# Mathematical Model of Competitive Impacts between Business Entities

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ABSTRACT

Presented here is the economical model with one commodity that is produced by two independent business entities. Investigated is the mutual impact of entities on each other, and formally described is the dynamics of competitive behavior. The research techniques and results are based on the author's Continuous-Time Model of Business Fluctuations. *Journal of Economic Literature* Classification Numbers: D 41; E 32.

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## 1. Introduction

In present paper I show how the techniques of Continuos-Time Model of Business Fluctuations (CTMBF) developed by me earlier (see Krouglov [2-8]) can be extended to investigate the economical interactions between multiple business entities producing the same (or similar) production.

I start the paper with the description of the basics of CTMBF, apply the model to the situation of economy consisting of two entities producing the same type of commodity, and consider three separate market scenarios.

The first one is the situation when one entity discontinues the production of commodity, and other entity fills up whole market for the commodity.

The second situation shows how both entities are adjusting to the change of commodity's demand on market, and new market shares for the entities are calculated.

In the third situation I describe the procedure how business entity with a stronger *production power* can take over the market share of weaker entity.

## 2. Basic Assumptions of CTMBF

(a) The commodity's price is growing / falling with a rate proportional to the amount of commodity's deficit / surplus (i.e. commodity's demand minus its supply) on the market.

(b) The commodity's production rate is increasing / decreasing with an acceleration / deceleration proportional to the rate of commodity's price.

(c) The demand on commodity is increasing / decreasing with a rate inverse proportional to the rate of commodity's price.

## 3. CTMBF Application to Two-Entity-One-Commodity Economy

Let me consider an economical model with one commodity, which is produced by two business entities. I denote $V_d(t)$ the volume of commodity's demand on market at time $t$, and $V_p(t)$ the volume of commodity's supply on market at the same time. I designate $r_d(t)$ and $r_p(t)$ to be the commodity's demand rate and production rate respectively.

Demand and production rates relate with the commodity's demand and supply by the following expressions,

$$r_d(t) = \dot{V}_d(t) \text{ and } r_p(t) = \dot{V}_p(t).$$

At the equilibrium point the commodity's demand and production rates are constant and equal,

$$r_d^0(t) = r_p^0(t) = const \,, \tag{1}$$

and the volumes of demand and supply both equal to zero,

$$V_d^0(t) = V_p^0(t) = 0 \,. \tag{2}$$

We can rewrite (1) and (2) for each entity, therefore

$$r_d^0(t) = r_{p1}^0(t) + r_{p2}^0(t), \tag{3}$$

and

$$V_{p1}^0(t) = V_{p2}^0(t) = 0. \tag{4}$$

### 3.1. One entity drops commodity's production

In our first scenario I assume that one of the entities stops the commodity's production since $t = 0$.

Such action violates the balance between commodity's demand and supply, the commodity's price grows, and another entity is trying to fill up the gap for the commodity on market.

Actually,

$$r_{p2}(t) = \begin{cases} r_{p2}^0(t) = const & t < 0, \\ 0 & t \geq 0, \end{cases} \tag{5}$$

which creates the commodity's deficit on the market,

$$r_{df}(t) = r_d(t) - r_p(t) = r_d(t) - r_{p1}(t), \tag{6}$$

where $t \geq 0$ and $r_{df}(0) = r_{p2}^0$.

Therefore the amount of deficit $V_{df}(t)$ accumulated since $t = 0$ is driving the changes in the commodity's price,

$$\dot{P}(t) = \lambda \cdot V_{df}(t), \tag{7}$$

where $P(t)$ is the value of commodity's price at time $t$, and $\lambda > 0$ represents the coefficient of *price inertia* for the commodity.

Price changes boost the changes in commodity's production,

$$\dot{r}_{p1}(t) = \mu_1 \cdot \dot{P}(t), \tag{8}$$

where $\mu_1 > 0$ is the coefficient of *positive price inductance* on the production rate for the first entity.

Price changes also affect the commodity's demand on market,

$$r_d(t) = -\nu \cdot \dot{P}(t), \tag{9}$$

where $\nu > 0$ is the coefficient of *negative price inductance* on the commodity's demand.

Therefore, the following equation describes the dynamics of the commodity's deficit on market,

$$\ddot{V}_{df}(t) + \nu \cdot \lambda \cdot \dot{V}_{df}(t) + \mu_1 \cdot \lambda \cdot V_{df}(t) = 0, \tag{10}$$

where $\dot{V}_{df}(0) = r_{p2}^0$ and $V_{df}(0) = 0$.

Such equation can be easily solved by the standard mathematical methods (for example, see Arnol'd [1]).

I note here that $V_{df}(t) \to 0$ for $t \to +\infty$, and eventually economy again reaches the balance between the commodity's demand and supply (mathematically speaking, system is stable).

We can derive the equation for the dynamics of commodity's price,

$$\ddot{P}(t) + \lambda \cdot \nu \cdot \dot{P}(t) + \lambda \cdot \mu_1 \cdot P(t) + C = 0, \tag{11}$$

where $\dot{P}(0) = 0$, $P(0) = P^0$ and $C = \lambda \cdot r_{p1}^0 - \lambda \cdot \mu_1 \cdot P^0$.

Note that $P(t) \to \left( P^0 - \frac{1}{\mu_1} \cdot r_{p1}^0 \right)$ for $t \to +\infty$.

The most remarkable result is $r_{p1}(t) \to r_d^0 = (r_{p1}^0 + r_{p2}^0)$ for $t \to +\infty$ (see calculation details in Krouglov [5]) that agrees with our expectations: first business entity fulfils the commodity's gap on market.

### 3.2 Demand on commodity grows

In this situation I assume that commodity's demand is increasing at the time $t = 0$ with a constant rate $\Delta_r$,

$$r_d(t) = \begin{cases} r_d^0(t) & t < 0\,, \\ r_d^0(t) + \Delta_r & t = 0\,. \end{cases} \tag{12}$$

Therefore we can observe the commodity's deficit on market,

$$r_{df}(t) = r_d(t) - r_p(t), \tag{13}$$

where $r_{df}(0) = r_d(0) - r_p(0) = \Delta_r$.

Similarly to the previous scenario, the dynamics of commodity's deficit is as follows,

$$\ddot{V}_{df}(t) + \nu \cdot \lambda \cdot \dot{V}_{df}(t) + (\mu_1 + \mu_2) \cdot \lambda \cdot V_{df}(t) = 0\,, \tag{14}$$

where $\dot{V}_{df}(0) = \Delta_r$, $V_{df}(0) = 0$, $\lambda > 0$ is the coefficient of *price inertia* for the commodity, $\mu_1 > 0$ and $\mu_2 > 0$ are the coefficients of *positive price inductance* on the production rates for the first and second entities respectively, and $\nu > 0$ is the coefficient of *negative price inductance* on the commodity's demand.

Here again $V_{df}(t) \to 0$ for $t \to +\infty$.

For the dynamics of commodity's price, it follows,

$$\ddot{P}(t) + \lambda \cdot \nu \cdot \dot{P}(t) + \lambda \cdot (\mu_1 + \mu_2) \cdot P(t) + C = 0\,, \tag{15}$$

where $\dot{P}(0)=0$, $P(0)=P^0$ and $C=\lambda\cdot r_p^0-\lambda\cdot(\mu_1+\mu_2)\cdot P^0$.

Here it is fulfilled, that $P(t)\to\left(P^0-\frac{1}{(\mu_1+\mu_2)}\cdot r_p^0\right)$ for $t\to+\infty$.

It is interesting to observe that

$$r_{p1}(t)\to r_{p1}^0+\frac{\mu_1}{\mu_1+\mu_2}\cdot\Delta_r \text{ for } t\to+\infty,$$

$$r_{p2}(t)\to r_{p2}^0+\frac{\mu_2}{\mu_1+\mu_2}\cdot\Delta_r \text{ for } t\to+\infty,$$

that is the business entities fill up the commodity's deficit by distributing market shares according to their *production powers*.

### 3.3 Strongest entity takes over

This scenario consists of the following steps: strongest entity produces an excessive amount of commodity for some period of time that drives the commodity's price down, and second entity begins to decrease its share of commodity's production. Then stronger entity follows the market forces and eventually enjoys the bigger market share.

Formally,

$$r_{p1}(t)=\begin{cases} r_{p1}^0(t) & t<0, \\ r_{p1}^0(t)+\Delta_r & 0\le t<\tau. \end{cases} \tag{16}$$

We have the following equations for the dynamics of commodity's deficit,

$$\ddot{V}_{df}(t)+\nu\cdot\lambda\cdot\dot{V}_{df}(t)+\mu_2\cdot\lambda\cdot V_{df}(t)=0, \tag{17}$$

where $0\le t<\tau$, $\dot{V}_{df}(0)=-\Delta_r$, and $V_{df}(0)=0$,

$$\ddot{V}_{df}(t)+\nu\cdot\lambda\cdot\dot{V}_{df}(t)+(\mu_1+\mu_2)\cdot\lambda\cdot V_{df}(t)=0, \qquad (18)$$

where $\tau \le t < +\infty$.

As in Krouglov [7], I will show here the ultimate result for the *critically damped case* (i.e. for $\frac{\nu}{2}=\sqrt{\frac{\mu_2}{\lambda}}$). Other two cases provide analogous results with more calculation efforts.

We can obtain for $0 \le t < \tau$,

$$\begin{aligned} V_{df}(t) &= -\Delta_r \cdot t \cdot e^{-\frac{\nu\cdot\lambda}{2}\cdot t} \\ r_{df}(t) &= \Delta_r \cdot \left(\frac{\nu\cdot\lambda}{2}\cdot t-1\right)\cdot e^{-\frac{\nu\cdot\lambda}{2}\cdot t} \end{aligned} \qquad (19)$$

Therefore, initial conditions for (18) are as follows,

$r_{df}(\tau)=\Delta_r \cdot \left(\frac{\nu\cdot\lambda}{2}\cdot\tau-1\right)\cdot e^{-\frac{\nu\cdot\lambda}{2}\cdot\tau}$ and $V_{df}(\tau)=-\Delta_r\cdot\tau\cdot e^{-\frac{\nu\cdot\lambda}{2}\cdot\tau}$.

For these initial conditions, we can find,

$$r_{p1}(t)\to r_{p1}^0+\Delta_r-\frac{\mu_1}{\mu_1+\mu_2}\cdot\Delta_r\cdot\left(\frac{\nu\cdot\lambda}{2}\cdot\tau+1\right)\cdot e^{-\frac{\nu\cdot\lambda}{2}\cdot\tau} \text{ for } t\to+\infty,$$

$$r_{p2}(t)\to r_{p2}^0-\Delta_r+\frac{\mu_1}{\mu_1+\mu_2}\cdot\Delta_r\cdot\left(\frac{\nu\cdot\lambda}{2}\cdot\tau+1\right)\cdot e^{-\frac{\nu\cdot\lambda}{2}\cdot\tau} \text{ for } t\to+\infty.$$

In this scenario *production powers* represented by the values $\mu_i > 0$ are playing the reverse roles during the last stage (i.e. for $\tau \le t < +\infty$). Here entities distribute between themselves the commodity's surplus accumulated during the time $0 \le t < \tau$.

## 4. Conclusion

Thus the present paper has been achieving two important goals.

At first, it shows how CTMBF can be extended to describe inter-business activities inside a particular industry. Therefore, the domain constrained by CTMBF applications is not limited to the field of Macroeconomics.

Secondly, we validated CTMBF for three distinct economic scenarios. Results produced by the CTMBF have coincided with our expectations based on the surrounding economic reality.

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