section 2 uses these approaches to evaluate pollution prevention.

1. Production Models

Here we present the production models that will be used in Section 2 for evaluating pollution prevention. We start by studying some theoretical concepts and then show how these can be incorporated into the DEA or activity analysis model discussed by Haynes, Ratick, Bowen and Cummings-Saxton (1993).

We distinguish between good or desirable and bad or undesirable outputs. The former are denoted by $y = (y_1, \ldots, y_M) \in \Re_+^M$ and the latter by $z = (z_1, \ldots, z_L) \in \Re_+^L$. These outputs are produced by the input vector $x = (x_1, \ldots, x_N) \in \Re_+^N$. The technology is represented by the output sets

(1) $$P(x) = \{(y,z) : x \text{ can produce } (y,z)\},\ x \in \Re_+^N.$$

In order to model jointness between the desirable and undesirable outputs, we adopt the notion of nulljointness introduced by Shephard and Färe (1974). They say that the subvector y is nulljoint with z if for all $(y,z) \in P(x)$ and $x \in \Re_+^N$, $z=0$ implies $y=0$. In words, if there is no production of undesirable outputs then there can be no desirable outputs. Or put differently, to produce good outputs one will have to produce some bad outputs.

Prior to illustrating an output set with one output nulljoint to another, we may benefit from defining weak and (subvector) strong output disposability. Following Shephard (1970), we say that outputs are weakly disposable if

(1.2) $$(y,z) \in P(x) \text{ and } 0 \le \theta \le 1,\ (\theta y, \theta z) \in P(x),\ x \in \Re_+^N.$$

Weak disposability allows proportional reduction of all outputs, good and bad. However, nonproportional reduction of outputs may not be feasible. To allow for nonproportional reduction in outputs, one needs to introduce the idea of free or strong disposability, i.e., outputs are strongly disposable if

(1.3) $$(y,z) \in P(x) \text{ and } (\bar{y},\bar{z}) \le (y,z) \text{ then } (\bar{y},\bar{z}) \in P(x),\ x \in \Re_+^N.$$

In this paper, we make the assumption that only the desirable outputs are strongly disposable, i.e., if

(1.4) $$(y,z) \in P(x) \text{ and } \bar{y} \le y \text{ then } (\bar{y},z) \in P(x),\ x \in \Re_+^N.$$

In this case, the desirable outputs y can be disposed of without any resource cost, i.e., disposal is "free".

In order to illustrate an output set P(x) that satisfies the two disposability assumptions (1.2) and (1.4) and which has a desirable output nulljoint with an undesirable output, suppose only two outputs are produced using an input vector $x \in \Re^N_+$.

Figure 1. Disposability and Nulljointness

The output set P(x) in Figure 1 is bounded by (0abcd0). It satisfies weak disposability. This is illustrated by the proportional contraction of (y,z). Moreover, the desirable output y is freely disposable, but the undesirable output z is not. In addition, Figure 1 exemplifies that y is nulljoint with z, since if z=0 the only feasible y is y=0. Clearly if all outputs y and z were freely disposable then y would not be nulljoint with z. We note that the DEA model discussed by Haynes et. al., (1993, p. 272) satisfies (1.3), see Charnes, Cooper and Rhodes (1978). This in turn implies that it does not allow for nulljointness of outputs.

To formulate a production model that has good outputs nulljoint with bads and that satisfies the disposability assumptions (1.2) and (1.4). We assume that there are k=1, ..., K observations. We denote goods, bads and inputs by y_{km}, z_{kl} and x_{kn}, m=1, ... M, l=1, ..., L and n=1, ..., N, respectively. The piecewise linear output set may be written as

(1.5) $$P(x) = \{(y,z) : \sum_{k=1}^{K} \lambda_k y_{km} \geq y_m, m=1, ..., M,$$

$$\sum_{k=1}^{K} \lambda_k z_{kl} = z_l, \ l=1, ..., L,$$

$$\sum_{k=1}^{K} \lambda_k x_{kn} \le x_n, \ n=1, ..., N,$$

$$\lambda_k \ge 0, \ k=1, ..., K\}.$$

We note that (1.5) satisfies Constant Return to Scale (CRS) i.e., $P(\mu x) = \mu P(x), \mu \ge 0$. To see that it satisfies weak disposability of outputs assume that $(y^o, z^o) \in P(x)$. Then there are $\lambda_k^o \ge 0$ such that

$$\sum_{k=1}^{K} \lambda_k^o y_{km} \ge y_m^o, \ m=1, ..., M$$

$$\sum_{k=1}^{K} \lambda_k^o z_{kl} = z_l^o, \ l=1, ..., L,$$

$$\sum_{k=1}^{K} \lambda_k^o x_{kn} \le x_n, \ n=1, ..., N.$$

Now define $\tilde{\lambda}_k = \theta \lambda_k^o, 0 \le \theta \le 1$, then $\tilde{\lambda}_k \ge 0$, k=1, ..., K, and

$$\sum_{k=1}^{K} \tilde{\lambda}_k y_{km} \ge \theta y_m^o, \ m=1, ..., M,$$

$$\sum_{k=1}^{K} \tilde{\lambda}_k z_{kl} = \theta z_l^o, \ l=1, ..., L,$$

$$\sum_{k=1}^{K} \tilde{\lambda}_k z_{kn} \le \theta x_n, \ n=1, ..., N,$$

proving our claim. It follows from the m=1, ..., M inequalities that P(x) satisfies (1.4) for the desirable outputs.

To model nulljointness between good and bad outputs, we essentially have to assume that no good output can be produced without some bad output. This assumption can be associated with the observed outputs by:

(1.6) For any k, if $y_{km} > 0$ for some m, then there is $z_{kl} > 0$, for some l.

This assumption states that if one observation k produces some positive amount of a desirable output it will also produce some undesirable output. To show that (1.6) implies nulljointness, suppose $z_l = 0$, l=1, ..., L. Then in the piecewise linear specification of technology in (1.5), we have $\lambda_k = 0$ for all k such that $z_{kl} > 0$, for the equality to hold in (1.5). Now if $y_m > 0$ for some m, then some $\lambda_k > 0$, for the associated inequality to hold in (1.5). This contradiction proves that (1.6) is sufficient for nulljointness between the good and the bad outputs in our activity analysis model.

Finally, we recall that the output sets associated with the DEA model (5) in Haynes, et. al. (1993) can be written as:[2]

$$(1.7) \qquad P^{CCR}(x) = \{(y,z) : \sum_{k=1}^{K} \lambda_k y_{km} \geq y_m, \ m=1, ..., M,$$

$$\sum_{k=1}^{K} \lambda_k z_{kl} \geq z_l, \ l=1, ..., L,$$

$$\sum_{k=1}^{K} \lambda_k x_{kn} \leq x_n, \ n=1, ..., N,$$

$$\lambda_k \geq 0, \ k=1, ..., K\}.$$

The difference between (1.7) and (1.5) is that (1.7) allows the bads to be freely disposable, (the equality is replaced with an inequality in the z constraints) which is not the case in (1.5).

[2]This follows from Charnes, Cooper and Rhodes (1978).

2. Measures of Pollution Prevention

Different measures of pollution prevention are discussed next. Färe, Grosskopf and Pasurka (1986)[3] measured the impact of pollution prevention as the ratios of efficiency relative to the model (1.5) and to a model allowing free disposability of both good and bad outputs. The following diagram explains.

Figure 2

The output set P(x) in Figure 2 is bounded by (0abcd0). If one would allow the bad output to be freely disposable, then the output set would be bounded by (0ebcd0). Färe, Grosskopf and Pasurka (1986) computed the radial distances 0A and 0B using linear programming methods. They used the ratio

(2.1) 0B/0A

to measure the impact of pollution prevention or environmental regulations in terms of proportion of output foregone. The more stringent the regulation, the larger one would expect the ratio (2.1) to be. That is, by measuring the deviation between the weak and strong disposable technologies, (0B/0A) "picks up" the impact of regulation measured at the observed mix of goods and bads.

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[3] Färe, Grosskopf and Pasurka (1989) use an input oriented measure which does not apply here.

A nonradial hyperbolic measure was introduced by Färe, Grosskopf, Lovell and Pasurka (1989) as an alternative to the radial approach discussed above. This measure applied a common scalar to expand the desirable and contract the undesirable outputs. The hyperbolic path allows for explicitly contraction of the bad and expansion of the good outputs. It does this by abandoning the connection with the output distance function (Shephard, 1970) and hence to the duality between the revenue function and technology.

Haynes, et. al. (1993, p. 273) suggest two measures of pollution prevention, (i) "the amount of chemical waste per unit of chemical input to the process ..." and (ii) "... the amount of chemical input used to produce the products ..." Both measures can be seen in terms of (1.7). The first measure is the maximal expansion of chemicals used given the related chemical wastes. Thus if y in (1.7) is the vector of the chemicals used and x is the vector of chemical wastes, then for observation k' their measure is

(2.2) $$H(x^{k'},y^{k'}) = \max \theta$$

$$\text{s.t.} \sum_{k=1}^{K} \lambda_k y_{km} \geq \theta y_{k'm}, \; m=1, ..., M,$$

$$\sum_{k=1}^{K} \lambda_k x_{kn} \leq x_{k'n}, \; n=1, ..., N,$$

$$z_k \geq 0, \; k=1, ..., K.$$

Here the pollution prevention frontier would be determiend by the observations that had the highest ratio of chemical (y) to chemical waste (x), i.e., those with $\theta = 1$. A drawback here is that other inputs (like labour and capital) and outputs (like good outputs) are not included in the model. Thus (2.2) does not describe the complete production model and may not accurately reflect the resource use required to prevent pollution. Introduction of additional constraints for outputs and inputs could result in a totally different distribution of θ's.

In the second measure, y is the desirable output and x is the vector of chemicals, and the measure for k' is

(2.3) $$G(x^{k'},y^{k'}) = \max \theta$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \lambda_k y_{km} \geq \theta y_{k'm}, \; m=1, \ldots, M,$$

$$\sum_{k=1}^{K} \lambda_k x_{kn} \leq x_{k'n}, \; n=1, \ldots, N,$$

$$z_k \geq 0, \; k=1, \ldots, K.$$

Here the pollution prevention frontier is determined by the firms with the highest ratio of good output to total chemical input, and waste is ignored. Other inputs are excluded, as in the previous model.

It is clear that Haynes et al. exclude other inputs and outputs because the data are not readily available. However, their approach also does not account for nulljointness or weak disposability. It also does not explicitly model pollution abatement as maximum feasible reduction in undesirable outputs.

In order to avoid some of these shortcomings, we suggest a measure that maximizes each desirable output and at the same time minimizes each undesirable output and also accounts for all inputs. Such a measure can be written for k' as

(2.4) $$E(x^{k'}, z^{k'}, y^{k'}) =$$

$$\text{s.t.} \quad \theta_m y_{k'm} \leq \sum_{k=1}^{K} \lambda_k y_{km}, \; m=1, \ldots, M,$$

$$\mu_l z_{k'l} = \sum_{k=1}^{K} \lambda_k z_{kl}, \; l=1, \ldots, L,$$

$$\sum_{k=1}^{K} \lambda_k x_{kn} \leq x_{k'n}, \; n=1, \ldots, N,$$

$$\lambda_k \geq 0, \; k=1, \ldots, K.$$

here $\tilde{K}$ denotes the number of positive $y_{k'm}$, and K denotes the number of positive z_{kl}. (We are assuming the positive valued observations are ordered first.) Here we would include chemical wastes as our measures of jointly produced undesirable outputs (the z's), and total chemicals used as the inputs (x's) employed to produce both desirable (y) and undesirable (z) outputs.

This measure is nonradial, however, unlike the hyperbolic measure used in FGLP (1986), this measure does not require that all goods (bads) be increased (decreased) by the same proportion. Rather, like the Russell measure introduced in Färe and Lovell (1978), the individual goods (bads) can be increased (decreased) at different rates, providing more flexibility.

To sum up, we believe that frontier approaches (like DEA) are an appropriate way to think about something like a "pollution prevention frontier." Haynes et al. have provided a point of departure. We suggest augmenting their model to:

1. explicitly account for the basic pollution problem, namely that effluents are byproducts or (null)jointly produced with desirable outputs, using a variety of inputs,
2. model the restrictions imposed by regulation on the free disposability of (bad) outputs,
3. explicitly seek reductions in "bads" and enhancements in goods,
4. allow the aforementioned reductions/endorsements to vary across affluent type and type of desirable output.

References

Charnes, A., Cooper, W.W., Rhodes, E. "Measuring efficiency of decisionmaking units," *European Journal of Operational Research* 2(4), 429-444, 1978.

Färe, R., Grosskopf, P., Lovell, C.A., Pasurka, C. "Multilateral productivity comparisons when some outputs are undesirable: A nonparametric approach," *The Review of Economics and Statistics* LXXI(1), 90-98, 1989.

Färe, R., Grosskopf, P., Pasurka, C. "Effects on relative efficiency in electric power generation due to environmental controls," *Resources and Energy* (8), 167-184, 1986.

Färe, R., Grosskopf, P., Pasurka, C. "The effect of environmental regulations on the efficiency of electric utilities: 1969 vs. 1975," *Applied Economics* (21), 225-235, 1989.

Färe, R., Lovell, C.A.K. "Measuring the technical efficiency of production," *Journal of Economic Theory* 19(1), 150-162, 1978.

Haynes, K., Ratick, S., Bowen, W., Cummings-Saxton, J. "Environmental decision models: U.S. experience and a new approach to pollution management," *Environmental International* (19), 261-275, 1993.

Shephard, R.W. *Theory of Cost and Production Functions*, Princeton University Press, Princeton, 1970.

Shephard, R.W., Färe, R. "The law of diminishing returns," *Zeitschrift für Nationalökonomie* (34), 69-90, 1974.