Vector Analysis

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Abstract

The principles of resource allocation are applied in number of environments, including biological systems, computer equipment, and stellar phenomena. Therefore, economics can be considered a branch of mathematics, like geometry or calculus.

In this paper, a framework for approaching economics from this direction is introduced.

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Outline

Introduction

Section 1: Platform

Let an element be defined as a phenomenon that is, or is treated as, irreducible.
Let a compound be defined as a set of elements.

Let composition be defined as the set of elements that constitute a compound.
Let structure be defined as the arrangement of elements in a compound.
Let state be defined as a compound's composition and structure.

Let a transformation be defined as an alteration in a compound's composition.
Let a translation be defined as an alteration in a compound's structure.
Let a transition be defined as a transformation or translation.

Let a metric be defined as a set of elements that define an aspect of a state or transition.
Let a dynamic be defined as a relationship between a set of states and transitions.
Let a principle be defined as a concept that defines a dynamic.

Section 2: Platform Analysis

Let detection be defined as the identification of a compound's composition or structure.
Let inspection be defined as the identification of a transformation or translation.
Let observation be defined as the identification of a dynamic.

Let cartography be defined as the derivation of a compound's composition or structure.
Let tomography be defined as the derivation of a transformation or translation.
Let deduction be defined as the derivation of a dynamic.

Let induction be defined as the derivation of a principle.

Let data be defined as records pertaining to a compound.

Infrastructure

Section 1: Composition

Let infrastructure be defined as a set of compounds.

Let an array be defined as a compound that connects a subset of the compounds in an infrastructure.
Let an ether be defined as a set of arrays.
Let a transporter be defined as a compound that transports a set of compounds through an ether.
Let a path be defined as the route a transporter travels.

Let a card be defined as a compound that describes an aspect of another compound.
Let a deck be defined as a set of cards.

Section 2: Structure

Let the identifier of a compound in an infrastructure be of the form

z(am)-(em)-(im)-(om)-(um) (2.1)

where z is the name applied to the compound,
(am) is its classification,
(em) is its serial code,
(im) is the list of its technical specifications,
(om) is the list of transitions it is involved in, and
(um) is a set of co-ordinates or equations describing its position with respect to a reference point or another compound.

An infrastructure can be expressed as a set of identifiers or a map. The properties of an infrastructure can be expressed in a separate matrix or a set of equations.

Depending on the set of compounds that define an infrastructure, it can be convenient to let z represent a set of compounds.

Processes

Section 1: Composition

Let a process be defined as a set of transitions involving a given set of compounds.

Let a reactant be defined as a compound that undergoes a set of transitions.
Let an accelerant be defined as a compound that accelerates a set of transitions but is not a reactant.
Let a decelerant be defined as a compound that decelerates a set of transitions but is not a reactant.
Let a regulator be defined as an accelerant or decelerant.

Let an input be defined as a reactant or regulator.

Let a product be defined as a compound that is formed by a set of transitions.
Let a residual be defined as an input that is intact after a process has been executed.
Let an output be defined as a product or by-product.

Let a resource be defined as an input or output.

Section 2: Structure

Let α_γ be a 1 x A matrix of inputs for the γ^{th} transition in a process.
Let β_γ be a 1 x B matrix of outputs for the γ^{th} transition in a process.

Let vec(γ) be the identifier of the γ^{th} transition in a process.

Let co(vec(γ)) denote a progression to the γ^{th} transition in a process.
Let au(vec(γ)) denote synchronization with the γ^{th} transition in a process.
Let fb(vec(γ)) denote a regression to the γ^{th} transition in a process.

From these definitions, a given process, *PS*, can be expressed as

$$PS = [\text{vec}(1); \text{co}(\text{vec}(\gamma)) \ \ldots \\ \text{vec}(\Gamma); \text{fb}(\text{vec}(\gamma))] \quad (2.1)$$

where *PS* contains Γ transitions.

The properties of a process can be expressed in a separate matrix or set of equations.

Depending on the set of outputs used to define a process, it may be convenient to let a given vec(γ) represent a set of transitions.

Vectors

Section 1: Composition

Let a vector be defined as the set of processes pertaining to a given set of outputs.

Let search (se) be defined as the process of locating a set of resources that satisfy a given set of criteria.
Let acquisition (aq) be defined as the process of obtaining the set of resources located in a search.
Let evaluation (ev) be defined as the process of expending the set of resources obtained in an acquisition.

Let notification (nt) be defined as the process of presenting a deck pertaining to a set of resources.
Let distribution (di) be defined as the process of transporting a set of resources.
Let engineering (en) be defined as the process of constructing a set of resources.

The classification of a process can vary depending on the set of outputs that define a vector.

Section 2: Structure

The structure of a vector can be shown as a set of paths on a map or expressed as a set of processes using process notation. In either case, the classification of a process in a vector can be denoted using identifiers.

Any given process can be involved in more than one vector at a point in time.

<u>First-Order Principles (Profit)</u>

Let cost, C_t, be defined as the set of inputs required to execute a process at time t, such that

$$C_t = \alpha_t \tag{1.1}$$

Let revenue, R_t, be defined as the set of outputs formed by a process at time t, such that

$$R_t = \beta_t \tag{1.2}$$

Let profit, π_t, be defined as the returns to executing a process at time t, such that

$$\begin{aligned} \pi_t &= R_t - C_t \\ &= \beta_t - \alpha_t \end{aligned} \tag{1.3}$$

Where resources present in R_t or C_t are absent in the other matrix, empty columns can be inserted in the appropriate matrix to make the calculation possible.

Alternatively, let currency be defined as a resource used to measure the value of a set of resources. Using a currency, profit can be calculated using the per unit value of each resource involved in a process, such that

$$\pi_t = \beta_t\varepsilon_t - \alpha_t\delta_t \tag{1.4}$$

where ε_t is the B x 1 matrix of per unit prices that corresponds to β_t, and
δ_t is the A x 1 matrix of per unit prices that corresponds to α_t.

Let

$$\pi_T = \sum_{t=1}^{T} \pi_t \tag{1.5}$$

for t = 1...T periods.

Second-Order Principles (Technical Constraints)

Section 1: Scale

Let scale be defined as the relationship between the price, P, and quantity, Q, of a resource, such that

$$P_{\theta t} = x_{\theta t} Q_{1t} + x_{1t} \tag{1.1}$$

where x_{1t} is a Q_{1t} independent component of $P_{\theta t}$.

Let increasing returns to scale (IRS) be defined as a scale where $x_{\theta t} < 0$. Let constant returns to scale (CRS) be defined as a scale where $x_{\theta t} = 0$. Let decreasing returns to scale (DRS) be defined as a scale where $x_{\theta t} > 0$.

Section 2: Elasticity

Let elasticity be defined as the relationship between the quantity and price of a resource, such that

$$Q_{\iota t} = x_{\iota t} P_{\theta t} + x_{2t} \tag{2.1}$$

where x_{2t} is a $P_{\theta t}$ independent component of $Q_{\iota t}$.

Let inelastic elasticity (IE) be defined as an elasticity where $x_{\iota t} < -1$. Let neutral elasticity (NE) be defined as an elasticity where $x_{\iota t} = -1$. Let elastic elasticity (EE) be defined as an elasticity where $x_{\iota t} > -1$.

Section 3: Symmetry

Let symmetry be defined as the relationship between the profits of separate processes, such that

$$\pi_{\kappa t} = x_{\kappa t} \pi_{\iota t} + x_{3t} \tag{3.1}$$

where x_{3t} is a $\pi_{\iota t}$ independent component of $\pi_{\kappa t}$.

Let increasing symmetry (IS) be defined as a symmetry where $x_{\kappa t} > 0$. Let neutral symmetry (NS) be defined as a symmetry where $x_{\kappa t} = 0$. Let decreasing symmetry (DS) be defined as a symmetry where $x_{\kappa t} < 0$.

Symmetry is caused by any of progression, syncronization, or regression.

Section 4: Conclusion

Equation 3.1 can be expressed as

$$\begin{aligned} \pi_{\kappa t} &= x_{\kappa t}(q_{\zeta\iota t} p_{\zeta\iota\theta t} - q_{\eta\iota t} p_{\eta\iota\theta t}) + q_{\zeta\kappa t} p_{\zeta\kappa\theta t} - q_{\eta\kappa t} p_{\eta\kappa\theta t} \\ &= x_{\kappa t}(\beta_{\iota t}\varepsilon_{\iota\theta t} - \alpha_{\iota t}\delta_{\iota\theta t}) + \beta_{\kappa t}\varepsilon_{\kappa\theta t} - \alpha_{\kappa t}\delta_{\kappa\theta t} \end{aligned} \tag{4.1}$$

Let $\Pi_{\kappa t}$ be defined as

$$\Pi_{\kappa t} = \sum_{t=1}^{T} \pi_{\kappa t} \tag{4.2}$$

for t = 1...T periods.

Third-Order Principles (General Constraints)

Given second-order principles, there are nine types of general constraints:

$x_{\kappa t} = \upsilon_t$ (1.1)

$\beta_{\iota t} = \phi_{\beta t}$ (1.2)

$\varepsilon_{\iota\theta t} = \phi_{\varepsilon t}$ (1.3)

$\alpha_{\iota t} = \chi_{\alpha t}$ (1.4)

$\delta_{\iota\theta t} = \chi_{\delta t}$ (1.5)

$\beta_{\kappa t} = \psi_{\beta t}$ (1.6)

$\varepsilon_{\kappa\theta t} = \psi_{\varepsilon t}$ (1.7)

$\alpha_{\kappa t} = \omega_{\alpha t}$ (1.8)

$\delta_{\kappa\theta t} = \omega_{\delta t}$ (1.9)

Using Lagrange multipliers,

$$\Pi_{\kappa\tau} = \sum_{t=1}^{T} [\pi_{\kappa\tau} - \lambda_{1t}(x_{\kappa t} - \upsilon_t) - \lambda_{2t}(\beta_{\iota t} - \phi_{\beta t}) - \lambda_{3t}(\varepsilon_{\iota\theta t} - \phi_{\varepsilon t}) - \lambda_{4t}(\alpha_{\iota t} - \chi_{\alpha t}) - \lambda_{5t}(\delta_{\iota\theta t} - \chi_{\delta t}) - \lambda_{6t}(\beta_{\kappa t} - \psi_{\beta t}) - \lambda_{7t}(\varepsilon_{\kappa\theta t} - \psi_{\varepsilon t}) - \lambda_{8t}(\alpha_{\kappa t} - \omega_{\alpha t}) - \lambda_{9t}(\delta_{\kappa\theta t} - \omega_{\delta t})] \quad (1.10)$$

<u>Fourth-Order Principles (Resolution)</u>

Where the set of processes a process is symmetric with can vary, different sets of outputs can result. Let each possible set of symmetries be called a symmetry set.

Let $\mu_{\nu t}$ be the probability of the μ^{th} symmetry set occurring at time t. Let $\Pi_{\nu t}$ be the profit associated with the ν^{th} symmetry set. Let $\xi_{\nu t}$ be the probability that $\Pi_{\nu t}$ will be realised at time t. From these definitions,

$$\mu_{\nu t} \geq \xi_{\nu t} \tag{1.1}$$

Where a set of resources can form more that one process, multiple outcomes are possible. Let $\rho_{\sigma t}$ be the probability of the σ^{th} process occurring at time t, such that

$$\rho_{\sigma t} = x_{\sigma t}[\sum_{t=1}^{T}\sum_{\nu=1}^{N}\xi_{\nu\tau}\Pi_{\nu t}] + x_{\tau t} \tag{1.2}$$

where $x_{\tau t}$ is the $\xi_{\nu t}\Pi_{\nu t}$ independent component of $\rho_{\sigma t}$.

For a set of possible processes,

$$P_t = \sum_{\sigma=1}^{\Omega} p_{\sigma t} \tag{1.3}$$

$$\leq 1$$

for $\sigma = 1...\Omega$ possible processes.