

# **Asymmetric Information in a Competitive Market Game: Reexamining the Implications of Rational Expectations**

by

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*Abstract:* We examine price formation in a simple static model with asymmetric information, an infinite number of risk neutral traders and no noise traders. Here we re-examine four results associated with rational expectations models relating to the existence of fully revealing equilibrium prices, the advantage of becoming informed, the costly acquisition of information, and the impossibility of having equilibrium prices with higher volatility than the underlying fundamentals.

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## 1. Introduction

Milgrom (1981) and Dubey, Geanakoplos, and Shubik (1987) offer powerful critiques of rational expectations equilibrium as a tool for investigating price behavior and information acquisition. The core of the criticism is that, in order to understand the value of and incentives to acquire asymmetric information, one has to model the mechanism through which prices are formed. Well known paradoxes, such as those discussed by Grossman (1976) and Grossman and Stiglitz (1980), are symptomatic of the fact that rational expectations equilibrium is not well suited for understanding how prices come to incorporate information. Rather, it is a limiting notion, capturing the situation in which no further adjustment in actions or prices is necessary. If one takes the rational expectations definition literally, then one must reconcile the assumption that all traders can observe the price realization before taking actions with the fact that prices result from those actions.

Here, following this criticism, we examine price formation in a simple, static model. Trade occurs according to the rules of the market game of Shapley and Shubik (1977). Specifying the trading rules allows us to explore price behavior, information acquisition, and efficiency. The point of our analysis is to reconsider several of the results which are characteristic of a (noiseless) rational expectations approach and to show that these results can change when price formation is modeled explicitly.

To be more specific, let us begin by identifying results which are true in a (noiseless) rational expectations framework but which need not hold when price formation is explicit.

**R1:** Equilibrium prices reflect the underlying expected value of the economic fundamentals (suitably weighted for time discounting and risk aversion).<sup>1</sup>

**R2:** There generically exist equilibria in which (i) equilibrium prices reveal all payoff-relevant information that the traders could have by pooling their signals, and (ii) an uninformed trader faces no disadvantage in trading with informed traders.<sup>2</sup>

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<sup>1</sup> See, for instance, Tirole (1982).

<sup>2</sup> The generic existence of fully revealing equilibria in rational expectations models, (i), is shown in Radner (1979) and Allen (1981), among others. These results rely on some assumptions about the structure of the assets in the economy, as discussed by several authors, and surveyed by Jordan and Radner (1982). Part (ii) of R2 is a consequence of the price taking assumption and fully revealing prices, since all agents may condition on the same price function. See, for example, the discussion following Theorem 3 in Grossman (1981).

**R3:** In models where buyers can acquire information by paying a small fixed cost, and where exogenous supply is noiseless, equilibrium fails to exist. <sup>3</sup>

**R4:** Equilibrium prices cannot be more volatile than dividends. <sup>4</sup>

At first glance, R1 and R2 seem quite attractive, since they imply that prices are efficient in communicating information. However, when one takes a closer look they can be problematic, especially in light of R3. The fact that prices reflect the information present in an economy is of fundamental importance in efficiently allocating resources. However, the fact that prices reveal all the available information, tied into a rational expectations model where agents may invert prices and condition on them, renders gathering information unprofitable (hence R3). The nonexistence of equilibria with costly information acquisition is a problem since it is not clear whether or not information will be gathered and eventually disseminated. To understand information acquisition and its welfare properties, one has to overcome the equilibrium non-existence problem (R3), and the ability of agents to condition on fully revealing equilibrium prices (R1 and R2). The motivation for worrying about excess volatility (R4) is tied directly to the data, which suggests that the simple models which have been well analyzed cannot generate some of the basic characteristics of observed prices.

Of course, the difficulties associated with results R1-R4 have been recognized and analyzed in previous papers. Grossman and Stiglitz offered what has subsequently become a popular method of handling the problems associated with R3. That method is to directly alter the rational expectations model so that prices are not fully revealing, and so R1 and R2 do not hold. This can be done by adding noise to the rational expectations framework. Noise can come in many forms, including an exogenously given noisy supply of an asset

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<sup>3</sup> The seminal articles on this are Grossman (1976) and Grossman and Stiglitz (1980).

<sup>4</sup> This was raised in papers by Shiller (1979) and LeRoy and Porter (1981) that suggested that the observed variability of stock and bond prices cannot be matched with a model built on the expected discounted stream of dividends. [For a survey, and discussion of some debates surrounding this issue, see Flood and Hodrick (1991).] More recent studies [see for example Campbell and Shiller (1988), West (1988), and Cochrane (1992)] admit richer models and use stronger econometric techniques. Particular focus has been on finding a way to generate a time varying discount rate which would generate sufficient volatility in prices. The requirements on the volatility of this discount rate seem to be too large to justify with reasonable parameters in a symmetric information competitive model (and thus a fully revealing rational expectations model-given the relation to the artificial full information model).

and ‘noise trading’ (see, for instance, Grossman and Stiglitz (1980), Verrechia (1982), Kyle (1989)). Noisy equilibrium prices are generally not fully revealing. Once prices are not fully revealing, information can become valuable, and so one can address the costly information equilibrium existence problem, R3, and establish existence of equilibria in which agents pay to become informed (see, for instance, Grossman and Stiglitz (1980)).<sup>5</sup> More recently, variations on models with noisy supply and noise traders have also been used to generate excess volatility, overcoming R4 (see Delong, Shleifer, Summers, and Waldman (1992), Campbell and Kyle (1993) and Wang (1993)).

A second approach to dealing with the difficulties associated with R1-R4 also results in imperfectly revealing prices, but does not introduce the noise into the prices directly. It is to model the possibility of sunspot equilibria and/or equilibria with price bubbles, where traders’ actions are tied to non-fundamental events. The original papers on excess volatility, R4, argue that the data provide evidence for bubbles (again, see Flood and Hodrick (1991) for a survey). The sunspots literature also admits excess volatility (and is consistent with Shiller’s (1981) alternative hypothesis). See, for instance, Shell (1977), Azariadis (1981), and Cass and Shell (1983). Sunspot equilibrium and bubbles loosen the tie between prices and fundamentals, thus avoiding R1 and overcoming R4 since the variability of prices may be tied to events which all traders rationally expect to influence prices, but which are extraneous. More recent papers on this subject allow for private information, which can then also address some of the difficulties associated with perfect revelation and information acquisition, R2 and R3. In Jackson and Peck (1991), it is shown that there exist speculative equilibria in which fully rational agents use private information that is uncorrelated with dividends in bidding for assets. Agents earn strictly positive rents from their information and so R2 and R3 are overcome, in addition to R1 and R4.

The approaches of noisy equilibrium, sunspot equilibria, and bubbles to resolving the difficulties associated with R1-R4 differ in important ways from the approach of the present paper. In the above approaches, the informativeness of prices, the value of information,

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<sup>5</sup> Another approach is to examine imperfectly revealing rational expectations equilibria where no noise is added. [See Ausubel (1991) for an example of such an equilibrium.] When such equilibria exist R1, R2(ii), and R3 can be overcome. The approach of modeling price formation explicitly, however, has advantages including tractability, understanding how information is incorporated into prices, and handling R2(i) and R4.

and the level of excess volatility are all tied to the amount of noise (either exogenous or endogenous) present in the economy. Here, we find that these price characteristics depend instead directly on the structure of the information associated with the underlying dividends and the cost of gathering information. Also, there are environments where there may not be sufficient noise to generate excess volatility, and situations where sunspot equilibria do not exist. Here, we have no noise or extrinsic correlating devices, and yet still exhibit costly information acquisition and excess volatility. In this way the approach of explicitly modeling price formation offers a completely complementary approach and insight. Our analysis suggests a format that allows us to separate effects due to the fundamental information structure and trading rules from effects attributable to the inherent noise in an economy. Also, we can examine welfare implications and compare welfare across institutions, without having noise traders that are required to absorb losses.

The key to overcoming the difficulties associated with R1-R4 without introducing exogenous noise comes from two sources. First, agents cannot condition directly on the price. They make bids and offers in ignorance of what the eventual market clearing price will be (much like in a sealed bid auction). In this way, an agent's own information is valuable. Second, when these bids and offers are aggregated, they do not necessarily result in a price that reflects any conditional expected value of the asset. We will call this the "uncoupling" effect and analyze it some detail. Equilibrium prices may turn out, ex post, to be invertible and fully revealing; but will not reflect the expected value of the dividend. The ex post invertibility of prices is not helpful to the agents at the interim stage at which they must choose bids and offers.

Let us now discuss the relationship of our work to previous models with explicit price formation. Milgrom (1981) and Matthews (1984) both examine information acquisition in auctions. These papers deal directly with R3, showing that there exist equilibria with costly information acquisition. Jackson (1991) shows that R3 can also be overcome in a demand submission game, which differs from the rational expectations paradigm only in that the market clearing price is required to be determined by (measurable with respect to) the demand functions of the agents. Regarding R1 and R2 (i) on the informativeness of the resulting prices, results by Wilson (1977) and Milgrom (1979) show that with large numbers of bidders the price begins to approximate the conditional expected value of the

asset (thus R1 and R2 (i) hold). Matthews (1984) shows that this result may depend on the cost and structure of information if information acquisition is endogenous. On R2 (ii), there are several results such as one by Milgrom and Weber (1982) that show that uninformed bidders may have to abstain from bidding and have lower expected utilities in equilibrium than informed bidders.

Thus, the previous literature has shown that the difficulties associated with R3 and R2 (ii) can be overcome with explicit models of price formation. However, with large numbers of bidders the existing results from the auctions literature tend to show that R1 and R2 (i) hold, at least approximately. Finally, excess volatility (R4) has not been examined in an auction setting or any model which pays careful attention to price formation.<sup>6</sup>

We follow Dubey, Geanakoplos, and Shubik (1987), in using the market game of Shapley and Shubik (1977) as a model of price formation. The motivation of Dubey, Geanakoplos, and Shubik is similar to Milgrom's for overcoming R3 and R2 (ii). They also relate the equilibria of the market game to Walrasian equilibria under certain conditions, and study efficient market hypotheses. Our focus here will be more exclusively on the standard results, R1-R4 above.

Our contributions in this paper may thus be summarized as follows.

First, we show that excess volatility (overturning R4) is possible within an extremely simple competitive model with an infinite number of risk neutral traders. The key lies in explicitly modeling price formation. The intuition and sources of volatility complement previous studies of excess volatility.

Second, the understanding of why excess volatility is possible is due to an uncoupling of prices and dividends, which happens since agents must decide on bids and offers based on their own information and the anticipated (Bayesian) equilibrium actions of others, but without being able to fully condition on the realized price. This uncoupling of prices and dividends yields counter examples to R1. Indeed, equilibria exist in which the price can be higher than the highest possible dividend. For our environment, R1 and R2 together would imply that the price should equal the dividend in each state of nature. This conclusion is obviously inconsistent with a positive cost of information acquisition. Even when

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<sup>6</sup> We report below an example that shows that excess volatility is possible in an auction with small numbers of agents. The excess volatility, however, does not remain in that example with larger numbers of agents.

information is endowed (costlessly), we show that, for an open set of parameter values, no equilibrium has prices which equal the dividend, state by state.

Third, one can interpret parts (i) and (ii) of R2 as two expressions of the efficient markets hypothesis. The auctions literature makes clear that both parts of R2 may be false, although uninformed traders recognize their disadvantage and stay on the sidelines. We provide a counter example to R2 in which uninformed traders participate alongside informed traders, yet receive lower returns (which balance the cost of becoming informed). In this counter example, part (i) holds but part (ii) does not, so the two expressions of the efficient markets hypothesis are not even equivalent.

In examining the price characteristics in our model we also find two other interesting results. The first is that, with costly information acquisition, the price exhibits a “V”-shape as a function of the cost of information, where prices are highest if all agents are informed or all are uninformed, but lower when only some agents become informed.<sup>7</sup> The resulting welfare implications follow the same V-shape, since sellers care about expected prices and buyers receive zero expected profits, ex ante. Resources spent on information are lost to the economy and have no beneficial redistributive effect, so welfare is highest when information is either costless or so costly that no one chooses to acquire it. We discuss how this can change if price information is important in directing investment decisions.

Finally, there is a result that we call “good news can be bad news.” Suppose that a particular signal is only received when the realized dividend takes on high values. It would therefore be called “good news.” However in states where many agents receive the good news, the bidding will actually drive the price above the level of the dividend. Thus, good news can be bad news. This overshooting can happen in equilibrium since it is balanced by the possibility that fewer traders receive good news so that the price undershoots the dividend. The good news can be bad news effect can cause dividend volatility to be amplified by the price formation process, providing a concrete and intuitive example of why the variance of prices can exceed the variance of dividends.

The Shapley-Shubik market game is a framework that allows us to overcome the difficulties associated with R1-R4 (with new insights) and identify interesting price characteristics.

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<sup>7</sup> This supports a concluding remark of Matthews (1984) who found a V-shape as a function of the level of proprietary information in discriminatory auctions. We show that the discounting of the price exactly balances the costs of acquiring information.

These results suggest the need to reassess our notions of what theory implies about equilibrium in markets with many traders and asymmetric information. We do not claim that the simple model corresponds to any real-world trading mechanism. However it does suggest that it is worthwhile to venture beyond this tractable model to games corresponding to actual or proposed trading institutions.

## 2. The Basic Model with Endowed Information

In working with the market game, we consider an infinite number of agents. This assures that price formation is competitive: no single agent can influence the price. We also consider risk neutral agents. This provides us with closed form solutions and guarantees that the results do not rely on risk sharing. The terminal payoff (dividend) of the risky asset is unknown, and individuals observe a signal which is correlated with the payoff. Signals are independently and identically distributed across individuals, conditional on the dividend.

We begin with an analysis where the information structure is exogenously given. Agents privately observe signals which are independently and identically distributed. In subsequent sections (5 and 6), we will extend the model so that information is costly and only some agents will choose to become informed in equilibrium.

There is a countable number of risk-neutral agents, indexed by  $i \in \{1, 2, 3, \dots\}$ . The countable number assures that markets are competitive, without running into the technical problems with regards to the law of large numbers experienced by models with a continuum of agents. There is a single consumption good. A risky asset pays dividends of the consumption good. Agents are each endowed with 1 unit of the consumption good and 1 unit of the risky asset.

INFORMATION.

There is a finite number,  $n$ , of possible dividend realizations. These are represented by the set  $D = \{d_1, \dots, d_n\}$  with generic element  $d \in [0, 1]$ . An element  $d$  is referred to as the dividend state. (This distinguishes it from a state of the world, which also specifies realizations of all private information.) The probability of dividend state  $d$  is  $\pi(d)$ . Information takes the form of signals  $s \in S = \{s_1, \dots, s_m\}$ . The probability of observing signal  $s$  conditional on  $d$  is  $\pi(s|d)$ . The probability of realizing  $d$  conditional on observing

signal  $s$  is denoted  $\pi(d|s)$ , and is defined according to Bayes' Rule. Let  $s^i$  denote the signal observed by the  $i$ -th agent. Signals are independently and identically distributed across agents, conditional on  $d$ .

Agents observe information at time 0, trade at time 1, and the state is revealed and agents consume at time 2.

#### THE MARKET GAME.

Trade occurs according to the market game of Shapley and Shubik (1977). Agents simultaneously make bids and offers in the market for the risky asset. Offers are amounts of the risky asset to be sold, and bids are amounts of the consumption good to be paid. The total amount of the consumption good bid is allocated to agents in proportion to the size of the offers they make. Similarly, the total amount of the risky asset offered is allocated to agents in proportion to the size of the bids they make.

The strategy space for agent  $i$  is  $(b^i, q^i) \in [0, 1] \times [0, 1]$ , where  $b^i$  denotes the amount of the consumption good bid by agent  $i$  and  $q^i$  denotes the amount of the risky asset offered by agent  $i$ . Notice that agents are permitted to bid and offer at the same time. We have limited bids and offers to be bounded simply for expositional convenience. The results still hold without this limit, as will become clear in the proofs.<sup>8</sup>

Given the infinite number of agents, we need a definition of the average amount bid and offered. We denote these by  $B$  and  $Q$  and define them as

$$B = \lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{b^i}{N}$$

and

$$Q = \lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{q^i}{N}.$$

No trade takes place if either  $B$  or  $Q$  is zero or not well defined. Otherwise, the price associated with the market game is

$$P = \frac{B}{Q}.$$

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<sup>8</sup> For more on this see an earlier version of this paper (Jackson and Peck (1993) where unlimited bids and offers were admitted.

An agent offering  $q^i$  units of the risky asset for sale receives  $q^i P$  units of the consumption good, and an agent bidding  $b^i$  units of the consumption good receives  $b^i/P$  units of the risky asset.<sup>9</sup>

We consider (Bayesian) Nash equilibria of the market game, where agents choose bids and offers after having observed information, and where the payoff to agent  $i$  is the utility of final consumption. Agents' strategies are functions from observed signals into bids and offers, which maximize expected utility conditional on information and the strategies of the other agents. We focus on pure strategies, so agent  $i$ 's strategy space is represented by  $b^i : S \rightarrow [0, 1]$  and  $q^i : S \rightarrow [0, 1]$ . Given risk-neutrality and the convexity of the action spaces, considering mixed strategies would not substantively change matters, but would lead to considerable technical complications.<sup>10</sup>

A strong version of the law of large numbers assures that prices have the following simple representation (almost surely) as a function of the dividend state  $d$ , whenever bids and offers are well defined for each  $s$ . (See the appendix for details.) That is, if  $B(s) = \lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{b^i(s)}{N}$  and  $Q(s) = \lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{q^i(s)}{N}$  are well defined for each  $s \in S$ , then

$$P(d) = \frac{\sum_s \pi(s|d)B(s)}{\sum_s \pi(s|d)Q(s)}. \quad (1)$$

### 3. Uncoupling Prices and Dividends.

In a rational expectations world, a common method for proving generic existence of fully revealing equilibrium prices is to consider the price corresponding to an artificial economy in which everyone has access to the full information of all agents. (See, for instance, Grossman (1981) and Allen (1981).) In our setting, as long as  $\pi(s|d)$  differs across dividend states, this full information situation corresponds to knowing the dividend state. The unique equilibrium prices<sup>11</sup> in such full information situations are those equal to the corresponding dividends. Thus, using this approach, one knows that in our setting there generically exist

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<sup>9</sup> For more discussion of this market game see Shapley (1976), Shubik (1973) (1977), Mas Colell (1982), Dubey, Geanakoplos, and Shubik (1987), and Peck and Shell (1991).

<sup>10</sup> A mixed strategy would map signals into the some set of probability measures over  $(b, q) \in [0, 1] \times [0, 1]$ . Careful restrictions on the admissible measures are necessary so that the resulting measures over the product space are well defined over particular realizations.

<sup>11</sup> There is always a no trade equilibrium as well, in which prices are not well defined. In such an equilibrium no agent makes any bids or offers, since they anticipate that no other agent will make a bid or offer.

fully revealing rational expectations equilibria, where the prices correspond to the realized dividend in each dividend state and all agents are indifferent between trading and not trading.

To show that there is an interesting contrast between rational expectations equilibria and the explicit price formation process, we show that for open sets of parameters it is impossible to have such rational expectations equilibrium prices be the outcome of the price formation process. We show that for an open set of parameters equilibrium prices of the market game must differ from the dividend values.

The key element of the rational expectations equilibrium existence argument is that agents are able to incorporate all the information that is contained in the price that is currently being formed. In the market game, such conditioning is not possible, as agents must make bids and offers without knowing what the realized price will turn out to be. Thus, prices can still end up being fully revealing *ex-post*, but are not available at the interim stage.<sup>12</sup> This makes it possible for all equilibrium prices to differ from dividends.

The uncoupling of prices and dividends is important, since it means that the equilibrium prices we find through the price formation process in some cases must differ significantly from the fully revealing rational expectations prices. This uncoupling leads to some of the properties we examine in the coming sections, such as excess volatility and endogenous acquisition of information in equilibrium.

For prices to equal dividends in each dividend state, it must be that for each  $d$

$$P(d) = \frac{\sum_s \pi(s|d)B(s)}{\sum_s \pi(s|d)Q(s)} = d$$

or that for each  $d$

$$\sum_s \pi(s|d)B(s) - \sum_s \pi(s|d)Q(s)d = 0 \tag{2}$$

Fix a number of dividend states  $n$  and a number of signals  $m$ . For the purposes of finding solutions to (2), our model is parameterized by the selections of  $\pi(s|d)$  and  $D$ . That is, one needs to specify  $d_1, \dots, d_n$ , and  $\pi(s|d)$  for each  $d \in D$  and each  $s \in S$ . Let  $\mathcal{A}$

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<sup>12</sup> There is some question of what would occur in our model if agents could place fully contingent limit orders. (Such orders are permitted in the games analyzed by Wilson (1979) and Jackson (1991).) We conjecture, but do not explore in this paper, that as long as there are some costs associated with fully contingent limit orders, or some limits to how contingent orders can be made, that uncoupling results would still hold.

denote the set of all such selections.  $\mathcal{A}$  can be viewed as the subset of  $[0, 1]^{nm}$  such that  $\sum_{i=(k-1)m+1}^{km-1} a_i \leq 1$  for each  $k \in \{1, \dots, n\}$ .

**Proposition 1.** UNCOUPLING PRICES AND DIVIDENDS. *If  $m \leq \frac{n}{2}$  then generically<sup>13</sup> there are no combinations of bids and offers where realized prices equal dividends. If  $m > \frac{n}{2}$ , then there is an open set in  $\mathcal{A}$  such that no selection of parameters in that open set has bids and offers for which realized prices equal dividends. If  $m > \frac{n}{2}$ , then there is also an open set in  $\mathcal{A}$  such that each selection of parameters in that open set has at least one equilibrium for which realized prices equal dividends.*

The first part of the proposition is fairly obvious. It applies to situations where there are few signals relative to dividend states. If there are too few, then it will clearly be impossible to have prices equal to dividends, simply because bids and offers cannot take on enough different values to adjust to reflect the different dividend realizations. The second part of the proposition is of more interest since it applies to situations where signals are rich enough relative to dividends to have the possibility of prices matching dividends.<sup>14</sup>

To get a feel for the proposition, we can consider a simple situation where the dividend can take on a value of 0 or 1 and that there are two signals, say “high” and “low”, which are imperfectly correlated with the dividend value. Given the imperfect correlation, there will always be some traders who see the high signal, even when the state is low. Their bids will thus result in a positive price, making it impossible for the price to match the dividend. This situation does not rely on the smallest dividend realization being 0. For instance, it is also true if there are two different dividend realizations which are relatively close to each other and a third which is far away. The basic intuition is that in any dividend state there will be agents observing different signals. Only in some circumstances do the probabilities and dividend states allow strategies to be found which, when weighted according to the appropriate conditional probabilities (of signals given dividends), yield prices that are precisely equal to the dividend realizations in each state.

One implication of Proposition 1 is that R1 fails. An argument along the lines of the proof of Proposition 1 shows that prices must generically reveal the dividend state (as in the

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<sup>13</sup> On an open set and dense set on  $\mathcal{A}$ .

<sup>14</sup> The reason for the comparison of  $m$  to  $\frac{n}{2}$  rather than  $n$  is that agents can make both bids and offers.

corresponding rational expectations model). That is, the expected dividend, conditional on the price, is the actual dividend. Therefore, Proposition 1 shows that for an open set of parameters prices are not equal to the expected value of the dividend, conditional on the price. The robust example in section 4 shows that the price may sometimes be higher than the highest possible dividend so in fact the price may not equal an expected value of the dividend, *conditional on anything*.

Consider the two strong versions of the efficient markets hypothesis given in R2. These statements are that (i) prices convey all relevant information, and (ii) uninformed traders are not at a disadvantage relative to informed traders. In a fully revealing rational expectations equilibrium, the two are equivalent. However, (i) and (ii) are not equivalent in this model of price formation. Prices can turn out to be fully revealing ex-post, but not to equal dividends state by state. This means that an uninformed trader could be at a disadvantage relative to an informed trader, as will be shown in more detail in Section 4. Thus, part (i) of R2 holds, while part (ii) can fail.<sup>15</sup>

#### 4. Excess Volatility and Comparative Statics.

The next property of prices that we consider is their variance. One anomaly in observed price patterns is that the variance of prices exceeds the the variance of the underlying dividends. This is not possible in a fully revealing rational expectations equilibrium. Here we show that in a market game it is possible for the variance of equilibrium prices to exceed the variance of the underlying dividends.<sup>16</sup> We will examine how excess volatility depends on the informativeness of signals.

In this section, we continue to assume that each agent is endowed with a signal and specialize the model as follows. There are three dividend states  $d \in \{0, \delta, 1\}$ . The probabilities of the states are  $\pi(d = 0) = \pi(d = 1) = .25$  and  $\pi(d = \delta) = .5$ . In this case, the expected dividend is  $\delta/2 + 1/4$ . There are two possible signals,  $s = 0$  and  $s = 1$ . The probability of seeing signals conditional on  $d$  is given by

<sup>15</sup> See Dubey, Geanakoplos, and Shubik (1987) for more on this point and a discussion in support of a weaker efficient markets hypothesis.

<sup>16</sup> As discussed in the introduction there are several other explanations that have been offered to explain the observed volatility. These are complementary to the approach of explicitly modeling price formation. We emphasize the simplicity of the price formation approach, and the fact that it works without having to change any of the standard assumptions (as here it works, in fact, with risk neutrality).

$$\pi(s = 0|d = 0) = z$$

$$\pi(s = 1|d = 0) = 1 - z$$

$$\pi(s = 0|d = \delta) = .5$$

$$\pi(s = 1|d = \delta) = .5$$

$$\pi(s = 0|d = 1) = 1 - z$$

$$\pi(s = 1|d = 1) = z,$$

where  $z \in [.5, 1]$ . Given this probability structure, Bayes' rule defines the following probabilities of a dividend state given a signal

$$\pi(d = 0|s = 0) = \frac{z}{2}$$

$$\pi(d = \delta|s = 0) = \frac{1}{2}$$

$$\pi(d = 1|s = 0) = \frac{1-z}{2}$$

$$\pi(d = 0|s = 1) = \frac{1-z}{2}$$

$$\pi(d = \delta|s = 1) = \frac{1}{2}$$

$$\pi(d = 1|s = 1) = \frac{z}{2}.$$

**Proposition 2.** EQUILIBRIUM AND EXCESS VOLATILITY. *There exists an equilibrium with well defined bids and offers. In any such equilibrium prices are uniquely determined, and aggregate equilibrium bids and offers contingent on any signal are unique up to a rescaling. Furthermore, the variance of the price exceeds the variance of the dividend for open sets of values of  $z$  and  $\delta$ .*

Before proceeding further, we provide the explicit expressions for the expected value and variance of the price.

For  $z = .5$  (uninformative signals), equilibrium prices are always equal to the unconditional expected dividend and there is no price variance.

For  $z > .5$  (informative signals):

If  $\delta \leq \frac{1-z}{2z-1}$ , then the (unique) equilibrium expected price and price variance are

$$E[P] = \frac{1 + \delta - z}{8z(1 - z)}$$

$$\text{Var}[P] = \frac{(1 + \delta - z)^2(2z - 1)^2(3 - 4z + 4z^2)}{(8z(1 - z))^2}.$$

If  $\delta \geq \frac{1-z}{2z-1}$ , then the (unique) equilibrium expected price and price variance are

$$E[P] = E[d] = 1/4 + \delta/2$$

$$\text{Var}[P] = \frac{7 + 2\delta^2 + 2z^2 - 6z + 6\delta - 8\delta z - 2(1 + \delta - z) \sqrt{(2 + \delta - z)^2 - 4\delta z}}{16}.$$

The proof of Proposition 2 is presented in the appendix.

The expressions presented above are hard to interpret directly. They are more easily understood graphically, as illustrated in Figures 1 and 2. Figure 1 shows that the equilibria described in Proposition 2 exhibit excess volatility for some values of  $\delta$  and  $z$ . We will consider how these values affect excess volatility, after we offer an explanation for the excess volatility.

### Good News can be Bad News

The key to the possibility of excess volatility is the uncoupling of prices and dividends. What is happening is that, conditional on their private information, agents are doing as well as they can. When agents' actions are aggregated, however, prices can overshoot or undershoot the actual values of the dividends. Since traders cannot know for certain the price or the value of the dividend at the interim stage when they choose their actions, traders cannot take advantage of the overshooting or undershooting.

Consider the case where  $z = .75$  and  $\delta = .5$ . In this case, signals are informative, but imperfectly so, and dividends are evenly distributed. Here,  $\delta = \frac{1-z}{2z-1}$ , so both subcases of the proposition apply and  $E[P] = .5$  and  $\text{Var}[P] = .140625$ . The price variance exceeds the dividend variance,  $\text{Var}[d] = .125$ . The prices are uncoupled from the dividends in the sense discussed in the previous section. In particular, the prices are .125 in the low dividend state, .375 in the middle dividend state, and 1.125 in the high dividend state. Notice that the price 'overshoots' in the high (and low) dividend states, and undershoots in the middle dividend state.

Of course, the price of 1.125 when  $d = 1$  cannot be a conditional expectation of the dividend, since the dividend is known to be less than or equal to 1. However, the expected price, conditional on either signal, is equal to the conditional expectation of the dividend. For instance, when someone sees a high signal, the dividend is 0 with probability .125, .5 with probability .5, and 1 with probability .375, yielding an expected dividend of .625. Also conditional on a high signal, the price is .125 with probability .125, .375 with probability .5, and 1.125 with probability .375, yielding an expected price of .625. Conditional on the high signal, traders cannot gain by changing bids and they know that half of the time they will be overbidding and half the time they will be underbidding.

This example illustrates the good news can be bad news effect. Traders receiving the

high signal increase their expectation from .5 to .625, which is good news. However, when the value of the dividend is high, 75% of the traders receive the high signal, which leads to a high aggregate bid and a low aggregate offer, resulting in the overshooting. Traders with the high signal share the same optimism, but are unsure about how many other traders have the same view. Traders are willing to risk this overshooting, because the middle dividend could arise yielding a price below the dividend.

It is important to notice is that prices can be either more volatile or less volatile than the underlying dividends.

The uncoupling means that the price behavior can be different from dividend behavior and the specifics of how it varies will depend on the structure of information and dividends. Looking at some comparative statics gives some intuition for how price volatility relative to dividend volatility should vary with  $\delta$  and  $z$ .

First let us consider varying  $z$  for a fixed  $\delta$ . (See Figure 2, where  $\delta = .5$ .) If  $z$  is very low (near .5), then signals are essentially uninformative. In such a case, prices are close to the unconditional expected value of the dividend, and so they do not vary much with signals (or the dividend state) and so the variance of prices is very low. At the other extreme, as  $z$  nears 1, the signals become very informative and the prices converge to the actual dividend values, and so the variance of prices is close to the variance of the dividends. The area with significant excess volatility is for  $z$  in the middle of the range between .5 and 1. Here, signals are somewhat informative so that bids and offers, and thus prices, vary substantially with signals; but not so informative that the bids and offers pinpoint the actual dividends. Next, let us fix  $z$  and consider how excess volatility varies with  $\delta$ . When  $\delta$  is very low, conditional on the high signal, the expected value of the dividend is still relatively low, and so the expected price, conditional on the high signal, is also relatively low. Prices are all relatively low so the variance is low. When  $\delta$  is close to 1, the variance of prices decreases with  $\delta$  even though the variance of dividends increases with  $\delta$ . It is when  $\delta$  is in the middle range that the potential for excess volatility is highest.

Notice that, although “no-trade” theorems (see, for example, Milgrom and Stokey (1982)) apply to the situation analyzed in this section, trade occurs. No-trade theorems state that it be common knowledge that some agents anticipate being strictly better off as

a result of voluntary trade. Here agents are indifferent between trading and not trading at the equilibrium prices. The no-trade theorem makes the excess volatility result all the more striking.

If one introduces transactions costs, then the structure of the equilibrium described above would change and also result in no-trade. However, this makes evident another distinction between price formation models and rational expectations-type models. When there is no trade in a model of price formation, there are no prices.

It is clear that in order to understand the role of a cost of information in price formation, we will have to move away from situations where endowments are initially Pareto optimal. This is done in the next section where we examine the value of information.

## **5. Acquisition of Costly Information.**

Information acquisition is a problem in models with fully revealing rational expectations equilibria, as discussed in Grossman (1976) and Grossman and Stiglitz (1981). The simple intuition is that no one is willing to pay for information that will become fully incorporated into the price. Once one models price formation, however, this seeming paradox goes away (see, for instance, Milgrom (1981), Dubey, Geanakoplos, and Shubik (1987), Matthews (1984), and Jackson (1991)). In this section, we analyze price formation when information is costly and acquired endogenously. In particular, we wish to explore how the number of agents who become informed, the expected price, and the variance of the price change with the cost of information.

One important aspect of this analysis is that we are able to do it without adding any noise to the system. In the rational expectations model of Grossman and Stiglitz (1981), equilibrium with costly information acquisition fails to exist unless supply is assumed to be noisy. We assume that the supply is noiseless and known and yet equilibrium still exists. Our suppliers are constrained to supply a fixed amount, but they are not noise traders in the usual sense. First, there is no noise in their behavior (it is perfectly predictable), and second, we can evaluate their utility and perform a welfare analysis. The approach of Grossman and Stiglitz (1981) and Kyle (1989) precludes such a welfare analysis.

An interesting outcome of our analysis is that there are equilibrium situations where some agents become informed and other agents remain uninformed, and yet both informed

and uninformed agents bid. This contrasts with auction markets with more informed traders than objects for sale, where uninformed traders stay out of the bidding to avoid the “winner’s curse.” When news is favorable, rather than outbid the uninformed traders and leave them with nothing (as in the auction model), here the informed traders push up the price and leave the uninformed traders with a smaller, yet positive, number of shares of stock.

In Jackson (1991) it is also true that some agents become informed and others remain uninformed, and yet both bid in equilibrium. There, uninformed traders can condition on the fully revealing prices and so there is no winner’s curse. The puzzle there is not why both informed and uninformed bid simultaneously, but why anyone chooses to become informed. Jackson’s result relies heavily on the fact that a finite number of agents become informed and that they take into account the effect that their information will have on subsequent prices. In our context, no single agent can affect the equilibrium price.

The analysis of information acquisition requires us to modify the basic model of previous sections. In order to have an equilibrium in which some agents pay to become informed, there must be some value to information in equilibrium. Previously, the initial allocation was Pareto optimal, so the no-trade theorem implied that no agent could expect to profit from equilibrium trade. There are several methods of introducing the gains from trade necessary for information to be valuable in equilibrium. One method is to consider an overlapping generations model, where risky assets are infinitely lived and sold from generation to generation. In such a model, there are always some agents constrained to sell their assets. As a proxy for the overlapping generations dynamics, but in keeping with a simple and static model, we assume that  $q$  units of the risky asset (per agent) are exogenously offered for sale and that agents can bid but not offer. Thus, one can think of our traders as the younger generation of an OLG model, and the risky asset as being sold by the older generation.<sup>17</sup>

Before we proceed with the equilibrium analysis, let us remark that if information is costly, then it will be impossible to have equilibrium prices equal to the realized value of dividends, for each possible dividend realization (assuming that dividends can take on at least two distinct values). This contrasts with the conclusion of Proposition 1 where, for

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<sup>17</sup> An alternative route would be to introduce risk aversion and heterogeneous endowments. The current method allows for more transparent solutions.

an open set of parameters, it was possible to have equilibrium prices equal to the realized value of dividends, for each possible dividend realization. The reasoning is straightforward. Suppose to the contrary that equilibrium prices equal the realized value of dividends, for each possible dividend realization. It is necessarily true that some agents are expending resources to acquire information. Since equilibrium prices are equal to the realized value of dividends, an agent is indifferent between bidding and not bidding. Any agent acquiring information could deviate and not acquire information nor bid, thereby raising utility by the cost of the information.

Let  $c \in (0, 1]$  denote the cost of information (in units of the consumption good). The timing of the game is as follows. First, traders simultaneously choose whether or not to become informed. Each trader starts with an endowment of 1 unit of the consumption good, so informed traders are left with resources of  $(1 - c)$  when the market opens. Next, traders bid any amount less than or equal to their resources, so  $p = B/q$ .<sup>18</sup> We assume that traders do not observe whether or not others are informed.<sup>19</sup>

To further simplify the analysis, we specialize to the signal structure of section 4, setting  $z = 1$ . Again, there are three dividend states  $d \in \{0, \delta, 1\}$ . The probabilities of the dividend states are  $\pi(d = 0) = \pi(d = 1) = .25$  and  $\pi(d = \delta) = .5$ . In this case, the expected dividend is  $E[d] = 1/4 + \delta/2$ . There are two possible signals  $s = 0$  and  $s = 1$ .

Let the fraction of informed traders (when it exists) be denoted as  $F$ . Thus  $F = \lim_{N \rightarrow \infty} \sum_{i \leq N} \frac{x^i}{N}$ , where  $x^i = 1$  if  $i$  is informed and 0 if  $i$  is not informed.

**Proposition 3.** EQUILIBRIUM WITH COSTLY INFORMATION ACQUISITION. *If the quantity of the asset offered is not too large*<sup>20</sup> ( $q < \frac{2(1+c)}{1+2\delta}$ ), *then there exists a unique*

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<sup>18</sup> Allowing short sales complicates the analysis, but does not change the flavor of the results, provided there is something which limits agents' desire to bid infinite amounts. For instance, risk aversion or imperfect competition will suffice.

<sup>19</sup> Thus, (Bayesian) Nash equilibrium is still our equilibrium notion. We could allow agent's to observe which other agents have become informed, in which case the game would be sequential. One would then use some notion of perfection. The equilibrium we are examining would still hold.

<sup>20</sup> If  $q$  is large, then given the cap on bids, prices will be low. This means that expected profits are possible, above the costs of becoming informed. Although such equilibria are interesting, they seem to be an artifact of the limits on bidding. For this reason, we do not focus on them. The proof makes clear how to find the (unique symmetric) equilibria for large  $q$ .

trading equilibrium with costly information acquisition which is symmetric among informed and symmetric among uninformed. This equilibrium is characterized as follows:

(i) If  $c \leq 1/(3 + 8\delta)$ , then  $0 < F < 1$ , uninformed bidders do not trade, and informed bidders bid only when they observe a high signal.

(ii) If  $1/(3 + 8\delta) < c \leq 1/(3 + 4\delta)$ , then  $0 \leq F < 1$ , uninformed bidders participate in the bidding, and informed bidders bid only when they observe a high signal.

(iii) If  $1/(3 + 4\delta) < c$ , then  $F = 0$  and aggregate uninformed bids (per share) equal the expected value of the dividend.

**Proposition 4.** “V”-SHAPED PRICES. *For the equilibria outlined in Proposition 3, the expected price has a “V” shape: when  $c = 0$ , the expected price equals the expected dividend; as  $c$  increases, expected price falls, reaches a minimum (at a kink), and then rises until it again equals the expected dividend. Furthermore, for small enough  $c$  and large enough  $\delta$ , prices have higher variance than dividends.*

The basic intuition behind the “V” shape of expected prices is simple. The discount in prices exactly balances the resources spent on information acquisition. This compensates agents who become informed, keeping them indifferent between becoming informed and staying uninformed.

Uninformed traders are at a disadvantage relative to the informed, even though the expected price is discounted below the expected dividend. In equilibrium, the informed are bidding when they receive the high signal, but not when they receive the low one (see the appendix for details). One should expect this result since, if the informed were always willing to bid, then they would be better off not paying for information. Thus, an uninformed bidder who bids unconditionally faces a winners curse effect. When an uninformed bidder receives many shares of the asset, most of the informed have stayed out of the bidding, so the dividend is low. Informed bidding is heavy in the high state so an uninformed bidder will receive fewer shares. When the cost of information is low, uninformed bidders stay out of the bidding since they face many informed bidders and this winners curse effect is dominant. (Again, see the appendix for details.) As the cost of information rises, the fraction of informed agents falls, and eventually the uninformed are willing to enter the bidding.

This points out a new aspect of the equilibrium identified in this section. For some cost levels, both informed and uninformed agents are bidding at the same time.

Since the discount in expected prices is compensating the informed for the resources they expend acquiring information, the “V” shape of expected prices is simply inversely related to the average expenditure on information. When costs are low, everyone chooses to become informed, but the average expenditure on information is low. As the cost of information increases, the fraction of agents who become informed falls off. However, at first the fraction falls off slowly enough so that the average expenditure of resources on information acquisition is increasing in cost. Eventually, the fall off in the fraction of agents comes to dominate and the average expenditure of resources on information acquisition becomes decreasing in cost. Finally, after some level ( $c > 1/(3 + 4\delta)$ ) no one chooses to become informed, as the cost exceeds the maximal potential gain from becoming informed.

The resulting welfare implications are straightforward: since resources spent on information are lost to the economy, and since the equilibrium has no redistributive effect on resources, it is best to either have costless information, or information so costly that no one chooses to acquire it. This would suggest taxing information acquisition to make it prohibitively costly, if it is not already so. This recommendation will change if there are actually increased efficiencies in investment decisions made in the presence of increased information acquisition.

Proposition 3 and 4 are illustrated in figures 3 through 5. The figures specialize to the case where  $\delta = 1$ . There, when costs are small enough, agents end up overbidding in the highest dividend state (the price in that dividend state is close to  $3/2$ ) and underbidding in the middle dividend state (the price in that dividend state is close to  $3/4$ ). Informed traders are actually hoping that it is the middle dividend state; they are willing to bid with the high signal because the losses in the high dividend state are compensated by gains in the middle dividend. As  $c$  increases, informed traders must be compensated for higher costs, so prices fall. Expected price and excess volatility therefore fall. When  $c = 1/11$ , there are few enough informed traders so that the uninformed start to enter. Here is where expected price is at its minimum. Naturally, price volatility continues to fall since the uninformed make the same bid in all states. As  $c$  continues to rise, fewer traders become informed, the “winner’s curse” effect becomes less severe, and expected price rises. When  $c > 1/7$ ,

the cost is too great to justify anyone becoming informed, and everyone bids the expected dividend for each share on the market,  $q(1 + 2\delta)/4$ .

### **Investment Decisions.**

Sellers receive the highest expected revenue when information is either costless or so costly that no one chooses to become informed. Information is not serving any productive purpose; resources spent gathering information are simply lost to the economy. When some agents pay a positive cost to become informed, total societal welfare would improve if the government placed a prohibitively high tax on information. This conclusion depends critically on the fact that information does not affect the resources available.

If instead information is valuable in determining the total resources available, then resources spent gathering information could allow managers to make better decisions and increase the fundamental value of a project. In such a situation the social benefit of improved decision making from informative prices can be compared to the expenditure on information acquisition. Making information prohibitively costly could interfere with the ability of markets to efficiently allocate resources.

Although we are hesitant to use this model to make policy recommendations on how to regulate information gathering on markets, we feel that the approach taken here, of specifying the price-formation mechanism, is the right way to pose the question. There is a tradeoff between allocative efficiency of information gathering and the cost of information gathering; a tradeoff that is impossible to capture in “price-taking” and “noise-trader” models. One direction for future work would be to model the information in a more sophisticated way. Here, a higher fraction of traders becoming informed does not affect the total information possessed by the market. An implication is that the smallest positive  $F$  is welfare-maximizing. A richer information structure, such as one where (a finite number of) agents choose the precision of their information, would allow the optimal amount of information gathering to continuously influence payoffs, and the discontinuity problem mentioned above would probably not arise.

## **6. Choices in Modelling**

We wish to emphasize that it is the methodology of this paper, rather than the particulars of the model choice, which is responsible for the results we have obtained. Qualitatively

similar results hold if the particular rules of the game are changed, or if we allow risk aversion, a different signal structure, or a finite number of agents.<sup>21</sup> We chose to use the market game, because it models price formation and still allows a competitive analysis when information is asymmetric<sup>22</sup>, it has unique symmetric trading equilibria, allows for bids and offers, and permits agents to buy or sell variable amounts.<sup>23</sup> The risk neutrality of agents, and the three dividend state uncertainty structure were chosen for their analytic tractability. These assumptions have allowed us to perform comparative statics on the excess volatility, endogenize information acquisition. In sum, the specificity of the model allows for a very extensive analysis. What we have lost, of course, is the ability to develop a general understanding of how sensitive these results are to broader changes in the structure of the environment. This leaves ample room for future research.

The model we have discussed in this paper is a static one. This provides for unique equilibria, simplifying the analysis. We have argued that the results obtained extend to dynamic models. We should clarify some of the issues which would arise in such a dynamic version. If dividends are independent across time, and agents only have information about the next period's dividend, the model extends most easily. When there is serial correlation, the model becomes more complicated, since price histories convey information. In any case, as long as information is arriving over time, the price phenomena we have outlined here are likely to persist.

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<sup>21</sup> A simple example of a second price auction with two bidders illustrates these changes all at once. The terminal value of the risky asset, denoted  $d$ , is equal to the realization of the average of two random variables  $X^1$  and  $X^2$ , which are independently and identically, uniformly distributed over the interval  $(0,1)$ . Agent 1 knows the value of  $X^1$  (but not the value of  $X^2$ ). Similarly, agent 2 knows the value of  $X^2$  (but not the value of  $X^1$ ). There is a symmetric equilibrium in which agent  $i$  bids  $b^i = x^i$ , where  $x^i$  is the realized value of  $X^i$ . The (unconditional) expected value  $d$  is  $E(d) = 1/2$  and the price of the asset is  $p = \min(X^1, X^2)$ , where  $E(p) = 1/3$  (1/6 less than the expected dividend). The ratio of the variance of the price to the variance of the dividend is 4/3. Thus, there is excess volatility, since the price is more variable than the payoff of the underlying asset. This also raises some interesting questions, since the excess volatility in this example is sensitive to the number of bidders and the structure of information.

<sup>22</sup> McAfee and McMillan (1987) provide a nice motivation for paying close attention to price formation when there is asymmetric information, and for being able to do so in a competitive setting.

<sup>23</sup> Alternatives to the market game, such as a share auction (see Wilson (1979)), or a game where agents submit demand schedules (as in Jackson (1991)), generally have an enormous multiplicity of equilibria.

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## Appendix

### Derivation of (1)

Suppose that dividend state  $d$  is realized and that  $B(s)$  and  $Q(s)$  are well defined for each  $s$ . By definition

$$\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[b^i|d]}{N} = \lim_{N \rightarrow \infty} \sum_{i=1}^N \sum_s \frac{\pi(s|d)b^i(s)}{N}$$

and thus

$$\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[b^i|d]}{N} = \lim_{N \rightarrow \infty} \sum_s \pi(s|d) \sum_{i=1}^N \frac{b^i(s)}{N}.$$

Since  $b^i(s) \in [0, 1]$  for all  $i$  and  $s$  and the number of  $s$  is finite,

$$\lim_{N \rightarrow \infty} \sum_s \pi(s|d) \sum_{i=1}^N \frac{b^i(s)}{N} = \sum_s \pi(s|d) \lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{b^i(s)}{N} = \sum_s \pi(s|d)B(s).$$

Thus

$$\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[b^i|d]}{N} = \sum_s \pi(s|d)B(s). \quad (*)$$

Similarly,

$$\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[q^i|d]}{N} = \sum_s \pi(s|d)Q(s). \quad (**)$$

Given that  $B(s)$  and  $Q(s)$  are well defined for each  $s$ , it follows from (\*) and (\*\*) that  $\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[b^i|d]}{N}$  and  $\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[q^i|d]}{N}$  exist and are finite. Given that  $q_i$  and  $b_i$  are in  $[0,1]$ , it follows that  $\sum_{i=1}^{\infty} \frac{\text{var}[b^i|d]}{i^2} < \infty$ , and  $\sum_{i=1}^{\infty} \frac{\text{var}[q^i|S]}{i^2} < \infty$ . By Theorem D of Halmos (1974), page 204, it follows that  $\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{b^i - E[b^i|d]}{N} = 0$  almost surely and  $\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{q^i - E[q^i|d]}{N} = 0$  almost surely. Let  $b_d^i$  denote the random variable that takes on value  $b^i(s)$  with probability  $\pi(s|d)$ , and similarly define  $q_d^i$ . It follows that the price in dividend state  $d$  is given by

$$P(d) = \frac{\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{b_d^i}{N}}{\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{q_d^i}{N}} = \frac{\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[b^i|d]}{N}}{\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[q^i|d]}{N}}.$$

Then from (\*) and (\*\*) we know that

$$P(d) = \frac{\sum_s \pi(s|d)B(s)}{\sum_s \pi(s|d)Q(s)},$$

almost surely.

**Proof of Proposition 1:** (2) can be written as  $AX = 0$  for some  $X \geq 0$  such that  $X \neq 0$ , where  $A$  and  $X$  are defined by

$$A = \begin{pmatrix} \pi(s=1|S=1) & \dots & \pi(s=m|S=1) & -d^1\pi(s=1|S=1) & \dots & -d^1\pi(s=m|S=1) \\ \pi(s=1|S=2) & \dots & \pi(s=m|S=2) & -d^2\pi(s=1|S=2) & \dots & -d^2\pi(s=m|S=2) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \pi(s=1|S=n) & \dots & \pi(s=m|S=n) & -d^n\pi(s=1|S=n) & \dots & -d^n\pi(s=m|S=n) \end{pmatrix}$$

$$X = \begin{pmatrix} B(1) \\ \vdots \\ B(m) \\ Q(1) \\ \vdots \\ Q(m) \end{pmatrix}$$

It follows that there exists an equilibrium for which realized prices equal dividends, if and only if the intersection of the null space of  $A$  and  $\mathbb{R}_+^{2m}$  is larger than  $\{0\}$ . Letting  $r$  denote the number of independent rows of  $A$ , the null space of  $A$  has dimension  $2m - r$  (recall that it is always true that  $r \leq 2m$ ).

The result for  $m \leq \frac{n}{2}$  follows from the fact that for a generic selection of parameters ( $d$  and  $\pi(s|d)$  in  $\mathcal{A}$ ) the number of independent rows  $r$  will be  $2m$  and so the null space will be of dimension 0.

Next, suppose that  $m > \frac{n}{2}$  and consider  $A$  with full row rank ( $r = n$ ). Also suppose that there exists a dividend state  $S'$  such that  $d^{S'} = 0$ , there exists a dividend state  $S''$  such that  $d^{S''} > 0$ , and that  $\pi(s|S) > 0$  for all  $s$  and  $S$ . For  $X \neq 0$  to solve  $AX = 0$  (in particular the equation corresponding to state  $S''$ ) it must be that  $X_h > 0$  for some  $h$ ,  $1 \leq h \leq m$ . Since  $d^{S'} = 0$  and  $A_{Sj} > 0$  for all  $j$ ,  $1 \leq j \leq m$ , we must have  $X_l < 0$  for some  $l$ ,  $1 \leq l \leq m$  (in particular, in order to solve the equation corresponding to state  $S'$ ). Since negative bids are not allowed, there is no equilibrium where prices equal dividends state by state. Given the full row rank of the matrix  $A$ , the null space varies continuously at  $A$  (see

below for details), so prices cannot equal dividends state by state, for a set of matrices  $A$  corresponding to an open set of parameters in  $\mathcal{A}$ .

Finally, suppose that  $m > \frac{n}{2}$  and construct  $A$  as follows. For  $i \leq m$ , let  $d^i = i$ ,  $\pi(s = i|S = i) = 1$  and  $\pi(s = j|S = i) = 0$  for  $j \neq i$ . For  $m < i$ , let  $d^i = (2i + 1)/2$ ,  $\pi(s = i - m|S = i) = 1/2 = \pi(s = i - m + 1|S = i)$  and  $\pi(s = j|S = i) = 0$  for  $j \neq i - m$ ,  $j \neq i - m + 1$ . (Since  $n < 2m$ ,  $i < 2m$  and so the above definitions make sense.) Consider  $X$  such that  $B(s) = s$  and  $Q(s) = 1$ . It is easily checked that  $AX = 0$ . Given the full row rank of the matrix  $A$ ,<sup>24</sup> the null space varies continuously at  $A$ . Thus there exists  $X'$ , which is positive in all elements, in the null space of each  $A'$  in some neighborhood of  $A$ .

### Continuity of the Null Space.

Consider  $A$ . First we show that the null space varies upper hemi-continuously at  $A$ . Consider a sequence  $A^k \rightarrow A$  and consider  $X^k \rightarrow X$  where  $X^k$  is in the null space of  $A^k$  for each  $k$ . Suppose that  $X$  is not in the null space of  $A$ , so  $AX = b \neq 0$ . Find  $\epsilon$  such that if  $\|b' - b\| \leq \epsilon$  then  $b' \neq 0$ . There exists  $K$  such that for  $k \geq K$ ,  $\|A^k X - b\| \leq \epsilon/2$  and  $\|A^k X^k - A^k X\| \leq \epsilon/2$ . Thus  $\|A^k X^k - b\| \leq \epsilon$  and so  $A^k X^k \neq 0$ , which is a contradiction. Next notice that the set of matrices with less than full row rank is a closed set. Thus, since  $A$  has full row rank, it follows that there is a neighborhood of  $A$  consisting of matrices of full rank. Thus all matrices in that neighborhood have the same dimensional null space. Given the upper hemi-continuity and the fact that the null spaces are of the same dimension, the null spaces must vary continuously at  $A$ .

### Proof of Proposition 2:

To characterize trading equilibria, we begin by examining agent  $i$ 's expected wealth,  $E[W^i]$ , as a function of  $b^i$  and  $q^i$ , conditional on seeing  $s = 0$  or  $s = 1$ . Under the convergent bids and offers condition  $P(0)$ ,  $P(1)$ , and  $P(2)$  are well defined and so

$$\begin{aligned} E[W^i|s^i = 0] &= 1 + \frac{1-z}{2} \left[ 1 - q^i + \frac{b^i}{P(2)} - b^i + q^i P(2) \right] \\ &\quad + \frac{1}{2} \left[ \delta \left( 1 - q^i + \frac{b^i}{P(1)} \right) - b^i + q^i P(1) \right] + \frac{z}{2} \left[ -b^i + q^i P(0) \right] \end{aligned}$$

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<sup>24</sup>  $A$  can be diagonalized through elementary operations. Rows  $i \leq m$  are already diagonalized. For each row  $i > m$  the leading entry is  $1/2$ , so subtract  $1/2$  of rows  $i - m$  and  $i - m + 1$ . The  $i$ -th entry (now the leading entry) of row  $i > m$  will be  $-1/4$ .

$$\begin{aligned}
E[W^i | s^i = 1] &= 1 + \frac{z}{2} \left[ 1 - q^i + \frac{b^i}{P(2)} - b^i + q^i P(2) \right] \\
&\quad + \frac{1}{2} \left[ \delta \left( 1 - q^i + \frac{b^i}{P(1)} \right) - b^i + q^i P(1) \right] + \frac{1-z}{2} [-b^i + q^i P(0)]
\end{aligned}$$

$P(0)$ ,  $P(1)$ , and  $P(2)$  are not affected by a change in  $b^i, q^i$  by a single agent. The first order conditions in bids and offers conditional on information lead to

$$\frac{(1-z)}{P(2)} + \frac{\delta}{P(1)} \leq 2 \quad (= \text{ if } B(0) > 0) \quad (3)$$

$$\frac{z}{P(2)} + \frac{\delta}{P(1)} \leq 2 \quad (= \text{ if } B(1) > 0) \quad (4)$$

$$(1-z)P(2) + P(1) + zP(0) \leq 1 - z + \delta \quad (= \text{ if } Q(0) > 0) \quad (5)$$

$$zP(2) + P(1) + (1-z)P(0) \leq z + \delta \quad (= \text{ if } Q(1) > 0) \quad (6)$$

Notice that if  $z = .5$  then by (1),

$$P(0) = P(1) = P(2) = \frac{B(0) + B(1)}{Q(0) + Q(1)}.$$

Also notice that if  $z = .5$  then (4) is equivalent to (3) and (6) to (5). Since there is trade, at least one  $B(0)$  and  $B(1)$  are greater than 0, and similarly at least one  $Q(0)$  and  $Q(1)$  are greater than 0. (3), (4), (5) and (6) become

$$(.5 + \delta) \frac{Q(0) + Q(1)}{B(0) + B(1)} = 2$$

or

$$\frac{B(0) + B(1)}{Q(0) + Q(1)} = \frac{.5 + \delta}{2}.$$

So from now on assume that  $z > .5$ . It must be that  $B(1) > 0$  and  $B(0) = 0$ , since (4) implies that (3) holds strictly. Normalize  $B(1) = 1$ . Thus (3), (4), (5), and (6) can be rewritten as

$$zQ(1) + (1-z)Q(0) + \delta(Q(1) + Q(0)) = 2 \quad (7)$$

$$\frac{(1-z)z}{zQ(1) + (1-z)Q(0)} + \frac{1}{Q(1) + Q(0)} + \frac{z(1-z)}{(1-z)Q(1) + zQ(0)} \leq 1 - z + \delta \quad (= \text{ if } Q(0) > 0) \quad (8)$$

$$\frac{z^2}{zQ(1) + (1-z)Q(0)} + \frac{1}{Q(1) + Q(0)} + \frac{(1-z)^2}{(1-z)Q(1) + zQ(0)} \leq z + \delta \quad (= \text{ if } Q(1) > 0) \quad (9)$$

where at least one of (8) and (9) must hold with equality. Trading equilibria can then be partitioned into situations where (i)  $Q(0) = 0$ ,  $Q(1) > 0$ , (ii)  $Q(0) > 0$ ,  $Q(1) = 0$ , or (iii)  $Q(0) > 0$ ,  $Q(1) > 0$ . We consider these cases in turn.

(i)  $Q(0) = 0$ ,  $Q(1) > 0$ . (8) and (9) become

$$\frac{2}{Q(1)} \leq 1 - z + \delta \quad (10)$$

$$\frac{2}{Q(1)} = z + \delta. \quad (11)$$

Since  $z > .5$ , then it is clear that there is no solution to (10) and (11).

(ii)  $Q(0) > 0$ ,  $Q(1) = 0$ . In this case, (7), (8), and (9) become

$$(1 - z + \delta)Q(0) = 2 \quad (12)$$

$$\frac{2}{Q(0)} = 1 - z + \delta \quad (13)$$

$$\frac{z^2}{(1-z)Q(0)} + \frac{1}{Q(0)} + \frac{(1-z)^2}{zQ(0)} \leq z + \delta. \quad (14)$$

To have a solution to these equations, it follows from (12) and (13) that  $Q(0) = \frac{2}{\delta + 1 - z}$ .

Verifying (14) then leads to

$$\frac{z^2}{(1-z)} + 1 + \frac{(1-z)^2}{z} \leq \frac{2(z+\delta)}{\delta+1-z}$$

which reduces to  $\delta \leq \frac{1-z}{2z-1}$ . Substituting equilibrium bids and offers into prices (1) provides  $P(0) = \frac{(1-z)(\delta+1-z)}{2z}$ ,  $P(1) = \frac{\delta+1-z}{2}$  and  $P(2) = \frac{z(\delta+1-z)}{2(1-z)}$ . Direct calculation of the expectation and variance leads to the claimed expressions.

(iii)  $Q(0) > 0$ ,  $Q(1) > 0$ . (7) implies that

$$Q(0) = \frac{2 - (z + \delta)Q(1)}{\delta + 1 - z}. \quad (15)$$

Since (8) and (9) hold with equality, substituting from (15), (8) and (9) become

$$\begin{aligned} \frac{(1-z)z}{zQ(1) + (1-z)\frac{2-(z+\delta)Q(1)}{\delta+1-z}} + \frac{1}{Q(1) + \frac{2-(z+\delta)Q(1)}{\delta+1-z}} \\ + \frac{z(1-z)}{(1-z)Q(1) + z\frac{2-(z+\delta)Q(1)}{\delta+1-z}} = 1 - z + \delta \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{z^2}{zQ(1) + (1-z)\frac{2-(z+\delta)Q(1)}{\delta+1-z}} + \frac{1}{Q(1) + \frac{2-(z+\delta)Q(1)}{\delta+1-z}} \\ + \frac{(1-z)^2}{(1-z)Q(1) + z\frac{2-(z+\delta)Q(1)}{\delta+1-z}} = z + \delta \end{aligned} \quad (17)$$

Solving (16) for  $Q(1)$  provides two possible solutions:

$$Q(1) = \frac{(2-\delta-z)(1+\delta-z) - 4\delta z + (1+\delta-z)\sqrt{(2+\delta-z)^2 - 4\delta z}}{2\delta(1+\delta)(2z-1)}$$

and

$$Q(1) = \frac{(2-\delta-z)(1+\delta-z) - 4\delta z - (1+\delta-z)\sqrt{(2+\delta-z)^2 - 4\delta z}}{2\delta(1+\delta)(2z-1)}.$$

The corresponding solutions for  $Q(0)$  according to (15) are

$$Q(0) = \frac{z(2\delta+2-z) - \delta(\delta+2) - (\delta+z)\sqrt{(2+\delta-z)^2 - 4\delta z}}{2\delta(1+\delta)(2z-1)}$$

and

$$Q(0) = \frac{z(2\delta+2-z) - \delta(\delta+2) + (\delta+z)\sqrt{(2+\delta-z)^2 - 4\delta z}}{2\delta(1+\delta)(2z-1)}.$$

Since the first of these two solutions for  $Q(0)$  is always negative, it follows that

$$Q(1) = \frac{(2-\delta-z)(1+\delta-z) - 4\delta z - (1+\delta-z)\sqrt{(2+\delta-z)^2 - 4\delta z}}{2\delta(1+\delta)(2z-1)}$$

and

$$Q(0) = \frac{z(2\delta+2-z) - \delta(\delta+2) + (\delta+z)\sqrt{(2+\delta-z)^2 - 4\delta z}}{2\delta(1+\delta)(2z-1)}.$$

A tedious but straightforward calculation shows that (17) is also satisfied. Notice that  $Q(1) \geq 0$  if and only if  $\delta \geq \frac{1-z}{2z-1}$ , while  $Q(0) \geq 0$  for any  $z \in [.5, 1]$ . To obtain the expected price and price variance expressions, one can substitute the equilibrium bids and offers into (1) to obtain equilibrium prices. An alternative (and computationally easier) method is to

simultaneously solve (4), (5) and (6) directly. (4) shows that  $P(2) = P(1)z/(2P(1) - \delta)$ . Substituting and solving (5) and (6) for  $P(0)$  and  $P(1)$  one finds

$$P(0) = \frac{-2 + \delta + z + \sqrt{(2 + \delta - z)^2 - 4\delta z}}{4}$$

$$P(1) = \frac{2 + 3\delta - z - \sqrt{(2 + \delta - z)^2 - 4\delta z}}{4}.$$

Then subtracting (6) from (5) provides  $P(2) = P(0) + 1$ , so

$$P(2) = \frac{2 + \delta + z + \sqrt{(2 + \delta - z)^2 - 4\delta z}}{4}.$$

Direct calculations of the expectation and variance lead to the claimed expressions.

**Proof of Proposition 3:** For trading equilibria that are symmetric among informed and symmetric among uninformed, denote each informed trader's bid function as  $b(1)$  and  $b(0)$  (conditional on  $s = 1$  and  $s = 0$ , respectively), and denote each uninformed trader's bid as  $b^u$ .

First, notice that  $F$  must be well-defined in any trading equilibrium which is symmetric among informed and symmetric among uninformed. Otherwise, for prices to be well defined it would have to be that the informed and uninformed are bidding the same whenever trade occurs, which is inconsistent with an equilibrium in which some agents pay to become informed.

Consider any agent who purchases information in equilibrium. The agent must bid  $1 - c$  conditional on one signal and zero conditional on the other signal. (Informed agents must expect to profit conditional on receiving at least one signal, or they would not acquire information. Since no single agent is large enough to affect the price, if they expected to profit conditional on one signal and not to lose conditional on the other signal, then an informed agent would be better off bidding all of his/her wealth without purchasing the signal.) Since the dividend has an equal chance of being  $\delta$  or  $1$  conditional on  $s = 1$  and the dividend has an equal chance of being  $\delta$  or  $0$  conditional on  $s = 0$ , uninformed agents' bids are the same in all states, informed agents always have a higher conditional expected return from investing conditional on  $s = 1$  than  $s = 0$ . Thus, informed agents must be bidding

$1 - c$  conditional on  $s = 1$  and bidding 0 conditional on  $s = 0$ . The expected utility of an uninformed agent who bids  $b^u$  is

$$U^{uninf} = 1 + b^u \left( \frac{1}{4P(2)} + \frac{\delta}{2P(1)} - 1 \right). \quad (18)$$

The ex ante expected utility of informed traders, who bid  $1 - c$  given the high signal and 0 given the low signal, is

$$U^{inf} = (1 - c) \left( \frac{1}{2} + \frac{1}{2} \left( \frac{\delta}{2P(1)} + \frac{1}{2P(2)} \right) \right). \quad (19)$$

We can represent prices as follows:

$$P(0) = (1 - F)b^u/q, \quad (20)$$

$$P(1) = (1 - c)F/2q + (1 - F)b^u/q, \quad (21)$$

and

$$P(2) = (1 - c)F/q + (1 - F)b^u/q. \quad (22)$$

We first divide potential equilibria into two categories. (I) where the (ex ante) expected utility of all agents is larger than one, and (II) where the (ex ante) expected utility of all agents is equal to one. First, we show that (I) is only possible when  $q > \frac{2(1+c)}{1+2\delta}$ . Then we will outline the types of equilibria (corresponding to those in Proposition 3) which satisfy (II).

We subdivide (I) into three categories. (a)  $F = 1$ ,  $U^{inf} > 1$ , and  $U^{uninf} < U^{inf}$  when  $b^u = 1$ . (b)  $F = 0$ ,  $U^{uninf} > 1$ ,  $U^{inf} < U^{uninf}$ , and  $b^u = 1$ . (c)  $0 < F < 1$ ,  $U^{uninf} > 1$ ,  $U^{inf} = U^{uninf}$ , and  $b^u = 1$ .<sup>25</sup>

Using (18)–(22), the requirement of  $U^{inf} > 1$  under (a) reduces to  $q > \frac{2(1+c)}{1+2\delta}$ . The requirement that  $U^{uninf} > 1$  when  $b^u = 1$  under (b) reduces to  $q > \frac{4}{1+2\delta}$  which is always as large as  $\frac{2(1+c)}{1+2\delta}$ . Under (c)  $P(1) > (1 - c)/2q$  and  $P(2) > (1 - c)/q$ . Thus  $U^{inf} < \frac{1-c}{2} + q \left( \frac{1+2\delta}{4} \right)$ . Since  $U^{inf} > 1$ , this implies that  $\frac{1-c}{2} + q \left( \frac{1+2\delta}{4} \right) > 1$  or that  $q > \frac{2(1+c)}{1+2\delta}$ .

Thus we have shown that under the requirement that  $q \leq \frac{2(1+c)}{1+2\delta}$ , only (II) is possible. Now let us examine the existence of equilibria under (II). Again we subdivide into three

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<sup>25</sup> Notice that for  $U^{uninf} > 1$ , one needs  $b^u > 0$ , and then it must be that the unconditional expected return from bidding is greater than zero so uninformed traders should set  $b^u = 1$ .

types of equilibria. Type (i):  $U^{inf} = 1$  and  $U^{uninf} < 1$  whenever  $b^u > 0$ . Type (ii):  $U^{inf} = 1 = U^{uninf}$  and  $b^u > 0$ . Type (iii):  $U^{inf} < 1 = U^{uninf}$  and  $b^u > 0$ .

For a type (i) equilibrium,  $b^u = 0$ ,  $U^{inf} = 1$  and  $U^{uninf} < 1$  whenever  $b^u > 0$  (thus when  $b^u = 1$ ). In this case from (20)–(22), we know that  $P(0) = 0$ ,  $P(1) = (1 - c)F/2q$  and  $P(2) = (1 - c)F/q$ . Solving  $U^{inf} = 1$  provides  $F = \frac{2q}{1+c} \left( \frac{\delta}{2} + \frac{1}{4} \right)$ . for  $F \leq 1$  we need  $\frac{2q}{1+c} \left( \frac{\delta}{2} + \frac{1}{4} \right) \leq 1$ , which is guaranteed when, as in Proposition 3,  $q < \frac{2(1+c)}{1+2\delta}$ . Solving  $U^{uninf} < 1$  when  $b^u = 1$ , provides  $c \leq 1/(3 + 8\delta)$ .

Type (ii):  $U^{inf} = 1 = U^{uninf}$  and  $b^u > 0$ . Solving  $U^{inf} = 1 = U^{uninf}$  from (18) and (19) provides  $P(1) = (1 - c)\delta/(2 - 6c)$  and  $P(2) = (1 - c)/8c$ . Then simultaneously solving (21) and (22) provides  $F = \frac{q}{c} \left( \frac{1-3c-4c\delta}{4-12c} \right)$  and  $b^u = \frac{(1-c)(1-3c-8c\delta)}{2(q-3cq-4c\delta q-4c+12c^2)}$ . The inequalities  $0 \leq F < 1$  and  $0 < b^u$ , are satisfied when  $\frac{1}{3+8\delta} < c \leq \frac{1}{3+4\delta}$ .  $b^u \leq 1$  corresponds to  $q \leq 8c(1 - 3c)/((1 + c)(1 - 3c) - 8c^2\delta)$ , which is guaranteed when, as in Proposition 3,  $q < \frac{2(1+c)}{1+2\delta}$ .<sup>26</sup>

For type (iii) equilibria,  $U^{inf} < 1 = U^{uninf}$  and  $b^u > 0$ . Here  $F = 0$  and  $P(0) = P(1) = P(2) = b^u/q$ . Solving  $1 = U^{uninf}$  provides  $b^u = q \left( \frac{1}{4} + \frac{\delta}{2} \right)$ . Thus  $b^u \leq 1$  whenever  $q \leq 1 / \left( \frac{1}{4} + \frac{\delta}{2} \right)$  which is guaranteed when, as in Proposition 3,  $q < \frac{2(1+c)}{1+2\delta}$ .  $U^{inf} < 1$  becomes  $(1 - c) \left( \frac{1}{2} + (\delta + 1)/(2\delta + 1) \right) < 1$  which simplifies to  $\frac{1}{3+4\delta} < c$ . We have thus established Proposition 3.

**Proof of Proposition 4:** The proof above provides expressions for  $P(0)$ ,  $P(1)$ , and  $P(2)$  in each of (i)–(iii). (In (ii),  $P(0)$  needs to be calculated using (20) and the expressions obtained for  $F$  and  $b^u$ . This provides  $P(0) = (1 - c)(8c\delta + 3c - 1)/(8c(1 - 3c))$ .) Straight-forward calculations show that the expected price always equals  $P(1)$ , and that for type (i) equilibria, the price variance equals  $\frac{(1-c)^2 \left( \frac{1}{4} + \frac{\delta}{2} \right)^2}{2(1+c)^2}$ , for type (ii) the price variance equals  $\frac{(4c\delta - (1-3c))^2}{2} \left( \frac{1-c}{8c(1-3c)} \right)^2$ , and for type (iii) equilibria, the price variance equals zero. These provide Proposition 4.

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<sup>26</sup>  $8c(1 - 3c)/((1 + c)(1 - 3c) - 8c^2\delta) > \frac{2(1+c)}{1+2\delta}$  reduces to  $c > \frac{1}{3+8\delta}$ .