Resolving the Identification Problem in Linear Social Interactions Models: Modeling with Between-Group Spillovers

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Abstract

The linear-in-means model has been a theoretical and empirical workhorse of the social interactions field. As was noted by Manski (1993), the collinearity between group-level "contextual" and "endogenous" effects leads to an inability to identify the structural parameters of this model. Manski called this the "reflection" problem. This paper suggests that Manksi's reflection problem is unique to a special case of a more general context in which agents care about multiple reference groups. Specifically, the identification problem is resolved through a model generalization to include between-group and within-group effects.

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"Functional relations between groups which are of consequence to the groups in question will tend to bring about changes in the pattern of relations within the in-groups involved"

"The course of relations between two groups ... in a state of competition and frustration will tend to produce an increase in in-group solidarity."

"From a theoretical point of view... events occurring between groups have consequences at both a group level (norms relating to the out-group) and at a psychological level (formation of negative attitudes toward the out-group)"

M Sherif, et al. (1961) Robbers Cave Experiment

1 Introduction

The purpose of this paper is to discuss a method for identifying endogenous social effects. These effects, where an individual is presumed to respond to the norms or expectations of behavior of his or her reference groups, are increasingly used by economists to explain phenomena not readily explained through pure individual effects. As in economic studies that focus on purely individual effects, many economists rely on the use of linear regression models to explain social effects. The most common formulation of such a model is the "linear-in-means" model, so dubbed because agents are assumed to include the mean behavior of their reference group in their utility function. Both the linear structure and the simple interpretation of the coefficient of this mean-behavior term (as the degree to which agents respond to social norms) lead to frequent use of this model.

An additional advantage of the model is that the three types of social interactions outlined by Manski (1993) can be simply encompassed. The first of these three types are endogenous effects, where an individual's actions vary with the expectation of those of his/her reference group. The second are contextual (exogenous) effects, where individual behavior varies with (exogenous) observed mean characteristics of a group. The final effects are correlated effects, where individuals act similarly due to shared institutional or individual characteristics.

In a linear model, any or all of these effects can be represented as additively separable components of an agents' utility function. Endogenous effects are of particular interest to researchers since they are a potential explanation of social multipliers (c.f. Durlauf 2004).

The difficulty that presents itself was noted in Manski's seminal 1993 paper: group-level "contextual" and "endogenous" effects are empirically indistinguishable – leading to an inability to identify the structural parameters of this model. Manski called this the "reflection" problem. As many researchers have discovered, the reflection problem makes many problems difficult or impossible to solve. A classic question in this literature addresses how students are affected by the actions of their peers. The reflection problem presents itself in that a researcher will not be able to distinguish between the influence of the mean behavior of other students in the class and the influence of a shared environment (such as the quality of classroom materials or teacher). As will be illustrated below, in a linear model, these effects are econometrically indistinguishable. To date, the two principle methods for answering this type of problem to assume either the ability to distinguish between endogenous and contextual effects due to some characteristic feature of the problem, or to introduce some type of non-linearity into the model. Doing so runs the obvious risk that inference is inaccurate both from the assumption itself and from ignoring the effect of students' reactions to other groups.

However, it is found that this "reflection" problem is unique to a limited special case of a more general context. That is, the inability to identify endogenous social interactions only exists when agents care exclusively about their own reference group. As the problem is generalized to allow agents to consider the behavior of other groups, it is found that inference is possible with only minor conditions. That is, once we consider how students react to the actions and context of students in the next classroom, we can isolate the two previously collinear effects. Specifically, the identification problem is resolved through a model generalization to include between-group effects (agents' concern for the behavior in other groups) in addition to within-group effects. Agents respond distinctly to others' actions within their own reference group as well as to those within other groups. The reflection problem manifests when agents are restricted to consider only the actions of a single group.

The work of justifying the inclusion of social effects into utility functions has been done thoroughly; however, a brief comment is due. The precedent for including social effects is long-standing, particularly in the sociology literature. This study incorporates mechanisms by which people respond to reference groups outside of their own. A famous example is Sherif's 1954 Robbers Cave experiment (c.f. Sherif, et al. (1961)). Sherif and his colleagues illustrated that groups of teenage boys with no prior connection could form both in and out-group stereotypes based solely on the researchers' experimental set-up. One of the implications is that people behave in part based on their beliefs about their own group and about other groups.

Similar research has increasingly been accepted in the economics field. Though long a subject of debate, a rising number of economics studies have now found evidence that social interactions exist within a person's reference group. A few well-known examples are Case & Katz (1991), Ioannides (2002), and Ioannides & Zabel (2002,2003). Note that, of these, only Ioannides & Zabel (2003) seek to distinguish endogenous effects from contextual ones (see Durlauf (2004) for a full review). And though the existence of multiple reference groups and between-group effects has been acknowledged, it is not been explored much empirically or theoretically. A partial exception to this is a recent paper (Iyer & Weeks 2004) that evaluates two dimensions of endogenous interactions in the fertility decisions of a group of women in Kenya. This first effort to expand the domain of interactions analyzes Kenyan women's reactions to their local village cluster as well as their ethnic group as a whole. Iyer and Weeks, however, do not provide additional insight into the resolution of the reflection problem. They use the reasonable assumption of Poisson non-linearity to describe fertility – thus allowing identification. Their paper is useful both in its thoughtful use of data and in that it discusses some additional rationale for the use of multiple social interactions. The study of these effects is important not simply for their ability to resolve the identification problem, but because they could provide further insight into the existence of social motivations for agent behavior that are not revealed in the context of single-effect models.

As a further example, a multiple-interactions models suggests that a teenager might care differently about what other young women do than about what young men do – the assumption is that the boys and girls are distinct reference groups. For example, a girl might be more prone to smoke if girls in her school do, but less likely to if the boys do so. Or she might care being more inclined to smoke based on the girls actions in her school, but less likely to based on those in other schools. The vast majority of existing models impose a simpler framework of decision making in which the girl's decision must be based on a single group (girls, school) or on a single metric of distance (distance from own school).

A third example of the relevance of multiple interactions is the case of ethnic conflict. Evaluating agents' behavior in the context of complex multi-ethnic environments requires an understanding of between-group perceptions. That is, one would have trouble evaluating Hutu-Tutsi conflict with endogenous effects constrained to respond only to the actions of an individual's own group.[1] Consider a single-group model to predict the occurrence of religious violence. A single-group model would suggest increases in violence by Hutus are a function of expectations of similar behavior or perceptions amongst this same group. It would, however, ignore the role that expectations of Tutsi behavior play in inciting this violence since the models by definition are focused on a single group.

As a note, a single-effect model would allow agents to consider actions by both groups if they were to be placed along some measure of distance. However, it nonetheless requires that this distance metric be defined and, that it constrains the responsiveness to be constant along this metric.

This paper's method of resolving the identification problem is to expand the rank of the regressors used by including beliefs about out-group effects. By including an agent's belief about other group's actions as well as observables

[1] The Hutus and Tutsi are the predomint ethnic groups in Rwanda, Burundi and Eastern Congo. All three areas have been riven with ethnic-based conflict for many years.

for each group, the model can be fully identified under the condition that the matrix of observable parameters is of sufficiently high rank. This increase in dimensionality allows one to distinguish between contextual and endogenous effects by utilizing variation in between-group contextual effects. For example, by incorporating a Hindu agent's reaction to both Hindu *and* Muslim contexts and behavior, one gains increasing degrees of freedom. With sufficient information on these contextual effects, one can disentangle the desired endogenous effects. When the model is collapsed into a single group, the between-group variation collapses to a single factor and can no longer be distinguished. Specifically, identification requires that the dimension of observable linearly-independent contextual effects be greater than or equal to the number of distinct groups being evaluated.

2 Background

Linear models have a long tradition in economics due to their simplicity of use and interpretation; these features are true for the linear-in-means model of social interactions as well. They have been used by a wide range of researchers in the search for evidence of social drivers of individual action. Some recent papers have explicitly distinguished between endogenous and contextual effects within a linear model. Ioannides (2002) uses an example in which contextual effects can reasonably be considered to be non-present, and Ioannides & Zabel (2003) use a case where they consider actual, rather than expected, behavior to be a measure of endogenous actions. Both of these side-step the identification problem and allow interpretation of results.

Recent work by Brock & Durlauf (2001a,b) and Glaeser, Sacerdone, & Scheinkman (1996) has introduced statistical mechanics models from physics to explain discrete choice models of social interactions. Discrete choice models are one of many possible non-linear models of interactions, and are intuitively appealing since many of the agent choices studied to date are binary ones (e.g. attend college, take drugs, join a neighborhood). Similar to Iyer and Weeks' use of a Poisson model, these models, though analytically more complex, benefit from non-linearity in that they avoid problems of multi-collinearity central to the identification failure of linear models.

These strands of research model reference groups within a single neighborhood. That is, once a particular driver of social action has been posited, its effect is a single function of social space between the agent and other agents in the designated space. Models differ in their definition of the function and in the relevant space, but all of them assume a single dimension. To make this more concrete, a model might assume that teenagers base their decision to smoke on whether other teenagers do likewise. A "local" model (c.f. Glaeser, Sacerdone, & Scheinkman (1996)) would specify that the agent cares only about the action of the individual "closest" to her in some notion of space. A "global" model would assume that the agent cares about all agents in her reference group. The degree to which she cares might be a function of the "distance" within this group, and the reference group might vary in size.

The goal of this paper is to allow general usage of the linear-in-means model without the need to resort to the creative solutions used to date. In the past, one needed to fit agents into a single metric of distance, find a appropriate non-linearity, or find solutions that allow exclusion restrictions on the basic linear model.

3 The Expanded Linear-in-Means Model

We assume in this paper that agents care about their own reference groups in the "global" sense defined above. In addition, they care about the actions taken in other reference groups - perhaps many of them. We further assume that contextual effects are measured on a relative basis. That is, when considering how to react to exogenous mean characteristics of a group, agents react to their groups' difference from a global mean. Referring to the smoking example, it would be assumed that the girls care about the *difference* in anti-smoking campaigns. That is, since there might be a social status-quo anti-smoking message, the ability of teachers to influence behavior is measured by their ability to differentiate their message from the status-quo. As well, it would be assumed that the girls care about the difference in action from the average behavior of girls in society. If the average behavior is to smoke, then the peer effect of all girls smoking in her own reference group is considered to be irrelevant.

3.1 Theoretical Basis

Beginning with a simple outline of the foundations of the model: Consider Φ individuals divided into J groups of various sizes. Each group has I_j individuals, such that $\Phi = \sum_{j \in J} I_j$. Every individual i in each j makes a choice ω_i^j (taken from the set of possible behaviors Ω_i^j). It is assumed here that all agents choose from the same set of actions. This set of individual-level choices can be aggregated across groups to produce probabilistic descriptions of social behavior. Since agents are influenced by others within their group and those without, define $\omega_{n,-i}^j$ as the vector of choices of agents within group j other than that of i, and $\omega_{n,i}^l$ as the vector of choices of all agents in group $l \neq j$. Durlauf (2004) identifies four basic influences on agent behavior. This project modifies them to incorporate multiple groups and adds a fifth.

1. X_i^j is a vector of deterministic (to the modeler) individual-specific characteristics associated with individual i in group j.

2. ε_i^j is a vector of random individual-specific characteristics associated with individual i in group j.

3. Y_j is a vector of predetermined group-specific characteristics for group j.

4. $\mu_i^e\left(\omega_{n,-i}^j\right)$ is the subjective beliefs individual i possesses about behaviors of others in his group (j), described as a probability measure over those behaviors.

5. (new) $\mu_i^e\left(\omega_{n,i}^j\right)$ is the subjective beliefs individual i possesses about behaviors of agents in group l, described as a probability measure over those behaviors.

Again following the literature (Manski 1993), this project will refer to Y_j as a "contextual" effect ; however, it will relabel $\mu_i^e\left(\omega_{n,-i}^j\right)$ as a "within-group endogenous" effect and dub $\mu_i^e\left(\omega_{n,i}^j\right)$ as a "between-group endogenous" effect.

This paper also assumes that beliefs are rational, leading to a simple modification of the standard as follows:

$$\mu_i^e\left(\omega_{n,-i}^j\right)=\mu\left(\omega_{n,-i}^j|\varepsilon_i^j,Y_j,X_k^j,\mu_k^e\left(\omega_{n,-k}^j\right),\forall k\neq i\in j\right)$$

and

$$\mu_i^e\left(\omega_{n,i}^l\right)=\mu\left(\omega_{n,-i}^l|\varepsilon_i^l,Y_l,X_k^l,\mu_k^e\left(\omega_{n,k}^l\right),\forall k\in j\right)$$

We can then make the standard assumption that individual choices follow:

$$\omega_i^j=\arg\max_{\omega\in\Omega}V\left(\omega_i^j,X_i^j,\varepsilon_i^j,Y_j,\mu_i^e\left(\omega_{n,-i}^j\right),\mu_i^e\left(\omega_{n,i}^{-j}\right)\right) \tag{1}$$

A relevant extension concerns the standard statement of mean group behavior: $\bar{\omega}_{-i}=(I-1)^{-1}\sum_{k\neq i}\omega_k$. Without modification, and in the context of multiple groups, this becomes $\bar{\omega}_{-i}=(I-1)^{-1}\sum_{j\in J}\sum_{k\neq i}\omega_k^j$ and $\frac{\partial\mu_i^e\left(\omega_{n,i}^j\right)}{\partial\omega_{n,i}^{-j}}=0,\forall i,j$; that is, the prototypical model of a single group necessarily assumes that there is no feedback between groups. This paper will specify more generally the vector: $\bar{\omega}_{-i}^j=(I-1)^{-1}\sum_{k\neq i}\omega_k^j,\forall j\in J$. With these in place, equation 1 can be expressed as:

$$\omega_i^j=\arg\max_{\omega\in\Omega}V\left(\omega_i^j,X_i^j,Y_j,\bar{\omega}_{-i}^j,\bar{\omega}_i^{-j}\right)$$

To fix ideas, note that self-selection into groups is ignored.

4 The Econometric Model

Moving to a discussion of econometrics, the expanded linear-in-means model is specified here. The extension to the classic linear model is the addition of between-group effects. As discussed above, agents have rational beliefs about the actions of individuals in their group and in others. They also have knowledge of the characteristics of all individuals, groups and neighborhoods. Explicitly, the extensions are two. One, agents care about the difference in contextual effects between their own group and others, and two, they care about the difference in average action between their own group and others. Note that this does not allow for individuals to have distinct reactions to different out-groups, and is made to simplify calculations below.

Note that for the model to be identifiable, there must be sufficient variation in group-level variables across neighborhoods: that is, girls in each school cannot behave identically in response to observed information.

4.1 Two Groups

Consider first the case of two groups: $1, 2$. In model form, this is expressed as the pair of equations:

$$\begin{aligned} \omega_i^1 &= k_1 + c_1 X_i^1 + d_1 Q_1 + g_1 Z_{1,k} + \alpha_1 m_1 + \varepsilon_i^1 \\ \omega_i^2 &= k_2 + c_2 X_i^2 + d_2 Q_2 + g_2 Z_{2,k} + \alpha_2 m_2 + \varepsilon_i^2 \end{aligned} \tag{2}$$

or

$$\begin{aligned} \omega_i^1 &= k_1 + c_1 X_i^1 + d_1 Q_1 + (g_1 + \alpha_1)\, m_1 + g_1 m_2 + \varepsilon_i^1 \\ \omega_i^2 &= k_2 + c_2 X_i^2 + d_2 Q_2 + (g_2 + \alpha_2)\, m_2 + g_2 m_1 + \varepsilon_i^2 \end{aligned} \tag{2$'$}$$

where k_j, c_j, d_j, g_j, and $\alpha_j, j = 1, 2$ are coefficients, X_i^j is an r-length vector of individual characteristics, Q_j is an s length vector of distinct contextual effect deviations: $Q_j = (Y_j - E_{k \neq j \in J} Y_k)$, Y_j specifies a group j contextual effect, $m_j = E_i \omega_i^j$, and $Z_{j,k} = E_i \omega_i^j - E_{i,k} \omega_i^k, \forall k \neq j \in J$ is a scalar measuring between-group endogenous effects. It should further be noted that to ensure that the models in 2 are valid regression models, the assumption is made that conditional on being part of group j, the errors, ε are independent. Further set $E\left[\mathbf{x}_i^j \varepsilon_i^j\right] = 0, j = 1, 2$, where $\mathbf{x} = \left[k_j, X_i^j, Q_j, Z_{j,k}, m_j\right], j = 1, 2$.

Assuming agents in each group observe rational expectations, and assuming that $g_j + \alpha_j \neq 1, j = 1, 2$, and writing $X_j = E\left[X_i^1 | i \in j\right]$, we have:

$$\begin{aligned} m_1 &= \frac{k_1 + c_1 X_1 + d_1 Q_1 - g_1 m_2}{1 - (g_1 + \alpha_1)} \\ m_2 &= \frac{k_2 + c_2 X_2 + d_2 Q_2 - g_2 m_1}{1 - (g_2 + \alpha_2)} \end{aligned} \tag{3}$$

Expanding produces:

$$
\begin{aligned}
m_1 &= \frac{k_1 + c_1 X_1 + d_1 \left(Y_1 - Y_2\right) - g_1 m_2}{1 - \left(g_1 + \alpha_1\right)} \\
m_2 &= \frac{k_2 + c_2 X_2 + d_2 \left(Y_2 - Y_1\right) - g_2 m_1}{1 - \left(g_2 + \alpha_2\right)}
\end{aligned}
$$

substituting back into (3):

$$
\begin{aligned}
\omega_i^1 &= k_1 + c_1 X_i^1 + d_1 \left(Y_1 - Y_2\right) - g_1 m_2 + \\
&\quad + \left(\alpha_1 + g_1\right) \frac{k_1 + c_1 X_1 + d_1 \left(Y_1 - Y_2\right) - g_1 m_2}{1 - \left(g_1 + \alpha_1\right)} + \varepsilon_i^1 \\
\omega_i^2 &= k_2 + c_2 X_i^2 + d_2 \left(Y_2 - Y_1\right) - g_2 m_1 + \\
&\quad + \left(\alpha_2 + g_2\right) \frac{k_2 + c_2 X_2 + d_2 \left(Y_2 - Y_1\right) - g_2 m_1}{1 - \left(g_2 + \alpha_2\right)} + \varepsilon_i^2
\end{aligned}
$$

Then noting that Y_j is econometrically indistinguishable from m_j, and X_j from Y_j:

$$
\begin{aligned}
\omega_i^1 &= \frac{k_1}{1 - \left(g_1 + \alpha_1\right)} + c_1 X_i^1 + \frac{\left(\alpha_1 + g_1\right)}{1 - \left(g_1 + \alpha_1\right)} c_1 X_1 + \\
&\quad + \frac{d_1 Y_1}{1 - \left(g_1 + \alpha_1\right)} - \left[\frac{\left(d_1 + g_1\right)}{\left(1 - \left(g_1 + \alpha_1\right)\right)}\right] Y_2 + \varepsilon_i^1 \\
\omega_i^2 &= \frac{k_2}{1 - \left(g_2 + \alpha_2\right)} + c_2 X_i^2 + \frac{\left(\alpha_2 + g_2\right)}{1 - \left(g_2 + \alpha_2\right)} c_2 X_2 + \\
&\quad + \frac{d_2 Y_2}{1 - \left(g_2 + \alpha_2\right)} - \left[\frac{\left(d_2 + g_2\right)}{\left(1 - \left(g_2 + \alpha_2\right)\right)}\right] Y_1 + \varepsilon_i^2
\end{aligned} \tag{4}
$$

or

$$
\begin{aligned}
\omega_i^1 &= \frac{k_1}{1 - \left(g_1 + \alpha_1\right)} + c_1 X_i^1 + \frac{\left(\alpha_1 + g_1\right) c_1 + d_1}{1 - \left(g_1 + \alpha_1\right)} Y_1 - \\
&\quad - \left[\frac{\left(d_1 + g_1\right)}{\left(1 - \left(g_1 + \alpha_1\right)\right)}\right] Y_2 + \varepsilon_i^1 \\
\omega_i^2 &= \frac{k_2}{1 - \left(g_2 + \alpha_2\right)} + c_2 X_i^2 + \frac{\left(\alpha_2 + g_2\right) c_2 + d_2}{1 - \left(g_2 + \alpha_2\right)} Y_2 - \\
&\quad - \left[\frac{\left(d_2 + g_2\right)}{\left(1 - \left(g_2 + \alpha_2\right)\right)}\right] Y_1 + \varepsilon_i^2
\end{aligned} \tag{4'}
$$

The econometric issue at this point is whether there are sufficiently many linearly independent coefficients in (4′) to identify the structural parameters in

(2). We can see that the pair of equations (4′) produces $r + 2s + 1$ for each of the two equations. The model (2) has $1 + r + (s + 1)$ structural parameters for each of the two equations. It is straightforward to see that in the absence of hairline collinearity between the regressors in (4′), the model is identified if $s \geq 2$.

4.2 Reduction to Manski's Classic Linear-in-Means Model

Notice that the key to resolution of the identification problem in this context is the addition of the between-groups effect. Without this effect, equation (2) reduces to

$$\omega_i^1 = k_1 + c_1 X_i^1 + d_1 Y_1 + \alpha_1 m_1 + \varepsilon_i^1$$

and the problem is the same as in Manski (1993). Apply rational expectations and then abbreviate equation (4) as follows: consider only group 1 and remove between group effects ($g_1 = 0, Y_2 = 0$). This reveals the well known equation (c.f. Durlauf 2004, p39) below.

$$\omega_i^1 = \frac{k_1}{1-\alpha_1} + c_1 X_i^1 + \frac{(\alpha_1)}{1-\alpha_1} c_1 X_1 + \frac{d_1 Y_1}{1-\alpha_1} + \varepsilon_i^1 \tag{5}$$

As is well known, identification fails in this case.

4.3 N Groups

Similar to the two-group case above, one can see the general multi-group case here. In model form, this is expressed as the system of equations for a collection of $J = N$ groups:

$$\begin{aligned} \omega_i^1 &= k_1 + c_1 X_i^1 + d_1 Q_1 + g_1 Z_{1,k} + \alpha_1 m_1 + \varepsilon_i^1 \qquad (6) \\ \omega_i^2 &= k_2 + c_2 X_i^2 + d_2 Q_2 + g_2 Z_{2,k} + \alpha_2 m_2 + \varepsilon_i^2 \\ &\quad ... \\ \omega_i^j &= k_N + c_N X_i^N + d_N Q_N + g_N Z_{N,k} + \alpha_N m_N + \varepsilon_i^N \end{aligned}$$

or

$$\begin{aligned}
\omega_i^1 &= k_1 + c_1 X_i^1 + d_1 Q_1 + (g_1 + \alpha_1)\, m_n^1 + g_1 A_{1,k} + \varepsilon_i^1 \\
\omega_i^2 &= k_2 + c_2 X_i^2 + d_2 Q_2 + (g_2 + \alpha_2)\, m_n^2 + g_2 A_{2,k} + \varepsilon_i^2 \\
&\quad ... \\
\omega_i^j &= k_N + c_N X_i^N + d_N Q_N + (g_N + \alpha_N)\, m_n^N + g_N A_{N,k} + \varepsilon_i^N
\end{aligned}$$

where k_j, c_j, d_j, g_j, and $\alpha_j, \forall j \in J$ are coefficients, X_i^j is an r-length vector of individual characteristics, Q_n^j is an $s\,(J-1)$ length vector of distinct contextual effect deviations: $Q_j = (Y_j - E_{k \neq j \in J} Y_k)$, Y_j specifies a $s\,(J-1)$ length contextual effect for group j, $m_j = E_i \omega_i^j$, $Z_{j,k} = E_i \omega_i^j - E_i \omega_i^k, \forall k \neq j \in J$ is a $J-1$ length vector of between-group endogenous effects, and $A_{1,k} \equiv \sum_{j \in J} m_j$. One again uses group-specific error independence here. Further setting $E\left[\mathbf{x}_i^j \varepsilon_i^j\right] = 0, j = 1, 2, ..N$, where $\mathbf{x} = \left[k_j, X_i^j, Q_j, Z_{j,k}, m_j\right], j = 1, 2, ...N$, we complete the model specification.

Assuming all agents observe rational expectations, allowing the standard regression assumptions of conditional mean-zero, and assuming that $(1-J)\, g_j + \alpha_j \neq 1, \forall j$ we have:

$$\begin{aligned}
m_1 &= \frac{k_1 + c_1 X_1 + d_1 Q_1 - g_1 A_{1,k}}{1 - ((J-1)\, g_1 + \alpha_1)} \\
m_2 &= \frac{k_2 + c_2 X_2 + d_2 Q_2 - g_2 A_{2,k}}{1 - ((J-1)\, g_2 + \alpha_2)} \\
&\quad ... \\
m_N &= \frac{k_N + c_N X_N + d_N Q_N - g_N A_{N,k}}{1 - ((J-1)\, g_N + \alpha_N)}
\end{aligned}$$

this can also be written as:

$$\begin{aligned}
m_1 &= \frac{k_1 + c_1 X_1 + d_1 Q_1 - g_1 \sum_{l \neq 1} m_l}{1 - ((J-1)\, g_1 + \alpha_1)} \\
m_2 &= \frac{k_2 + c_2 X_2 + d_2 Q_2 - g_2 \sum_{l \neq 2} m_l}{1 - ((J-1)\, g_2 + \alpha_2)} \\
&\quad ... \\
m_N &= \frac{k_N + c_N X_N + d_N Q_N - g_N \sum_{l \neq N} m_l}{1 - ((J-1)\, g_N + \alpha_N)}
\end{aligned}$$

Expanding this:

$$
\begin{aligned}
m_1 &= \frac{k_1 + c_1 X_1 + d_1 \left(Y_1 - \frac{1}{J-1} \sum_{l \neq 1} Y_l\right) - g_1 \sum_{l \neq 1} m_l}{1 - \left(\left(J-1\right) g_1 + \alpha_1\right)} \\
m_2 &= \frac{k_2 + c_2 X_2 + d_2 \left(Y_2 - \frac{1}{J-1} \sum_{l \neq 2} Y_l\right) - g_2 \sum_{l \neq 2} m_l}{1 - \left(\left(J-1\right) g_2 + \alpha_2\right)} \\
& ..., \\
m_N &= \frac{k_N + c_N X_N + d_N \left(Y_N - \frac{1}{J-1} \sum_{l \neq N} Y_l\right) - g_N \sum_{l \neq N} m_l}{1 - \left(\left(J-1\right) g_N + \alpha_N\right)}
\end{aligned}
$$

substituting back into the above:

$$
\begin{aligned}
\omega_i^1 &= k_1 + c_1 X_i^1 + d_1 \left(Y_1 - \frac{1}{J-1} \sum_{l \neq 1} Y_l\right) - g_1 \sum_{l \neq 1} m_l + \\
& + \left(\alpha_1 + \left(J-1\right) g_1\right) \frac{k_1 + c_1 X_1 + d_1 \left(Y_1 - \frac{1}{J-1} \sum_{l \neq 1} Y_l\right) - g_1 \sum_{l \neq 1} m_l}{1 - \left(\left(J-1\right) g_1 + \alpha_1\right)} + \varepsilon_i^1 \\
\omega_i^2 &= k_2 + c_2 X_i^2 + d_2 \left(Y_2 - \frac{1}{J-1} \sum_{l \neq 2} Y_l\right) - g_2 \sum_{l \neq 2} m_l + \\
& + \left(\alpha_2 + \left(J-1\right) g_2\right) \frac{k_2 + c_2 X_2 + d_2 \left(Y_2 - \frac{1}{J-1} \sum_{l \neq 2} Y_l\right) - g_2 \sum_{l \neq 2} m_l}{1 - \left(\left(J-1\right) g_2 + \alpha_2\right)} + \varepsilon_i^2 \\
& ..., \\
\omega_i^N &= k_N + c_N X_i^N + d_N \left(Y_N - \frac{1}{J-1} \sum_{l \neq N} Y_l\right) - g_N \sum_{l \neq N} m_l + \\
& \left(\alpha_N + \left(J-1\right) g_N\right) \frac{k_N + c_N X_N + d_N \left(Y_N - \frac{1}{J-1} \sum_{l \neq N} Y_l\right) - g_N \sum_{l \neq N} m_l}{1 - \left(\left(J-1\right) g_N + \alpha_N\right)}
\end{aligned}
$$

noting that Y_j, m_j and X_j, Y_j are econometrically indistinguishable:

$$\begin{aligned}
\omega_i^1 &= k_1 + c_1 X_i^1 + d_1 \left(Y_1 - \frac{1}{J-1} \sum\nolimits_{l \neq 1} Y_l \right) - g_1 \sum\nolimits_{l \neq 1} Y_l + \\
&+ (\alpha_1 + (J-1) g_1) \frac{k_1 + c_1 X_1 + d_1 \left(Y_1 - \frac{1}{J-1} \sum\nolimits_{l \neq 1} Y_l \right) - g_1 \sum\nolimits_{l \neq 1} Y_l}{1 - ((J-1) g_1 + \alpha_1)} + \varepsilon_i^1 \\
\omega_i^2 &= k_2 + c_2 X_i^2 + d_2 \left(Y_2 - \frac{1}{J-1} \sum\nolimits_{l \neq 2} Y_l \right) - g_2 \sum\nolimits_{l \neq 2} Y_l + \\
&+ (\alpha_2 + (J-1) g_2) \frac{k_2 + c_2 X_2 + d_2 \left(Y_2 - \frac{1}{J-1} \sum\nolimits_{l \neq 2} Y_l \right) - g_2 \sum\nolimits_{l \neq 2} Y_l}{1 - ((J-1) g_2 + \alpha_2)} + \varepsilon_i^2 \\
&..., \\
\omega_i^N &= k_N + c_N X_i^N + d_N \left(Y_N - \frac{1}{J-1} \sum\nolimits_{l \neq N} Y_l \right) - g_N \sum\nolimits_{l \neq N} Y_l + \\
&+ (\alpha_N + (J-1) g_N) \frac{k_N + c_N X_N + d_N \left(Y_N^1 - \frac{1}{J-1} \sum\nolimits_{l \neq N} Y_l \right) - g_N \sum\nolimits_{l \neq N} Y_l}{1 - ((J-1) g_N + \alpha_N)}
\end{aligned}$$

To simplify notation, grouping terms and defining the superparameters β, δ, γ,

$$\begin{aligned}
\beta_j &= \frac{k_j}{1 - ((J-1) g_j + \alpha_j)} \\
\delta_j &= \frac{(\alpha_j + (J-1) g_j) c_j + d_j}{1 - ((J-1) g_j + \alpha_j)} \\
\gamma_j &= \frac{(\alpha_j + (J-1) g_j) d_j}{1 - ((J-1) g_j + \alpha_j)} - \frac{d_j}{J-1},
\end{aligned}$$

we can then write:

$$\begin{aligned}
\omega_i^1 &= \beta_1 + c_1 X_i^1 + \delta_1 Y_1 + \gamma_1 \sum\nolimits_{l \neq 1} Y_l + \varepsilon_i^1 \qquad (7) \\
\omega_i^2 &= \beta_2 + c_2 X_i^2 + \delta_2 Y_2 + \gamma_2 \sum\nolimits_{l \neq 2} Y_l + \varepsilon_i^2 \\
&..., \\
\omega_i^N &= \beta_N + c_N X_i^N + \delta_N Y_N + \gamma_N \sum\nolimits_{l \neq N} Y_l + \varepsilon_i^N
\end{aligned}$$

The econometric issue at this point is whether there are sufficiently many linearly independent coefficients in (7) to identify the structural parameters in (6).

Proposition 1 *For a collection J with N groups, and under the condition that $\frac{\partial \omega_i^j}{\partial Y_k} \neq 0, \forall j \neq k \in J$, The linear-in-means model with between-group effects* (6) *can be identified if $s \geq N$.*

Proof. *The proof is immediate from equations* (6)*and* (7)*. Note that equation* (7) *has* $r + Ns + 1$ *coefficients in each of the* N *equations - leading to* $N(r + Ns + 1)$ *independent coefficients. The model* (6) *has* $N[r + (s + 1)(N - 1)]$ *structural parameters to identify. It is clear then that*

$$N(r + Ns + 1) \geq N[r + (s + 1)(N - 1)]$$

iff $s \geq N$*.* ■

The immediate question is whether the assumption that agents care about differences between own contextual effects and out-group ones is appropriate. Two justifications are in order. First, concern for out-group behavior naturally extends to both contextual and endogenous effects − if present at all. Second, we can see that this is a generalization of the linear-in-means model for finite numbers of out-groups. As the number of groups increases ($J \to \infty$), it is clear that the expectation term becomes a constant in the regression and the identification problem arises again. In fact, the result is analytically indistinguishable from the classic linear-in-means model.

Corollary 2 *As* $J \to \infty$*, identification fails and the model reduces to the linear-in-means model and cannot be fully identified.*

Proof. *Assume* J *is large, and note that in this case, the average out-group effects are constant across groups. That is, for all groups* j*,* $\sum_{l \neq j} Y = \hat{Y}$*. Thus* (7) *can be expressed as:*

$$\begin{aligned} \omega_i^1 &= \beta_1 + c_1 X_i^1 + \delta_1 Y_1 + \gamma_1 \hat{Y} + \varepsilon_i^1 \\ \omega_i^2 &= \beta_2 + c_2 X_i^2 + \delta_2 Y_2 + \gamma_2 \hat{Y} + \varepsilon_i^2 \\ &\ldots, \\ \omega_i^N &= \beta_N + c_N X_i^N + \delta_N Y_N + \gamma_N \hat{Y} + \varepsilon_i^N \end{aligned} \tag{8}$$

Then, the $\hat{Y}$ *term becomes part of the constant in the regression and this system only has* $N(r + s + 1)$ *coefficients - short of the* $N[r + (s + 1)(N - 1)]$ *needed for identification.* ■

5 Reference Groups

Hand-in-hand with the extension of the linear-in-means model is the expanded difficulty of parsing reference groups out from a population. The methods for identifying an individuals group have varied widely from geographical to sociological methods. A simple, but not completely satisfying approach is to use census-tract level geographics as the basis for neighborhood identification. Studies using this measure of neighborhoods include Corcoran, et al. (1992), Datcher (1982), Plotnick and Hoffman (1999). Turley (2003) used a census-tract residence measure in combination with other indicators. Census based grouping has clear problems, principally that it is doubtful that individuals consider all those that live in the census geographic construct to be their reference group.

A resolution for this has been to use individuals' self-declared reference groups. Some data collection exercises have called on youth to identify their best friends. Then groups of self-reported best friends can be used to test theoretical models. Though certainly more plausible than a geographic basis, this method calls into question whether the scope of suggested influence is sufficiently broad. For example, once the existence of social forces is acknowledged, a teenager's school performance is surely affected by more than a small set of self-reported friends.

Other authors use social measures of distance to indicate groups. Significantly, Conley & Topa (2002) used four measures including travel time, spatial distance, occupational distance, and ethnicity distance. The particularly promising component of this research vis-à-vis the project here is that it incorporates a metric of difference. The limiting feature of this model is the need to specify the distance metric.

Identification of multiple reference groups or neighborhoods adds new complexities. Among these are the need for collection of data regarding individuals' different types of acquaintances, "enemies", etc. It is not completely clear how to extract information from individuals about their levels and degrees of friendship (and/or envy, hate, etc.) that may delineate the groups that affect them.

In addition, the assumptions necessary for use of geographic data alone become even less plausible in the multiple groups case. For example, even if one were to accept that one's own census tract is the appropriate neighborhood for measurement, assuming that all other tracts have a measurable impact is a further stretch.

Perhaps most realistic is the use of ethnicity or age-cohort data as a measure. Each of are discrete, and readily accessible measures of reference – discreteness being essential to ensure that cross-group interactions can differ. In the Conley & Topa model distance is a one-dimensional concept – thus one cannot seek to be like one out-group and different than another one (i.e. Hispanic youth cannot seek to mimic some aspects of African American culture and reject similarly defined parts of "White" American culture).

6 Complementarity and Equilibrium

A key step in future research will be to investigate equilibria of these models. In particular, the notions of existence of multiple equilibria critical to the social interactions field have not been proven in the context of multiple reference groups. Some of the points to note include:

Within-group complementarity and the absence of between-group effects means that if $\omega_i^j\,(low) < \omega_i^j\,(high)\,, \forall j \in J$ and $\bar{\omega}_{-i}^j\,(low) < \bar{\omega}_{-i}^j\,(high)\,, \forall j \in J$, then

$$\begin{aligned} & V\left(\omega_i^j\,(high)\,, X_j, Y_j, \bar{\omega}_{-i}^j\,(high)\,, \bar{\omega}_i^{-j}\right) - \\ & V\left(\omega_i^j\,(low)\,, X_j, Y_j, \bar{\omega}_{-i}^j\,(high)\,, \bar{\omega}_i^{-j}\right) \\ > \; & V\left(\omega_i^j\,(high)\,, X_j, Y_j, \bar{\omega}_{-i}^j\,(low)\,, \bar{\omega}_i^{-j}\right) - \\ & V\left(\omega_i^j\,(low)\,, X_j, Y_j, \bar{\omega}_{-i}^j\,(low)\,, \bar{\omega}_i^{-j}\right) \end{aligned} \tag{9}$$

and $\frac{\partial \mu_i^e\left(\omega_{n,i}^j\right)}{\partial \omega_{n,i}^{-j}} = 0, \forall i, j$ (Durlauf 2004). This effectively states that holding the actions of out-group members constant, an agent experiences a greater increase in utility by following the behavior of others. Between-group complementarity without within-group effects suggests that if $\omega_i^j(low) < \omega_i^j(high), \forall j \in J$ and $\bar{\omega}_i^{-j}(low) < \bar{\omega}_i^{-j}(high), \forall j \in J$, (notice the change in sub and super-scripts to indicate out-group effects in this case) then

$$\begin{aligned} & V\left(\omega_i^j(high), X_j, Y_j, \bar{\omega}_{-i}^j, \bar{\omega}_i^{-j}(high)\right) - \\ & V\left(\omega_i^j(low), X_j, Y_j, \bar{\omega}_{-i}^j, \bar{\omega}_i^{-j}(high)\right) \\ > \; & V\left(\omega_i^j(high), X_j, Y_j, \bar{\omega}_{-i}^j, \bar{\omega}_i^{-j}(low)\right) - \\ & V\left(\omega_i^j(low), X_j, Y_j, \bar{\omega}_{-i}^j, \bar{\omega}_i^{-j}(low)\right) \end{aligned} \tag{10}$$

This suggests that holding own-group actions constant, an agent prefers to act like others than to act differently.

7 Bias in Single-Group Models

Note that neither within or between-group interactions require positive interactions. In fact, the study of between-group interactions is partially motivated by the appearance of negative reactions to out-group behavior. The interesting question here motivated by the inclusion of between-group effects is whether the study of a single neighborhood results in biased or attenuated conclusions. If between-group complementarity is high, a study of within-group effects can result in spurious conclusions about the appearance of neighborhood effects. To see this, consider the ethnic conflict case again. Imagine that a Hutu is more likely to commit an act of violence due to Tutsi violence than due to violent behavior by Hutus; that is, the between-group effect is larger. In this case, the coefficient on the between group effect will be large, but if not included, the resulting within-group coefficient will be too large. This can lead to spurious conclusions about the role of within-group effects.

We can conclude that an investigation into endogenous neighborhood social interactions which considers only within-group effects leads to biased results if the between-group effects are positive effects

$$
\begin{aligned}
& V\left(\omega_i^j, X_j, Y_j, \bar{\omega}_{-i}^j\left(high\right), \bar{\omega}_i^{-j}\left(high\right)\right) - \\
& V\left(\omega_i^j, X_j, Y_j, \bar{\omega}_{-i}^j\left(low\right), \bar{\omega}_i^{-j}\left(high\right)\right) \\
> \; & V\left(\omega_i^j, X_j, Y_j, \bar{\omega}_{-i}^j\left(high\right), \bar{\omega}_i^{-j}\left(low\right)\right) - \\
& V\left(\omega_i^j, X_j, Y_j, \bar{\omega}_{-i}^j\left(low\right), \bar{\omega}_i^{-j}\left(low\right)\right)
\end{aligned} \tag{11}
$$

and will lead to attenuated conclusions if the between group effects are negative:

$$
\begin{aligned}
& V\left(\omega_i^j, X_j, Y_j, \bar{\omega}_{-i}^j\left(high\right), \bar{\omega}_i^{-j}\left(high\right)\right) - \\
& V\left(\omega_i^j, X_j, Y_j, \bar{\omega}_{-i}^j\left(low\right), \bar{\omega}_i^{-j}\left(high\right)\right) \\
< \; & V\left(\omega_i^j, X_j, Y_j, \bar{\omega}_{-i}^j\left(high\right), \bar{\omega}_i^{-j}\left(low\right)\right) - \\
& V\left(\omega_i^j, X_j, Y_j, \bar{\omega}_{-i}^j\left(low\right), \bar{\omega}_i^{-j}\left(low\right)\right)
\end{aligned} \tag{12}
$$

This set of complementarity conditions differs from the last section. In the prior section (9)and (10) are equivalent to the cross-partials of $V()$ with respect to ω_i^j and either $\hat{\omega}_{-i}^j$ or $\hat{\omega}_i^{-j}$. These here, (11) and (12), instead compares the cross-partials of $V()$ with respect to $\hat{\omega}_{-i}^j$ and $\hat{\omega}_i^{-j}$.

Further work is necessary to parse out the importance of the various complementarity results here. The key implications here are twofold. First, the apparent need for a measure of between-group complementarity to generate multiple equilibria. Second, it is possible that sufficiently large between-group complementarity could swamp the within-group effect, thus generating multiple equilibria even in the absence of within-group interactions.

8 Conclusion

This paper has shown that a between-group generalized linear-in-means model solves the reflection problem while maintaining a simple-to-estimate and interpret linear form. However, it provides no insight into perhaps the greatest outstanding question in this literature: the foundations or sources of social interactions

In addition to the continued research into this source question, future possibilities include investigation of the universality properties of social interactions. It would be a further boon to the advocates of the presence of social interactions effects if a multiple groups model verified prior work.

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