DATA ENVELOPMENT ANALYSIS IN ENVIRONMENTAL VALUATION: ENVIRONMENTAL PERFORMANCE, ECO-EFFICIENCY AND COST-BENEFIT ANALYSIS

TIMO KUOSMANEN[*]
Environmental Economics and Natural Resources Group
Wageningen University
&
MIKA KORTELAINEN[†]
Department of Business and Economics
University of Joensuu

Abstract

Data Envelopment Analysis (DEA) is a linear programming based method for evaluating performance of comparable production units such as firms. Although the method is already extensively applied in many areas of economics, its use in environmental economics and related fields remains limited. The purpose of this paper is to present basic principles of DEA and evaluate its application possibilities for a range of environmental valuation problems. We show how DEA has to be adjusted to the context of environmental performance, eco-efficiency and Cost-Benefit analysis (CBA). By modifying the traditional DEA framework to the specific features and purposes of environmental application we show that the valuation principles to which DEA is based on can offer useful insights and complement the conventional toolbox of environmental economists in valuation of the environmental services in general.

Keywords: *Data Envelopment Analysis, performance measurement, production theory, environmental valuation, environmental performance, eco-efficiency, Cost-Benefit analysis*

JEL classification: Q51, D61, C61, D24

[*] Wageningen University, Environmental Economics and Natural Resources Group, Hollandseweg 1, 6706 KN Wageningen, The Netherlands, Tel. +31 317 484 738, Fax. +31 317 484 933, E-mail: Timo.Kuosmanen@wur.nl

[†] University of Joensuu, Department of Business and Economics, P.O. Box 111, FI-80101 Joensuu, Finland, Tel. +358 13 251 5225, Fax. +358 13 251 3290, E-mail: Mika.Kortelainen@joensuu.fi

1. Introduction

While goods and services exchanged in the market place have readily observable measures of their value, the market price, many environmental goods and services such as clean air and water resources are not generally valued at all. The absence of markets for environmental services is, in fact, one of the prime examples of the market failure. It is well known that the lack of economic value for environmental goods generally leads to over-exploitation and degradation of these resources. Therefore, economic valuation of the environment and its services is one of the most fundamental topics of environmental economics.

Standard valuation methods of environmental economics can be classified into two main categories: the *stated preference* methods and the *revealed preference* methods. The first category includes methods such as Contingent Valuation (CV) that addresses people directly to inquire about their willingness to pay for environmental goods, asking the respondents to describe their behavior in a hypothetical situation. While there are many different strategies to encourage the respondents to state their true willingness to pay, all approaches of this category rely on their subjective valuation of the environmental issue at hand. The second category rejects the idea of asking individuals' opinions, and instead, tries to infer their willingness to pay indirectly based on observed behavior. Notable examples of the revealed preference approaches include the Travel Cost Method and the hedonic estimation. While the revealed preference techniques stand on a more objective ground, their scope of environmental valuation tends to be more limited. The revealed preference approaches can be applied in situations where people already pay for an environmental good or service in one way or another, and this payment can be directly observed and associated with the use of that particular good or service. The relative strengths and weaknesses of the stated and the revealed preference methods have been subject to lively debate among academics, to which we take no position here.[1]

This paper presents *Data Envelopment Analysis* (DEA) as an alternative valuation technique, and evaluates its application possibilities for a range of environmental valuation problems. While DEA is not yet widely diffused into the field of environmental economics, it is frequently applied in many other areas of applied economic sciences, including agricultural economics, development economics, financial economics, public economics, and macroeconomic policy, among others, in addition to its traditional confinements in productivity and efficiency analysis. In its purest form, the unique valuation principle of DEA does not depend on either stated or revealed preferences. Rather, it turns the value problem other way around, and asks what kind of prices would favor this or that particular firm or project. Relying on the implicit preferences of firm managers and the project proponents that can be revealed by their observed emphases of different environmental aspects, DEA does share some common intellectual roots with the revealed preference valuation approaches. Thus, the basic DEA approach is most likely to appeal those who generally prefer the revealed preference approach to stated preference methods. On the other hand, the DEA framework is technically closely related with the Multi-Criteria Analysis (MCA), which is often mentioned as a "softer" alternative for the more traditional economic techniques. Like MCA, DEA approaches the valuation problem from a multi-dimensional perspective, and can be applied in combination of MCA or other valuation techniques that incorporate subjective judgments and stated preference information to the objective DEA assessment. Thus, DEA offers a flexible and general framework that can easily be adapted to the specific features and purposes of the application.

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[1]Further discussion about advantages and disadvantages of the stated and the revealed preference methods can be found in any textbook on environmental economics, e.g.

- Perman, R., Y. Ma, J. McGilvray, and M. Common (2003): *Natural Resource and Environmental Economics*, 3rd ed., Pearson Education.

The purposes of this paper are two-fold. The first purpose is to introduce the key insights and the basic principles of DEA to people in environmental economics and related fields. DEA should be particularly interesting for those who are interested in environmental valuation, but are not fully content with the standard toolbox of valuation techniques. The usual textbook treatments of DEA[2] places great emphasis on the abstract production theoretic or technical aspects of DEA, which makes them somewhat difficult to access if one is not accustomed to reading abstract mathematical expressions. The purpose here is to provide a more readily accessible introduction to this method, focusing on its potential in the context of environmental performance analysis.

The second purpose is to explore the possible uses of DEA in environmental valuation that extend beyond the traditional confinement of DEA in the comparative assessment of firm performance. We believe the valuation principles to which DEA is based on can offer useful insights and complement the conventional toolbox of environmental economists in valuation of the environmental services in general. To illustrate the potential, we show how DEA approach could be adjusted to the context of the Cost-Benefit Analysis (CBA). In contrast to traditional DEA that focuses on comparative performance assessment, the purpose of environmental CBA is to identify the socially optimal project or a basket of projects to be implemented from a set of available alternatives. Traditional DEA builds on relative prices, while the absolute prices are also needed in CBA to determine whether any project is profitable enough to be implemented. Despite these rather fundamental differences, we show that DEA can be modified to the context of CBA. In some situations, DEA can provide a clear-cut solution for the CBA valuation problem without a need to study stated or revealed preferences. Even if such clear-cut solution does not arise, a preliminary DEA assessment can help to structure the problem as well as identify the critical parameters that need to be estimated by other methods, which can save a considerable amount of time and costs when implementing the more demanding stated or revealed preference evaluation studies.

While we have tried to make the text accessible for readers lacking strong analytical skills by keeping the tedious technical details to the minimum, by explaining the technical material also in non-technical terms, and by illustrating the key results by graphical diagrams and numerical examples, we have also wanted to provide a sufficiently thorough treatment for those who seek an in-depth understanding of the method. However, we have avoided the use of elegant matrix algebra, which means that many of our mathematical formulas take a lot more space but are hopefully easier to read for those unaccustomed to reading matrix algebra. We should add that a large number of numerical examples are presented to facilitate better understanding of the general linear programming formulations. It is unnecessary to go through all numerical examples in detail, especially at the first reading; those are primarily offered for those who struggle with understanding the finer points of DEA.

The rest of the paper is organized as follows. In Section 2 we provide a concise introduction to the production theoretic underpinnings of DEA, which we find important for a through understanding of the method. Section 3 presents the basic DEA approach in light of this production theoretic framework. Section 4 then introduces the environment to the traditional production theoretic setting, which leads us to the literature of environmental performance analysis. Sections 2-4 represent the current state of the art, and should be read as one entity. These sections probably appear the most

[2] Standard textbook references include:

- Coelli, T., D.S. Prasada Rao and G.E. Battese (1998): *An Introduction to Efficiency and Productivity Analysis*, Kluwer Academic Publishers.
- Cooper, W.W., L.M. Seiford and K. Tone (2000): *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software*, Kluwer Academic Publishers.
- Färe, R., S. Grosskopf and C.A.K. Lovell (1994): *Production Frontiers*. Cambridge University Press.

“foreign” part of this paper for environmental economics oriented readers. Nevertheless, we chose to start with this production theoretic perspective to be able to proceed from the simple towards complex, and to better preserve the chronological order of developments in this field. The purpose of these sections is to summarize the current knowledge in a relatively concise but still accessible form. In our assessment, the most interesting (and original) material is presented in Sections 5 and 6. Section 5 applies DEA weighting for the measurement of eco-efficiency in production. Section 6 presents a novel DEA-based approach to Cost-Benefit Analysis. Section 6 is not very technical, and it may be possible to start from Section 6, or read Section 6 as an independent entity, although the reader will certainly benefit of understanding the more technical Sections 2-4. Section 7 presents brief review of the relevant literature and computational software. To keep the text readable, we avoid the use of references in the main text, and only provide some selected references in the footnotes.

2. Multi-output production theory

The traditional focus of DEA lies on firms[3] that transform multiple inputs to multiple outputs. Since the microeconomics textbooks usually ignore the general multi-output setting, a brief introduction to the key insights of the multi-output production theory is in order.[4]

Figure 1 illustrates the traditional setting of the production economics. A firm consumes inputs (e.g., labor capital, materials, energy) to produce economic outputs (i.e., goods and services). Let the number of inputs be $L > 0$ and the number of outputs be $M > 0$. Input usage is represented by vector $\mathbf{x} = (x_1, x_2, \ldots, x_L)$ and the generated outputs by vector $\mathbf{y} = (y_1, y_2, \ldots, y_M)$. Inputs and outputs are usually interpreted as flow variables (e.g. units of output per month), although capital input is often measured as the average stock during the time period under study. For transparency, we will adopt the notational convention that the upper case symbols $\mathbf{X}_k$ and $\mathbf{Y}_k$ refer to the observed input and output quantities of firm k while the lower case symbols refer to arbitrary (or theoretical) input and output vectors.

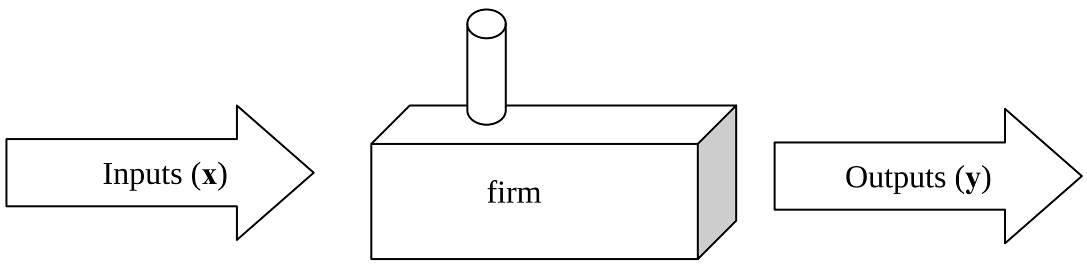

Figure 1: The traditional setting of production analysis

The multi-output production theory makes extensive use of mathematical notation of set theory.[5] The most general representation of technology is the production possibility set

[3] Hereafter, by ”firm” we refer to production units in general, without discriminating between private enterprises and other types of production organizations such as public sector firms and non-for-profit firms.
[4] For a more thorough (and technically demanding) presentation, see

- Färe, R., and D. Primont (1995): *Multi-Output Production and Duality: Theory and Applications*, Kluwer Academic Publishers.

(2.1) $T = \{(\mathbf{x}, \mathbf{y}) \,|\, \text{inputs } \mathbf{x} \text{ can produce outputs } \mathbf{y}\}.$

This set simply lists all technically feasible combinations of inputs and outputs.[6]

A firm k is said to be *technically efficient* if its input-output combination $(\mathbf{X}_k, \mathbf{Y}_k)$ lies on the boundary of set T, in other words, if it is not possible to reduce the use of any inputs without decreasing one of the outputs or increasing the use of another input, or conversely, if it is not possible to increase the amount of any output without decreasing the amount of another output or increasing the input usage. The classic Farrell[7] output efficiency measure indicates the potential increase of all outputs in equal proportions, and is formally defined as

$$\begin{aligned} &Oeff_k = \max \theta \\ (2.2) \quad &\textit{subject to } (s.t.) \\ &(\mathbf{X}_k, \theta \mathbf{Y}_k) \in T. \end{aligned}$$

In words, we try to maximize parameter θ, subject to the constraint that when all outputs of firm k are multiplied by parameter θ, the resulting output bundle can still be produced with the present input usage of this firm. If $Oeff_k$ is greater than one, then all outputs could be increased by factor $Oeff_k$, consuming the same amounts of inputs, and hence firm k is output inefficient. If $Oeff_k$ is equal to one, then firm k is output efficient: it is not possible to increase all outputs without increasing the input usage.[8]

An alternative way of looking at technical efficiency is to take the input perspective. The Farrell input efficiency measure is defined analogous to (2.2) as

$$\begin{aligned} &Ieff_k = \min \theta \\ (2.3) \quad &s.t. \\ &(\theta \mathbf{X}_k, \mathbf{Y}_k) \in T. \end{aligned}$$

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[5] A set is defined by using curly parentheses "{}", indicating the members of the set (e.g. set $A=\{a,b,c\}$ consists of alphabets a,b,c). We can specify conditions for the membership in the set by using symbol "|". For example, consider set B that includes natural numbers greater than or equal to five, i.e., $B = \{5, 6, 7, ...\}$ (a set may have an infinite number of elements). The set of natural numbers is $\mathbb{N} = \{0, 1, 2, 3, ...\}$. We can express B using the condition symbol "|" as $B = \{x \in \mathbb{N} \,|\, x \geq 5\}$; in words, set B consists of natural numbers x, which satisfy the condition $x \geq 5$.

[6] Set T is assumed to be closed and non-empty. Moreover, it is often assumed that point $(\mathbf{0},\mathbf{0})$ belongs to set T, which means that inactivity is possible. The points $(\mathbf{0},\mathbf{y})$, $\mathbf{y}>\mathbf{0}$ are assumed to lie outside the boundaries of T, because positive amounts of output cannot be created from nothing. This property is known as "no free lunch".

[7] The seminal article by Farrell presents the fundamentals of efficiency analysis in an insightful, non-technical manner, and is thus highly recommended reading:

- Farrell, M. J. (1957): The Measurement of Productive Efficiency, *Journal of Royal Statistical Society*, Series A, 120(3), 253 – 290.

[8] In the strict definition of technical efficiency, $Oeff = 1$ is a necessary but not sufficient condition for efficiency. Even if the Farrell measure indicates that production of *all* outputs cannot be increased with the given inputs, we might be able to increase production of *some* outputs, which can be interpreted as technical inefficiency as well. Efficiency concept implied by the Farrell measures is sometimes referred to as "weak efficiency", while the more strict definition (due to Tjalling Koopmans) is known as "strong efficiency.

In this problem we multiply inputs of firm *k* by parameter θ to scale down all inputs by the smallest possible factor, subject to the condition that these downsized inputs must still be able to produce the original output bundle. If $Ieff_k$ is less than one, then all inputs could be decreased by factor $Ieff_k$, while maintaining the same output level, and hence firm *k* is input inefficient. If $Ieff_k$ is equal to one, then firm *k* is input efficient: it is not possible to increase all outputs without increasing the input usage. In general, the input and output efficiency measures can give different results. Färe and Lovell have shown that $Ieff_k = 1/Oeff_k$ if and only if the production technology exhibits *constant returns to scale* (which prevail if $(\mathbf{x}, \mathbf{y}) \in T$ implies that also $(a\mathbf{x}, a\mathbf{y}) \in T$ for all values of parameter $a \geq 0$).[9]

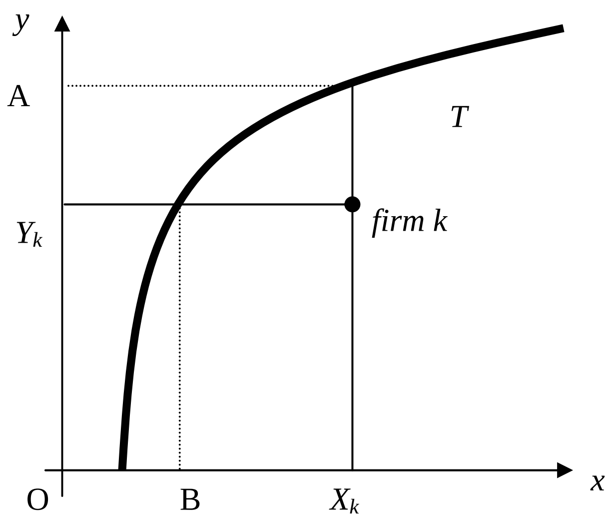

Figure 2: Illustration of production possibility set T and the input and output efficiency measures

Figure 2 illustrates the production possibility set *T* and the input and output efficiency measures in the simplest thinkable case of a single input and single output. The bold curved line in Figure 2 represents the efficient frontier of the production possibility set *T*; set *T* is the area that lies below this curve. The point in the middle of the diagram is the input-output vector of firm *k*. The output efficiency measure $Oeff_k$ can be read from the vertical axis *y* as $Oeff = |OA|/|Oy_k|$ (i.e., the length of line segment OA divided by the length of line segment Oy_k). Similarly, the input efficiency measure $Ieff_k$ can be read from the horizontal axis *x* as $Ieff = |OB|/|Ox_k|$ (i.e., the length of line segment OB divided by the length of line segment Ox_k).

The rationale for measuring efficiency as the equiproportionate decrease (increase) of all inputs (outputs) is not only based on the intuitive appeal of such measure; the Farrell efficiency measures (2.2) and (2.3) also have an attractive dual interpretation in terms of economic efficiency. Let the input prices be represented by vector $\mathbf{p}_x = (p_{x1}, p_{x2}, p_{x3}, ..., p_{xL})$ and the output prices by vector $\mathbf{p}_y = (p_{y1}, p_{y2}, p_{y3}, ..., p_{yM})$. Given output prices, the revenue efficiency of firm *k* is the ratio of maximum revenue obtainable by inputs of firm *k* to the actual observed revenue, that is,

[9] Axiomatic analysis of efficiency indices

- Färe, R., and C. A. K. Lovell (1978): Measuring the Technical Efficiency of Production, *Journal of Economic Theory* 19, 150-162.

(2.4) $$Reff_k = \max_{\mathbf{y}'} \frac{p_{y1}y'_1 + p_{y2}y'_2 + ... + p_{yM}y'_M}{p_{y1}Y_{k1} + p_{y2}Y_{k2} + ... + p_{yM}Y_{kM}} \text{ s.t. } (\mathbf{X}_k, \mathbf{y}') \in T .$$

The elements of output vector **y’** are the unknown variables of this problem. Output vector **y’** should maximize the revenue of firm *k* at the given output prices, and must be producible with the given inputs $\mathbf{X}_k$. The score of revenue efficiency $Reff_k$ can be interpreted in the same way as output efficiency measure; a value of one means that firm *k* is revenue efficient, and a value greater than one indicates the degree of revenue inefficiency.

Similarly, given input prices, cost efficiency of firm *k* is the ratio of minimum cost of producing the output of firm *k* to the actual observed cost, that is,

(2.5) $$Ceff_k = \min_{\mathbf{x}'} \frac{p_{x1}x'_1 + p_{x2}x'_2 + ... + p_{xL}x'_L}{p_{x1}X_{k1} + p_{x2}X_{k2} + ... + p_{xL}X_{kL}} \text{ s.t. } (\mathbf{x}', \mathbf{Y}_k) \in T .$$

Cost efficiency measure is bounded by zero and one; it attains a value of one for the cost efficient firm, and a value less than one for the cost inefficient firm.

It can be shown that if the production possibility set *T* satisfies free disposability and convexity (the meaning of which will be elaborated in the next paragraph), then the output efficiency measure can be interpreted as revenue efficiency at the ‘most favorable’ prices (which minimize the revenue efficiency measure) for firm *k*.[10] Specifically,

(2.6) $$Oeff_k = \min_{\mathbf{p}_y} Reff_k.$$

Similarly, the input efficiency can be interpreted as cost efficiency at the ‘most favorable’ input prices (which maximize the cost efficiency measure) for firm *k*. Specifically,

(2.7) $$Ieff_k = \max_{\mathbf{p}_x} Ceff_k.$$

Free disposability means that, given inputs **x**, it is possible to decrease the production of any output by any desired amount (i.e., get rid off any excess output free of charge), or conversely, it is possible to produce a given output **y** with more input resources than is absolutely necessary. More specifically, free disposability means that if $(\mathbf{x}, \mathbf{y}) \in T$ and $\mathbf{x}' \geq \mathbf{x}, \mathbf{y}' \leq \mathbf{y}$ then also $(\mathbf{x}', \mathbf{y}') \in T$. Since the free disposability assumption is of particular importance for the environmental performance analysis, we will devote further attention on this property as we proceed.

Convexity means that it is possible to combine firms: if we observe firms A and B, it is possible to take a half of both firms and produce the average output of these firms by the average input consumption (consider e.g. a merger of two companies). Since convexity is not as interesting an assumption as free disposability, for simplicity, we will follow the usual approach and take it as a maintained assumption.[11]

[10] If these assumptions fail to hold, then equalities (2.6) and (2.7) become inequalities (i.e., replace ”=” by ”≤”).

[11] The convexity assumption has attracted a lot of debate in the recent DEA literature, see e.g.

- Cherchye, L., T. Kuosmanen, and G.T. Post (2000): What Is The Economic Meaning of FDH? A Reply to Thrall, *Journal of Productivity Analysis* 13(3), 259-263.
- Kuosmanen, T. (2001): DEA with Efficiency Classification Preserving Conditional Convexity, *European Journal of Operational Research* 132(2), 83-99.

Alternatively interpreted, free disposability can be seen as a first-order curvature condition for the efficient frontier: the maximum output does not decrease if input usage increases (i.e., the marginal product of every input is non-negative). Convexity can be seen as the second-order condition: the maximum output increases at non-increasing rate as the inputs increases (i.e., the marginal product of every input is non-increasing).

The most favorable prices $\mathbf{p}_x^*$ and $\mathbf{p}_y^*$ are referred to as "shadow prices" or the "benefit-of-the-doubt" prices. If we are interested of economic efficiency but price information is unavailable, then we can use the Farrell technical efficiency measures to estimate conservative lower or upper bounds for economic efficiency.[12] These shadow prices can be also used for the valuation of environmental impacts and pressures as will be demonstrated in Sections 4, 5, and 6 below.

3. DEA efficiency analysis

This section introduces the traditional DEA analysis in the production economic setting discussed in the previous section. Thus far we have discussed about production possibility set *T* as if it was known to us. In practice, set *T* is usually estimated from the empirical input-output data. DEA has originally emerged as an approach for estimating *T*. The principle of DEA is to make minimal prior assumptions about the shape of *T*, and rather let the data 'speak for themselves'.

DEA is based on the comparison of a sample of *N* firms that are assumed to utilize the same production technology *T*. The critical assumption that almost all DEA studies make is that all relevant inputs and outputs are observed for all firms without error. Thus, all observed input-output vectors must be contained in *T*, otherwise they could not be observed to begin with. Other typical assumptions are free disposability and convexity introduced above.

Imposing the assumptions of free disposability and convexity (alternative sets of assumptions are also possible but we here restrict to the most standard ones), then DEA approximates the production possibility set *T* by the following set

$$(3.1) \qquad \hat{T}^{DEA} = \left\{ (\mathbf{x}, \mathbf{y}) \middle| \mathbf{x} \geq \sum_{n=1}^{N} \lambda_n \times \mathbf{X}_n ; \mathbf{y} \leq \sum_{n=1}^{N} \lambda_n \times \mathbf{Y}_n ; \sum_{n=1}^{N} \lambda_n = 1; \lambda_n \geq 0 \right\}.$$

This set satisfies the so-called minimum extrapolation principle, being the smallest set that contains all observed firms and satisfies the maintained assumptions of free disposability and convexity. Thus, the DEA production set (3.1) gives a conservative estimate of the true production possibilities *T*, (i.e., $\hat{T}^{DEA} \subseteq T$).

Firm-specific efficiency measures are calculated relative to DEA technology by solving a linear programming problem.[13] DEA output efficiency of firm *k* is obtained as the optimal solution to problem

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[12] For further details, see

- Kuosmanen, T., and G.T. Post (2001): Measuring Economic Efficiency with Incomplete Price information: With an Application to European Commercial Banks, *European Journal of Operational Research* 134, 43-58.

[13] Linear programming problem is characterized by a linear objective function (to be minimized or maximized) and a system of linear constraints. A classic text on linear programming (written by economists for economists) is

- Dorfmann R., P.A. Samuelson and R. Solow (1958): *Linear Programming and Economic Analysis*, McGraw Hill.

$$
\text{(3.2)} \quad
\begin{aligned}
& Oeff_k = \max_{\theta,\lambda} \theta \\
& s.t \\
& \left.\begin{aligned}
& Y_{k1}\theta \le Y_{11}\lambda_1 + Y_{21}\lambda_2 + \ldots + Y_{N1}\lambda_N \\
& Y_{k2}\theta \le Y_{12}\lambda_1 + Y_{22}\lambda_2 + \ldots + Y_{N2}\lambda_N \\
& \vdots \\
& Y_{kM}\theta \le Y_{1M}\lambda_1 + Y_{2M}\lambda_2 + \ldots + Y_{NM}\lambda_N
\end{aligned}\right\} \textit{outputs} \\
& \left.\begin{aligned}
& X_{k1} \ge X_{11}\lambda_1 + X_{21}\lambda_2 + \ldots + X_{N1}\lambda_N \\
& X_{k2} \ge X_{12}\lambda_1 + X_{22}\lambda_2 + \ldots + X_{N2}\lambda_N \\
& \vdots \\
& X_{kL} \ge X_{1L}\lambda_1 + X_{2L}\lambda_2 + \ldots + X_{NL}\lambda_N
\end{aligned}\right\} \textit{inputs} \\
& \left.\begin{aligned}
& \lambda_1 + \lambda_2 + \ldots + \lambda_N = 1 \\
& \lambda_k \ge 0 \ \forall k = 1,\ldots,N.
\end{aligned}\right\} \textit{weights}
\end{aligned}
$$

These problems are too complicated to solve analytically (i.e., by paper and pen) but standard PC equipped with widely available software will solve even a large problem in a split second. We return to the interpretation of this formulation shortly. But before that, let us also introduce the linear programming formulation for the input efficiency:

$$
\text{(3.3)} \quad
\begin{aligned}
& Ieff_k = \max_{\theta,\lambda} \theta \\
& s.t. \\
& \left.\begin{aligned}
& Y_{k1} \le Y_{11}\lambda_1 + Y_{21}\lambda_2 + \ldots + Y_{N1}\lambda_N \\
& Y_{k2} \le Y_{12}\lambda_1 + Y_{22}\lambda_2 + \ldots + Y_{N2}\lambda_N \\
& \vdots \\
& Y_{kM} \le Y_{1M}\lambda_1 + Y_{2M}\lambda_2 + \ldots + Y_{NM}\lambda_N
\end{aligned}\right\} \textit{outputs} \\
& \left.\begin{aligned}
& X_{k1}\theta \ge X_{11}\lambda_1 + X_{21}\lambda_2 + \ldots + X_{N1}\lambda_N \\
& X_{k2}\theta \ge X_{12}\lambda_1 + X_{22}\lambda_2 + \ldots + X_{N2}\lambda_N \\
& \vdots \\
& X_{kL}\theta \ge X_{1L}\lambda_1 + X_{2L}\lambda_2 + \ldots + X_{NL}\lambda_N
\end{aligned}\right\} \textit{inputs} \\
& \left.\begin{aligned}
& \lambda_1 + \lambda_2 + \ldots + \lambda_N = 1 \\
& \lambda_k \ge 0 \ \forall k = 1,\ldots,N.
\end{aligned}\right\} \textit{weights}
\end{aligned}
$$

In both these problems, the first set of M constraints run through all outputs (as indicated on the right), the second set of L constraints run through all inputs. The right-hand sides of the first M+L constraints form linear combinations of the observed input and output quantities. This effectively represents the production possibilities (compare with (3.1)). Observe that these right-hand sides are exactly the same for the both output and input efficiency formulations. The difference of orientation reveals itself on the left-hand sides of these constraints. In the output efficiency problem we try to increase all outputs of firm k by the same factor θ. In the input efficiency problem we try to reduce all inputs of firm k by factor θ.

The assumption of free disposability reveals itself in problems (3.2) and (3.3) in the form of inequality constraints for inputs and outputs: not all of these constraints have to be binding in the optimal solution. The assumption of free disposability could be easily relaxed in problems (3.2) and (3.3) by replacing the inequality signs of the input and output constraints by equalities (=).

Like all linear programming problems, problems (3.2) and (3.3) have equivalent dual formulations. We can calculate the efficiency measures equivalently by solving the following linear programming problems:

$$Oeff_k = \min_{\hat{\mathbf{p}}_x, \hat{\mathbf{p}}_y, f} X_{k1}\hat{p}_{x1} + X_{k2}\hat{p}_{x2} + \ldots + X_{kL}\hat{p}_{xL} + f$$

$$\text{(3.4)} \quad \left.\begin{array}{l} s.t. \\ (Y_{11}\hat{p}_{y1} + Y_{12}\hat{p}_{y2} + \ldots + Y_{1M}\hat{p}_{yM}) - (X_{11}\hat{p}_{x1} + X_{12}\hat{p}_{x2} + \ldots + X_{1L}\hat{p}_{xL} + f) \le 0 \\ (Y_{21}\hat{p}_{y1} + Y_{22}\hat{p}_{y2} + \ldots + Y_{2M}\hat{p}_{yM}) - (X_{21}\hat{p}_{x1} + X_{22}\hat{p}_{x2} + \ldots + X_{2L}\hat{p}_{xL} + f) \le 0 \\ \vdots \\ (Y_{N1}\hat{p}_{y1} + Y_{N2}\hat{p}_{y2} + \ldots + Y_{NM}\hat{p}_{yM}) - (X_{N1}\hat{p}_{x1} + X_{N2}\hat{p}_{x2} + \ldots + X_{NL}\hat{p}_{xL} + f) \le 0 \end{array}\right\} \textit{firms}$$

$$\left.\begin{array}{l} Y_{k1}\hat{p}_{y1} + Y_{k2}\hat{p}_{y2} + \ldots + Y_{kM}\hat{p}_{yM} = 1 \\ \hat{\mathbf{p}}_x, \hat{\mathbf{p}}_y \ge 0. \end{array}\right\} \textit{prices}$$

$$Ieff_k = \max_{\hat{\mathbf{p}}_x, \hat{\mathbf{p}}_y, f} Y_{k1}\hat{p}_{y1} + Y_{k2}\hat{p}_{y2} + \ldots + Y_{kM}\hat{p}_{yM} + f$$

$$\text{(3.5)} \quad \left.\begin{array}{l} s.t. \\ (Y_{11}\hat{p}_{y1} + Y_{12}\hat{p}_{y2} + \ldots + Y_{1M}\hat{p}_{yM} + f) - (X_{11}\hat{p}_{x1} + X_{12}\hat{p}_{x2} + \ldots + X_{1L}\hat{p}_{xL}) \le 0 \\ (Y_{21}\hat{p}_{y1} + Y_{22}\hat{p}_{y2} + \ldots + Y_{2M}\hat{p}_{yM} + f) - (X_{21}\hat{p}_{x1} + X_{22}\hat{p}_{x2} + \ldots + X_{2L}\hat{p}_{xL}) \le 0 \\ \vdots \\ (Y_{N1}\hat{p}_{y1} + Y_{N2}\hat{p}_{y2} + \ldots + Y_{NM}\hat{p}_{yM} + f) - (X_{N1}\hat{p}_{x1} + X_{N2}\hat{p}_{x2} + \ldots + X_{NL}\hat{p}_{xL}) \le 0 \end{array}\right\} \textit{firms}$$

$$\left.\begin{array}{l} X_{k1}\hat{p}_{x1} + X_{k2}\hat{p}_{x2} + \ldots + X_{kL}\hat{p}_{xL} = 1 \\ \hat{\mathbf{p}}_x, \hat{\mathbf{p}}_y \ge 0. \end{array}\right\} \textit{prices}$$

Observe that, in contrast with the standard problem of the firm, in this efficiency evaluation problem the input and output quantities are known parameters, while the shadow prices $(\hat{\mathbf{p}}_x, \hat{\mathbf{p}}_y)$ and the fixed cost/benefit (*f*) are unknown variables. In problem (3.4) the objective is to set the prices such that the total cost of firm *k* is minimized. The *N* firm-specific constraints impose that all firms make a nonpositive profit. The output prices are normalized such that the revenue of firm *k* is one. If we compare this with the general economic efficiency interpretation of the output efficiency measure given in (2.6), we can interpret the optimal solution as revenue efficiency of firm *k* at the most favorable prices for firm *k*. Analogous interpretation applies to input efficiency problem (3.5), where the total cost of firm *k* appears in the price normalization constraint, and the total revenue is taken to the objective function.

It is important to note that in both these problems the shadow prices should be interpreted as *relative* values of inputs and outputs, not absolute prices. The inequality constraints essentially normalize the prices such that the efficiency measure can be read directly from the optimal solution.

In problems (3.4) and (3.5) the free disposability assumption is enforced by the non-negativity constraint for prices $\hat{\mathbf{p}}_x, \hat{\mathbf{p}}_y$. From economic perspective, free disposability can be interpreted as the requirement that input usage incurs an outflow of funds and the output production yields an inflow of funds to the firm. Relaxation of free disposability means that prices can become negative.

To illustrate the DEA approach, we consider a simple numerical example involving four firms labeled as A, B, C, and D, which produce a single output using two inputs. The input and output data of these firms are reported by Table 1 below.

Table 1: Numerical example with four firms, a single output and two inputs

	Firm A	Firm B	Firm C	Firm D
Y	8	3	5	5
X_1	5	1	4	3,5
X_2	6	4	1	3,5

Consider input efficiency of Firm D. Problem (3.3) can be written in this example as

$$
\text{(3.6)} \quad
\begin{aligned}
& Ieff_D = \max_{\theta,\lambda} \theta \\
& s.t. \\
& 5 \le 8\lambda_A + 3\lambda_B + 5\lambda_C + 5\lambda_D && \} \textit{output} \\
& \left.\begin{aligned} & 3.5\theta \ge 5\lambda_A + 1\lambda_B + 4\lambda_C + 3.5\lambda_D \\ & 3.5\theta \ge 6\lambda_A + 4\lambda_B + 1\lambda_C + 3.5\lambda_D \end{aligned}\right\} \textit{inputs} \\
& \left.\begin{aligned} & \lambda_A + \lambda_B + \lambda_C + \lambda_D = 1 \\ & \lambda_A, \lambda_B, \lambda_C, \lambda_D \ge 0. \end{aligned}\right\} \textit{weights}
\end{aligned}
$$

Similarly, problem (3.5) reads

$$
\text{(3.7)} \quad
\begin{aligned}
& Ieff_D = \max_{\hat{p}_y, \hat{p}_{x1}, \hat{p}_{x2}, f} 5\hat{p}_y + f \\
& s.t. \\
& \left.\begin{aligned} & (8\hat{p}_y + f) - (5\hat{p}_{x1} + 6\hat{p}_{x2}) \le 0 \\ & (3\hat{p}_y + f) - (1\hat{p}_{x1} + 4\hat{p}_{x2}) \le 0 \\ & (5\hat{p}_y + f) - (4\hat{p}_{x1} + 1\hat{p}_{x2}) \le 0 \\ & (5\hat{p}_y + f) - (3.5\hat{p}_{x1} + 3.5\hat{p}_{x2}) \le 0 \end{aligned}\right\} \textit{firms} \\
& \left.\begin{aligned} & 3.5\hat{p}_{x1} + 3.5\hat{p}_{x2} = 1 \\ & \hat{p}_y, \hat{p}_{x1}, \hat{p}_{x2} \ge 0. \end{aligned}\right\} \textit{prices}
\end{aligned}
$$

The optimal solutions to both problems give 0.9121, which means that firm D could reduce both its inputs by 8.79 percent and produce five units of outputs by using 3.192 units of both inputs.

Problem (3.6) finds this reduction potential by assigning weights $\lambda_A^* = 0.23$, $\lambda_B^* = 0.35$, $\lambda_C^* = 0.42$, and $\lambda_D^* = 0$. The optimal prices of the dual problem (3.7) are $\hat{p}_y^* = 0.20, \hat{p}_{x1}^* = 0.21, \hat{p}_{x2}^* = 0.08$, and $f^* = -0.09$.

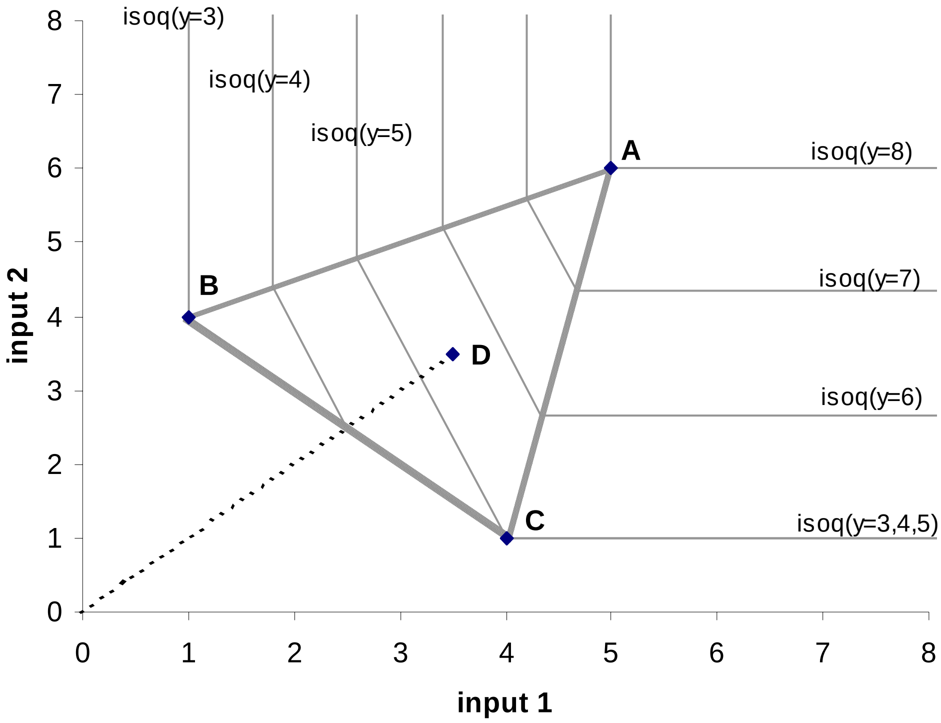

Figure 3: Isoquant map of the numerical example

The example is further illustrated graphically by means of an isoquant map in Figure 3. The horizontal axis represents the quantity of input 1 and the vertical axis represents the quantity of input 2. Points A,B,C,D indicate the input usage of the corresponding firm. Triangle ABC represents the efficient frontier of production possibility set $\hat{T}^{DEA}$, as seen from above from the bird perspective. The isoquant map can be read like an ordinary map. The isoquant lines (i.e., isoq(.)) indicate all input combinations that can produce (according to the DEA model) the indicated output quantity. Point B lies at the lowest level (i.e., firm B produces the smallest amount of output), point A lies on the top of the hill. Since points A, B, and C lie on the isoquants corresponding to the output level of the firm, these firms are all technically efficient. Firm D lies above the input isoquant associated with output level of five. This means that firm D uses an excessive amount of inputs to produce its output and is hence inefficient. Technical efficiency is measured by decreasing both inputs in the same proportions: we move along the broken line from point D towards the origin, until we hit the isoquant line of five units of output and thus achieve full efficiency.

Now that we have a basic understanding about the DEA methodology, we are ready to proceed to environmental applications.

4. Environmental Performance Analysis

The literature of environmental performance (or environmental efficiency) analysis is closely oriented towards production theory and the intellectual roots of DEA. Most environmental applications of DEA adhere to this perspective, which basically just integrates the undesirable outputs into this classic Farrell framework of efficiency analysis. This extension is thematically illustrated by Figure 4 below.

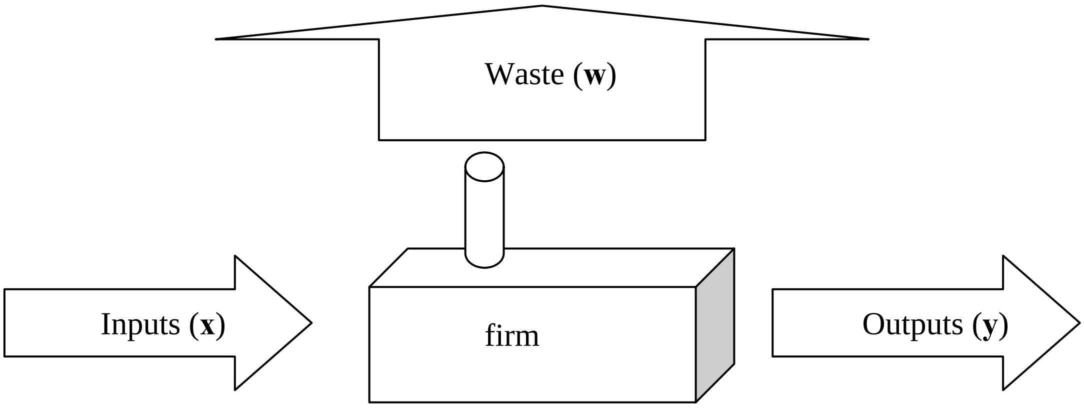

Figure 4: The basic setting of environmental efficiency analysis

It is straightforward to redefine the production technology T such that it takes into account the undesirable side-products and side-effects of production, including the generation of (non-recycled) solid waste, emission of substances to air and water, as well as non-material undesirable outputs such as noise, in addition to solid waste. For brevity, we shall refer to all these undesirable side-effects by "waste", and denote the quantities of these detrimental variables by vector **w**. Integrating waste in the production theory, we redefine the production possibility set as

(4.1) $$T^{ENV} = \{(\mathbf{x}, \mathbf{w}, \mathbf{y}) \mid \text{inputs } \mathbf{x} \text{ can produce outputs } \mathbf{y} \text{ and waste } \mathbf{w}\}.$$

The usual approach is then to measure environmental performance of the firm as a distance to the environmental technology T^{ENV}. There are many different ways of measuring the distance to the frontier, as illustrated by Table 2. Some authors consider reduction of waste, keeping the inputs and outputs constant; others reduce waste and inputs simultaneously, keeping the outputs constant. Also simultaneous changes in all variables have been considered, by means of the so-called hyperbolic and directional distance function measures. In the directional distance function approach we need to choose a pre-determined direction, represented by vectors $(\mathbf{g}^x, \mathbf{g}^w, \mathbf{g}^y)$, along which the firms move towards the efficient frontier.[14]

[14] For further details, see e.g.

- Färe, R., and S. Grosskopf (2004): *New Directions: Efficiency and Productivity*, Kluwer Academic Publishers.

Table 2: Alternative orientations for environmental performance measurement

Orientation of measurement	Formal expression
Environmental	$\min \theta$ s.t. $(\mathbf{x}, \theta\mathbf{w}, \mathbf{y}) \in T^{ENV}$
Input-environmental	$\min \theta$ s.t. $(\theta\mathbf{x}, \theta\mathbf{w}, \mathbf{y}) \in T^{ENV}$
Output-environmental	$\min \theta$ s.t. $(\mathbf{x}, \theta\mathbf{w}, \mathbf{y}/\theta) \in T^{ENV}$
Hyperbolic	$\min \theta$ s.t. $(\theta\mathbf{x}, \theta\mathbf{w}, \mathbf{y}/\theta) \in T^{ENV}$
Directional	$\min \theta$ s.t. $(\mathbf{x} - \theta\mathbf{g}^x, \mathbf{w} - \theta\mathbf{g}^w, \mathbf{y} + \theta\mathbf{g}^y) \in T^{ENV}$

The choice of the efficiency measure essentially depends on the purposes of the application. As a rule of thumb, it is advisable to take variables that the firms cannot influence as given constants, and increase or decrease those variables that are under direct control of the firm. While the directional distance function approach is very general, the interpretation of the resulting efficiency measure can be difficult. Of course, it is also possible to report multiple complementary efficiency indicators.

As for the DEA estimation of the efficient frontier, the linear programming approach easily extends to the more general case that takes into account waste variables **w**. However, the specific treatment of waste as either input or output has attracted considerable debate in this literature. Perhaps the most traditional approach is to model waste (technically) as an input, because both waste and inputs incur costs for the firm, and hence the firm generally tries to avoid the excessive consumption of inputs and the generation of waste.[15] Other authors have argued that waste is technically an output, and hence should be modeled as such. Of course, standard treatment of outputs does not apply. As an answer to this problem, an alternative property of *weak disposability* has been suggested.[16]

Suppose we treat waste as inputs, then free disposability means that if $(\mathbf{x}, \mathbf{w}, \mathbf{y}) \in T^{ENV}$ and $\mathbf{x}' \geq \mathbf{x}, \mathbf{w}' \geq \mathbf{w}, \mathbf{y}' \leq \mathbf{y}$ then also $(\mathbf{x}', \mathbf{w}', \mathbf{y}') \in T$. The free disposability condition for waste **w** is not physically realistic because it implies that a finite quantity of input **x** could (in theory) produce an infinite amount of waste. (Yet, from the economic perspective this need not be a problem as we will argue below.) The alternative weak disposability property is formally defined as follows: if $(\mathbf{x}, \mathbf{w}, \mathbf{y}) \in T^{ENV}$ and $0 \leq d \leq 1$, then $(\mathbf{x}, d\mathbf{w}, d\mathbf{y}) \in T^{ENV}$. In words, it is possible to reduce the original activity level by factor *d*, which simultaneously reduces both outputs and waste by that factor. We next illustrate the difference between the free versus weak disposability perspectives by means of a numerical example.

Since there are a large number of different approaches for measuring environmental performance, and the modifications to the standard DEA formulations are relatively straightforward, we do not present general formulations for the environmental performance measurement but merely illustrate the difference between the use of waste as input and the weak disposability approaches by means of numerical examples. To allow for comparison with Section 3, let us simply modify the previous numerical example involving firms A, B, C, and D such that output 2 of the previous example is considered to be waste, and the same numerical values apply. Table 3 summarizes the data set.

[15] See e.g.:

- Cropper, M.L. and W.E. Oates (1992): Environmental Economics: A Survey, *Journal of Economic Literature* 30, 675-740.
- Hailu, A. and T.S. Veeman (2001): Non-Parametric Productivity Analysis with Undesirable Outputs: An Application to the Canadian Pulp and Paper Industry. *American Journal of Agricultural Economics* 83, 605-616.

[16] The most notable proponents of the weak disposability approach are Rolf Färe and Shawna Grosskopf, see e.g.:

- Färe, R., and S. Grosskopf (2004): *New Directions: Efficiency and Productivity*, Kluwer Academic Publishers.

Table 3: Numerical example with four firms, a single output, a single input and a single waste

	Firm A	Firm B	Firm C	Firm D
Y	8	3	5	5
X	5	1	4	3,5
W	6	4	1	3,5

We will focus on the case of environmental efficiency measure that minimizes the amount of waste at the given level of inputs and outputs (i.e., $\min \theta$ s.t. $(\mathbf{x}, \theta\mathbf{w}, \mathbf{y}) \in T^{ENV}$). Let us first consider that traditional approach of treating the waste as an input. In this case, the DEA distance function problem becomes

$$(4.2) \qquad \begin{aligned} & ENVeff_D = \max_{\theta,\lambda} \theta \\ & s.t. \\ & 5 \le 8\lambda_A + 3\lambda_B + 5\lambda_C + 5\lambda_D \quad \} \textit{output} \\ & 3.5 \ge 5\lambda_A + 1\lambda_B + 4\lambda_C + 3.5\lambda_D \quad \} \textit{input} \\ & 3.5\theta \ge 6\lambda_A + 4\lambda_B + 1\lambda_C + 3.5\lambda_D \quad \} \textit{waste} \\ & \left. \begin{aligned} & \lambda_A + \lambda_B + \lambda_C + \lambda_D = 1 \\ & \lambda_A, \lambda_B, \lambda_C, \lambda_D \ge 0 \end{aligned} \right\} \textit{weights} \end{aligned}$$

This is identical to problem (3.6) above, except that we now only decrease the amount of waste by factor θ, keeping the input at its current level. The dual pricing problem becomes

$$(4.3) \qquad \begin{aligned} & ENVeff_D = \max_{\hat{p}_y, \hat{p}_x, \hat{p}_w, f} 5\hat{p}_y + f - 3.5\hat{p}_x \\ & \left. \begin{aligned} & s.t. \\ & (8\hat{p}_y + f) - (5\hat{p}_x + 6\hat{p}_w) \le 0 \\ & (3\hat{p}_y + f) - (1\hat{p}_x + 4\hat{p}_w) \le 0 \\ & (5\hat{p}_y + f) - (4\hat{p}_x + 1\hat{p}_w) \le 0 \\ & (5\hat{p}_y + f) - (3.5\hat{p}_x + 3.5\hat{p}_w) \le 0 \end{aligned} \right\} \textit{firms} \\ & \left. \begin{aligned} & 3.5\hat{p}_w = 1 \\ & \hat{p}_y, \hat{p}_x, \hat{p}_w \ge 0 \end{aligned} \right\} \textit{prices} \end{aligned}$$

The only difference to the DEA problem (3.7) is that we shift of the input cost $3.5\hat{p}_x$ from the price normalization constraint to the objective function.

The optimal solutions to both problems are equal to 0.6735, meaning that firm D could reduce its waste by 32.65 percent to 2.357 units and produce five units of outputs by using 3.5 units of input. Problem (4.2) finds this reduction potential by assigning weights λ_A^* = 0.14, λ_B^* = 0.21, λ_C^* =0.64, and λ_D^* = 0.

Figure 5 illustrate the case by means of the input-waste isoquant map. Notice that the isoquants identical to those of Figure 3, and thus the efficiency status of every firm remains the unchanged. The only difference lies in the direction of measurement. We now only decrease amount of waste generated by the inefficient firm D, keeping the input level fixed. Thus, we move from point D downwards towards the *x* axis, until we reach the isoquant corresponding to output level of five units.

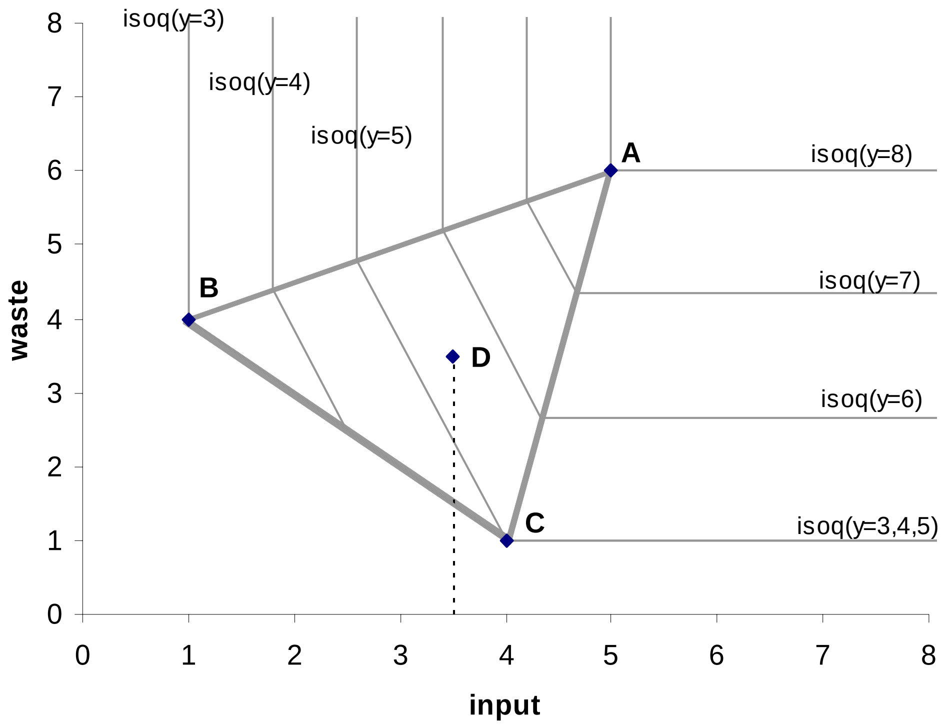

Figure 5: Isoquant map of the numerical example

The isoquants illustrate the theoretical problem with free disposability: a finite amount of input could generate an infinite amount of waste, which is a physical impossibility. In practice, however, this hardly matters because we are usually interested of decreasing the amount of waste anyway. Whatever happens at the backward bending part of the true isoquant is not economically very interesting.

Let us next consider the alternative approach of treating the waste as an output by assuming weak disposability. Since the DEA production set is in mathematical sense a convex closure of its extreme points, the simplest and the most effective way of implementing the weak disposability is to add to the data set the new extreme points that arise if the firms would exploit the weak disposability property.[17] For example, firm A, could ultimately stop the production, that is, use five units input to produce zero units of waste and zero units of output. Let us label this "inactive" variant of firm A by lower case *a*. Similarly, the inactive reincarnations of firms B, C and D that dispose the inputs and produce nothing are labeled as *b*, *c* and *d*, respectively. Introducing these inactive pseudo-firms, our DEA model becomes

[17] To the best of our knowledge, the inactive firms have not been taken into account in this literature before. The DEA formulations which build on the weak disposability property usually only relax the free disposability condition, but fail to exploit the weak disposability assumption in full.

(4.4)

$$\begin{aligned}
&\max_{\theta,\lambda} \theta \\
&s.t. \\
&5 \le 8\lambda_A + 3\lambda_B + 5\lambda_C + 5\lambda_D + 0\lambda_a + 0\lambda_b + 0\lambda_c + 0\lambda_d \quad \} \textit{output} \\
&3.5 \ge 5\lambda_A + 1\lambda_B + 4\lambda_C + 3.5\lambda_D + 5\lambda_a + 1\lambda_b + 4\lambda_c + 3.5\lambda_d \} \textit{input} \\
&3.5\theta = 6\lambda_A + 4\lambda_B + 1\lambda_C + 3.5\lambda_D + 0\lambda_a + 0\lambda_b + 0\lambda_c + 0\lambda_d \} \textit{waste} \\
&\left.\begin{aligned}&\lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_a + \lambda_b + \lambda_c + \lambda_d = 1 \\ &\lambda_A, \lambda_B, \lambda_C, \lambda_D, \lambda_a, \lambda_b, \lambda_c, \lambda_d \ge 0.\end{aligned}\right\} \textit{weights}
\end{aligned}$$

Note that free disposability of waste is now eliminated by writing the waste constraint in equality form. The opportunities for weak disposability are accounted for by taking convex combinations of both existing, observed firms and inactive pseudo-firms. These are the two key differences between formulations (4.2) and (4.4).

The dual pricing problem is simply obtained by introducing the inactive firms in problem (4.3), specifically:

(4.5)

$$\begin{aligned}
&\max_{\hat{p}_y, \hat{p}_x, \hat{p}_w, f} 5\hat{p}_y + f - 3.5\hat{p}_x \\
&s.t. \\
&\left.\begin{aligned}
&(8\hat{p}_y + f) - (5\hat{p}_x + 6\hat{p}_w) \le 0 \\
&(3\hat{p}_y + f) - (1\hat{p}_x + 4\hat{p}_w) \le 0 \\
&(5\hat{p}_y + f) - (4\hat{p}_x + 1\hat{p}_w) \le 0 \\
&(5\hat{p}_y + f) - (3.5\hat{p}_x + 3.5\hat{p}_w) \le 0
\end{aligned}\right\} \textit{firms} \\
&\left.\begin{aligned}
&(0\hat{p}_y + f) - (5\hat{p}_x + 0\hat{p}_w) \le 0 \\
&(0\hat{p}_y + f) - (1\hat{p}_x + 0\hat{p}_w) \le 0 \\
&(0\hat{p}_y + f) - (4\hat{p}_x + 0\hat{p}_w) \le 0 \\
&(0\hat{p}_y + f) - (3.5\hat{p}_x + 0\hat{p}_w) \le 0
\end{aligned}\right\} \textit{inactive firms} \\
&\left.\begin{aligned}
&3.5\hat{p}_w = 1 \\
&\hat{p}_y, \hat{p}_x \ge 0.
\end{aligned}\right\} \textit{prices}
\end{aligned}$$

The two key differences between problems (4.3) and (4.5) are the introduction of inactive firms to the production possibility set and the relaxation of the non-negativity constraint for the price of waste. In general, prices of some waste variables can now be negative, but the total cost of waste must still add up to one. Since here we have only one waste, the relaxation of the non-negativity constraint does not have any effect, which confirms the earlier conclusion that when efficiency is measured in terms of waste reduction, the free disposability assumption is not that restrictive after all.

Numerical calculations verify that the input efficiency does not change for any of these four firms when weak disposability of waste is assumed. This, however, is not a general result. Let us compare the input-waste isoquant map of Figure 5 with the map that is based on weak disposability, as presented in Figure 6. Introducing the inactive firm *b*, we get a new efficient frontier segment represented by triangle bBC. This changes the shape of all isoquants corresponding to output levels of less than five units. For firm D there is no effect since it operates at the output level of five units; the weak disposability would have an effect on efficiency of firm D if it produced less than five units of output. The other obvious difference between Figures 5 and 6 is that in Figure 6 the isoquants do not raise up to infinity, but stop after the line-segment AB, continuing horizontally under the frontier towards infinite input levels (as free disposability of inputs is still assumed). (To keep the figure readable, the horizontal segments of isoquants that fall below the frontier are omitted.) It should also be noted that any combinations between points a and A, d and D, and c and C are also technically feasible. We have not connected these points by lines simply because these lines lie in the interior of the production possibility set (as line aA) or do not contribute to reshaping the frontier (as line cC).

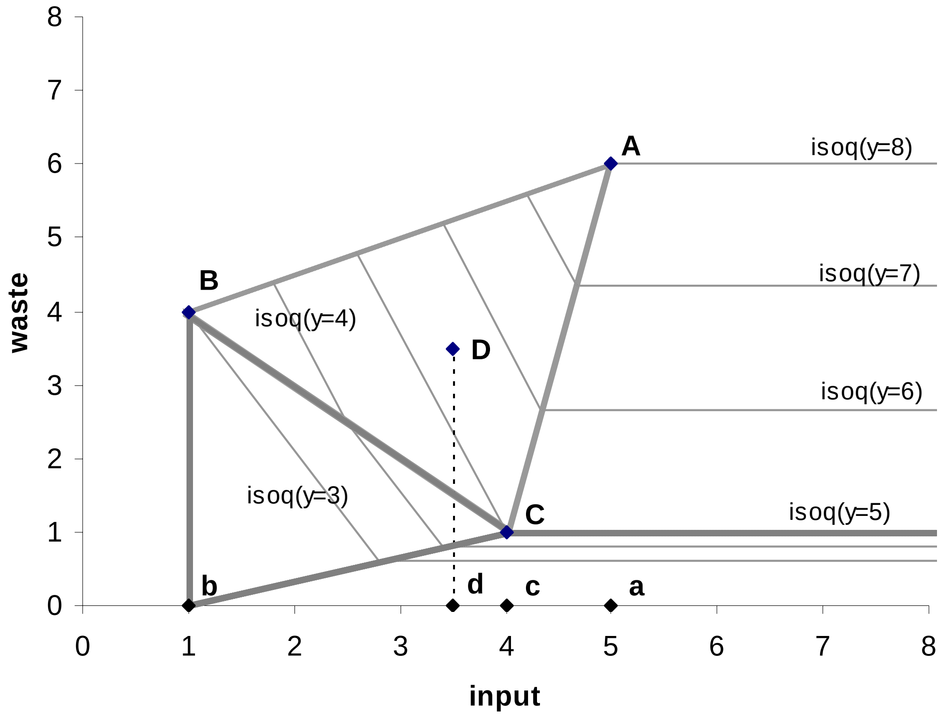

Figure 6: Isoquant map of weakly disposable DEA technology

Observe that the environmental performance analysis defines efficiency in purely technical terms, without taking any reallocation possibilities into account. The efficiency measures considered thus far only capture the waste reduction potential which can be achieved virtually free of charge by improving operational efficiency. These measures do not necessarily measure how "environmentally friendly" the firms are. For example, firm B that produces a relatively large amount of waste per unit of output is considered to be efficient, since reduction of waste would require further input usage and thus increase the costs for the firm. Still, firm B can hardly be viewed as an environmentally friendly company; saving inputs is not a valid excuse for polluting the environment. By these observations, we next turn to applying the DEA tools to eco-efficiency analysis.

5. Eco-efficiency analysis

Environmental sustainability is a widely acknowledged as major challenge for today's world. However, making transparent decisions on development and investment with an optimal balance for economy, ecology and social development is not self-evident. The concept of sustainability is so vaguely defined that no sound, generally accepted operational methods for quantifying sustainability currently exist. Therefore, it seems practical to focus on quantifying more narrowly confined components of sustainability. Ecological efficiency or eco-efficiency is one such component.

Eco-efficiency is usually defined as the ratio of economic value added to the index of environmental pressures:

$$(5.1) \qquad \textit{Eco-efficiency} = \frac{\textit{Economic value added}}{\textit{Environmental pressures}}.$$

Before we can measure this ratio, we have to take a more detailed look at the numerator and the denominator of this ratio.

Let us remain in the context of production, as in previous sections. For firm *k*, the economic value added V_k is simply the profit:

$$(5.2) \qquad V_k = \sum_{m=1}^{M} p_{km}^{y} Y_{km} - \sum_{l=1}^{L} p_{kl}^{x} X_{kl} .$$

Note that economic value added encompasses the technical efficiency of producing maximal output with given inputs, as well as allocative or economic efficiency of producing the right mix of outputs with the least expensive combination of inputs. While traditional DEA analysis is typically focused on situations where input-output prices are unknown, we here assume that the total value added is known for all firms in the sample, even if the specific prices might be unknown to the analyst. More detailed data of individual inputs and outputs (or their prices), as assumed in standard DEA, is unnecessary in this section.

As for the denominator of (5.1), we will deviate from most DEA approaches in that we focus on *environmental pressures* rather than specific undesirable outputs per se. Undesirable outputs of production might include air emissions such as carbon-dioxide and methane. Both these emissions contribute to the same environmental problem: the green house effect. Numerous studies have investigated the effects of different green house gases, and conversion factors are available for translating the amounts of different green house gases into carbon-dioxide equivalents. Since we are ultimately concerned about the green house effect rather than the amount of carbon-dioxide in the atmosphere per se, and since different green house gases can be aggregated based on scientifically sound conversion factors, we believe it is most appropriate to use aggregated measures of environmental themes such as climate change as inputs for the eco-efficiency analysis.

The aggregated carbon-dioxide equivalents do not adequately capture the environmental impact, measured by the social costs of climate change. Rather, such measures only represent the pressure on the ecosystem. To a certain extent, the forests are capable of sequestrating the extra carbon-dioxide emitted to the atmosphere. The problem occurs when the green house gas emissions exceed the carrying capacity of the ecosystem, and extra carbon-dioxide stocks start to accumulate causing drastic, unpredictable changes in climate conditions. The relationship between the environmental pressure and the environmental impact is often complex, nonlinear, and difficult to predict.

Moreover, it seems practically impossible to attribute the effects of climate change (such as loss of life due to heavy storms or flooding) to specific firms that have emitted a certain amount of green house gases. Therefore, we do not attempt to measure the ultimate environmental impacts, but find it most appropriate to work at the level of environmental pressures.

The most significant problem in eco-efficiency measurement concerns the aggregation of various environmental pressures. Like in the case of green house gases, we believe that it is often possible and meaningful to aggregate individual pollutants that contribute to the same environmental theme in the same aggregate measure for the overall environmental pressure using some *a priori* conversion factors. By contrast, pressures on different environmental themes referring to different types of environmental deterioration or resource depletion are incommensurable, meaning that these effects that cannot be aggregated unanimously by using objective conversion factors.

To illustrate the relationship between "environmental pressures" and "pollutants", consider the main environmental pressures due to road transportation listed in Table 4. Some environmental effects of road transportation (e.g. dispersion of particles) are directly measurable by a single indicator, while other effects (e.g. climate change, acidification) are influenced by several emissions. To assess a given effect, different undesirable outputs arising in the production activity can often be aggregated by using well-defined conversion factors. For example, different types of air emissions contribute to the climate change, but their effects can be fairly accurately summarized by converting different emissions to CO_2 equivalents. By contrast, there is no unambiguous way of summarizing all the different environmental pressures in a single overall environmental index. For example, we cannot simply add green-house gases measured in CO_2 equivalents to particle emissions measured in tons of TPM. Moreover, it is very difficult (if not impossible) to express some generally accepted weights that would reflect the relative importance of the effects. While this example pertains to the case of road transportation, which in industrialized countries is one of the main sources of air emissions, the similar type of aggregation possibilities and problems are faced equally well in other industries and at all levels of aggregation.

Table 4: The main environmental pressures due to road transportation

Environmental pressure	Specific emission /resource	Unit of measurement
Climate change	CO_2, CH_4, N_2O, CO	CO_2 equivalents
Acidification	NO_x, SO_2	acid equivalents
Smog formation	HC	tons of HC
Dispersion of particles	TPM	tons of TPM
Degradation of natural resources	Oil	tons of gasoline
Noise	Urban noise level	dB

To overcome this incommensurability problem, we propose to apply the DEA approach. To this end, it is necessary to introduce some notation. Suppose the production activity under consideration induces R different environmental pressures, the severity of which is measured by variables $\mathbf{z} = (z_1 \ldots z_R)$. For simplicity, all environmental pressures are assumed to be harmful (i.e., $\mathbf{z} \geq 0$). We will assume that these pressures can be unambiguously quantified, and the pressures caused by firm k can be numerically represented by vector $\mathbf{Z}_k = \left(Z_{k1} \ldots Z_{kR}\right)'$. As before, we shall refer to observed data for firms by capital symbols V_k, $\mathbf{Z}_k$, and reserve the lower case symbols v, and $\mathbf{z}$ for arbitrary (theoretical) values.

Even though we do not observe prices for these environmental pressures, those prices do exist (in Platonian world of ideas). These unknown prices are defined as the marginal social cost of the impact, and are represented by the vector $\mathbf{p} = \left(p_1, ..., p_R \right)'$. The problem of how to estimate the unknown unobservable prices **p** has been one of the key issues in environmental economics. We here deviate from the conventional approaches in that we do not try to "parameterize" the prices based on stated or revealed preferences, but instead, treat the prices as model variables as in standard DEA.

Given the more precise notation, eco-efficiency ratio (5.1) can now be formally expressed in absolute terms as

$$(5.3) \qquad EE_k \equiv \frac{V_k}{\sum_{r=1}^{R} p_r Z_{kr}}.$$

Note that this absolute value is not very informative as such: if eco-efficiency of firm k is 3.67, then how should we interpret that? Is firm k environmentally friendly or not? To interpret the eco-efficiency score, we have to compare it with the best performers in the sector. To this end, we introduce the notion of *relative eco-efficiency* as the ratio of eco-efficiency measure (5.3) to the maximum observed eco-efficiency in the sample, formally defined as

$$(5.4) \qquad E/E_k \equiv \frac{EE_k}{\max\limits_{n \in \{1,...,N\}} EE_n}.$$

Now we are ready to apply the DEA tools for the pricing of the environmental pressures. We may normalize prices **p** such that the maximum eco-efficiency ratio becomes equal to unity. Applying the DEA weighting, we obtain the following optimization problem

$$(5.5) \qquad \begin{aligned} &\widehat{E/E}_k = \max_{\hat{\mathbf{p}}} \frac{V_k}{\hat{p}_1 Z_{k1} + \hat{p}_2 Z_{k2} + ... + \hat{p}_R Z_{kR}} \\ &s.t. \\ &\frac{V_1}{\hat{p}_1 Z_{11} + \hat{p}_2 Z_{12} + ... + \hat{p}_R Z_{1R}} \le 1 \\ &\frac{V_2}{\hat{p}_1 Z_{21} + \hat{p}_2 Z_{22} + ... + \hat{p}_R Z_{2R}} \le 1 \\ &\vdots \\ &\frac{V_N}{\hat{p}_1 Z_{N1} + \hat{p}_2 Z_{N2} + ... + \hat{p}_R Z_{NR}} \le 1 \\ &\hat{p}_1, \hat{p}_2, ..., \hat{p}_R \ge 0. \end{aligned}$$

The objective function is the eco-efficiency measure of firm k at the normalized prices $\hat{\mathbf{p}}$. Observe that the price normalization does not change the relative eco-efficiency measure (5.4). If both the numerator and denominator change by factor g, then these changes cancel out as we take the ratio of the two. The first N constraints guarantee that the eco-efficiency measures of all N firms in the sample must be less than or equal to one. Thus, the objective function directly indicates the value of relative eco-efficiency. Like in traditional DEA, the valuation is based on the most favorable prices for firm k; prices that maximize the eco-efficiency ratio. If this "benefit-of-the-doubt" eco-efficiency ratio is low, then we can be sure that the 'true' eco-efficiency (in terms of unknown prices **p**) must be even lower. If this eco-efficiency ratio is high, however, it does not necessarily guarantee that the true efficiency is high.

The technical difficulty with problem (5.5) is that it now includes ratios (or fractions) both in the objective function and in the constraints. The problem is thus nonlinear with regard to shadow prices, which makes it computationally demanding. However, this difficulty is easily eliminated by solving the reciprocal (or inverse) problem (exploiting the fact that economic benefits V_k are simply given constants):

(5.6)
$$
\begin{aligned}
&\widehat{E/E}_k^{-1} = \min_{\hat{\mathbf{p}}} \frac{1}{V_k}(\hat{p}_1 Z_{k1} + \hat{p}_2 Z_{k2} + \ldots + \hat{p}_R Z_{kR}) \\
&s.t. \\
&\frac{1}{V_1}(\hat{p}_1 Z_{11} + \hat{p}_2 Z_{12} + \ldots + \hat{p}_R Z_{1R}) \geq 1 \\
&\frac{1}{V_2}(\hat{p}_1 Z_{21} + \hat{p}_2 Z_{22} + \ldots + \hat{p}_R Z_{2R}) \geq 1 \\
&\vdots \\
&\frac{1}{V_N}(\hat{p}_1 Z_{N1} + \hat{p}_2 Z_{N2} + \ldots + \hat{p}_R Z_{NR}) \geq 1 \\
&\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_R \geq 0.
\end{aligned}
$$

The eco-efficiency measure $\widehat{E/E}_k$ is obtained by taking the inverse of the optimal solution to (5.6). This measure has a clear interpretation as a degree of efficiency: the value of $\widehat{E/E}_k$ lies between zero and one, and the higher the value, the more eco-efficient the firm. The result $\widehat{E/E}_k = 1$ implies the best eco-efficiency in the sample.

In analogy with the previous DEA formulations, also (5.6) has an equivalent dual formulation, which can be written as

(5.7)
$$
\begin{aligned}
&\widehat{E/E}_k = \min_{\theta,\boldsymbol{\lambda}} \theta \\
&s.t. \\
&\left.\begin{aligned}
&\theta Z_{k1} \geq \lambda_1 Z_{11} + \lambda_2 Z_{21} + \ldots + \lambda_N Z_{N1} \\
&\theta Z_{k2} \geq \lambda_1 Z_{12} + \lambda_2 Z_{22} + \ldots + \lambda_N Z_{N2} \\
&\vdots \\
&\theta Z_{kR} \geq \lambda_1 Z_{1R} + \lambda_2 Z_{2R} + \ldots + \lambda_N Z_{NR}
\end{aligned}\right\} \textit{pressures} \\
&V_k \leq \lambda_1 V_1 + \lambda_2 V_2 + \ldots + \lambda_N V_N \quad \} \textit{value added} \\
&\lambda_n \geq 0 \ \forall n = 1, \ldots, N.
\end{aligned}
$$

It is illustrative to compare this linear programming problem with the input oriented DEA problem (3.3) discussed in Section 3. From technical point of view, we can interpret (5.7) as the standard DEA input efficiency problem which uses environmental pressures as inputs and economic value added as output. The only technical difference we observe is the fact that weights $\boldsymbol{\lambda}$ are required to sum to one in (3.3) but not in (5.7). This means that the size of the firm, measured by absolute levels of value added and environmental pressures, does not matter in eco-efficiency problem (5.7);

we are only interested about the ratio of the value added to the environmental pressure. In DEA literature (5.7) is interpreted as a constant returns to scale model.

Notwithstanding these technical similarities, the economic interpretation of (5.7) bears many important differences to standard DEA as well as to its applications to environmental performance analysis. Firstly, this measure does not only capture differences in operational efficiency across firms, it also captures allocative efficiencies – how valuable the goods produced and the inputs consumed are at the market. Secondly, the severity of different emissions is accounted for by aggregating emissions that contribute to environmental theme r into the pressure indicator z_r based on scientifically sound conversion factors. Thus, our measure accounts not only the quantity of emissions but also their quality. Thirdly, the DEA weighting is here applied not to the technical problem of estimating the production possibility frontier, but the economic problem of finding prices for the environmental pressures. Although we can construct a set of feasible ($\mathbf{z}$,v) combinations and measure efficiency as a distance to the frontier of that set, the mathematical structure of problem (5.7) does not arise from technical assumptions concerning the shape of that set. The constant returns to scale property of that set is not imposed by assumption; it arises from the definition of eco-efficiency as a ratio in (5.1). Convexity is not imposed by assumption; it arises from the economic structure of our environmental damage indicator defined as the economic cost of environmental pressure (i.e., using $\sum_{r=1}^{R} p_r Z_{kr}$ in (5.3)). Finally, free disposability is not imposed; it follows from the assumption that every environmental pressure has a non-negative economic cost (i.e., $\mathbf{p} \geq 0$).

Let us illustrate the presented technique by considering a simple example from the area of road transportation. For simplicity, let us assume that there are only three cities whose eco-efficiency has to be evaluated. Table 5 presents the data of economic value added and four environmental pressures. In the case of road transportation, there is not self-evident measure for the net economic benefit. One good candidate is the measure V_k = mileage price (€/km) × total road transportation mileage (km) – fuel price (€/l) × fuel consumption (l). This measure yet demands information about mileage and fuel prices.[18] The environmental pressures can be estimated based on more detailed data of different emissions caused by a certain type of vehicle and the data concerning the vehicle fleet and the mileage. As discussed above, different emissions are aggregated to environmental pressures using well established conversion factors.

Table 5: Example data related to eco-efficiency of road transportation in three cities

	City 1	**City 2**	**City 3**
Economic value added			
Net economic benefit (thousand dollars)	100	80	140
Environmental pressures:			
Climate change (CO_2 equivalents/year)	1200	300	2600
Acidification (acid equivalents/year)	39	10	44
Smog formation (tons of HC/year)	42	4	23
Dispersion of particles (tons of TPM/year)	143	20	87

Consider the eco-efficiency measure of City 3. Eco-efficiency problem (5.6) becomes in this case

[18] In principle, these prices can also be estimated (or replaced by shadow prices).

(5.8)

$$\widehat{E/E_3}^{-1} = \min_{\hat{\mathbf{p}}} \frac{1}{140}(2600\hat{p}_1 + 44\hat{p}_2 + 23\hat{p}_3 + 37\hat{p}_4)$$
$$s.t.$$
$$\frac{1}{100}(1200\hat{p}_1 + 39\hat{p}_2 + 42\hat{p}_3 + 143\hat{p}_4) \geq 1$$
$$\frac{1}{80}(300\hat{p}_1 + 10\hat{p}_2 + 4\hat{p}_3 + 20\hat{p}_4) \geq 1$$
$$\frac{1}{140}(2600\hat{p}_1 + 44\hat{p}_2 + 23\hat{p}_3 + 37\hat{p}_4) \geq 1$$
$$\hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4 \geq 0$$

and the dual problem (5.7) reads as

(5.9)

$$\widehat{E/E_3} = \min_{\theta,\boldsymbol{\lambda}} \theta$$
$$s.t.$$
$$2600\theta \geq 1200\lambda_1 + 300\lambda_2 + 2600\lambda_3$$
$$44\theta \geq 39\lambda_1 + 10\lambda_2 + 44\lambda_3$$
$$23\theta \geq 42\lambda_1 + 4\lambda_2 + 23\lambda_3$$
$$87\theta \geq 143\lambda_1 + 20\lambda_2 + 87\lambda_3$$
$$140 \leq 100\lambda_1 + 80\lambda_2 + 140\lambda_3$$
$$\lambda_1, \lambda_2, \lambda_3 \geq 0.$$

Table 6 reports the optimal solutions to both these problems, as well as the similar problems for the other two cities. This table shows that city 2 is the only eco-efficient city in this sample. However, this does not yet mean that city 2 is absolutely eco-efficient, since eco-efficiency scores are relative. Hence, there may be some other cities, which were not included in this sample, but are more eco-efficient compared to city 2. In any case, cities 1 and 3 have considerable room for improvement in their eco-efficiency ratios, compared with city 2.

Table 6: Optimal solutions

	City 1	**City 2**	**City 3**
Eco-efficiency	0.321	1.000	0.402
Prices $\hat{\mathbf{p}}^*$	$\hat{p}_1^* = \hat{p}_3^* = \hat{p}_4^* = 0,$	$\hat{p}_1^* = \hat{p}_2^* = \hat{p}_4^* = 0,$	$\hat{p}_1^* = \hat{p}_2^* = \hat{p}_3^* = 0,$
	$\hat{p}_2^* = 7.96$	$\hat{p}_3^* = 20$	$\hat{p}_4^* = 0.02$
Weights $\boldsymbol{\lambda}^*$	$\lambda_1^* = \lambda_3^* = 0, \lambda_2^* = 1.25$	$\lambda_1^* = \lambda_3^* = 0, \lambda_2^* = 1$	$\lambda_1^* = \lambda_3^* = 0, \lambda_2^* = 1.75$

In the pricing problem (5.8) cities assign a positive price for themes in which they perform relatively well, assigning price of zero for those themes they perform poorly. Such extremely "unrealistic" valuations often occur in DEA models. Note, however, that the eco-efficiency scores of cities 1 and 2 can only further decrease if more balanced relative prices are applied. This example demonstrates that resorting to the 'extremist' DEA pricing approach may suffice to reveal significant inefficiencies without resorting to expensive valuation studies to find out more "realistic" economic values.

If certain relative prices are considered unacceptable for the purposes of the analysis, it is straightforward to exclude such prices from the DEA valuation by imposing additional linear constraints (called weight restrictions in the DEA literature). Since only the relative prices matter, the price restrictions must also be formulated in relative terms. For example, if we want to impose (just hypothetically) that the pressure on climate change is always at least twice as costly as pressure on acidification, then this constraint should be expressed as $\hat{p}_1 \geq 2\hat{p}_2$. It does not make sense to impose absolute restrictions (such as $\hat{p}_2 \geq 1$) because we work with normalized prices; the absolute level of prices does not have any meaning, only the relative price level matters.

The relative valuation is one of the key ideas behind the success of DEA. It works well in the context of eco-efficiency analysis where we are primarily interested in the economic value added per environmental pressure types of ratio measures. However, absolute prices of environmental impacts are of decisive importance in many type of environmental-economic analyses, such as Cost-Benefit Analysis. Can DEA approach work in the context of absolute pricing problems? This is what we are going to find out next.

6. Cost-Benefit Analysis

Environmental Cost-Benefit Analysis (CBA) typically concerns social evaluation of an investment project that involves significant environmental impacts, for example, construction of a new highway. Suppose there are *N* alternative candidate projects (e.g., different highway configurations). In contrast to the eco-efficiency analysis, which is especially useful for revealing improvement potential in the most poorly performing units, CBA focuses on identifying one optimal project (or the optimal combination of projects) to be implemented. Without a loss of generality, we will henceforth assume projects to be mutually exclusive, that is, only one of the projects can be implemented.[19]

A CBA consists of a multiple stages, which usually include:

1) the problem definition (i.e., what are the objectives, what are the alternatives, whose welfare is considered, and over what time period),
2) identification of the physical impacts of each project (i.e., environmental impact analysis), valuation of the impacts,
3) discounting of cost and benefit flows,
4) selection of the project to be implemented based on the net present value test, and
5) sensitivity analysis (i.e., is the result robust to small changes in parameter values).

In this paper the focus will be on the valuation stage, which we consider to be the most challenging stage of the analysis.

Assume that stages 1) and 2) have been completed: the setting has been defined and the economic costs and benefits of each project have been estimated. Let the net economic benefit of project *n* in time period *t* be denoted by B_{nt}. The net benefit is the difference of economic revenues and costs; it has a positive value in the periods where the total revenue exceeds the total cost, and a negative

[19] If combinations of several projects can be implemented, we can always treat such combinations as ”new” alternative candidate projects. For example, if three projects A, B, C and any combinations thereof are possible, we can treat the model combinations (A, B, C, AB, AC, ABC) as mutually exclusive “projects”.

value when the costs exceed revenues. Suppose further that there are M relevant environmental impacts that should be considered. We will assume that the environmental impacts can be unambiguously quantified, and the impacts of project n in period t can be numerically represented by vector $\mathbf{Z}_{nt} = \left(Z_{1nt} \ldots Z_{Mnt} \right)'$. As before, we shall refer to quantified data of the project n by capital symbols B_{nt}, $\mathbf{Z}_{nt}$, and reserve the lower case symbols b, and z for arbitrary (theoretical) values net benefits and environmental impacts, respectively.

Before proceeding, the meaning of "environmental impact" is worth elaborating. By impact we here refer both to direct impacts do to the project (for example, loss of forest land due to highway, extinction of certain species) as well as pressures that contribute indirectly and over longer time scale to environmental problems (for example, emission of green house gases, depletion of natural resources). By impact we also refer to broader environmental themes such as acidification, not to specific substances that cause it. In case of acidification, for example, the different emissions (e.g. nitrogen oxides, sulphur dioxide) should be first converted to acid equivalents and then summed together to get an overall measure for the acidification pressure due to the project. Some harmful substances may contribute to several impacts, for example, carbon monoxide from traffic has direct health effects in humans and it also contributes to the climate change in the atmosphere.

As in the previous sections, we denote the unknown prices for the environmental impacts by $\mathbf{p} = \left(p_1,..., p_M \right)$. The problem of how to estimate these prices has been one of the key issues in environmental economics, and constitutes the stage 3) of the usual CBA routine. We here deviate from the conventional approaches in that we do not try to "parameterize" the prices based on stated or revealed preference information, but rather treat the prices as unknown variables of the model. Therefore, we next proceed to stage 4) and leave the determination of prices to later stages.

Usually the economic benefits and environmental impacts vary over time. Discounting the costs and benefits that occur over time, to express them in net present value terms, is important because most project have considerable economic set-up costs while the benefits and the environmental impacts accumulate over a longer period. For example, a lump sum payment of one million dollar today is worth more than a million dollars of benefits accumulating over the next ten years due to the opportunity cost of the foregone interest revenue. Discounting forms the step 4) of the usual CBA routine.

Denoting the interest rate by r, the net present value of the economic benefits of project n can be calculated as

(6.1) $$NPV(B_n) = \sum_{t=0}^{\infty} (1+r)^{-t} B_{nt}.$$

Similarly, the net present value of the environmental costs of project n can be calculated as

(6.2) $$NPV(C_n) = \sum_{t=0}^{\infty} \sum_{m=1}^{M} (1+r)^{-t} p_m Z_{nmt}.$$

If the prices of the environmental impacts (**p**) are constant over time (as we assume here), then we may first discount the impacts and make the conversion to economic costs later. Observe that identity (6.1) can be re-written as

(6.3) $$NPV(C_n) = \sum_{m=1}^{M} p_m \left(\sum_{t=0}^{\infty} (1+r)^{-t} Z_{nmt} \right).$$

The sum expressed in the parentheses is the discounted total environmental impact *m* for project *n*. Note that although we discount the physical impacts, the environmental impacts of the future are considered to be equally valuable as environmental impacts of today (i.e., we do not assume any time preference for the environmental impacts). The rationale for the discounting lies in the necessity to discount the monetary costs due to the opportunity cost of the foregone interest. As equation (6.2) shows, discounting costs or impacts yields the same net present value when the price vector p is constant over time. For consistency, the same interest rate *r* should be applied in discounting of both economic benefits and environmental impacts.

The discounting stage 4) provides us with the discounted total environmental impacts denoted by Z_{nm} (i.e., the time index *t* is eliminated). Similarly, we use B_n for the total (discounted) net present value of the net economic benefits. If B_n has a negative value, then project *n* does not make economic sense even if we disregard the environmental impacts. Such projects can be safely discarded at this stage. All remaining candidate projects are assumed to yield a strictly positive net economic benefit.

We are now ready to proceed to the most interesting stage of the analysis, namely, the selection of the socially optimal project to be implemented. First, however, we further illustrate the setting by considering a stylized example from the area of transportation.

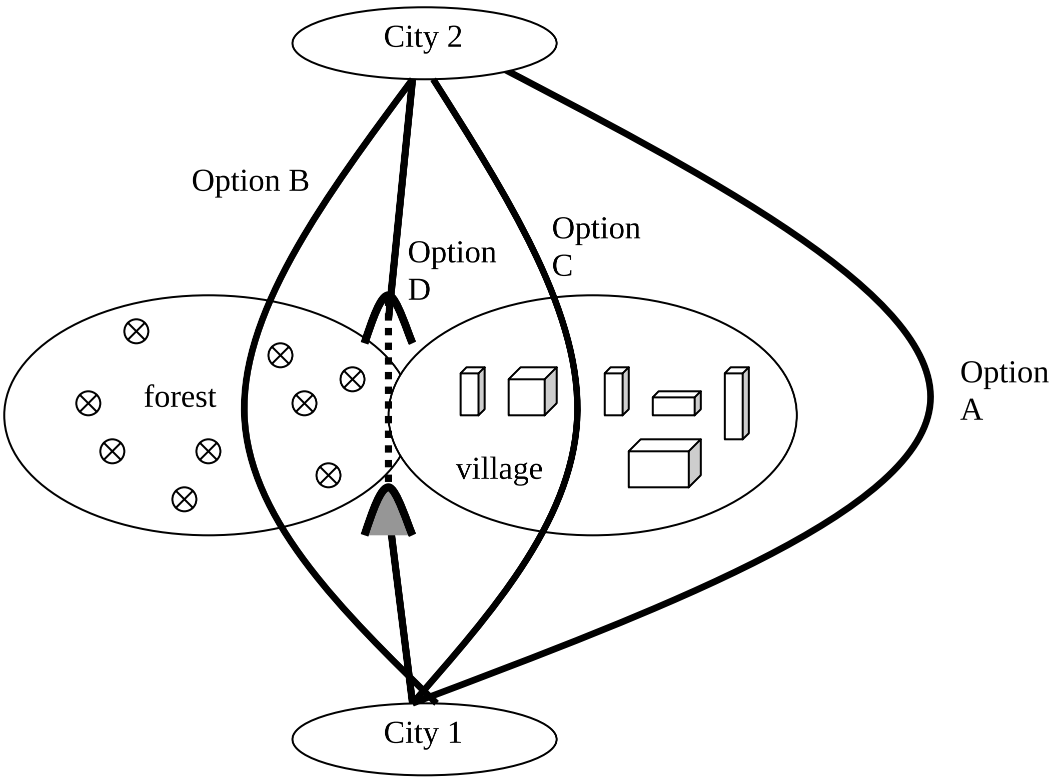

Figure 7: Illustration of the alternative highway configurations

Suppose for simplicity that the municipal authorities consider implementing one of the four alternative highway projects labeled as A, B, C, and D connecting two major cities. Option A is to improve the existing highway, and hence involves only small costs and benefits. Option B is to build a new, more direct highway connection through a pristine forest that lies between the two cities. This option has high economic benefits but large environmental costs. Option C is to by-pass the forest and build the road through urban areas. This option has almost equally high economic benefits but involves a high cost of deteriorating urban environment. Option C is to build a tunnel for the critical part of the highway. This option has high economic costs and benefits but smaller environmental impacts. The alternative highway configurations are illustrated by Figure 7.

The environmental impact assessment indicates only two major environmental impacts that should be considered: the loss of forest-land and the deterioration of urban air quality. The urban air quality is influenced by a number of harmful substances such as particles and carbon monoxide, as well as noise and the risk of traffic accidents. To keep the example simple, we here assume that these diverse human effects can be aggregated by using meaningful and well-defined conversion factors (in reality, keeping such diverse impacts disaggregated would be advisable).

As noted already many times in the previous sections, the results of the economic and environmental impact assessments are summarized in Table 7. We have aggregated the economic costs and revenues to net present value of the benefits. In environmental impacts, we express the loss of forest-land in hectares and the deterioration in urban air quality as an index where zero is the current value. For the urban air quality we take into account the estimated number of people affected, the seriousness of effect, and then applied the same discount factors as for economic net benefits.

The key problem is that the economic benefits and environmental impacts are not further commensurable. Standard approach of the cost-benefit analysis is to use either stated preference techniques (e.g. contingent valuation) or revealed preference techniques (e.g. travel cost method) to estimate "economic" prices for the environmental impacts for which market prices do not exist. Alternative approaches include multi criteria analysis and participatory methods. We next examine how the valuation problem can be addressed using DEA.

Table 7: Results of the economic and environmental impact assessments

	A: Improve the existing highway	B: New highway through the forest	C: New highway through an urban area	D: New highway through a tunnel
Discounted economic effects				
Benefits (time saving, etc)	50	200	190	250
- Costs (construction and maintenance)	-10	-30	-60	-200
= Net economic benefit (mill. dollars)	= 40	= 170	= 130	= 50
Total discounted environmental effects:				
1. Loss of forest land (ha)	1	120	2	10
2. Deterioration of urban air quality	2	3	60	12

The net social benefit of project *n* (SB_n) is the monetary benefit that is left after subtracting the cost of environmental impacts from the net economic benefits (both expressed in terms of the net present value). Formally, SB_n can be expressed as

(6.4) $$SB_n \equiv B_n - \sum_{m=1}^{M} p_m Z_{nm}.$$

In stage 4) we need to identify the project that offers the highest net social benefit. Now the price variables p must be determined.

In the present context it is illustrative to view the DEA method from the game-theoretic perspective. Let us evaluate project *k*. Suppose the proponents of project *k* exhibit opportunistic, strategic behavior. Suppose further that the project proponents can order a bogus valuation study where they can manipulate the price estimates $\hat{\mathbf{p}}$ to show the project *k* in the best possible light (e.g. by paying bribes for the respondents). How would such aggressively opportunistic project proponents valuate the environmental impacts? What is the maximum comparative advantage that the proponents of project *k* can demonstrate over competing projects if they could choose the prices $\hat{\mathbf{p}}$ at will?

While the previous questions are extremely cynical, we believe these questions are still worth asking. The answers to these questions can guide us to more objective policy recommendations in the sense that subjective valuation of prices **p** is not required. In particular, if the most aggressively opportunistic project proponents cannot demonstrate their project to offer the social optimum, then we have a strong argument for rejecting this project. If the proponents successfully demonstrate the benefits, we can objectively identify a range of prices under which project *k* is the socially optimal choice.

The problem of the opportunistic project proponent can be addressed by using DEA. Specifically, we calculate the maximum comparative advantage of project *k* (CA_k) that the proponents of this project can demonstrate over competing projects if they can choose non-negative prices $\hat{\mathbf{p}}$ subject to the condition that project *k* must be socially beneficial. Formally, the optimal CA^* and $\hat{\mathbf{p}}^*$ are obtained as the optimal solution to the following linear programming problem

(6.5)
$$\begin{aligned}
&\max_{\hat{\mathbf{p}}} CA_k \\
&s.t. \\
&CA_k \le \left[B_k - (\hat{p}_1 Z_{k1} + \hat{p}_2 Z_{k2} + ... + \hat{p}_M Z_{kM})\right] - \left[B_1 - (\hat{p}_1 Z_{11} + \hat{p}_2 Z_{12} + ... + \hat{p}_M Z_{1M})\right] \\
&CA_k \le \left[B_k - (\hat{p}_1 Z_{k1} + \hat{p}_2 Z_{k2} + ... + \hat{p}_M Z_{kM})\right] - \left[B_2 - (\hat{p}_1 Z_{21} + \hat{p}_2 Z_{22} + ... + \hat{p}_M Z_{2M})\right] \\
&\vdots \\
&CA_k \le \left[B_k - (\hat{p}_1 Z_{k1} + \hat{p}_2 Z_{k2} + ... + \hat{p}_M Z_{kM})\right] - \left[B_{k-1} - (\hat{p}_1 Z_{k-1,1} + \hat{p}_2 Z_{k-1,2} + ... + \hat{p}_M Z_{k-1,M})\right] \\
&CA_k \le \left[B_k - (\hat{p}_1 Z_{k1} + \hat{p}_2 Z_{k2} + ... + \hat{p}_M Z_{kM})\right] - \left[B_{k+1} - (\hat{p}_1 Z_{k+1,1} + \hat{p}_2 Z_{k+1,2} + ... + \hat{p}_M Z_{k+1,M})\right] \\
&\vdots \\
&CA_k \le \left[B_k - (\hat{p}_1 Z_{k1} + \hat{p}_2 Z_{k2} + ... + \hat{p}_M Z_{kM})\right] - \left[B_N - (\hat{p}_1 Z_{N1} + \hat{p}_2 Z_{N2} + ... + \hat{p}_M Z_{NM})\right] \\
&B_n - (\hat{p}_1 Z_{n1} + \hat{p}_2 Z_{n2} + ... + \hat{p}_M Z_{nM}) \ge 0 \\
&\hat{p}_1, \hat{p}_2, ..., \hat{p}_M \ge 0.
\end{aligned}$$

The "estimated" prices $\hat{\mathbf{p}} \ge 0$ are the unknown variables of problem (6.5), and the net economic benefits $B_1, B_2, ..., B_N$ and the vectors of environmental impacts $\mathbf{Z}_1, \mathbf{Z}_2, ..., \mathbf{Z}_N$ are known parameters of this linear programming problem. The first *N*-1 constraints compare in the pair-wise fashion the net benefits of project *k* relative to all competing projects. Because only one of the competing projects is chosen, the competitive advantage *CA* that is maximized in the objective function depends on how well project *k* performs relative to its best competitor. Thus, only the smallest value of the net benefit differences counts. To qualify as the socially optimal choice, the net benefit of project *k* must be greater than (or equal to) zero. The *N*th constraint of (6.5) ensures that net benefit is non-negative at the estimated prices $\hat{\mathbf{p}}$.

If the optimal solution CA_k^* to this problem is a negative number or zero, then such non-negative prices $\hat{\mathbf{p}}^* \geq 0$ for environmental impacts at which project *k* can yield the highest social net benefit *do not exist*. Whatever the prices of environmental impacts might be, there exists another project that yields a higher social net benefit. Therefore, projects with negative score in (6.5) can be discarded as "inefficient".

On the other hand, if the optimal solution CA_k^* is a positive number, then there exist non-negative prices $\hat{\mathbf{p}}^* \geq 0$ at which project *k* is socially optimal. In this case, project *k* is potentially an attractive investment project, so we diagnose it as "efficient". The objective function CA_k^* indicates the maximum monetary net benefit that this project can offer over the second best candidate (prices $\hat{\mathbf{p}}^*$ maximize this comparative advantage). If CA_k^* is large, then project *k* can present itself as be a superior candidate at certain prices. If CA_k^* is small, then project *k* can provide, even at best, merely a modest advantage over the competing candidates.

When ranking the projects, the *CA* scores are not the only information to consider. In general, it is more important to assess if the prices $\hat{\mathbf{p}}^*$ are realistic or not. In practice, there typically exist several candidate projects that can demonstrate a positive *CA* at some prices. In any case, all projects become unprofitable at some point when prices **p** are increased. To make the final choice of which project –if any- will be implemented, the DEA offers a platform for number of alternative approaches.

The first approach is to impose domain restrictions on the admissible prices **p**, as in the weigh-restricted DEA approaches (See Section 7.2 for references). In problem (6.5) we only postulated that prices should be somewhere between zero and plus infinity. It is often possible to narrow down this interval to a more specific range on objective or subjective grounds. Typically, specifying a certain range for the admissible price is considerably easier than finding a specific point estimate. If the lower bound for price of impact *m* is L_m and the upper bound is U_m, we can simply insert in (6.5) the additional linear constraints:

(6.6) $$L_m \leq p_m \leq U_m.$$

When the price ranges are gradually narrowed down, then at some one of the projects distinguishes itself as the only project that can show a positive *CA* score.

The second approach is to directly present the decision-makers the entire range of prices at which a given project is the socially optimal choice. Presenting such objective price ranges would enable the decision-makers to weigh the potential comparative advantages of the projects against the robustness regarding the choice of prices. However, identifying and presenting the supporting price domains can become technically demanding especially when there are multiple environmental impacts.

The third approach is to combine the DEA evaluation with the more traditional valuation techniques. We can check which of the objective price ranges the prices estimated by some other technique(s) fall into. In this sense, DEA can be a supportive tool for sensitivity analysis in the traditional valuation approaches: we can see if a small change in the estimated prices changes the policy recommendation. DEA could also save the costs of the traditional valuation studies. If the DEA analysis is conducted prior to the valuation study, we can differentiate between those environmental impacts that are critically important for the decision and should be evaluated using more expensive valuation techniques (such as CV), and those impacts which are unimportant for the decision.

To illustrate the presented technique, let us return to the highway construction example. Inserting numerical values from Table 1 in problem (6.5) to evaluate highway B (through the forest), we get

$$\widehat{CA}_B = \max_{\hat{\mathbf{p}}} CA_B$$
$$s.t.$$
$$CA_B \leq [170 - (120 \times \hat{p}_1 + 3 \times \hat{p}_2)] - [40 - (1 \times \hat{p}_1 + 2 \times \hat{p}_2)]$$
(6.7)
$$CA_B \leq [170 - (120 \times \hat{p}_1 + 3 \times \hat{p}_2)] - [130 - (2 \times \hat{p}_1 + 6 \times \hat{p}_2)]$$
$$CA_B \leq [170 - (120 \times \hat{p}_1 + 3 \times \hat{p}_2)] - [50 - (10 \times \hat{p}_1 + 12 \times \hat{p}_2)]$$
$$170 - (120 \times \hat{p}_1 + 3 \times \hat{p}_2) \geq 0$$
$$\hat{p}_1, \hat{p}_2 \geq 0.$$

This optimization finds shadow prices $\hat{\mathbf{p}} \geq 0$ that make $\widehat{CA}_B$, i.e., the comparative advantage of highway B, as large as possible. The optimal solution to this problem gives the comparative advantage of 128.4 million dollars, which is based on the prices of zero for the loss of forest land and 1.55 dollars for unit of urban air quality. For comparison, the optimal solutions for all projects are reported in Table 8.

Table 8: Results of DEA-based CBA

	A	B	C	D
CA	300	128.4	87.8	-15.9
p_1	27.50	0.00	1.08	1.07
p_2	6.25	1.55	0.00	1.51
SB	0.0	165.3	127.8	21.1

The first row of Table 8 reports the comparative advantages (CA) scores obtained as the optimal solution to problem (6.5) for each project. We observe that project D has a negative CA score. This means that project D (i.e., the tunnel option) will never prove the optimal solution whichever prices we assign for the environmental impacts; one of the competing projects will always provide a greater social net benefit. Thus, we could discard project D by objective grounds without saying anything about the prices of the environmental impacts.

For alternatives A, B and C the CA score is a positive number. Project A can boast with the largest comparative advantage of 300 million dollars, but it is based on such large prices for the environmental impacts that its social net benefit falls to zero; the social net benefit is indicated on the fourth row labeled as SB. Project B can show considerable comparative advantage if the loss of forest area is considered to be inexpensive. Similarly, project C shows a large advantage if the urban air quality is of little concern.

In this case it is relatively simple to identify the entire range of prices at which project A, B, or C provides the highest social net benefit (subject to the constraint that this benefit must be non-negative). These price ranges are illustrated in the two-dimensional price space in Figure 8. The horizontal axis represents the price of forest loss (dollars per hectare) and the vertical axis represents the price of urban air quality (dollars per index number). An estimate or a subjective judgment of prices (p_1, p_2) can be viewed as a point in this two dimensional diagram. If the prices of the environmental impacts coincide to triangle A, then project A should be implemented. Similarly, if the prices coincide in area B, then project B is the optimal one; if in triangle C, then project C should be chosen. Finally, if the prices of both impacts are both sufficiently large that they fall outside areas A,B and C, then none of the projects should be implemented.

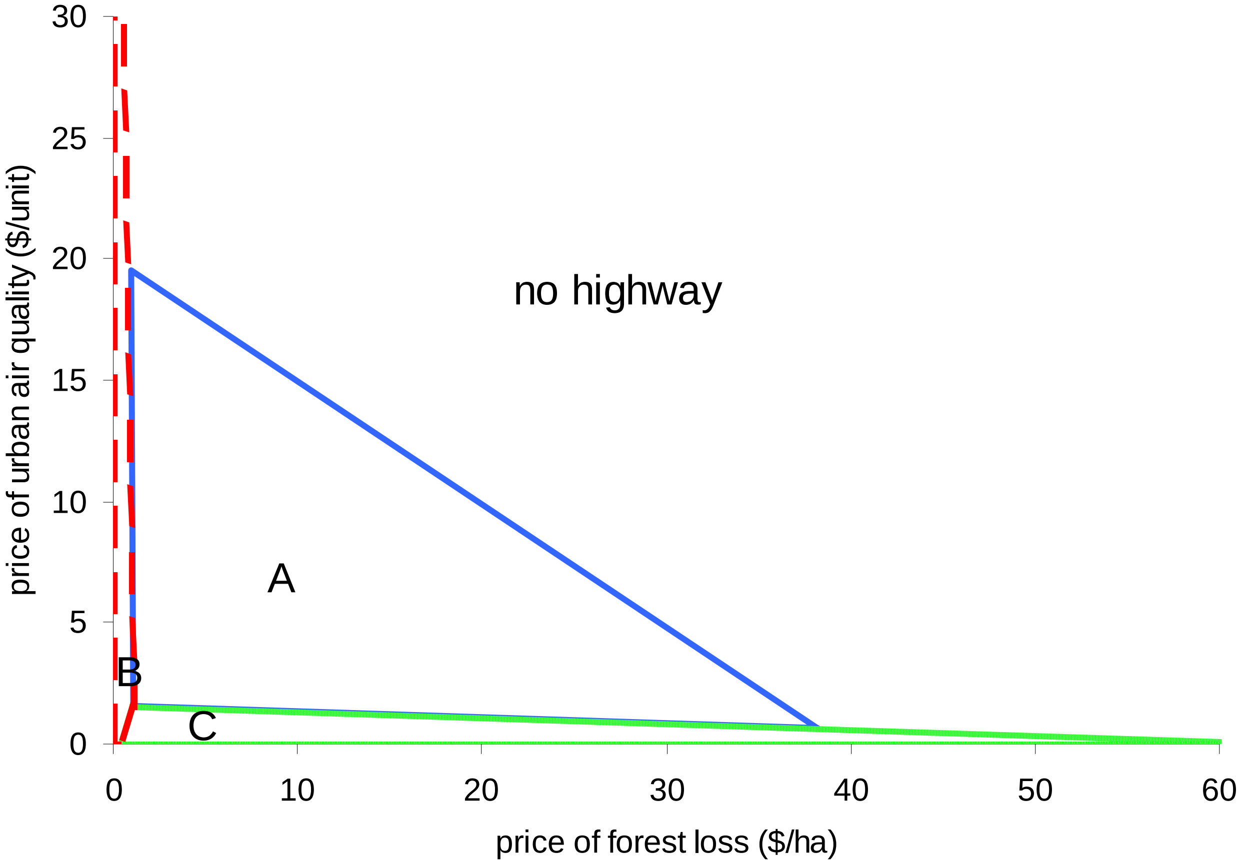

Figure 8: CBA in the price space[20]

This diagram presents the objective picture based on the impact assessment, without making any judgment or estimation of the prices of environmental impacts. We see that the highway A is a very robust choice, offering the highest social net benefit with a large range of prices. Alternative B becomes optimal if we give a very small price for the loss of forest land (less than ninety cents per hectare). Similarly, alternative C is the optimal choice if the price of urban air quality is very low (less than 1.5 dollars per index number). Recall that we have already ruled out alternative D, does not provide the maximal social net benefit at any prices. This kind of diagram may help the decision maker to rule out some other alternatives. For example, if we consider the loss of forest land to be certainly more expensive than one dollar per hectare, then we can safely rule out alternative B. Further studies might be implemented to learn more about the price of urban air quality. Since

[20] Areas B and C have been truncated to make the figure more informative. Area B continues upwards to point (0, 56.7), and area C continues to the right to point (65, 0).

valuation studies such as contingent valuation are expensive to implement, this kind of a prior DEA study could save time and money by enabling the further valuation study to focus resources on the most critical dimensions of the problem. After the price estimates have been obtained, the DEA approach can provide a useful framework for sensitivity analysis.

7. Further reading

This section presents a brief, structured literature review of a selection of topics that are useful for applying the approaches discussed above.

7.1 DEA: history and the state of the art

The intellectual roots of DEA lie in the activity analysis of Koopmans (1951), production theory of Shephard (1953, 1970), and Linear Programming (Danzig, 1949). The key ideas of DEA date back to the seminal paper by Farrell (1957), but the real breakthrough of DEA occurred with Charnes et al. (1978), who coined the name and popularized the approach in the field of operations research, which resulted in rapid technical development of the method.

- Charnes, A., W.W. Cooper and E. Rhodes (1978): Measuring the Efficiency of Decision-Making Units, *European Journal of Operational Research* 2, 429-444.
- Dantzig, G.B. (1949): Programming of Interdependent Activities II: Mathematical Model, *Econometrica* 17, 200 – 211.
- Farrell, M.J. (1957): The Measurement of Productive Efficiency, *Journal of the Statistical Society* Series A, General, 120, 253-281.
- Koopmans, T.C. (Ed.) (1951): *Activity Analysis of Production and Allocation*. Cowles Commission for Research in Economics, Monograph No. 13, Wiley, New York.
- Shepard, R.W. (1953): *Cost and Production Functions*, Princeton University Press, Princeton.
- Shepard, R.W. (1970): *Theory of Cost and Production Functions*, Princeton University Press, Princeton.

Recent textbook treatments of DEA include Färe et al. (1994), Coelli et al. (1998) and Cooper et al. (2000). The survey articles by Seiford (1996) and Taveres (2002) list thousands of references to articles published on DEA.

- Coelli, T., D.S. Prasada Rao and G.E. Battese (1998): *An Introduction to Efficiency and Productivity Analysis*, Kluwer Academic Publishers, Boston.
- Cooper, W.W., L.M. Seiford and K. Tone (2000): *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software*, Kluwer Academic Publishers, Boston.
- Färe, R., S. Grosskopf and C.A.K. Lovell (1994): *Production Frontiers*. Cambridge University Press.
- Seiford, L.M. (1996): Data Envelopment Analysis: The Evaluation of the State of the Art (1978-1995). *The Journal of Productivity Analysis* 7, 99-137.
- Tavares, G. (2002): *A Bibliography of Data Envelopment Analysis*, Rutcor Research Reports 1/2002.

7.2 Computation

Algorithms for solving linear programming problems are widely available as part of any optimizing software (e.g., GAMS, GAUSS). Modern spreadsheet solvers (e.g. Excel solver) can also handle increasingly large problems. The advantage of using some general purpose software is the flexibility to adjust the model specification to meet the purposes of analysis. Also tailor made DEA software are available for those who do not want to write their own computer code. A drawback of these software packages is that they only facilitate a limited selection of possible model specifications.

The most widely used shareware software (free for academic use) include

- The Efficiency Measurement System (EMS) by Holger Scheel: http://www.wiso.uni-dortmund.de/lsfg/or/scheel/ems/
- A Data Envelopment Analysis (Computer) Program (DEAP) by Tim Coelli: http://www.uq.edu.au/economics/cepa/software.htm

Popular commercial software packages include:

- OnFront: http://www.emq.com/software.html
- The Warwick software: http://www.deazone.com/software/
- FrontierAnalyst: http://www.banxia.com/

Two recent DEA textbooks include add-ins for the Microsoft Excel spreadsheet

- Cooper, W.W., L.M. Seiford, and K. Tone (2000) (See Section 7.1).
- Zhu, J. (2002): *Quantitative Models for Performance Evaluation and Benchmarking: DEA with Spreadsheets and DEA Excel Solver*, Kluwer Academic Publishers.

7.3 Weight restrictions

Earlier in this paper we presented weight restrictions (or price ranges) and discussed shortly how they can be used in the eco-efficiency and cost-benefit analysis. Weight restrictions can also be exploited when applying DEA for measuring environmental performance. In general, different kinds of restrictions can be particularly useful in environmental applications of DEA. The following list introduces some of the most important references concerning weight restrictions.

- Allen, R., A. Athanassopoulos, R.G. Dyson and E. Thanassoulis (1997): Weights Restrictions and Value Judgements in Data Envelopment Analysis: Evolution, Development and Future Directions, *Annals of Operations Research* 73, 13 -34.
- Dyson, R.G. and E. Thanassoulis (1988): Reducing Weight Flexibility in Data Envelopment Analysis, *Journal of the Operational Research Society* 39, 563-576.
- Kuosmanen, T., and G.T. Post (2001): Measuring Economic Efficiency with Incomplete Price Information: With an Application to European Commercial Banks, *European Journal of Operational Research* 134, 43-58.
- Pedraja-Chaparro, F., J. Salinas-Jimenez, and P. Smith (1997): On the Role of Weight Restrictions in Data Envelopment Analysis, *Journal of Productivity Analysis* 8, 215-230.

7.4 DEA in environmental economics

Cost-benefit analysis by using DEA is a completely new area of study and hence, there are no references for it in the literature of environmental economics. As yet, only a few papers have concentrated on measuring eco-efficiency by DEA as we have defined it. There are, however, number of studies in which environmental performance is measured in the DEA framework. The following lists (or classifications) include only part of them.

Eco-efficiency

- Haynes, K.E., S. Ratick and J. Cummings-Saxton (1994): Toward a Pollution Abatement Monitoring Policy: Measurements, Model Mechanics and Data Requirements. *Environmental Professional* 16, 292-303.

Environmental performance: surveys

- Allen, K. (1999): DEA in the Ecological Context – An Overview. In G. Westerman (ed.), Measuring the Efficiency in the Private and Public Service Sector. Gabler, Wiesbaden, 203-235.
- Dyckhoff, H. and K. Allen (2001): Measuring Ecological Efficiency with Data Envelopment Analysis (DEA), *European Journal of Operational Research* 132, 312-325.
- Tyteca, D. (1996): On the Measurement of the Environmental Performance of Firms – A Literature Review and a Productive Efficiency Perspective, *Journal of Environmental Management* 46, 281-308.

Environmental performance: applications

Agriculture

- Ball, V.E., C.A.K. Lovell, R. Nehring and A. Somwaru (1994): Incorporating Undesirable Outputs into Models of Production: An Application to U.S. Agriculture, *Cahiers d'economie et sociologie rurales* 31, 60-74.
- Ball, V.E., R. Färe, S. Grosskopf and R. Nehring (2001): Productivity of the U.S. Agricultural Sector: The Case of Undesirable Outputs. Chapter 13 in C.R. Hulten, E.R. Dean and M.J. Harper (eds.), New Developments in Productivity Analysis, University of Chicago Press for the National Bureau of Economic Research, Chicago.
- Piot-Lepetit, I., D. Vermersch, R.D. Weaver (1997): Agriculture's Environmental Externalities: DEA Evidence for French Agriculture, *Applied Economics* 29, 331-338.
- Reinhard, S., C.A.K. Lovell, G.J. Thijssen (2000): Environmental Efficiency with Multiple Environmentally Detrimental Variables; estimated with SFA and DEA, *European Journal of Operational Research* 121, 287-303.

Forest and paper industry

- Brännlund, R., R. Färe and S. Grosskopf (1995): Environmental Regulation and Profitability: An Application to Swedish Pulp and Paper Mills, *Environmental and Resource Economics* 12, 345-356.
- Färe, R., S. Grosskopf, C.A.K. Lovell and C. Pasurka (1989): Multilateral Productivity Comparisons When Some Outputs are Undesirable: A Non-Parametric Approach, *The Review of Economics and Statistics* 71, 90-98.
- Färe, R., S. Grosskopf, C.A.K. Lovell and S. Yaisawarng (1993): Derivation of Shadow Prices for Undesirable Outputs: A Distance Function Approach, *The Review of Economics and Statistics* 75, 374-380.
- Hailu, A. and T.S. Veeman (2001): Non-Parametric Productivity Analysis with Undesirable Outputs: An Application to the Canadian Pulp and Paper Industry, *American Journal of Agricultural Economics* 83, 605-616.
- Hailu, A. (2003): Pollution Abatement and Productivity Performance of Regional Canadian Pulp and Paper Industries, *Journal of Forest Economics* 9, 5-25.

Other industries

- Courcelle, C., M.P. Kestemont, D. Tyteca, M. Installé (1998): Assessing the Economic and Environmental Performance of Municipal Solid Waste Collection and Sorting Programmes, *Waste Management and Research* 16, 253-263.

- Färe, R., S. Grosskopf and D. Tyteca (1996): An Activity Analysis Model of the Environmental Performance of Firms – Application to Fossil-Fuel-Fired Electric Utilities, *Ecological Economics* 18, 161-175.
- Tyteca, D. (1997): Linear Programming Models for the Measurement of Environmental Performance of Firms – Concepts and Empirical Analysis, *Journal of Productivity Analysis* 8, 183-197.
- Yaisawarng, S. and J.D. Klein (1994): The Effects of Sulfur Dioxide Controls on Productivity Change in the U.S. Electric Power Industry, *The Review of Economics and Statistics* 76, 447-460.