Dynamic Parties and Social Turnout: An Agent-Based Model*

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Abstract
We develop an agent-based model of dynamic parties with social turnout built upon developments in different fields within social science. This model yields significant turnout, divergent platforms, and numerous results consistent with the rational calculus of voting model and the empirical literature on social turnout. In a simplified version of the model we show how a local imitation structure inherently yields dynamics that encourage positive turnout. The model also generates new hypotheses about the importance of social networks and citizen-party interactions.

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For the past half century social scientists - political scientists, economists, sociologists, and psychologists - have been intrigued by two important empirical regularities: why people vote and why political parties behave the way they do. The fact that millions of people vote may not seem to be puzzling (Berelson et al. 1954; Mackie & Rose 1997). However, given standard assumptions about rationality voter turnout cannot be easily explained. Numerous formal attempts to explain turnout predict vanishingly small turnout since the probability of affecting the outcome of an election approaches zero in large populations (Palfrey and Rosenthal 1985; Aldrich 1993; Myerson 1998). In fact, people going to polls have a much higher chance of getting into a car accident. This has led many scholars to infer that rational explanations of turnout must rely on an additional benefit derived from fulfilling a sense of duty or a general taste for voting (Riker and Ordeshook 1968).

The second interesting empirical regularity is the way political parties choose their electoral platforms – candidates offer voters policies that diverge significantly from the median voter and remain relatively stable over time (Peltzman 1984; Grofman et al 1990; Poole and Rosenthal 1984). Again, given standard assumptions about rationality this is not obvious since early models of party platforms predicted convergence to the median voter (Downs 1957; Davis et al 1970) or divergence across the entire policy space with any platform possible (McKelvey 1976; Plott 1967; Schofield 1983).[1] Subsequently, scholars explored the impact of uncertainty on policy-motivated parties (Wittman 1977; Calvert 1985; Roemer 2001). These models do yield equilibria with divergent policies, but analysis in a closed-form is rather complex. It quickly becomes intractable under all but the most basic assumptions (Roemer 2001).

Because of the complexity involved in modeling both parties and voters, past efforts have not combined them (Osborne 1995). Models of voter turnout have usually relied on assumptions of fixed party platforms, while models of platform choice have assumed a fixed level of voter turnout (usually 100%). The interdependence between people and politicians also has a dynamic character that is missing from many models because they consider a single election in isolation. Most elections are, in fact, part of a longer process of party competition and take place in a context of information about previous elections.

Economists and political scientists have also frequently abstracted away from elements that sociologists and psychologists believe to be critical for determining electoral behavior. For example, many models of elections have avoided situating voters in social networks, or social context in general. Voters are often assumed to exist independently of one another in spite of a growing body of sociological evidence suggesting that how they are situated in relation to one another plays a critical role in the decision to vote (Lazarsfeld et al 1948; Berelson et al 1954; Campbell et al 1954; Glaser 1959; Huckfeldt and Sprague 1995; Straits 1990; Knack 1992; Kenny 1992; Beck et al 2002).

Most models of elections also make typical cognitive assumptions about information and individual rationality, in spite of the evidence from psychology that both may be severely limited (e.g. Simon 1982; Quattrone and Tversky 1988). Instead, people might use “fast and frugal heuristics” to deal with informational limitations and strategic complexities but still achieve relatively good results (cf Gigerenzer 1999; Cosmides and Tooby 1996; Lupia and McCubbins 1998).

The complexity of including all these features in a formal analytical model would overwhelm it. A closed-form solution would probably not be tractable. However, leaving some or all of these features out may yield incomplete inferences about voter and party behavior. Therefore, we develop a formal model using an alternative methodology: agent-based modeling (ABM). Like analytical models, ABMs are built on formal assumptions about agents (players in games) and how they interact. Similar to the standard analytical models, the assumptions are clearly defined, the results are stated in precise terms, and are typically easy to replicate (Gilbert and Troitzsch 1999). Unlike most analytical models, however, ABMs are usually analyzed computationally, which means they are less elegant but also less susceptible to problems of tractability. Computational models generate data to show the relationships between variables of interest. Moreover, agent-based models may make it easier to analyze paths to equilibrium, to recognize emergent patterns of interaction, and to quickly generate models like this one where interaction is especially complicated (Johnson 1998). In other words, computational modeling provides an insight into not only the outcome of a process, but the dynamics of the process itself without sacrificing the rigor of formal modeling (Nelson and Winter 2002).

In this article we describe and analyze an agent-based model of repeated elections in which voters and parties behave simultaneously. We place voters in a social context and let them interact with one another when choosing whether or not to vote. We also let parties choose the platforms they offer, and these choices may change from election to election depending on feedback from the electorate. This allows us to explore the endogenous interaction of dynamic platforms and costly turnout. In the process we relax standard assumptions of unlimited information-processing capacities and individual hyper-rationality. Citizens are limited to information they can get from their immediate neighbors. They are boundedly rational agents who use simple heuristics to make the turnout decision. Parties are assumed to be more sophisticated, optimizing their choices given their beliefs about the expected behavior of voters and their opponents. However, they form these beliefs based on limited information—they only know the results of past elections.

The computational model that we analyze generates a number of results that contribute to the interdisciplinary literature on voting. First, the average level of aggregate turnout is empirically realistic and it varies from election to election within a stable range. Second, we show that the model is consistent with much of the empirical evidence generated to test the rational calculus of voting. Turnout increases as the cost of voting decreases, the stakes of the election increase, and the margin of victory declines. Thus even though citizens have very limited information and use a very simple learning rule, they are able to respond as though they were prospectively rational to variation in the incentive to vote. Third, the model is consistent with empirical results from the literature on the social context of voting. In particular, turnout correlates highly between neighbors, and citizens who discuss politics with more neighbors are more likely to vote. Fourth, the model also generates a surprising result: when citizens are situated near people with similar preferences they are less likely to vote. In short, segregation depresses turnout. Finally, we explain why a local imitation structure inherently yields dynamics that encourage positive turnout.

The model also generates a number of results that contribute to the literature on party behavior. First, consistent with Wittman equilibrium under uncertainty, policy-motivated parties offer divergent platforms. In this setting, citizens – free to vote or abstain – serve as a source of uncertainty since the location of the median voter is changing all the time. Second, parties adjust their platforms in direct response to the vote share in the previous election. Both parties move in the direction of the previous winner and in proportion to the previous margin of victory. Third, parties are drawn not only to the median voter, but also to the median citizen since she represents the median of the pool of potential future voters. Finally, the model generates another surprising result: electorates with higher local correlation of preferences lead to a greater divergence of party platforms. This suggests that parties polarize as neighborhoods become more segregated.

In the following section we describe most the general structure and most important elements of the agent-based model of elections: how voters make their decisions and how parties choose their platforms. Then we proceed with analysis of the main results of our model: most notably, why people vote despite the cost of voting, and what electoral aspects influence party platforms. In the final section we summarize our findings and discuss application of computational models of elections in the future research.

The Model

In this section we describe a simplified version of our computational model.[2] As in the standard political science model of elections, we assume that each citizen in a population has some preferred policy point on a one-dimensional left-right scale, which one can think of as liberal-conservative issue space. Two parties compete in elections, and these parties have fixed left and right preferences. The parties choose electoral platforms (see below) and each citizen chooses to vote or abstain. If a citizen turns out, she chooses the party offering the platform closest to her own preference. Votes for the left and right are counted and the election winner is determined by the majority rule. After each election, a citizen's utility is simply the negative squared distance between her preferred policy and the platform implemented by the winning party, minus the cost of voting.[3] Parties are assumed to be policy-motivated: they have the same preferences and utility over the policy space as voters (a party prefers to win the election with a policy closer to its ideal point).

Parties only know their own preference point and the results of past elections. They do not know the distribution of voter preferences and, therefore, they do not know the exact location of the median voter. Moreover, some of the former voters may abstain and some of the former abstainers may vote, meaning the location of the median voter may change from election to election (Brody and Page 1973). To deal with this uncertainty we assume that parties use previous election results to learn about the voter distribution. First, they use the results of the past election to estimate the location of the median voter.[4] For example, if the left party wins in a landslide, both parties can infer that the median voter was located closer to the left platform than the right platform. Second, they use Bayesian inference to update their beliefs about the expected median voter in the coming election.[5] Given these beliefs about the electorate, the parties choose platforms by mutually optimizing their expected payoffs.[6]

Unlike parties, citizens employ a less sophisticated decision-making mechanism. We model citizens as boundedly rational agents with access to limited information. In the model, they only know the utility and turnout behavior of their immediate neighbors. This means they also do not know the true preferences of any other citizens or parties. One might argue that this is unnecessarily naive—surely people think for themselves! However, we know from much of the empirical literature on contextual effects that local information has a powerful influence on individual voter behavior (Beck et al 2002; Fotos and Franklin 2002). Imitation has been shown to be an extremely cost-effective strategy in complex environments, even if it does not necessarily lead to the best possible outcome (e.g. Boyd and Richerson 1985).

To model local interaction we endow citizens with preferences and place them randomly on a grid.[7] We then allow them to have political discussions with other people in their neighborhood.[8] Given the constraints on information and the enormous complexity of maximizing utility over some set of future elections, citizens adopt the most successful strategies from past elections. We assume that there is an information flow among immediate neighbors with respect to the past election, in particular, whether or not they voted and how satisfied they were with the results. Since voters can learn about the turnout behavior and relative satisfaction of their neighbors they can use this information to decide whether or not to vote in the next election. Specifically, they divide people in their neighborhood between voters and abstainers, decide which type is more satisfied, and then imitate the behavior of the most satisfied group.[9]

Results: General Dynamics

To analyze computational results from the model we employ three strategies. First, we develop a graphical user interface (GUI) for the model so we can watch what happens to voter utilities, turnout, platforms, and other variables of interest. Computational modeling is unique in this respect because it allows us to inspect visually what is happening to our model as it progresses. This sometimes leads to hypotheses about the dynamic processes that might not otherwise have been obvious using different methodologies (Gilbert and Troitzsch 1999). Second, we produce graphs of several runs of consecutive elections. These graphs are snapshots of the dynamic behavior of one or two variables from the model and they are useful for characterizing typical boundaries and changes in the values for a given set of model assumptions. Third, we conduct multiple runs and collect data at the end of each run. This allows us to see how changes in assumptions affect how the model behaves.

In Figure 1 we present some results from a typical run of 100 elections. The lower left graph shows that turnout varies between 35% and 55%. When we let the simulation run for thousands of elections turnout never jumps out of this range: turnout seems to be significant and stable even when it is costly. The upper right graph shows how the model generates instability in the location of the median voter. Even though the preference of the median citizen remains fixed for a given run (represented by the straight horizontal line in the graph), the preference of the median voter depends on who decides to vote and changes from election to election. Notice especially that the median voter can remain to the left or right of the median citizen for several elections, indicating a period when one party's supporters are more active than the other's.

The upper left graph shows how party platforms change over time to adapt to these circumstances. After a brief convergence from initial conditions and a period of instability the platforms tend to oscillate in a stable range that remains significantly far from the center. This oscillation seems to vary with the location of the median voter as parties attempt to adjust their platforms in the median voter's direction. Constant adjustment by the parties also generates variation in the margin of victory in the lower right graph as parties alternate winning and losing elections.

Why So Much Turnout?

The main source of turnout in our model has to do with imitation in a social context. We assume that citizens are boundedly-rational, acquiring information only from their neighbors in order to decide whether to vote in the next election. In the extreme cases in which everyone votes or everyone abstains, the citizen simply repeats her prior action. In other cases we can derive the expected probability that the voters in a randomly sampled neighborhood will happen to do better than the abstainers because of the random location of their preferences.[10] Figure 2 shows the probability that a randomly sampled citizen will vote given the number of her neighbors who voted in the previous election and the cost of voting.

From top to bottom, each curve in Figure 2 represents a higher cost of voting. Note that increasing the cost of voting decreases the probability of voting for all neighborhood types. This is because the cost of voting directly decreases the average satisfaction of voters in all neighborhoods. This effect is intuitive and conforms to other theoretical and empirical models. Note also that when voting is costless (the top curve), the probability of voting is about 0.5 when about half the neighborhood votes and half abstains (between 4 and 5 voters in a 9 person neighborhood). The expected utility to voters and nonvoters is the same if there is no cost to voting, so the odds that one group does better than another should be the same for both at 0.5. However, this is only true when the number of voters and abstainers is about the same. The downward slope in the curves in Figure 2 indicates that citizens with *fewer* voters in their neighborhood are *more* likely to vote and citizens with *more* voters in their neighborhood are *less* likely to vote. This suggests a negative reinforcement effect that encourages turnout. As the probability of turnout declines, so does the expected number of voters in a given neighborhood, but the probability of turnout for these neighborhoods increases as the number of voters in the neighborhood decreases.

Negative reinforcement may seem counterintuitive, but consider the fact that each citizen is essentially sampling from the population. When one sampled group is substantially larger, it is more likely to yield an average satisfaction level that is close to the population average. The smaller group is privileged because there is a better chance that they will happen to have preferences very close to the winning platform. For example, suppose that half the citizens in a neighborhood vote in the first election and voting is costly. After that, citizens decide whether or not to vote by comparing average utilities of voters and abstainers. It is likely that eventually the number of voters in the neighborhood will decrease to 1 or 2 since the cost of voting is positive.

However, if one of the few remaining voters happens to have a preference that is relatively close to the platform of the winning party, the voter will be more satisfied than the abstainers. Since the number of voters in the neighborhood is small, her satisfaction will dominate the average satisfaction of turnout. As a result, her neighbors will imitate her turnout behavior. Of course, the local surge of voting will be quickly suppressed by the cost of voting and, thus, we have a local turnout-abstention cycle. The global dynamic is a combination of all the overlapping local neighborhoods, all of which experience periods of turnout and abstentions at different moments of the time. Hence a local imitation structure inherently yields dynamics that encourage turnout.[11]

The Rational Calculus of Voting

The rational calculus of voting model assumes that voters think *prospectively* about the impact of their actions on their own utility. Advocates of this model cite several empirical regularities predicted by the model as evidence that these assumptions are correct. In contrast, our model assumes that voters *adapt* to past outcomes. In Figure 3 we see that our model generates the same empirical regularities. For example, turnout is sensitive to the cost of voting. An increase from nothing to 0.1 depresses turnout by about 4 percentage points.[12] The tendency of voters to respond to higher costs with lower turnout is consistent with a broad empirical literature on the subject. For example, restrictive registration laws clearly discourage voting (Rosenstone and Wolfinger 1978; Squire et al 1987; Nagler 1991; Rhine 1995; Knack 1997, 2001; Franklin and Grier 1997; Fenster 1994; Highton 1997; Knack and White 2000; Highton 2000; Huang and Shields 2000), while liberal absentee ballot laws and all-mail elections encourage it (Oliver 1996; Karp and Banducci 2000; Southwell and Burchett 2000b).

The rational calculus of voting literature also posits that voters should be influenced by the expected benefits from voting expressed as a function of the distance between the parties and the probability of influencing the outcome of the election. Our model produces both of these relationships. In the upper right graph of Figure 3 turnout increases with the distance between party platforms, consistent with empirical work that suggests that turnout is somewhat higher in elections with higher stakes (Wolfinger and Rosenstone 1980; Boyd 1989; Hansen, Palfrey, and Rosenthal 1987; Forgette and Sala 1999; Jackson 2000) and a larger distance between the parties (Kaempfer and Lowenberg 1993).

In the lower graph turnout varies inversely with the closeness of the election: participation decreases as the margin of victory by one of the parties increases. This effect is consistent with an empirical literature that has tried to use the closeness of an election as a proxy for how voters perceive the likelihood of affecting the outcome. Though the relationship has been questioned by some (Key 1949; Matsusaka 1993; Ansolabehere and Iyengar 1994; Kirchgassner and Himmern 1997; Kunce 2001), the weight of the evidence seems to point to a small but significant correlation between closeness and turnout (Cox and Munger 1989; Berch 1993; Jackson 1983; Hanks and Grofman 1998; Grofman et al 1998; Nalebuff and Shachar 1999; Alvarez and Nagler 2000).

The fact that the model produces results consistent with the rational calculus of voting suggests that the adaptation model for citizens is sufficiently sophisticated that they are able to learn to vote more often when it would make them better off—that is, when costs are low, stakes are high, and elections are close. However, turnout is still quite high relative to a model in which citizens are perfectly informed and strictly utility-maximizing. To see if this discrepancy is associated with limited information, we alter our model slightly by endowing citizens with memory.

Memory permits citizens to combine information from previous elections with new information about the merits of voting and not voting. Specifically, a memory parameter governs how new information is weighed relative to previous information.[13] If this parameter is set to zero, then citizens only remember the results of the past election. As the parameter increases towards one, they remember more and more of the past and as a consequence the relevance of the current election decreases. The lower right graph in Figure 2 shows the effect of increasing citizen memory. As voters acquire more information about the relative merits of voting and abstaining, they choose to abstain in greater numbers.

The negative relationship between memory and turnout suggests that limited information about the costs and benefits of voting plays an important role in supporting high levels of participation. To make sense of this, think of the extreme case. Without memory, the only information citizens have is the relative satisfaction levels of their neighbors and themselves for the most recent election. With memory, citizens have access to all this information, plus some of the information they acquired in previous elections. As memories lengthen, the number of individual satisfaction levels that go into the average satisfaction level increases, improving the estimate of the relative costs and benefits of participation.

Social Networks and Turnout

Our model produces results that are consistent with findings related to social networks. At the level of the individual voter we find correlation in vote strategies between neighbors. For the baseline simulation, this correlation is about $\rho = 0.29$ and it does not change much when we try different combinations of parameters. This result conforms to the finding that turnout is correlated between friends, family, and co-workers (Lazarsfeld et al 1948; Berelson et al 1954; Campbell et al 1954; Glaser 1959; Huckfeldt and Sprague 1995; Straits 1990; Knack 1992; Kenny 1992, 1993). One might argue that this is a trivial result. After all, the model assumes that voters imitate their neighbors, so we should expect to find some correlation in turnout behavior. However, we emphasize that this is the only theoretical model we are aware of that generates correlated turnout. What it suggests is that models that do not embed their citizens in a social network context may be omitting an important feature of the real world that is relevant to turnout behavior.

The social network context we have supposed so far is artificial in a very important way. We assume that individual preferences are not correlated. The probability of a liberal speaking to another liberal in our model is the same as the probability of a liberal speaking to a conservative. However, a consistent finding in the social voting literature is that people tend to segregate themselves into like-minded groups. As a result, most social ties are between people who share the same interests. Even when people with ideological or class-based interests are not surrounded by like-minded individuals in their *physical* neighborhoods and workplaces they tend to withdraw and form relationships *outside* those environments (Huckfeldt, Johnson, and Sprague 2003; Huckfeldt and Sprague 1987, 1988; Noelle-Neumann 1984; Gans 1967; Berger 1960). Thus preferences between acquaintances tend to be highly correlated. For example, in the Indianapolis-St. Louis Election Study the correlation in liberal-conservative ideology is ρ=0.66, while the correlation in party preference is ρ=0.54.

What effect does the concentration of shared interests have on our model? Figure 5 shows that preference correlation has a dramatically negative effect on turnout. When a citizen has discussions with a diverse group, it is more difficult to discern the costs and benefits of voting. However, when all a citizen's neighbors are just like her, she is more likely to free ride. To see why, suppose an extreme case in which everyone in a citizen's neighborhood has the same preference. When comparing the average satisfaction level of voters and abstainers, the benefits will be exactly the same for everyone. The only thing that differentiates the voters from the abstainers is the cost of voting. Thus, it will be easy to figure out that free riding makes sense. Now suppose the opposite case in which neighbors have heterogeneous preferences. Even though all voters pay a cost of voting, some voters will be very satisfied because they happen to be located close to the winning candidate. Conversely, even though abstainers do not pay a cost of voting, some will be very dissatisfied because they have preferences that are far away from the winning candidate. Thus, as preference correlation decreases, the relationship between satisfaction level and turnout behavior breaks down and it becomes more difficult to discern the advantage of free riding. In short, social segregation hurts participation.

Party Behavior

Turning to party behavior, we note that the model generates a substantial degree of platform divergence (see Figure 1 above). The game-theoretic literature suggests that uncertainty is a necessary condition for platform divergence (Wittman 1977; Calvert 1985). These models introduce an exogenous source of uncertainty, but in our model uncertainty is generated endogenously by variation in voter turnout. The location of the median voter changes from election to election as new sets of voters show up to the polls. Figure 6 compares results when we fix voter turnout to those when we allow it to vary. When we fix turnout and the location of the median voter is constant, the parties quickly infer its location and converge. When we allow voters to choose whether or not to vote, the platforms diverge. Clearly, parties behave differently when turnout behavior is allowed to vary, suggesting that it may be important to model both voters and parties simultaneously as we do here.

The model also suggests that platform divergence may result from parties choosing strategies that react positively to the margin of victory. Figure 7 shows that both parties typically move their platforms in the direction of the winning candidate and in proportion to the margin of victory. For example, if the left wins a close election, both parties will shift slightly to the left. If the left wins in a landslide, both parties will shift a lot to the left. This is because a landslide victory causes the winning party to infer that it can win with a platform that is closer to its own preferences. It also causes the losing party to learn that it must moderate in order to be competitive in the next election. The relationship between platforms and vote share is consistent with the literature on Presidential mandates (Conley 2002; Kingdon 1966), a more detailed analysis of Wittman equilibrium (Smirnov and Fowler 2003), and recent evidence that shows past vote share affects the ideology of US Senate candidates (Fowler 2003).

The effect of these strategic interactions is that parties try to adapt to the (unknown) positions of the median voter and the median citizen. In Figure 1 we showed that in a given run the median voter changes frequently while the median citizen remains constant. Parties have a short-term incentive to exploit the former if there tends to be some persistence in the set of voters who turnout from one election to the next. However, they also have a long-term incentive to stay close to the median citizen since this represents the pool of all possible voters in future elections. Figure 8 shows that platforms tend to track *both* changes in the location of the median voter and the fixed location of the median citizen. Interestingly, the parties are more sensitive to the location of the median citizen than the median voter, which implies that parties pay more attention to the long-term shape of the electorate rather than the short-term changes.

Finally, we highlight a surprising interaction between parties and voters. Figure 9 shows that increasing preference correlation among voters dramatically increases platform divergence. This is because preference correlation tends to increase variance in the vote share. Heterogeneous neighborhoods will have one or two citizens switching their behavior when the parties adjust slightly to the left or right, but homogeneous neighborhoods will have several citizens switching together—small changes in the location of the parties can quickly lead to waves of imitation among supporters of one of the parties. Whole neighborhoods teeter on the brink of voting or not and the result is to increase swings in electoral outcomes. This increases uncertainty about the location of the median voter and has a corresponding effect on the parties. In short, self-segregation yields party polarization.

Summary

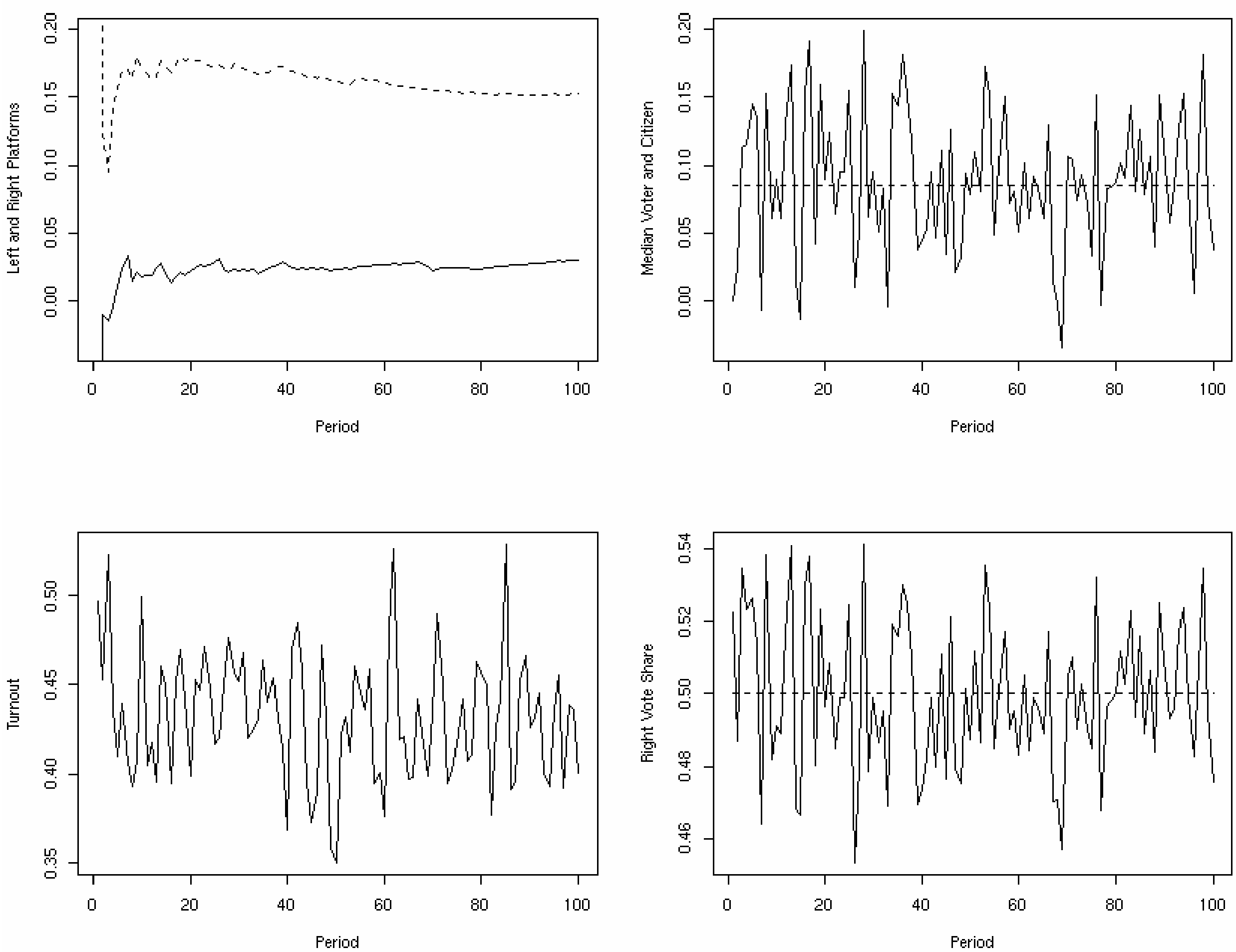
The subject of elections, including turnout and platforms dynamics, is challenging for all social scientists. One of the main reasons for this difficulty lies in the fact that various elements of the electoral process are easier to study separately. We believe that an interdisciplinary approach, built upon contributions from several social science disciplines, will lead us to a better understanding of the subject. The agent-based model we propose is built upon a number of important contributions by sociologists (social context of voters), psychologists (bounded rationality and use of heuristics), economists (platforms dynamics and turnout decision), anthropologists (cultural influence exemplified by imitation), and last, but not least, political scientists (interdependence of voters and candidates, dynamic nature of the electoral competition, empirical analysis of observations).

Our model yields several findings consistent with the empirical literature on parties and voters and suggests some relationships that have not yet been tested (see Table 1 for a summary). The central result is that turnout is significant, platforms diverge, and they both vary over time in an empirically realistic way. These phenomena emerge when we allow both turnout and platform strategies to adapt to one another over time. Making citizens boundedly rational and placing them in a social context turns out to be important. A closer looks at the model neighborhoods shows that local imitation in a social network inherently yields negative feedback dynamics that encourage turnout. The effect is further amplified by the natural limits on the information-processing capacities of the citizens such as the length of memory. On the other hand, local correlation of preferences appears to decrease individual propensity to turn out, which implies that ideologically homogenous communities are least likely to vote. The model also conforms to findings from the social voting literature. Citizens appear to be affected by the turnout decisions of their neighbors.

Turning to parties, the model yields several empirical implications. Allowing turnout to vary endogenously generates uncertainty about the location of the median voter and causes party platforms to diverge. We also note that parties pay attention to electoral mandates as they try to estimate the location of the median voter to remain competitive. This could help to explain empirical work that shows the ideology of US Senate candidates and expectations of economic policy are sensitive to previous vote share (Fowler 2002; Fowler 2003). The model also shows that party platforms tend to correlate with changes in the position of both the median voter and the median citizen, with parties being more sensitive to the latter. Finally, we find that a higher degree of local preference correlation among voters leads to greater platform divergence. Voter segregation yields party polarization.

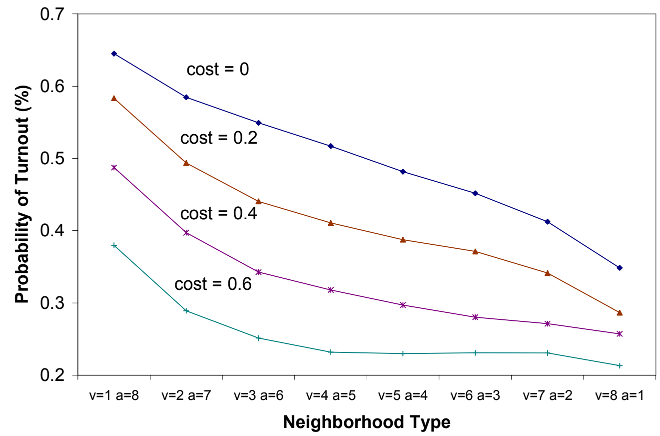
In conclusion, though our model generates relationships that correspond to much of what we know about turnout and platforms, it is important not to read too much into the results. There are many factors that we have not included here that may affect turnout and platforms such as socioeconomic status, endogenous voter and party preferences, multidimensional issue space, multiple parties, multiple districts, different electoral institutions, political institutions like legislatures, and so on. Agent-based modeling makes it easy to add such factors quickly to see if and how they are relevant, but we believe that initial modeling efforts for problems like these should remain simple to provide a bridge to what may already be an extensive analytical effort. Our hope is that this approach will not only provide good predictive models of electoral politics, but also generate hypotheses that inspire future analytical efforts to find related closed-form solutions and empirical efforts to test relationships suggested by the model.

Figure 1. Results of a Single Run of 100 Elections



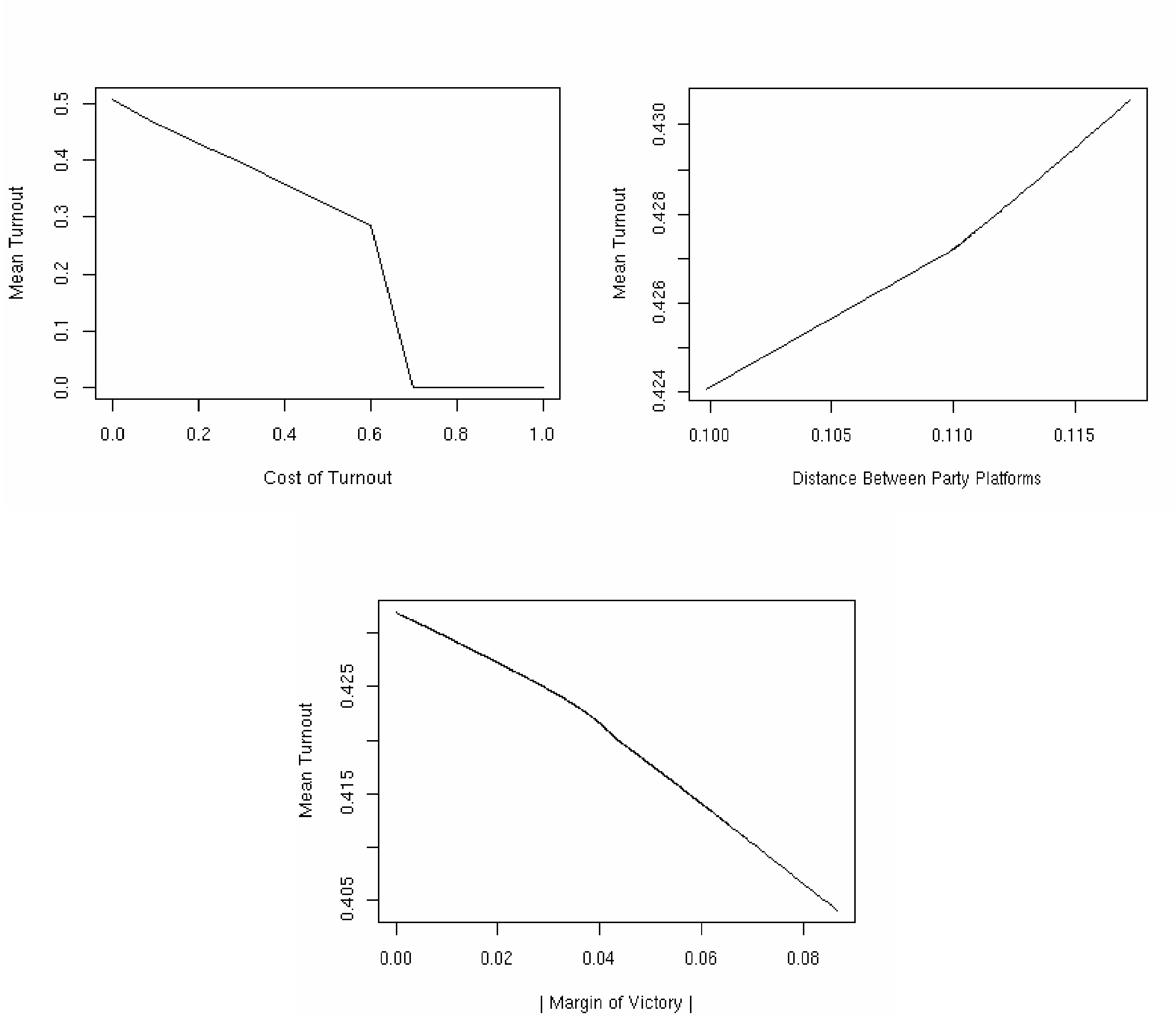
Note: For this run, we test a population of 1024 voters with independent preferences drawn from a standard normal distribution, party preferences at -1 and 1, cost of voting of 0.1, and initial probability of turnout of 0.5. In the upper left graph, solid line is the left party and dotted line is the right party. In the upper right graph solid line is the median voter and dotted line is the median citizen. In the lower right graph the dotted line marks the location of a tie (right vote share = 0.50).

Figure 2. Theoretical Impact of Cost and Neighborhood Type on Turnout



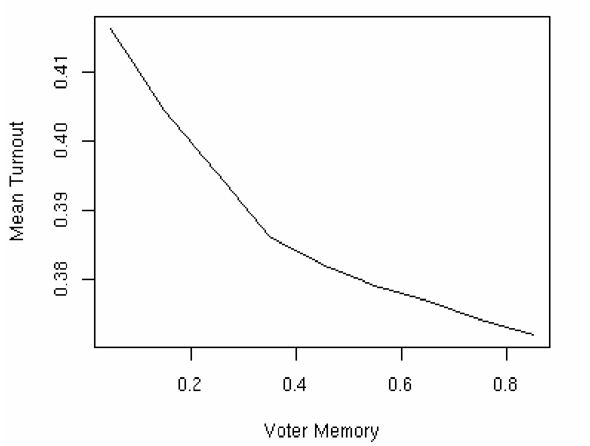
Note: a = number of abstainers in a neighborhood in previous election; v = number of voters in previous election (e.g. v=1 a=8 is a neighborhood with one voter and eight abstainers in the previous election. Citizens in a neighborhood like this have a 65% chance of voting if the cost of voting is 0, 58% if the cost is 0.2, 49% if the cost is 0.4, and 38% if the cost is 0.6). Probabilities are based on assumption that neighbors have preferences that are randomly drawn from a standard normal distribution.

Figure 3. Determinants of Voter Behavior



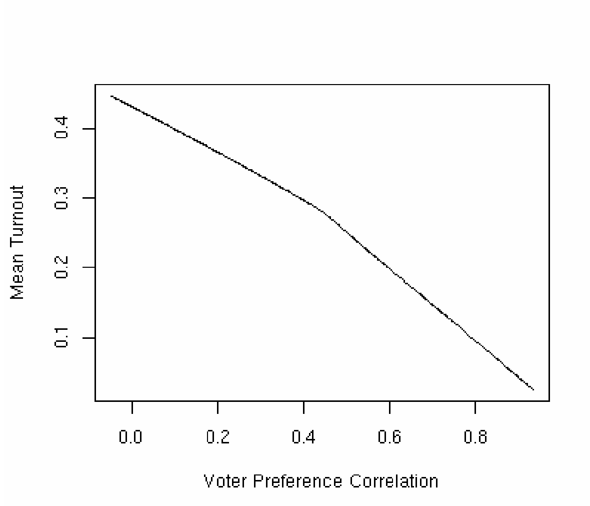
Note: Each graph based on 1000 simulations of a population of 1024 voters with independent preferences drawn from a standard normal distribution, party preferences at -1 and 1, and initial probability of turnout of 0.5. Upper right and lower graph assume a cost of voting of 0.1. Cost of turnout was varied from 0 to 1 in the upper left graph. Means calculated using full-bandwidth LOWESS.

Figure 4. Effect of Memory on Turnout



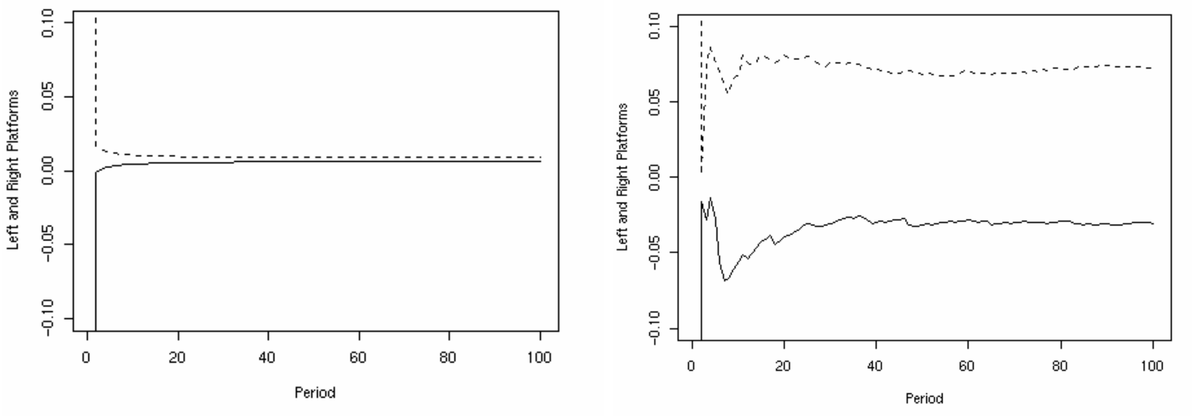
Note: Based on 1000 simulations of a population of 1024 voters with independent preferences drawn from a standard normal distribution, party preferences at -1 and 1, and initial probability of turnout of 0.5. Voter memory was varied from 0 (least weight on past information) to 0.9 (most weight on past information). Means calculated using full-bandwidth LOWESS.

Figure 5: The Effect of Preference Correlation on Turnout



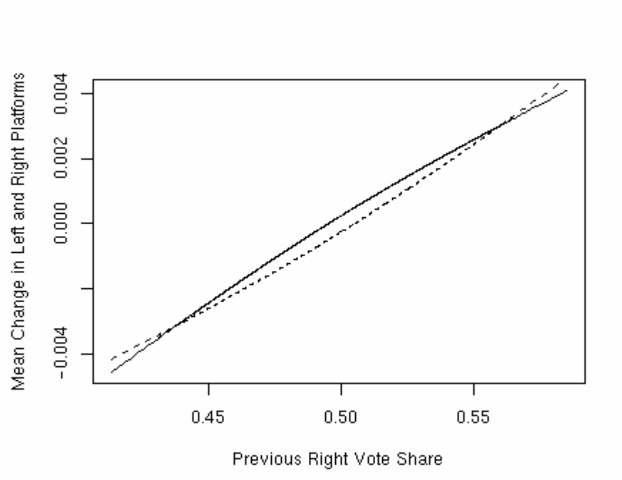
Note: Based on 1000 simulations of a population of 1024 voters with independent preferences drawn from a standard normal distribution, party preferences at -1 and 1, cost of voting of 0.1, and initial probability of turnout of 0.5. Preference correlation was varied from 0 to 0.95. Means calculated using full-bandwidth LOWESS.

Figure 6. Effect of Fixed and Variable Turnout on Party Behavior



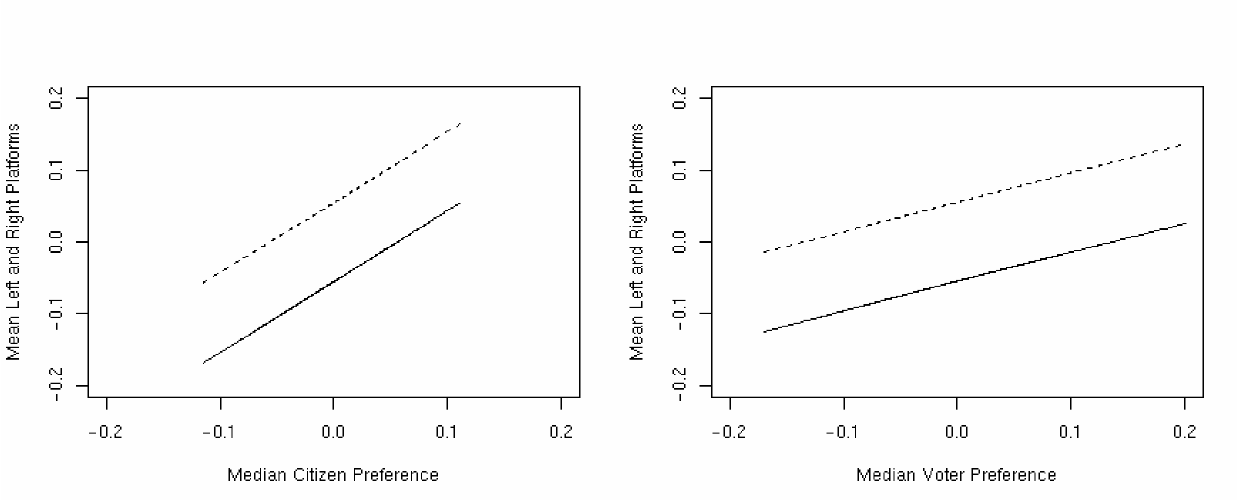
Note: Example based on a population of 1024 voters with independent preferences drawn from a standard normal distribution, party preferences at -1 and 1, and cost of voting of 0.1. Solid line is the left party and dotted line is the right party. The left graph assumes fixed voter turnout and the right graph assumes variable voter turnout.

Figure 7. Effect of Vote Share on Party Behavior



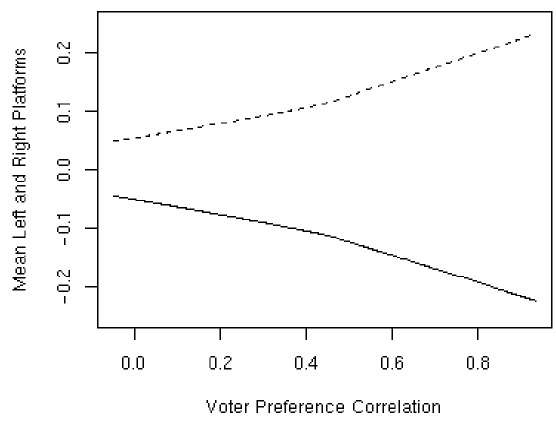
Note: Based on 1000 simulations of a population of 1024 voters with independent preferences drawn from a standard normal distribution, party preferences at -1 and 1, cost of voting of 0.1, and initial probability of turnout of 0.5. Solid line is the left party and dotted line is the right party. Means calculated using full-bandwidth LOWESS.

Figure 8: The Effect of Median Citizen and Median Voter Preferences on Party Behavior



Note: Based on 1000 simulations of a population of 1024 voters with independent preferences drawn from a standard normal distribution, party preferences at -1 and 1, cost of voting of 0.1, and initial probability of turnout of 0.5. Solid line is the left party and dotted line is the right party. Means calculated using full-bandwidth LOWESS.

Figure 9: The Effect of Preference Correlation on Party Behavior



Note: Based on 1000 simulations of a population of 1024 voters with correlated preferences drawn from a standard normal distribution, party preferences at -1 and 1, cost of voting of 0.1, and initial probability of turnout of 0.5. Voter preference correlation was varied from 0 to 0.95. Solid line is the left party and dotted line is the right party. Means calculated using full-bandwidth LOWESS.

Table 1. Summary of Results

Result	Consistent with
Turnout is significant and stable	Mackie & Rose 1997
Turnout depends negatively on voting costs	Rosenstone & Wolfinger 1978; Squire et al 1987; Nagler 1991; Rhine 1995; Knack 1997, 2001; Franklin & Grier 1997; Fenster 1994; Highton 1997; Knack & White 2000; Highton 2000; Huang & Shields 2000; Oliver 1996; Karp & Banducci 2000; Southwell & Burchett 2000b
Party divergence increases turnout	Wolfinger & Rosenstone 1980; Boyd 1989; Hansen et al 1987; Jackson 2000; Kaempfer & Lowenberg 1993
Turnout increases with the closeness of the election	Cox & Munger 1989; Berch 1993; Jackson 1983; Hanks & Grofman 1998; Grofman et al 1998; Nalebuff & Shachar 1999; Alvarez & Nagler 2000
Longer voter memories reduce turnout	*Original result*
Decision to vote depends on turnout behavior of socially-connected peers	Lazarsfeld et al 1948; Berelson et al 1954; Campbell et al 1954; Glaser 1959; Huckfeldt & Sprague 1995; Straits 1990; Knack 1992; Kenny 1992, 1993
Local imitation yields positive feedback for turnout	*Original result*
Ideological segregation reduces turnout	*Original result*
Parties diverge	Peltzman 1984; Grofman et al 1990; Jung et al 1991; Poole & Rosenthal 1984; Hansson & Stuart 1984; Lindbeck & Weibull 1993; Wittman 1977
Parties respond to past margins of victory	Conley 2001; Kingdon 1966; Fowler 2002; Smirnov and Fowler 2003
Parties respond *both* to median voter and median citizen	*Original result*
Ideological segregation yields polarized parties	*Original result*

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Endnotes

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[1] The idea of platform convergence on a single issue space was borrowed from economics. Two shops fighting for customers on a single street will choose their locations in the middle of the street in order to minimize the average distance to the shop for all potential customers (Hotelling 1929).

[2] Code for the R implementation can be found at http://jhfowler.ucdavis.edu

[3] $u_i = -(x_W - p_i)^2 - c$, where u_i is the utility of voter i, x_W is the platform of the winning party, p_i is the preference of voter i, and $c > 0$ is the cost of voting. If a citizen abstains, $c = 0$.

[4] The location of the median voter *m* is the solution to the equation $S = \int_{-\infty}^{(x_L + x_R)/2} f(v \mid m) dv$ where S is the vote share for the left party, x_L and x_R are the party platforms, and $f(v)$ is the voter distribution (which we assume to be normal with variance 1).

[5] We assume that parties model the location of the median voter as though it were drawn from a normal distribution with unknown mean and variance. It is well known that under these conditions the expected median voter will be the sample mean of all previous observations of the location of the median voter, and the variance in the expected median voter will be the sample variance.

[6] Following Wittman (1977) we assume that the expected payoff of each party is the probability of winning times the winning payoff plus the probability of losing times the losing payoff. Parties choose a set of equilibrium policies in which neither party can achieve a higher expected payoff by changing its platform. For a detailed description of the dynamic model of policy-motivated candidates under uncertainty and Wittman political equilibrium see Smirnov and Fowler (2003).

[7] We assume voter preferences are independent and drawn from a standard normal distribution. Later in the article we will relax the independence assumption by assuming preferences are correlated between neighbors.

[8] We assume a Moore neighborhood structure, which means individuals typically have 8 neighbors (top, bottom, left, right, top left, top right, bottom left, and bottom right). We also assume the grid is bounded, so individuals on the edges have fewer neighbors (e.g. an individual on the left edge has five neighbors—top, bottom right, top right, and bottom right).

[9] There are several learning algorithms that we could choose to model this behavior, so we have deliberately chosen a simple one. Citizens have discussions with each of their neighbors and learn how satisfied they were (i.e. their utility) with the results of the previous election. Each citizen then estimates the average satisfaction with voting s^{vote} by summing the satisfaction of all voters in the neighborhood (including themselves, if applicable) and dividing by the number of voters. Similarly, they find the average satisfaction with abstaining $s^{abstain}$ by summing the satisfaction of all abstainers in the neighborhood (including themselves, if applicable) and dividing by the number of abstainers. If the number of voters or abstainers in the neighborhood is zero, then the individual repeats her action from the previous election.

[10] It is important to emphasize here that while citizen decisions are deterministic in our model, the distribution of preferences is stochastic. Thus utility itself is a random variable: a citizen in an *n* person neighborhood with v voters will vote in the next election with probability

$$\Pr\left(\frac{1}{v}\sum_{i=1}^{v}u^i > \frac{1}{n-v}\sum_{j=1}^{n-v}u^i\right).$$

[11] The negative feedback mechanism not only leads to turnout when it is costly but also to abstention when it is not costly – in fact, even if we make the cost of voting negative – turnout will still be significantly less than 100% for this reason.

[12] This may seem like a trivially small cost of voting, but consider the fact that the mean distance between the left and right party platforms in our sample is about 0.2. If voters must bear a cost of 0.1 in order to vote, they are paying one half of the total benefit they would receive if they could choose the election winner. For most of the formal models cited above, the highest cost-benefit ratios that would yield positive turnout are typically several orders of magnitude smaller than this.

[13] Let *M* be a memory parameter. As above, citizens find the average satisfaction level of voting and abstaining for the current election, but they now weight the new information with previous estimates of the average satisfaction levels for voting and abstaining:

$$s_{t+1}^{vote} = (1-M)\frac{1}{v}\sum_{i=1}^{v} u_t^i + Ms_t^i \,,\; s_{t+1}^{abstain} = (1-M)\frac{1}{a}\sum_{i=1}^{a} u_t^i + Ms_t^i$$