Reputation and Imperfectly Observable Commitment: The Chain Store Paradox Revisited[1]

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Abstract

In their seminal solution of the chain store paradox Kreps and Wilson assumed that the incumbent monopolist is predisposed, with a small probability, to fight entry. Milgrom and Roberts suggested to view this predisposition to fight as a result of precommitment to an aggressive course of action. However, they did not examine whether such an ability to make commitments is actually chosen by a rational incumbent monopolist.

The present paper fills this gap. We assume that the monopolist has access to an appropriate commitment mechanism, with a small probability. Due to the possibility of misunderstanding or communication error, commitments are not perfectly observable. Otherwise, the assumptions of Kreps and Wilson are maintained.

These plausible modifications have drastic implications: Precommitment becomes useless, and reputation effects break down; Selten's chain store paradox comes back in full force.

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1 Introduction

This paper reconsiders the solution of the chain store paradox by Kreps and Wilson (1982). This solution assumed that the incumbent monopolist is predisposed, with a small probability, to fight entry. Amazingly, this addition to the original chain store game was sufficient to give rise to reputation effects that prevent entry, except in the last stages of the game.

However, this explanation of reputation and entry deterrence begs the question: where does the assumed predisposition to fight come from? Milgrom and Roberts (1982) suggested an explanation of how such a predisposition to fight may emerge. Apart from the possibility of "irrational" play, they emphasized that the incumbent may have entered precommitments that changed his payoffs appropriately:

> *"In the game actually being played, the established firm may be able to precommit itself to an aggressive course of action and may have done so."* Milgrom and Roberts (1982, p. 303)

Such precommitments might take the form of a build–up of capacity[1] or of contractual obligations. For example, the incumbent firm may be run by managers who operate on the basis of forcing contracts that gives them strong incentives to fight entry. The uncertainty concerning the type of incumbent monopolist would then reflect potential entrants' doubts about the kind of managerial contract or the possibility of contract renegotiations.[2]

However, Milgrom and Roberts did not examine whether such an ability to make firm commitments is actually chosen by a rational incumbent monopolist.

The present paper attempts to fill this gap. We assume that the monopolist has access to an appropriate commitment mechanism, with a small probability. Due to the possibility of misunderstanding or communication error, commitments are not perfectly observable. Otherwise, the assumptions of Kreps and Wilson are maintained. We show that these plausible modifications have drastic implications. In particular, precommitment becomes useless, and reputation effects break down. Selten's (1978) chain store paradox comes back in full force.

Our results complement a recent contribution by Bagwell (1995) who showed that in games of complete information the power of precommitment

[1] Milgrom and Roberts refer to Dixit (1980) and Spence (1977).

[2] Incentive contracts are typically not renegotiation–proof (see for example Matthews (1995)).

has no value if the commitment to a certain action is imperfectly observable.[3] Bagwell's result has the potential to launch a major attack on the use of stage games in economics, which have been exceedingly popular in economics, particularly in industrial organization. However, caution is advised already because the problem might vanish in games of incomplete information. Our results indicate that Bagwell's criticism also applies to sequential games of incomplete information.

The plan of the paper is as follows. In Section 1 we state the game. The analysis begins, in Section 2, with the extreme case where the incumbent monopolist's commitment is unobservable (or completely uninformative). This analysis is then extended in Section 3 to allow for imperfect observability. The paper closes with a discussion of limitations and extensions.

2 The Game

Consider the chain store game by Kreps and Wilson (1982). We modify it in two regards: the tough incumbent monopolist chooses a particular commitment, and entrants cannot perfectly observe that commitment.

Players The players are the incumbent monopolist m that serves a fixed sequence of markets: $N, N-1, \dots, 1$, where $N \geq 2$, and a set of potential entrants denoted by the market into which they may enter, $n \in \{N, N-1, \dots, 1\}$, in this sequence. (Like in Kreps and Wilson, time is indexed backwards; entrant $n-1$ succeeds n, and N is the first and 1 the last market.)

The incumbent monopolist m is either "weak" (w) or "tough" (t). The only difference between the two types is that t makes an irreversible commitment to a complete sequence of actions at the outset of the game, whereas w optimizes in each market. The monopolist knows his type; entrants do not.

Actions/Strategies Entrants either "enter" (E) or "stay out" (O) of their respective market. The monopolist responds to entry with either "fight" (F) or "accommodate" (A). The corresponding action sets are $A_n := \{E, O\}$, $n = N, \dots, 1$, and $A_m := \{F, A\}$. The entrants' and $w's$ strategies are reactions to the history of the game h, which is the sequence of past actions. In addition, the monopolist chooses a plan of action, as explained below.

[3] If one permits commitment to a random selection rule, this statement needs to be qualified; see the discussion in Sect. 5.

Sequence of Moves

Nature draws type t with probability $p^0 \in (0,1)$ and type w with probability $1 - p^0$.

Stage 1 The monopolist chooses an action plan $a^i := (a^i_N, a^i_{N-1}, \dots, a^i_1)$, $i \in \{t, w\}$, from the product set $\mathcal{A} := A^N_m$; t is irreversibly committed to act accordingly in each market. Commitments to random action selection rules are not permitted.

Stage 2 All players observe a blurred signal $s \in \mathcal{A}$ of the action plan a^i. N either enters or stays out of market N; mixed strategies are permitted. The entry decision is observed by all players.

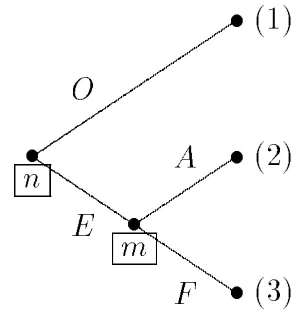
Stage 3 If entry has occurred, monopolist w responds with either F or A; mixed strategies are permitted. Monopolist t executes his plan of action, a^t. If no entry has occurred, the monopolist is not called upon to move. The monopolist's action is observed by all players.

Stages 2–3 are repeated in each successive market, from $N - 1$ to 1.

Payoffs Players' payoffs in each market n depend upon their moves in that market, as summarized in Fig. 1. Notice in that table, without the entrant present the incumbent earns monopoly profits of $c > 1$, whereas with the entrant present he can either accommodate and split the market (earning 0 each) or respond aggressively and force losses upon entrants ($b - 1 < 0$) at a cost (-1). Therefore, fighting only pays if it deters entry in at least one subsequent market (due to $c - 1 > 0$), and entry pays only if the monopolist accommodates with sufficiently high probability (due to $b - 1 < 0$). For simplicity, discounting is ignored.

Beliefs At the outset of the game, potential entrants assess the monopolist to be tough with probability $p^0 := \Pr\{t\}$ which is common knowledge. This probability is sufficiently small so that entry is never deterred in a one–shot version of the game ($p^0 \in (0, b)$). (Note: if $p^0 > b$, then the entrant would stay out already in the one–shot game.)

As is well known, the chain store game by Kreps and Wilson has an unique equilibrium only if one adopts certain equilibrium refinements that eliminate certain "implausible" off–equilibrium beliefs. In this regard we follow Kreps and Wilson and make the following assumptions.



	Payoff Entrant (n)	Payoff Monopolist (m)
(1)	0	c
(2)	b	0
(3)	$b-1$	-1

$c > 1,\ b \in (0,1)$

Figure 1: Stage Payoffs.

Assumption 1 *Once the monopolist is recognized with certainty as w, this belief remains unshakeable.*

Assumption 2 *Whenever an action is observed that deviates from t's equilibrium plan a^t, all subsequent entrants infer that they face w with certainty.*

Finally note, if t is committed to fight entry in every market, our game coincides with Kreps and Wilson (1982). However, in equilibrium t may choose another commitment or no commitment at all.[4]

3 Commitment without Observability

In this section we consider the extreme case in which entrants do not observe the plan of action or, equivalently, that the signal is completely uninformative.

To understand the complexity of that game, suppose for the moment that it is played only once. Then the entrant is unsure whether he plays a simultaneous moves game — which is the case if he deals with t — or a sequential moves game — which is the case if he faces the monopolist w that reacts to entry. The simultaneous moves game has two Nash equilibria: (O, F) and (E, A), whereas the sequential moves game has only one subgame perfect equilibrium: (E, A).

[4]Of course, t may make a commitment to the time consistent action plan; in this sense, we do not exclude that t makes essentially no commitment.

Now add finitely many repetitions. Then, the simultaneous moves game has a plethora of subgame perfect equilibria[5] whereas the sequential moves game still has exactly one.

Entrants are unsure which game is actually played. Together with Assumptions 1 and 2 (which eliminate certain implausible beliefs), this uncertainty gives rise to an unique sequential equilibrium.

Lemma 1 *Suppose $t's$ commitment is not observable. Then the chain store game has no sequential equilibrium where t fights entry in some market.*

Proof Suppose, *per absurdum,* that $t's$ equilibrium plan of action commits him to fight entry in some market $n \geq 1$. Let k be the last market where he is committed to fight. Then the equilibrium strategies must have the following additional properties that will be proved below:

1) In all markets $n < k$, entry occurs with certainty regardless of entrants' beliefs concerning the monopolist's type.

2) In market k, the weak monopolist w accommodates entry with certainty.

3) Entrant k stays out with certainty, provided the monopolist had responded to entry according to $t's$ equilibrium plan in all preceding markets.

4) In all markets $n > k$, monopolist w mimics t and behaves exactly according to $t's$ equilibrium plan of action.

Putting pieces together, one easily arrives at a contradiction: By 4) on the equilibrium path entrants do not update their prior beliefs concerning the monopolist's type until market k. Given these beliefs and the fact that w accommodates entry in market k by 2), it follows that k enters with certainty if in all preceding markets the monopolist responded to entry according to $t's$ equilibrium plan — which however contradicts property 3).

We now prove properties 1) to 4).

1) Since t does not fight in markets $n < k$, monopolist w reveals his type if he fights in any of these markets. Fighting entry pays only if entry is deterred in at least one subsequent market. Hence, by a standard backward induction argument, it follows that w accommodates entry in all markets $n < k$. Therefore, in all markets $n < k$, both w and t accommodate and entry occurs with certainty, regardless of beliefs.

[5] As Trockel (1986) pointed out, the finitely repeated simultaneous moves game has subgame perfect equilibria where actions are chosen that differ from the equilibrium actions of the associated one–shot game at least in some periods. Some of these equilibria can be viewed as reputational. This reflects a general property of finitely repeated games with multiple equilibria of the associated one–shot game.

2) Since entry occurs anyway in all markets $n < k$, w could only loose by fighting entry in market k.

3) Suppose k enters with positive probability, even though the monopolist had always behaved according to $t's$ equilibrium plan. Then, t would be better–off with a different plan which prescribes accommodation in market k, since entry occurs anyway thereafter, by 1). This contradicts the assumption that the given plan is an equilibrium plan.

4) If monopolist w has mimicked t up to market $k+2$, then he will also mimic in market $k+1$ because that prevents entry in market k, by 3). Repeated application of this argument to market $k+3$ etc. up to market N proves the assertion. ■

Proposition 1 (No Observability) *Assume $t's$ commitment is not observable. Then, the chain store game has an unique sequential equilibrium outcome: entry occurs in all markets and is never fought.*

Proof By Lemma 1 we know that in a sequential equilibrium t accommodates entry in all markets. Using a standard backward induction argument implies that w also accommodates entry in each market and that each entrant enters with certainty, regardless of beliefs concerning the monopolist's type. In turn, a commitment to "always accommodate" is $t's$ best reply to this strategy of entrants. ■

4 Commitment with Imperfect Observability

The above result also holds if commitment is imperfectly observable. In the following we formalize the notion of imperfect observability, introduce some assumptions concerning the signal quality, and then generalize Proposition 1.

The monopolist has chosen a plan of action $a \in \mathcal{A}$ at the outset of the game. This plan is imperfectly observable in the sense that after a is chosen all players observe a signal $s := (s_N, s_{N-1}, \ldots, s_1) \in \mathcal{A}$, which is common knowledge. That signal conveys information concerning the monopolist's plan of action subject to some imperfection. It has the following properties:

Assumption 3 (Full Support) *For each given action plan $a^i \in \mathcal{A}$, players observe each conceivable plan with positive probability (the support of the probability distribution of s is independent of a^i):*

$$\Pr\{S = s \mid a^i\} > 0, \quad \forall s \in \mathcal{A}, \forall a^i \in \mathcal{A}. \tag{4.1}$$

Assumption 4 (Small Error) *The signal is almost perfect in the sense that the true action plan is observed almost with certainty:*

$$\Pr\{S = a^i \mid a^i\} = 1 - \epsilon, \quad \forall a^i \in \mathcal{A}, \tag{4.2}$$

where ϵ is positive but close to zero.

Of course, only t is bound by his choice of actions. Therefore, s can only be indicative of the actions to be executed by t; the action plan of w is purely "cheap talk".

The signal s is not only indicative of the $t's$ actions but also of the monopolist's type. Entrants' assessment of the monopolist's type, evaluated after the signal is observed, depends upon s and the prior belief p^0. Consistency of beliefs with the underlying equilibrium action plan (a^w, a^t) requires

$$\begin{aligned} p_N(s) :&= \Pr\{\text{monopolist is type t} \mid S = s\} \\ &= \frac{p^0 \Pr\{s \mid a^t\}}{p^0 \Pr\{s \mid a^t\} + (1 - p^0) \Pr\{s \mid a^w\}}. \end{aligned} \tag{4.3}$$

By the full support assumption this posterior probability is defined everywhere and $p_N(s) \in (0, 1), \forall s$.

Similarly, define entrants' beliefs concerning $t's$ plan of action

$$\mu(a, s) := \Pr\{\text{monopolist t is committed to plan a} \mid S = s\}. \tag{4.4}$$

Consistency of beliefs requires that they confirm on the equilibrium path.

Lemma 2 (Action Plan Inference) *Entrants' beliefs concerning $t's$ plan of action is independent of the observed signal:*

$$\mu(a, s) = \begin{cases} 1 & \textit{if} \quad a = a^t \\ 0 & \textit{otherwise} \end{cases} \quad \forall s \tag{4.5}$$

(where a^t denotes $t's$ equilibrium plan of action).

Proof Suppose, in equilibrium t has made a commitment to the action plan a^t. Due to the full support assumption, entrants observe each possible signal $s \in \mathcal{A}$ with positive probability. Hence, μ is confirmed on the equilibrium path only if entrants apply probability 1 to the event that t has chosen a^t for each possible signal. This proves (4.5). ∎

Since the signal is costless, it is plausible that w will always mimic t with regard to his choice of "announced" plan of action.

Lemma 3 (Type Inference) *Suppose t's equilibrium action plan prescribes to fight entry in some market, and assume that the signal distortion ϵ is sufficiently small. Then, w announces the same plan as t, and entrants' beliefs concerning the monopolist's type are independent of the observed signal, $p_N(s) = p^0 \ \forall s$.*

Proof Let

$$\epsilon < \min\{\frac{p^0(1-b)}{p^0(1-b)+b(1-p^0)}, \frac{b(1-p^0)}{p^0(1-b)+b(1-p^0)}\}. \tag{4.6}$$

Assume, *per absurdum,* that $a^w \neq a^t$. In conjunction with (4.1), (4.2), and (4.3) one obtains (note: the inequality in the second line holds since $\Pr\{a^t \mid a^w\} + \Pr\{a^w \mid a^w\} < 1$)

$$\begin{aligned} p_N(a^t) &= \frac{p^0 \Pr\{a^t \mid a^t\}}{p^0 \Pr\{a^t \mid a^t\} + (1-p^0)\Pr\{a^t \mid a^w\}} \\ &> \frac{p^0 \Pr\{a^t \mid a^t\}}{p^0 \Pr\{a^t \mid a^t\} + (1-p^0)(1-\Pr\{a^w \mid a^w\})} \\ &= \frac{p^0(1-\epsilon)}{p^0(1-\epsilon)+(1-p^0)\epsilon} \\ &> b \end{aligned} \tag{4.7}$$

$$\begin{aligned} p_N(a^w) &= \frac{p^0 \Pr\{a^w \mid a^t\}}{p^0 \Pr\{a^w \mid a^t\} + (1-p^0)\Pr\{a^w \mid a^w\}} \\ &< \frac{p^0(1-\Pr\{a^t \mid a^t\})}{p^0(1-\Pr\{a^t \mid a^t\}) + (1-p^0)\Pr\{a^w \mid a^w\}} \\ &= \frac{p^0\epsilon}{p^0\epsilon + (1-\epsilon)(1-p^0)} \\ &< b. \end{aligned} \tag{4.8}$$

Suppose $s = a^t$ is observed. Then $p_N > b$ and w can prevent entry in each market where t fights simply by accommodating whenever t accomodates — without ever having to fight. Given $t's$ plan of action, this is the best that can possibly happen to w.

Suppose $s = a^w$ is observed. Then, $p_N < b$, by (4.8). Let k be the last market where t fights. Then, by an argument similar to the reasoning in Lemma 1, w accommodates in market k, and k stays out only if his probability assessment of facing t has increased sufficiently so that $p_k \geq b$. But such updating of beliefs can only have happened if entry occurred in some market to which w responded with a mixed strategy. (Otherwise, there is no updating of beliefs in the right direction, in which case $p_k \le p_N < b$ and k enters with certainty.) Responding to entry with a mixed strategy entails the risk that w reveals his type in which case entry occurs with certainty in all subsequent markets. Therefore, $w's$ payoff is lower than in the case when $s = a^t$ is observed.

Since the signal is almost perfect (Assumption 4), w can gain by switching from a^w to a^t — which contradicts the assumption that a^w is an equilibrium plan of action. ∎

Proposition 2 (Imperfect Observability) *Suppose $t's$ commitment is imperfectly observable in the sense of Assumptions 3–4. Then, the chain store game has an unique sequential equilibrium outcome: entry occurs in all markets and is never fought.*

Proof It is obvious that $a^t = a^w = (A, \dots, A)$, $a^w_n = A, \forall n$, $e_n = E, \forall n$ (for all histories) is a sequential equilibrium. In order to show that it is the only one, suppose $t's$ action plan prescribes fight in at least one market. By Lemma 2 and Lemma 3 both w and t make the same announcement, and the observed signal of their announced action plan is completely ignored. Therefore, the game with imperfectly observable signals collapses to the game without observability, which was already solved in Proposition 1. ∎

5 Discussion

The present paper has reconsidered the solution of the chain store paradox by Kreps and Wilson and Milgrom and Roberts. We introduced two plausible modifications: First, rather than assuming that the tough monopolist is predisposed to fight all entrants we assumed that he has access to a commitment mechanism and rationally chooses among different commitments. Second, we assumed that the commitment to a certain plan of action is imperfectly observable, due to a small probability of misunderstanding or communication error. These modifications were shown to completely erode the value of reputation mechanism and bring back Selten's chain store paradox in full force.

One limitation of our analysis is that we excluded the possibility of commitment to a *random* plan of action. In his analysis of the role of observability in complete information commitment games Bagwell pointed out that his game has several equilibria if one allows for randomized commitments. One of these mixed strategy equilibria preserves the value of commitment (but not the other). A similar multiplicity issue may come up in our framework. However, since our pure strategy equilibrium is strict whereas mixed strategy equilibria are necessarily weak, common equilibrium refinements are biased in favor of the equilibrium presented here.[6]

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[6] In the framework of Bagwell's model, Hurkens and van Damme (1994) have developed an equilibrium selection principle that actually favors that particular mixed strategy equilibrium that preserves the value of commitment. However, this selection principle is "unusual" already because it eliminates a strict equilibrium in favor of a weak.