MARKET BEHAVIOR AND FORMATION OF VALUE UNDER ABSENCE OF INFORMATION ON PRICES

Nikolai M. Svetlov

The intent of this paper is to stress on the three following items:

(a) the general character of the methodology which is expected to be appropriate for the development of the theory of value;

(b) the process of formation of the economic value under the most general assumptions on making economic decisions;

(c) the problem of a measurement for the economic value.

The most of the results presented here come from the article which is expected to be published in Russian in The Proceedings of the Independent Agroeconomic Society of the Russian Federation, issue 2, Moscow, Izdatelstvo MSHA, 1998.

In the majority of economic models (in particular of the Walrasian type) the concept of value is external in relation to models. It does not result from their character. Value variables are comprised in them with no regard to what should be an object of the model for value is inherent in its commodities. The reason to enter the value in the model is just existence of the value in the reality. Such models are suitable for research of properties, but not of essence of the value.

Leonid V. Kantorovich used alternative approach to research the value. He applied this approach to economic interpretation of a problem of linear programming and afterwards expanded it on the general optimization problem. In these problems, value is a result of the nature of the object. Being an inevitable property of the optimal solution it appears when solving for optimum rather than at a stage of the formal description.

The model described below follows this approach in research of decentralized economic systems. The purpose of modeling is a study of formation of value in conditions, when the economic agents have no information on the prices and on the intents of other agents. Therefore behavior of the agents and, in particular, the supply and demand depend neither on the prices, nor on the fact of their existence in the market. The preferences of systems are assumed given, transitive and complete. No other assumptions on preferences needed. The agents have opportunities of free interchange.

The economy is represented in the form of the *economic* system $H = \{H_k \mid k \in K\}$. H_k is an *elementary* system (in economic interpretation it corresponds to an agent). Each the elementary system follows the rules[1]

$$\begin{cases} \mathbf{a}_{kt+1} = \mathbf{a}_{kt} + \mathbf{x}_{kt} + \mathbf{e}_{kt}, \\ -\mathbf{x}_{kt}^{-} \leqslant \mathbf{a}_{kt}, \\ \mathbf{x}_{kt} \in X_k. \end{cases} \quad (1)$$

Under these constrains the elementary system H_k selects the preferred state given the preferences $\succsim_k$ which orders the field $(h_k, \succsim_k)$ of its possible states $h_k = (\mathbf{a}_k, \mathbf{x}_k)$. Here and in the rest of the paper K means the set of indices of elementary systems; $\mathbf{x}_k$, $\mathbf{x}_{kt}$ — an array of outputs and inputs[2] (positive and negative components respectively) of the system H_k; $\mathbf{a}_k$, $\mathbf{a}_{kt}$ — non-negative array of commodities (stock) under the ownership of H_k; $\mathbf{e}_{kt}$ — an array of net result of interchange operations of H_k; t — time index; X_k — the production set of H_k. Each H_k given any instant of t aims to achieve the most preferable state h_{kt+1} by means of both commodities in ownership and exchange. The $\#I\times\#K$-matrix $\mathbf{E}_t$ consists of the columns $\mathbf{e}_{kt}$ such that $\sum_k \mathbf{e}_{kt} = \mathbf{0}$, and represent some interchange between some subset of elementary systems.

The preferences $\succsim_k$ set relations between two given states h'_k and h''_k of the system H_k. One can easily see that this concept is an equivalent to ordering the arrays of stock: under the constraints of their stocks the elementary systems ever select the state which is the best from their points of view. So, it is reasonable to let the notation $\mathbf{a}_{k1} \succsim_k \mathbf{a}_{k2}$ have the same meaning as $\bar{h}_{k1} \succsim_k \bar{h}_{k2}$, where $\bar{h}_k$ stands for the best (the most preferable) state of H_k given the commodities $\mathbf{a}_k$ under its ownership. Speaking about some array $\mathbf{e}_k$ which is complementary to $\mathbf{a}_k$,

[1] The list of rarely used mathematical notations is at the end of this paper.

[2] Here inputs and outputs mean any change in system's environment due to the existence of the system. Thus they do not correspond to the system's intent (if any) to produce some goods.

its desirability in comparison with other arrays complementary to the same $\mathbf{a}_k$ depends on both the nature of $\succsim_k$ and the current stock $\mathbf{a}_k$. This kind of desirability of any change in the stock (I suggest the term "*marginal preferences*") can be formally described in the following way:

$$\mathbf{e}_{k1} \ggcurlyeq_k \mathbf{e}_{k2} \Leftrightarrow (\mathbf{a}_k + \mathbf{e}_{k1}) \succsim_k (\mathbf{a}_k + \mathbf{e}_{k2}).$$

Here $\ggcurlyeq_k$ denotes the relation of marginal preferences of H_k. To simplify notation it is supposed that H_k is currently in the state h_k and, therefore, owns $\mathbf{a}_k$. So the base to compare different changes in the stock is the stock $\mathbf{a}_k$. As far as $\succsim_k$ is transitive, $\ggcurlyeq_k$ is also transitive. The corresponding relations of the strict marginal preferences $\gg_k$ and the marginal indifference $\approx_k$ are also transitive.

Assume the set I of all the commodities in the economy H consists of two subsets, namely I_n and I_r, so that $I_n \cap I_r = \varnothing$, $I_n \cup I_r = I$. The former embodies the commodities that can be measured with non-negative integers, the latter — with non-negative real numbers. Now I'll introduce the set $I_\$ \subseteq I_r$ of the commodities having the following attributes despite of k:

a) given the complementary stock $\mathbf{y}_k$ (which, generally, can have negative components assuming $\mathbf{a}_k + \mathbf{y}_k$ belongs to the set of the possible states of H_k) one can nominate a real number c so that $c\mathbf{i}_i \approx_k \mathbf{y}_k$;

б) $c > 0$ purports $\mathbf{y}_k + c\mathbf{i}_i \gg_k \mathbf{y}_k$, and $c < 0$ purports $\mathbf{y}_k + c\mathbf{i}_i \ll_k \mathbf{y}_k$.

Generally $I_\$$ can be empty.

Interchange $\mathbf{E}$ is considered to be *expedient for* H_k, when the following is true (assuming $\mathbf{e}_k$ is a column of $\mathbf{E}$):

$$\begin{cases} \mathbf{x}_k + \mathbf{a}_k + \mathbf{e}_k \geqslant \mathbf{0}, \\ -\mathbf{e}_k \leqslant (\mathbf{a}_k + \mathbf{x}_k), \\ \mathbf{e}_k \gg_k \mathbf{0}. \end{cases} \qquad (2)$$

H_k *blocks* the interchange $\mathbf{E}$, if the following conditions are false:

$$\begin{cases} \mathbf{x}_k + \mathbf{a}_k + \mathbf{e}_k \geqslant \mathbf{0}, \\ -\mathbf{e}_k \leqslant (\mathbf{a}_k + \mathbf{x}_k), \\ \mathbf{e}_k \ggcurlyeq_k \mathbf{0}. \end{cases} \qquad (3)$$

Interchange $\mathbf{E}$ is *possible*, if no one of H_k blocks it. Possible interchange should be qualified as *expedient* if it is *expedient for* at least one H_k.

Notation A_k means the set of such the arrays of stock, that the preferences of H_k are defined for every two arrays of this set. No assumptions made of the topology of the set. Let $A_k^{\succsim} \subset A_k$ be the set of the arrays $\mathbf{a}_{k0}$ of commodities under the ownership of system H_k, for which $\mathbf{a}_{k0} \succsim_k \mathbf{a}_k$ where $\mathbf{a}_k$ denotes some array from A_k. E_k means the set of all the net interchanges meeting the condition $-\mathbf{e}_k \leqslant (\mathbf{a}_k + \mathbf{x}_k)$. Hence, H_k do not have enough commodities to participate in any interchange but in that belonging to E_k. If there is such an interchange that all the elementary systems have enough commodities to take part in it then this interchange is a member of the set $E = \bigcap_{k \in K} E_k$ of all the interchanges which meet the conditions $-\mathbf{e}_k \leqslant (\mathbf{a}_k + \mathbf{x}_k)$ for any $k \in K$.

Considering the aim of this study it is reasonable to make an assumption according to which the interchange $\mathbf{E}$ to be actualized in the economy H is a random member of E. The fact that we make no additional assumptions on the distribution of probabilities of different interchanges of E may mean that $\mathbf{E}$ is really selected by chance; or that we do not know how it is selected so it seems to us that this is a random process; at last, that we do know the rule of selection but ignore it in our study. Probably taking this rule into consideration is a way to achieve additional significant results.

In the context of economy H the following statements are true.

1. If exists some chain of interchanges due to which the economy H shifts from the state h_1 to the state h_2 then in the state h_1 a possible interchange exists which shifts H directly to the state h_2.

2. Suppose that some elementary system H_k have shifted from h_{k1} to h_{k2} due to some interchange. Then the set of states, which H_k can reach due to another interchange starting from the state h_{k2}, must be the subset of the set of states which H_k can reach due to an interchange starting from the state h_{k1}. If the interchange moving H_k from h_{k1} to h_{k2} is expedient then the former and the latter sets of reachable states must not be the same.

The two statements directly follow from the assumption of transitive character of preferences.

The relation $\mathbf{a}_k + \mathbf{e}_k \in A_k^{\succsim}$ is a condition under which H_k does not block the interchange $\mathbf{E}$. This condition leads to the following definition: the set $\bar{E}_k$ of all the arrays of net results of interchanges which are not blocked by H_k in the state h_k is $(A_k^{\succsim} - \mathbf{a}_k) \cap E_k$.

3. Assume the following: (a) in each set $\mathbf{a}_k + E_k$ regardless of k there is an array the most preferable for H_k; (b) in the economy H there are the possible interchanges. Then among these possible interchanges at least one interchange exists which shifts to the Pareto snare where there are no expedient interchanges. This conclusion follows out from statements 1 and 2.

In particular the most preferable array in $\mathbf{a}_k + E_k$ exists when: (a) preferences are continuous and all the sets E_k are compact or (b) when $I = I_n$. If the preferences are continuous and the stock exists which is more preferable than any array in any $\mathbf{a}_k + E_k$ but E_k are not sure compact then the Pareto optimum can not exist. In this case among the variety of interchanges one can find such interchange that shifts the economy to the state in which all the components of any expedient interchange are less than a given very small number.

The net interchange result $\mathbf{e}_k$ may be represented in a form $\mathbf{e}_k = \mathbf{e}_k^- + \mathbf{e}_k^+$, where $\mathbf{e}_k^-$ means the quantities of commodities sold, $\mathbf{e}_k^+$ — bought. It is natural that $(-\mathbf{e}_k^-) \prec\!\!\precsim_k \mathbf{e}_k^+$. If the economy persists in the state of the Pareto optimum then, if there are possible interchanges, it must be $(-\mathbf{e}_k^-) \approx_k \mathbf{e}_k^+$. Let's study the interchange between two participants, namely H_k and H_l (where $k, l \in K$) in the state of Pareto snare. To simplify notation, I shall denote the state of H_k before exchange $\mathbf{E}$ as $(\mathbf{a}_k + \mathbf{e}_k^-, \mathbf{x}_k)$, and of H_l — as $(\mathbf{a}_l + \mathbf{e}_l^-, \mathbf{x}_l)$. Here it must be $(-\mathbf{e}_k^-) = \mathbf{e}_l^+$ and $\mathbf{e}_k^+ = (-\mathbf{e}_l^-)$. Then one can easily achieve the following result.

4. Two statements are not jointly feasible in the state of the Pareto snare regardless of k:

(a) there exist semi-positive $\mathbf{e}_k^* \leqslant (\mathbf{a}_k + \mathbf{x}_k + \mathbf{e}_k^-)$ и $\mathbf{e}_l^* \leqslant (\mathbf{a}_l + \mathbf{x}_l + \mathbf{e}_l^-)$ such that $(-\mathbf{e}_k^-) \approx_k \mathbf{e}_l^*$, $-\mathbf{e}_l^- \approx_l \mathbf{e}_k^*$;

(b) the relation $\mathbf{e}_k^* \succ\!\!\succ_l \mathbf{e}_l^*$ is true.

In the Pareto optimum it is obvious that $(-\mathbf{e}_k^-) \approx_l (-\mathbf{e}_l^-)$. Furthermore the sentence (a) implies $(-\mathbf{e}_l^-) \approx_l \mathbf{e}_k^*$, hence it is true that $(-\mathbf{e}_k^-) \approx_l \mathbf{e}_k^*$. Sentence (b) implies that each one of the arrays $(-\mathbf{e}_k^-)$, $(-\mathbf{e}_l^-)$, and $\mathbf{e}_k^*$ represents the better suit for H_l than $\mathbf{e}_l^*$. Therefore H_l is interested in changing $\mathbf{e}_l^*$ for each of them, or else the three corresponding interchanges should be qualified as *expedient* *for* this elementary system. Among them at least change $(-\mathbf{e}_k^-)$ for $\mathbf{e}_l^*$ is not blocked by another side, namely H_k. The conclusion is that there is at least one expedient interchange in the economy H. This sentence is in contradiction with the assumption that H persists in the Pareto optimum. This proves the statement 4.

Assume $I_\$ \neq \varnothing$. This immediately implies the cardinal character of all the preferences of elementary systems in the economy. If $c\mathbf{i}_i \sim_k \mathbf{a}_k$ where $i \in I_\$$ then c perfectly characterizes the rate of desirability of the stores $\mathbf{a}_k$. Note that the assumption of non-empty $I_\$$ requires, in turn, a variety of formal conditions which are the subject of separate study. In particular, this assumption is not contradictory if the conditions of Debreu theorem on the cardinal measurement of desirability take place in H.

Now let again H_k persist in a state $(\mathbf{a}_k + \mathbf{e}_k^-, \mathbf{x}_k)$, and H_l in a state $(\mathbf{a}_l + \mathbf{e}_l^-, \mathbf{x}_l)$.

5. If one can qualify some $i \in I_\$$ and non-negative c_k, c_l such that $c_l\,\mathbf{i}_i \leqslant \mathbf{a}_k + \mathbf{e}_k$, $c_k\,\mathbf{i}_i \leqslant \mathbf{a}_l + \mathbf{e}_l$, $(-\mathbf{e}_k^-) \approx_k c_k\,\mathbf{i}_i$, $(-\mathbf{e}_l^-) \approx_l c_l\,\mathbf{i}_i$, then $c_k = c_l$.

This statement is directly follows from the statement 4.

As a consequence, if in the Pareto snare one can qualify some stock $\mathbf{y} \geqslant \mathbf{0}$ such that:

(a) the given H_k has the property $\mathbf{y} + \mathbf{a}_k$, $\mathbf{a}_k \geqslant \mathbf{0}$;

(b) some other elementary systems H_l, $l \in K' \subseteq K$, have the properties $\mathbf{y} + \mathbf{a}_l$, $\mathbf{a}_l \geqslant \mathbf{0}$;

(c) $\mathbf{y} \approx_k c\mathbf{i}_i$, $i \in I_\$$;

(d) $c\mathbf{i}_i \leqslant (\mathbf{y} + \mathbf{a}_l)$ $\forall\, l \in K'$;

then it can be easily proved that for each H_l the relation $\mathbf{y} \approx_l c\mathbf{i}_i$ is true. The way to prove that is a study of a specific possible interchange, namely changing $\mathbf{y}$ for $\mathbf{y}$.

The properties of the formal economy H can be transferred to the real economy in the case of:

(a) absence of appreciable externalities;

(b) the conditions under which the preferences can be treated as typically transitive.

To achieve wider generality, an additional study needed, which is expected not to be very complex. In the framework of real equivalent of H, in this study it is shown that assuming other factors are the constants every interchange reduces the further interchange opportunities (this results from statement 2).

In the reality, it is reasonable to consider the terms of statement 3 acceptable. As a result among the variety of possible interchanges (if possible interchanges exist) there is at least one interchange (maybe between more than two agents) which shifts the economy in the Pareto optimum. In this particular state, if reached, in accordance to statement 4, two sets of commodities are marginally equivalent if each of these sets is indifferent relative to the sets supplied by the two agents taking part in two-side interchange.

If in the formal economy persisting in the Pareto optimum the following statements take place:

(a) there exists a commodity i which due to its nature is able to be a measure of value (that means $i \in I_{\$}$);

(b) there is a possible interchange $\mathbf{e}_k = \mathbf{e}_k^- - \mathbf{e}_l^-$ between exactly two elementary systems H_k and H_l;

(c) both H_k and H_l have the commodity i in amount exceeding some minimum;

then both $(-\mathbf{e}_k^-)$ and $(-\mathbf{e}_l^-)$ are the equivalents to the same amount of the commodity i from the point of view of both H_k, and H_l. This consequence from the statement 5 is a subject to be treated as the possibility of formation of the value having the following properties:

(a) it is formed as a result of random expedient interchanges;

(b) it requires a commodity (at least one) which is potentially able to play a role of measure of the value;

(c) given the store of commodities its value is common for all the agents in the market.

Some agent who does not have enough commodity i to buy some store of commodities worth the amount c of commodity i is able to consider it more preferable than the amount c of commodity i (unlike every agent having enough i). Nevertheless the only real opportunity selling this store is to gain the only c of i or equivalent. Thus the "effective" value, in other words the value which control prices, is the equivalent of the opportunity cost and is the same for each participant of a possible interchange.

If more than two sides participate in an interchange then $\mathbf{e}_k^-$ and $\mathbf{e}_l^+$ may differ (sure $k \neq l$). But nevertheless $\mathbf{e}_k^-$ and $\mathbf{e}_k^+$ must be equivalent if the Pareto optimum persists. This easily implies expanding properties of value on the case of interchanges with more than two participants.

For the sake of interpretation, it is natural to treat the statement 5 in terms of value. The statement 4 has, generally, the same meaning as the statement 5 though one meets difficulties trying to directly interpret it. If one introduces a value as some reality which does not necessary require quantification but is able to be reflected by means of stores of commodities then she can treat the statement 4 as the possibility of formation of the same value for all the economic agents (and hence for the economy as a whole) due to any chain of expedient interchanges (suppose there is enough time to complete all the possible interchanges) under the conditions accepted here with no regard to the existence of a potential measure of value.

Mathematical notations used in this paper

$\mathbf{x}^+ = \frac{\mathbf{x} + |\mathbf{x}|}{2}$ — an array resulting from $\mathbf{x}$ my means of replacing negative components with zeroes.

$\mathbf{x}^- = \frac{\mathbf{x} - |\mathbf{x}|}{2}$ — an array resulting from $\mathbf{x}$ my means of replacing positive components with zeroes.

$\#X$ — the number of elements of a countable set X.

$\mathbf{i}_k$ — the k-th column of the unit matrix $\mathbf{I}$.