Risk and Evolution

Theodore To*

November 7, 1995

Abstract

I examine a Knightian model of entrepreneurial risk and investment where in addition to the self-selection process for choosing entrepreneurs, there is an evolutionary selection process over the representation of various risk attitudes. Under a standard evolutionary dynamic, rather than converging to a population of risk-neutrals (fitness maximizers), the population converges to a stationary distribution where both risk-averse and risk-loving types are represented and where only the risk-loving types invest. Many types are represented in stationary population distributions because an evolutionary market environment protects and encourages diversity with different types specializing in different activities and in the steady state each type earns, on average, the same objective payoff.

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*Department of Economics, University of St. Andrews, St. Andrews, Fife KY16 9AL, Scotland, (e-mail: `Theodore.To@st-and.ac.uk`). I thank Masaki Aoyagi, Helmut Bester, V. Bhaskar, John Duffy, Nick Feltovich, Werner Güth, Rick Harbaugh, Lau Ka-sing, Werner Ploberger, Karl Wärneryd, and seminar participants at the CentER for Economic Research and the 1994 North American Winter Meetings of the Econometric Society for helpful comments and suggestions on earlier versions of the paper.

1 Introduction

I examine a Knightian (1921) model of entrepreneurial risk and investment where in addition to the self selection process for choosing entrepreneurs, there is an evolutionary selection process over the relative representation of various risk attitudes. To do this, I construct a general equilibrium model of investment and trade where agents' preferences evolve over time. This framework has similarities to both Blume and Easley (1992) and Kihlstrom and Laffont (1979). Blume and Easley study an evolutionary model of an asset market where fitness is measured by 'expected growth rates of wealth share accumulation.' Kihlstrom and Laffont examine a model where individuals that differ in their attitudes towards risk, can choose to either supply labor or run a firm, however, they assume that the distribution of various risk attitudes is held fixed.

The investment model of the present paper can be interpreted as one where each agent has an endowment of labor which can be 'invested' in a risky activity (hunting or operating a firm) or a safe activity (gathering or working for a wage). It can also be interpreted as one where agents are initially endowed with a good which can be transformed, using a risky technology, into another good. For example, money held in a bank account provides a sure return. However, money can also be transformed into a risk-bearing asset which provides an uncertain return. In equilibrium, the propagation of each type will depend on its endogenously determined average monetary payoff. Hence the model can be broadly interpreted to be applicable to situations in which agents face a choice between a risky and a safe activity and where each type's future representation in the population depends on the average monetary return induced by its preferences.

Several recent papers study the evolutionary survival properties of various behaviors in economic settings (see Hansen and Samuelson (1988), Qin and Stuart (1993), and Wright (1993) for examples). However, like Blume and Easley and many other recent papers on evolutionary economics, agents actions are dictated by their strategy type and strategy types that do well, propagate.

More recently, there have been several papers that maintain the standard economic assumption that individuals behave rationally given their preferences and instead study the survival properties of those preferences themselves (examples include Balvers and Acharya (1994), Güth and Yaari (1992), Rogers (1994), and Waldman (1994)). Güth calls this the 'indirect evolutionary approach' in contrast to the standard or 'direct' approach. Using this approach, I consider a model of investment and trade in a setting where the distribution over various risk attitudes changes over time.

Robson (1994a, 1994b) and Wärneryd (1995) also consider evolution over preferences in situations in which agents choose between activities with differing degrees of risk. Robson's work considers the genetic evolution of risk preferences. In his first paper (1994b), he looks at a mating game where, under some circumstances, males pursue risk-taking activities (even with concave return functions) in the hopes of attracting more mates. In the second paper (1994a), using a model where the random payoff to an individual is independent of the actions of other agents, he derives expected and non-expected utilities as a result of the evolutionary selection process. Wärneryd (1995), like the current paper, considers the non-genetic evolution of risk preferences. Constructing an evolutionary model of rent seeking, he demonstrates that in the long-run, all rents are dissipated.

In order to consider evolution over preferences, one must have different measures for utility and fitness. In models employing the 'direct' evolutionary approach, a player's action is dictated by her strategy type and strategy types that do well earn greater representation in future generations. Since players here are not rational, maximizing agents, the difference between utility and fitness is unimportant. However, with models employing the 'indirect' evolutionary approach, players are free to make decisions based their on their own subjective preferences and preference types replicate based on an objective measure of fitness, common to all players. Separating fitness and utility has the additional advantage that one can now consider the idea that people often prefer to act in ways that are not necessarily 'good for them.' For example, there are numerous studies which have concluded that cigarette smoking or that high fat diets reduce life expectancy (one measure of fitness) and yet there are people who enjoy smoking and eating fatty foods.

I assume that preferences change over time through a process of cultural and social evolution that rewards successful preferences. This is in contrast to Robson's (1994a, 1994b) interpretation that preferences change as a result of genetic evolution. While there are arguments which support this interpretation, environment is also an important factor which shapes preferences.[1] Like biological traits, cultures and social conventions evolve over time to fit new and changing circumstances. The environment in which a person grows and lives includes her surrounding culture and the training she receives as a child. For example, women have gained more rights and greater financial independence and as a result, many prefer to marry later in life or not at all. Through imitation, people can 'learn to like' or 'acquire a taste for' beer, sushi, cigarettes, etc. Even animals can be trained to behave counter to how their instincts (i.e., their 'hardwired' preferences) dictate.

The main result then, is that this evolutionary system converges to a stationary population distribution where both risk-averse and risk-loving types are represented and where only the risk-loving types invest. These stationary distributions are stable in the sense that if mutations occur, the population will converge to another stationary distribution where all types are represented and only the risk-loving types invest. Furthermore, since average population fitness is maximized at these stationary distributions, they are 'optimal.' Hence, although non-risk-neutral preferences are inefficient, together the evolutionary and market mechanisms allow the existence of a diverse population where different types specialize in different activities and yet an objective measure of population welfare is maximized.

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[1] For several views on the long standing nature vs. nurture debate, see Jensen (1981), Lewontin, Rose and Kamin (1984), and Plomin (1990).

2 The Model

Consider the following three-stage model. In the first stage, individuals make investment decisions. In the second stage, after the outcomes of their investments have been realized, they participate in a competitive exchange market. Finally, in the last stage, through the process of cultural evolution described in the previous Section, agents that do well earn greater representation in the succeeding generation.

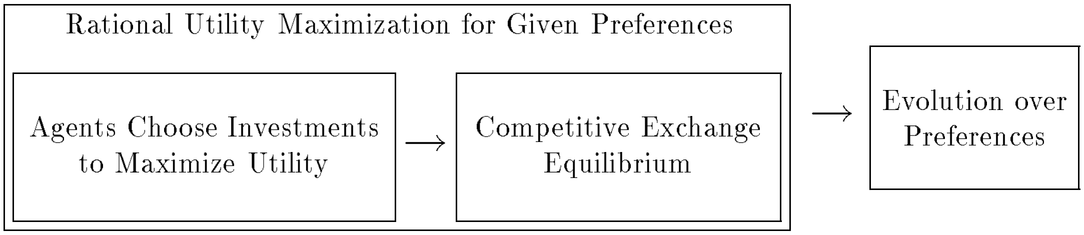

Figure 1: Structure of Model

There are two goods, 1 and 2. Assume that there is a continuum of types of agents, $\bar{A} = [\underline{\alpha}, \bar{\alpha}]$ where $0 < \underline{\alpha} < 1 < \bar{\alpha} \leq \infty$. Let $(\bar{A}, \mathcal{F}, \mu)$ be a probability space where $\mathcal{F}$ is the collection of Borel measurable sets generated by $\bar{A}$ and μ is an atomless probability measure with support $\bar{A}$.[2] Probability measure μ describes the distribution of agents of each type. Use i to denote goods 1 and 2.

All individuals have an endowment (e.g., time, money, etc.) which can be either used to produce a single unit of good 1 with certainty or invested to produce a single unit of good 2 with probability σ and nothing with probability $(1 - \sigma)$. Assume that the realization of each individual's investment is independent of the realizations of all other investments. Furthermore, since there is a continuum of i.i.d. random variables, assume that the law of large numbers holds,[3] so that a proportion σ of the investments undertaken by agents

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[2] When μ has atoms, results are analogous to those found with atomless μ. The introduction of distributions with atoms complicates the notation because agents that are indifferent may have mass and therefore one would have to keep track of the mass of those that choose each action. For this reason, I only consider specific examples of distributions with atoms. One such example is considered in Section 6 and I briefly discuss a population consisting entirely of risk-neutral individuals on page 19.

[3] Judd (1985) showed that when there is a continuum of i.i.d. random variables, in order to work with measures that have desirable properties, one can assume that the distribution is such that 'almost all

of each type succeed.

Assume that there are functions $f : \mathbb{R}^2_+ \to \mathbb{R}$ and $u_\alpha : \mathbb{R}^2_+ \to \mathbb{R}$. The function $f(x_1, x_2)$ is the objective measure of evolutionary fitness an individual derives from consuming x_1 and x_2, and $u_\alpha(x_1, x_2)$ is the subjective measure of utility an individual of type $\alpha \in \bar{A}$ derives from consuming x_1 and x_2. The objective measure of well-being, f, is independent of α and hence common to all agents while the subjective measure of well-being, u_α, is dependent on α.

Fitness determines how the population evolves over time and utility is the measure upon which rational agents base their decisions. I assume the population evolves following a natural generalization of the standard discrete time replicator dynamic. Let $f_t(\alpha)$ be the average fitness achieved by type α agents. Using t to denote time, the population evolves according to:

$$\mu_{t+1}(A) = \frac{\int_A f_t(\alpha) d\mu_t(\alpha)}{\int_{\underline{\alpha}}^{\bar{\alpha}} f_t(\alpha) d\mu_t(\alpha)} \tag{1}$$

for all $A \in \mathcal{F}$. A population evolving according to (1) has the property that types which achieve fitness greater than the average population fitness earn greater representation in the next generation. While there are disadvantages in using the replicator dynamic to consider the complex process of cultural and social evolution, it's use greatly simplifies the analysis of population dynamics. Dawkins (1989) argues that cultural evolution takes place via a process of learning and imitation. Börgers and Sarin (1993), Hopkins (1995), and Schlag (1994), among others, have demonstrated that dynamics much like replicator dynamics can arise from models of learning or imitation. As such, I use replicator dynamics to consider the evolution over risk preferences in a model of investment and trade.

For the purposes of this paper, assume that u_α is a monotonic transformation of f. Under this formulation, agents have utilities over fitness and agents with concave (convex)

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realizations are measurable and that a law of large numbers holds.'

utility functions are risk-averse (risk-loving) over various fitness gambles. Further assume that $u_\alpha = f^\alpha$ and that f is homogeneous of degree one. The rate of relative risk aversion exhibited by u_α is $(1-\alpha)$ so agent α is risk-averse if $\alpha < 1$, risk-neutral if $\alpha = 1$ and risk-loving if $\alpha > 1$. Constant rates of relative risk aversion provide a convenient means for characterizing agents with different attitudes towards risk. Homogeneity of fitness has the advantage that an agent's risk behavior is due solely to her utility function (i.e., a concave (convex) fitness function would tend to induce risk-averse (risk-loving) behavior). Furthermore, with homogeneous fitness, evolutionary fitness becomes a multiple of wealth or income so that in an environment of cultural or social evolution, evolutionary change has the appealing feature (or crass depending on your point of view) that it is driven by the relative wealth generated by the activities induced by different preferences.

In order to ensure that consumption of both goods is positive in equilibrium assume that $f(x_1, x_2) = 0$ only if $x_1 = 0$ or $x_2 = 0$. Since u_α is a monotonic transformation of f, the utility maximizing demand of every type is identical. Given the ratio of the price of good 1 to the price of good 2, p, and wealth, m, let demand for good i be $x_i^*(p, m)$. Assume that goods 1 and 2 are gross substitutes (i.e., $\partial x_2^*(p, m)/\partial p \geq 0$). Gross substitutability further implies that $\partial x_1^*(p, m)/\partial p < 0$.

3 Market Equilibrium

As is standard in multi-stage games, I work backwards and first solve for the competitive exchange equilibrium given investment decisions. Let $e_1, e_2 : \bar{A} \to \{0, 1\}$ summarize each type's investment decision with $e_1(\cdot) + e_2(\cdot) = 1$ where $e_2(\alpha) = 0$ if α type individuals do not invest and $e_2(\alpha) = 1$ if they do. I assume that all individuals of the same type choose the same investment levels and that $e_2(\cdot)$ is measurable. As will be seen in Section 4, both of these presumptions must be true when agents choose investments to maximize expected utility.

Let good 2 be the numeraire and $p = p_1/p_2$ be the price ratio. Given agents' investment decisions, $e_1(\cdot)$ and $e_2(\cdot)$, a competitive exchange equilibrium is a triplet

$(x_1^*(\cdot,\cdot), x_2^*(\cdot,\cdot), p^e)$ such that i) $(x_1^*(p,m), x_2^*(p,m))$ maximizes utility given price, p, and wealth, m, ii) p^e is such that aggregate demand for each good is equal to the aggregate amount of each good available.

Since all agents' utility functions are monotonic transformations of the same function, their demands are identical functions of price and wealth. Furthermore, since the function underlying utility is homogeneous, utility maximizing demands are:

$$x_i^*(p,m) = x_i^*(p,1)m.$$

Let $x_i^*(p) = x_i^*(p,1)$.

Given the law of large numbers and since $m(\alpha) = pe_1(\alpha) + e_2(\alpha)$ if an individual's investment paid off and $m(\alpha) = pe_1(\alpha)$ if it did not pay off, the average wealth of type α agents is $pe_1(\alpha) + \sigma e_2(\alpha)$. Aggregate wealth is thus $\int_{\underline{\alpha}}^{\bar{\alpha}} (pe_1(\alpha) + \sigma e_2(\alpha))d\mu(\alpha)$. Similarly, the aggregate supply of goods 1 and 2 are $\int_{\underline{\alpha}}^{\bar{\alpha}} e_1(\alpha)d\mu(\alpha)$ and $\sigma \int_{\underline{\alpha}}^{\bar{\alpha}} e_2(\alpha)d\mu(\alpha)$. Therefore the market clearing conditions for goods 1 and 2 are:

$$x_1^*(p^e) \int_{\underline{\alpha}}^{\bar{\alpha}} (p^e e_1(\alpha) + \sigma e_2(\alpha))d\mu(\alpha) = \int_{\underline{\alpha}}^{\bar{\alpha}} e_1(\alpha)d\mu(\alpha) \tag{2}$$

$$x_2^*(p^e) \int_{\underline{\alpha}}^{\bar{\alpha}} (p^e e_1(\alpha) + \sigma e_2(\alpha))d\mu(\alpha) = \sigma \int_{\underline{\alpha}}^{\bar{\alpha}} e_2(\alpha)d\mu(\alpha). \tag{3}$$

Gross substitutability implies uniqueness of the competitive exchange equilibrium.

Remark 1 *There is a unique price ratio p^e such that the markets for both goods clear.*

A unique equilibrium is required in order to consider evolution over preferences.

4 Investment Equilibrium

Given the anticipated outcome of the second stage exchange market, each individual makes her investment decision to maximize expected utility. Expectations about the price in the second stage exchange market are consistent in that given the aggregate equilibrium investment levels, these expectations are realized.

To get agents' expected utilities, I first substitute their demands into the fitness function to get an indirect fitness function, $g(p,m) = f(x_1^*(p,m), x_2^*(p,m))$. Since the fitness function is homogeneous, the indirect fitness function has the property that it can be expressed as $g(p,m) = g(p)m$.[4] Hence type α agents have indirect utility function $v_\alpha(p,m) = [g(p)m]^\alpha$. Using the indirect utility function, a type α individual's expected utility is:

$$g(p)^\alpha[\sigma(pe_1(\alpha) + e_2(\alpha))^\alpha + (1-\sigma)(pe_1(\alpha))^\alpha] \tag{4}$$

where,

$$e_1(\cdot) + e_2(\cdot) = 1, \tag{5}$$

and

$$e_1(\cdot), e_2(\cdot) \in \{0, 1\}. \tag{6}$$

A Nash equilibrium in the investment stage is given by $(e_1^*(\cdot\,,\cdot), e_2^*(\cdot\,,\cdot), p)$ if i) given p $(e_1^*(\alpha,p), e_2^*(\alpha,p))$ maximizes (4) subject to (5) and (6), for any α and ii) $p = p^e$ where p^e satisfies (2) and (3) given $(e_1(\cdot), e_2(\cdot)) = (e_1^*(\cdot\,,p), e_2^*(\cdot\,,p))$. In other words, no individual has an incentive to deviate from the given investment strategies when the price is p and p clears the market for these investment strategies.

Before characterizing the equilibria of the investment stage, define $n^{RL} = \mu[1,\bar{\alpha}]$ (i.e., n^{RL} is the mass of the risk-loving agents).

Theorem 1 *Equilibria of the investment stage,* $(e_1^*(\cdot\,,\cdot), e_2^*(\cdot\,,\cdot), p)$*, exist and satisfy:*

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[4] I.e., $g(p,m) = f(x_1^*(p,m), x_2^*(p,m)) = f(x_1^*(p)m, x_2^*(p)m) = f(x_1^*(p), x_2^*(p))m = g(p,1)m$. Let $g(p) = g(p,1)$.

i) $p \in (\underline{p}, \bar{p})$ *where* $\underline{p} = \sigma^{1/\underline{\alpha}}$ *and* $\bar{p} = \sigma^{1/\bar{\alpha}}$,

ii) $p \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \sigma$ *as* $n^{RL} \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} x_2^*(\sigma)$ *and*

iii) *if* $p < \sigma^{1/\alpha}$ *then* $e_2^*(\alpha, p) = 1$, *if* $p > \sigma^{1/\alpha}$ *then* $e_2^*(\alpha, p) = 0$.

Furthermore, the equilibria of the investment stage are unique in the sense that the equilibrium price and the aggregate level of investment are identical.

Proof: I first show that any equilibrium of the investment stage must satisfy iii) and then show that given iii) the investment stage has an equilibrium with a unique price and aggregate investments. Finally, I prove i) and ii).

To prove iii), consider each agent's expected utilities from investing and not investing: $g(p)^\alpha \sigma$ and $g(p)^\alpha p^\alpha$. For each α, $p = \sigma^{1/\alpha}$ is the price at which an individual of type α is indifferent between investing and not investing. Hence if $p < \sigma^{1/\alpha}$ then $e_2^*(\alpha, p) = 1$ and if $p > \sigma^{1/\alpha}$ then $e_2^*(\alpha, p) = 0$.

Now, to prove the existence of an equilibrium and the uniqueness of price and aggregate investments, note that *any* equilibrium of the investment stage must satisfy iii). It will be useful here to define the critical value $\alpha^*(p) = \ln \sigma / \ln p$. Given p, any agent of type $\alpha < \alpha^*(p)$ prefers not to invest and any agent of type $\alpha > \alpha^*(p)$ prefers to invest (i.e., $\alpha^*(p)$ solves $p = \sigma^{1/\alpha}$). For a given population distribution, this implies that the mass of agents who invest, $\mu[\alpha^*(p), \bar{\alpha}]$, is decreasing in p. Hence the aggregate investment function, $I(p) = \int_{\underline{\alpha}}^{\bar{\alpha}} e_2^*(\alpha, p) d\mu(\alpha) = \mu[\alpha^*(p), \bar{\alpha}]$, must be decreasing and continuous in p. Next, from the market clearing condition for good 2 (equation (3)) it can be seen that for a given aggregate investment level, $I = \int_{\underline{\alpha}}^{\bar{\alpha}} e_2(\alpha) d\mu(\alpha)$, the implicit function $p^e(I)$ must satisfy:

$$\frac{x_2^*(p^e) p^e}{(1 - x_2^*(p^e))\sigma} = \frac{I}{1 - I}.$$

Since $\partial x_2^* / \partial p^e \geq 0$, $p^e(I)$ is continuous and increasing in aggregate investment. The composite function $I \circ p^e : [0,1] \rightarrow [0,1]$ is thus continuous and decreasing and hence has a unique fixed point. Therefore an equilibrium exists and price and aggregate investment in every equilibrium are unique.

To prove i), note that $\bar{p} = \sigma^{1/\bar{\alpha}}$ is the price at which agents of type $\bar{\alpha}$ are indifferent between investing and not investing. If $p \geq \bar{p}$ then almost all agents strictly prefer to invest nothing and hence everyone gets zero utility. All agents thus have an incentive to deviate by investing something since they could then achieve positive expected utility. Therefore an upper-bound on the equilibrium price is $\bar{p}$. By a similar argument, the equilibrium price is bounded below by $\underline{p} = \sigma^{1/\underline{\alpha}}$. Clearly, $\underline{p} < \bar{p}$ and hence $p \in (\underline{p}, \bar{p})$.

Finally, I show that $p < (>)\sigma$ if and only if $n^{RL} < (>) x_2^*(\sigma)$. First note from the market clearing condition for good 2 (equation (3)) and the optimal investment strategies, iii), that when $p = \sigma$, $n^{RL} = x_2^*(\sigma)$. Without loss of generality suppose that $p < \sigma$. Again using the equilibrium investment strategies, I solve the market clearing condition for good 2 at price p and at price σ for $x_2^*(p)$ and $x_2^*(\sigma)$.

$$x_2^*(p) = \frac{\sigma(\int_{\underline{\alpha}}^1 e_2^*(\alpha, p) d\mu + n^{RL})}{\int_{\underline{\alpha}}^1 (p e_1^*(\alpha, p) + \sigma e_2^*(\alpha, p)) d\mu + \sigma n^{RL}} \tag{7}$$

$$x_2^*(\sigma) = \frac{\sigma x_2^*(\sigma)}{\sigma} \tag{8}$$

Suppose that $n^{RL} \geq x_2^*(\sigma)$. This implies that the top of the right hand side of (7) is strictly greater than the top of the right hand side of (8) since for $\alpha \in (\alpha^*(p), 1]$, $e_2^*(\alpha, p) = 1$. Furthermore, since $p < \sigma$, the bottom of the right hand side of (7) is strictly less than σ, the bottom of the right hand side of (8). Hence, the right hand side of (7) is greater than the right hand side of (8). Since $\partial x_2^* / \partial p \geq 0$ it must be that $p > \sigma$, a contradiction. Therefore $n^{RL} < x_2^*(\sigma)$. To prove the converse is similar. ■

Parts i) and ii) of the Theorem characterize the equilibrium prices and indicate first, that as the likelihood of a successful investment decreases, the band in which the equilibrium price ratio must reside moves downwards. That is, the reward for investing must be sufficiently high, relative to the reward for not investing, in order to induce investment. Second, that the value of good 1 is low when there are relatively few risk-loving types and high when there are relatively few risk-averse types. Furthermore, $\bar{p} \leq 1$ so that the return from a successful investment is always strictly greater than that from not investing. However, the *expected* return of investment may be greater or less than that of not investing, depending on the relative numbers of agents participating in each activity. If one considers the return from the safe activity to represent a 'normal' rate of return, then this result would be in contrast to Knight's (1921, page 365) argument that entrepreneurs would in the aggregate earn negative economic profits.

Part iii) characterizes individual investment decisions given price. (I.e., risk-neutral agents invest or don't invest based on which action yields the greatest objective reward, risk-averse agents invest only if the return from investing is relatively high, and risk-loving agents invest unless the return from investing is relatively low.) In general, part iii) implies that risk-loving types are more likely to invest than risk-averse types. This is similar in spirit to the Kihlstrom and Laffont (1979) result which says that the more risk-averse individuals become workers while the less risk-averse become entrepreneurs. Finally, although equilibrium investment strategies are not unique (i.e., one measure-zero type is indifferent between investing and not investing), price and aggregate investment are unique and hence the equilibrium is 'almost' unique.

Also note that since, for a given price p, the functions $e_1^*(\cdot\,, p)$ and $e_2^*(\cdot\,, p)$ are discontinuous at most at one point, they are measurable. In equilibrium, $e_i(\cdot) = e_i^*(\cdot\,, p)$, confirming the assumption of Section 3.

5 The Evolution of Risk Attitudes

So far, the above analysis has been purely static. Recall that the population distribution evolves according to the discrete time replicator dynamic. Employing the properties of the investment and exchange equilibrium and the law of large numbers, type α average fitness in period t is $f_t(\alpha) = g(p_t)(p_t e_1^*(\alpha, p_t) + \sigma e_2^*(\alpha, p_t))$. Hence the evolutionary population

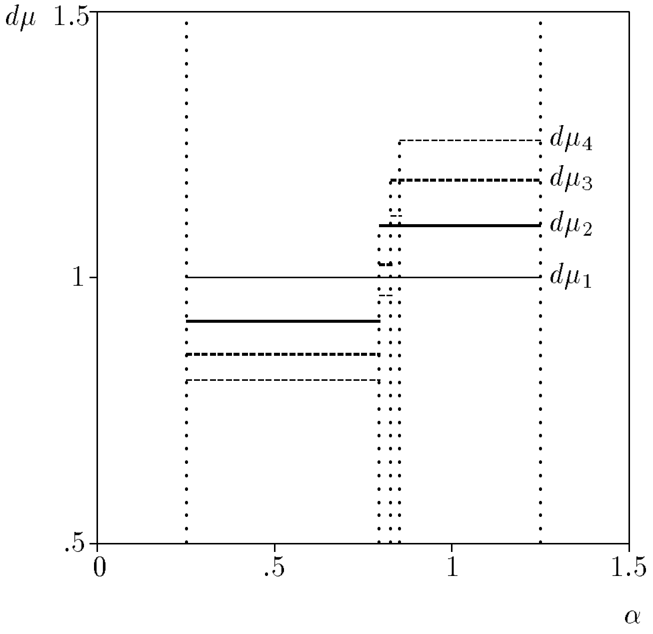

Figure 2: Evolution with Uniform Initial Distribution

dynamics equation (1) becomes:

$$\begin{aligned}\mu_{t+1}(A) &= \frac{\int_A g(p_t)(p_t e_1^*(\alpha,p_t)+\sigma e_2^*(\alpha,p_t))d\mu_t(\alpha)}{\int_{\underline{\alpha}}^{\bar{\alpha}} g(p_t)(p_t e_1^*(\alpha,p_t)+\sigma e_2^*(\alpha,p_t))d\mu_t(\alpha)} \\ &= \frac{\int_A (p_t e_1^*(\alpha,p_t)+\sigma e_2^*(\alpha,p_t))d\mu_t(\alpha)}{\int_{\underline{\alpha}}^{\bar{\alpha}} (p_t e_1^*(\alpha,p_t)+\sigma e_2^*(\alpha,p_t))d\mu_t(\alpha)} \qquad (9)\end{aligned}$$

for all $A \in \mathcal{F}$. Since $e_1^*(\alpha,p_t), e_2^*(\alpha,p_t) \geq 0$, $e_1^*(\alpha,p_t)+e_2^*(\alpha,p_t)=1$ and p_t is bounded, it is clear from this definition that if μ_t is a probability measure on $(\bar{A},\mathcal{F})$, then μ_{t+1} is also a probability measure on $(\bar{A},\mathcal{F})$, (i.e., $\mu_{t+1}(A_k) \in [0,1]$, $\mu_{t+1}(\emptyset)=0$, $\mu_{t+1}(\bar{A})=1$ and $\mu_{t+1}(\bigcup_{k=1}^{\infty} A_k) = \sum_{k=1}^{\infty} \mu_{t+1}(A_k)$ where the A_k are disjoint and $A_k \in \mathcal{F}$). Furthermore, if μ_t is atomless then μ_{t+1} is also atomless.

Notice that since the term $g(p_t)$ factors out of both integrals, it cancels and the propagation of each type depends only on the average value of its 'endowment' or wealth. Under this dynamic, the relative frequency of a type increases if its wealth is greater than the average population wealth, stays the same if it is equal to average wealth and falls if it is less than average wealth.

Before considering the convergence or non-convergence properties of the continuous

t	p_t	$\alpha^*(p_t)$
1	0.418	0.795
2	0.433	0.827
3	0.444	0.853
4	0.452	0.873
5	0.459	0.889
6	0.464	0.903
7	0.468	0.914
8	0.472	0.924
9	0.475	0.932
10	0.478	0.939

Table 1: Prices and Investment Cutoff

distribution replicator dynamic, it will be helpful to examine a graphic illustration of its mechanics. In Figure 2 the initial given distribution over types, μ_1, is uniform over $[.25, 1.25]$, the underlying fitness function is $f(x_1, x_2) = (x_1 x_2)^{1/2}$, and the probability of successful investment is $\sigma = 0.5$. With Cobb-Douglas fitness, steady-state population distributions depend only on the share-parameter—in this case one-half of any steady-state population must be risk-averse and the other half risk-loving. Given the initial distribution, there are 'too few' risk-loving agents hence the anticipated relative return from investing is high, with all of the risk-loving agents and some of the less-risk-averse agents investing. In period 2, the representation of agents who invested increases and that of agents who did not decreases. This process proceeds successively with the representation of the risk-loving slowly increasing towards one-half of the population. The equilibrium prices, p_t, and investment cutoffs, $\alpha^*(p_t)$,[5] from this example are presented in Table 1 for the first ten periods. From these values, the density functions from each period can be computed.

A population distribution, μ_t on $(\bar{A}, \mathcal{F})$ is **stationary** if and only if $\mu_{t+1}(A) = \mu_t(A)$ for all $A \in \mathcal{F}$, where μ_{t+1} follows (9). Let $\mathcal{M}$ be the family of such probability measures

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[5] The term $\alpha^*(p_t) = \ln\sigma / \ln p_t$ as defined in the proof of Theorem 1. This is the equilibrium cutoff point where if $\alpha < \alpha^*(p_t)$ then agents of type α do not invest and if $\alpha > \alpha^*(p_t)$ then agents of type α invest.

that are atomless. Define p_μ to be the equilibrium price, given population distribution μ.

Theorem 2 *An atomless probability measure, μ, on $(\bar{A}, \mathcal{F})$ is an element of $\mathcal{M}$ if and only if $p_\mu = \sigma$ or equivalently if and only if $\mu[1, \bar{\alpha}] = x_2^*(\sigma)$. Furthermore, if μ_1 is atomless, and μ_t follows (9) then $\mu_t \to \mu$ for some $\mu \in \mathcal{M}$.*

Proof: Let μ_1 be an atomless probability measure on $(\bar{A}, \mathcal{F})$ and let μ_t follow (9). Define p_t to be the equilibrium price that obtains if the population distribution is μ_t.

The strategy of the proof is as follows. 1) Characterize the set of stationary population distributions—every element of this set will have the property that the mass of the risk-loving types is equal to $x_2^*(\sigma)$. 2) Show that the mass of the risk-loving types, n_t^{RL}, converges. 3) Show that $p_t \to \sigma$ and $n_t^{RL} \to x_2^*(\sigma)$. 4) Show that $\mu_t \to \mu \in \mathcal{M}$.

1) Suppose μ is stationary. By examining (9) it must be that either $p_\mu = \sigma$ or else for some i, $e_i(\alpha, p_\mu) = 1$ for almost all α. Since not all types choose the same action (from Theorem 1), it must be that $p_\mu = \sigma$. Conversely, if $p_\mu = \sigma$ then by (9), μ must be stationary.

Using Theorem 1 and examining the market clearing condition for good 2 (equation (3)), it can be seen that if μ is such that $p_\mu = \sigma$ then $\mu[1, \bar{\alpha}] = x_2^*(\sigma)$. Conversely, suppose that $\mu[1, \bar{\alpha}] = x_2^*(\sigma)$ but $p_\mu \neq \sigma$. Without loss of generality assume that $p_\mu < \sigma$. Then:

$$\begin{aligned}
x_2^*(p_\mu) \int_{\underline{\alpha}}^{\bar{\alpha}} (p_\mu e_1^*(\alpha, p_\mu) + \sigma e_2^*(\alpha, p_\mu)) d\mu(\alpha) &< \sigma \mu[1, \bar{\alpha}] && (10) \\
&< \sigma \mu[\alpha^*(p_\mu), \bar{\alpha}] && (11) \\
&= \sigma \int_{\underline{\alpha}}^{\bar{\alpha}} e_2^*(\alpha, p_\mu) d\mu(\alpha) && (12)
\end{aligned}$$

(10) follows by assumption because $x_2^*(p_\mu) < x_2^*(\sigma) = \mu[1, \bar{\alpha}]$ and because $\int_{\underline{\alpha}}^{\bar{\alpha}} (p_\mu e_1^*(\alpha, p_\mu) + \sigma e_2^*(\alpha, p_\mu)) d\mu(\alpha) < \sigma$, (11) follows because when $p_\mu < \sigma$, $\alpha^*(p_\mu)$ as defined in the proof of Theorem 1 is strictly less than 1, (12) also follows from Theorem 1 because for $\alpha > \alpha^*(p_\mu)$, $e_2^*(\alpha, p_\mu) = 1$ and for $\alpha < \alpha^*(p_\mu)$, $e_2^*(\alpha, p_\mu) = 0$. The end result is a contradiction of the market clearing conditions (i.e., the left hand side of (10) must equal (12)) and therefore $p_\mu = \sigma$.

The remainder proves the global stability of (9). Without loss of generality, I have assumed for the remainder of the proof that μ_1 is such that $p_t < \sigma$ and $n_t^{RL} < x_2^*(\sigma)$ for all t.

2) First I show that if μ_1 is such that $p_1 < (>)\sigma$ then n_t^{RL} is an increasing (decreasing) sequence bounded above (below) by $x_2^*(\sigma)$. Given the equilibrium investment strategies,

$$\begin{aligned} n_{t+1}^{RL} &= \mu_{t+1}[1, \bar{\alpha}] && (13) \\ &= \frac{\sigma n_t^{RL}}{\int_{\underline{\alpha}}^1 (p_t e_1^*(\alpha, p_t) + \sigma e_2^*(\alpha, p_t)) d\mu_t + \sigma n_t^{RL}} && (14) \\ &< \frac{\sigma(\int_{\underline{\alpha}}^1 e_2^*(\alpha, p_t) d\mu_t + n_t^{RL})}{\int_{\underline{\alpha}}^1 (p_t e_1^*(\alpha, p_t) + \sigma e_2^*(\alpha, p_t)) d\mu_t + \sigma n_t^{RL}} && (15) \\ &= x_2^*(p_t) && (16) \\ &\leq x_2^*(\sigma) && (17) \end{aligned}$$

(14) follows from the definition of the population dynamic (9) and Theorem 1, (15) comes from adding the term $\sigma \int_{\underline{\alpha}}^1 e_2(\alpha) d\mu_t(\alpha)$ to the numerator of (14), (16) follows from (7), and (17) follows from the fact that $x_2^*(\cdot)$ is increasing. Hence $n_{t+1}^{RL} < x_2^*(\sigma)$. Finally, (14) and the fact that $p_t < \sigma$ implies that $n_t^{RL} < n_{t+1}^{RL}$. Therefore n_t^{RL} is an increasing sequence bounded above by $x_2^*(\sigma)$. Since n_t^{RL} is monotonic and bounded, it converges.

3) By examining (14), it can be seen that since $e_1^*(\alpha, p_t) = 1$ for all $\alpha < \alpha^*(p_t)$, n_t^{RL} converges only if either $p_t \to \sigma$ or $n_t^{RL} \to 1$. Since part 2) of the proof implies that $n_t^{RL} < x_2^*(\sigma)$ for all t and that $x_2^*(\sigma) < 1$ (i.e., consumption of both goods must be positive in equilibrium, so $x_2^*(\sigma) < 1$), it must be that $p_t \to \sigma$. Noticing that the market clearing condition for good 2 (equation (3)) is continuous in price, if $p_t \to \sigma$ then $n_t^{RL} \to x_2^*(\sigma)$.

4) The strategy here will be as follows. First show that for any interval $\mathcal{I} \subset [1, \bar{\alpha}]$, $\mu_t(\mathcal{I})$ converges. Then for any $a < 1$ and any interval $\mathcal{I} \subset [\underline{\alpha}, a]$, show that $\mu_t(\mathcal{I})$ converges. Next, for any $A \in \mathcal{F}$, show that $\mu_t(A)$ converges. Call that convergent point $\mu(A)$. It is clear that μ so defined is a probability measure on $(\bar{A}, \mathcal{F})$ and that $\mu_t \xrightarrow{v} \mu$. Lastly, show that μ has support $\bar{\alpha}$. Since every μ_t is atomless, μ must be atomless and $\mu \in \mathcal{M}$.

Let $f_t(\alpha) = p_t e_1^*(\alpha, p_t) + \sigma e_2^*(\alpha, p_t)$ and $c_t = \int_{\underline{\alpha}}^{\bar{\alpha}} f_t(\alpha) d\mu_t(\alpha)$. Redefining (9) as a density function:

$$d\mu_{t+1}(\alpha) = \frac{f_t(\alpha)}{c_t} d\mu_t(\alpha).$$

Since $p_{t+n} < \sigma$, $f_{t+n}(\alpha) = \sigma$ for any $n \geq 0$ and any $\alpha > 1$. Hence,

$$\begin{aligned} \mu_{t+n}[1, \bar{\alpha}] &= \int_1^{\bar{\alpha}} \frac{f_{t+n-1}(\alpha) f_{t+n-2}(\alpha) \cdots f_t(\alpha)}{c_{t+n-1} c_{t+n-2} \cdots c_t} d\mu_t(\alpha) \\ &= \frac{\sigma^n}{c_{t+n-1} c_{t+n-2} \cdots c_t} \mu_t[1, \bar{\alpha}]. \end{aligned}$$

But $\mu_{t+n}[1, \bar{\alpha}] \to x_2^*(\sigma)$ as $n \to \infty$. Since $\mu_t[1, \bar{\alpha}] > 0$,

$$\lim_{n \to \infty} \frac{\sigma^n}{c_{t+n-1} c_{t+n-2} \cdots c_t} = \frac{x_2^*(\sigma)}{\mu_t[1, \bar{\alpha}]}.$$

Therefore for any interval $\mathcal{I} \subset [1, \bar{\alpha}]$,

$$\mu_{t+n}(\mathcal{I}) = \frac{\sigma^n}{c_{t+n-1} c_{t+n-2} \cdots c_t} \mu_t(\mathcal{I})$$

also converges as $n \to \infty$.

Now consider any $a < 1$ and any interval $\mathcal{I} \subset [\underline{\alpha}, a] \subset [\underline{\alpha}, 1)$. Since $\alpha^*(p_t) \to 1$, then for sufficiently large t, $e_1^*(\alpha, p_t) = 1$ for all $\alpha \in \mathcal{I}$, implying that $f_t(\alpha) = p_t$ or

$$\begin{aligned} \mu_{t+n}(\mathcal{I}) &= \int_{\mathcal{I}} \frac{p_{t+n-1} p_{t+n-2} \cdots p_t}{c_{t+n-1} c_{t+n-2} \cdots c_t} d\mu_t(\alpha) \\ &= \frac{p_{t+n-1} p_{t+n-2} \cdots p_t}{c_{t+n-1} c_{t+n-2} \cdots c_t} \mu_t(\mathcal{I}). \end{aligned}$$

Now, consider the sequence:

$$\left\{ \frac{p_{t+n-1} p_{t+n-2} \cdots p_t}{c_{t+n-1} c_{t+n-2} \cdots c_t} \right\}_{n=0}^{\infty}.$$

This is a monotone, decreasing sequence since for every $t' \geq t$, $p_{t'}/c_{t'} < 1$. Furthermore, it is bounded below by zero and hence converges. Therefore for any $a < 1$ and any

$\mathcal{I} \subset [\underline{\alpha}, a]$, $\mu_{t+n}(\mathcal{I})$ converges as $n \to \infty$.

Next, let $A \in \mathcal{F}$. Define a sequence of real numbers $\{z_m\}$ such that $z_1 = \underline{\alpha}$, $z_{m+1} > z_m$ and $z_m \to 1$. It clearly follows that the intervals $Z_1 = [z_1, z_2]$ and $Z_m = (z_m, z_{m+1}]$, $m \geq 2$, are disjoint and $\bigcup_{m=1}^{\infty} Z_m = [\underline{\alpha}, 1)$. For each m, $Z_m \subset [\underline{\alpha}, a]$ for some $a < 1$ and hence $\mu_t(Z_m)$ converges. By the reasoning in the previous paragraph, $\mu_t(A \cap Z_m)$ also converges. Similarly, for $\mathcal{I} \in [1, \bar{\alpha}]$, $\mu_t(A \cap \mathcal{I})$ converges. Therefore,

$$\begin{aligned}
\mu_t(A) &= \mu_t\left(\left(\bigcup_{m=1}^{\infty} Z_m \cup [1, \bar{\alpha}]\right) \cap A\right) \\
&= \sum_{m=1}^{\infty} \mu_t(Z_m \cap A) + \mu_t([1, \bar{\alpha}] \cap A)
\end{aligned}$$

converges. For all $A \in \mathcal{F}$ define $\mu(A)$ to be this limit. It is easy to see that μ so defined is a probability measure on $(\bar{A}, \mathcal{F})$. That is, $\mu(A_k) = \lim_{t\to\infty} \mu_t(A_k) \in [0, 1]$, $\mu(\emptyset) = \lim_{t\to\infty} \mu_t(\emptyset) = 0$, $\mu(\bar{A}) = \lim_{t\to\infty} \mu_t(\bar{A}) = 1$ and $\mu(\bigcup_{k=1}^{\infty} A_k) = \lim_{t\to\infty} \mu_t(\bigcup_{k=1}^{\infty} A_k) = \sum_{k=1}^{\infty} \lim_{t\to\infty} \mu_t(A_k) = \sum_{k=1}^{\infty} \mu(A_k)$ where the A_k are disjoint and $A_k \in \mathcal{F}$. Hence $\mu_t \xrightarrow{v} \mu$. Since every μ_t is atomless, vague convergence implies that μ is atomless (Chung (1974)).

Probability measure μ has support $\bar{A}$ if and only if for any $\alpha \in \bar{A}$ and any $\epsilon > 0$, $\mu(N(\alpha, \epsilon)) > 0$. First, consider $\alpha \geq 1$. It is clear from above that $\mu_t(N(\alpha, \epsilon) \cap [1, \bar{\alpha}]) > 0$ is an increasing sequence so $\mu(N(\alpha, \epsilon)) > 0$. Second, for $\alpha < 1$, the decreasing sequence

$$\left\{\frac{p_{t+n-1} p_{t+n-2} \cdots p_t}{c_{t+n-1} c_{t+n-2} \cdots c_t}\right\}_{n=0}^{\infty}$$

is part of

$$\begin{aligned}
\mu_{t+n}(N(\alpha, \epsilon) \cap [\underline{\alpha}, a]) &= \int_{N(\alpha,\epsilon)\cap[\underline{\alpha},1)} \frac{p_{t+n-1} p_{t+n-2} \cdots p_t}{c_{t+n-1} c_{t+n-2} \cdots c_t} d\mu_t(\alpha) \\
&= \frac{p_{t+n-1} p_{t+n-2} \cdots p_t}{c_{t+n-1} c_{t+n-2} \cdots c_t} \mu_t(N(\alpha, \epsilon) \cap [\underline{\alpha}, a])
\end{aligned}$$

for some $\alpha < a < 1$ and sufficiently large t. Hence if $\mu(N(\alpha, \epsilon) \cap [\underline{\alpha}, 1)) = 0$ for any $\alpha < 1$ it must be true for every $\alpha < 1$. (I.e., the convergence of any sequence is driven by its tail and the tail of every sequence here is identical.) Since μ is atomless, this would contradict

$\mu[\underline{\alpha}, 1) = 1 - n^{RL} > 0$. Therefore μ has support $\bar{A}$.

Since $\mu[1, \bar{\alpha}] = \lim_{t \to \infty} \mu_t[1, \bar{\alpha}] = x_2^*(\sigma)$, $\mu \in \mathcal{M}$. ■

Theorem 2 demonstrates that all stationary distributions have the property that the masses of the risk-averse types and the risk-loving types depend only on the fitness function and the probability of a successful investment and where the risk-averse do not invest and the risk-loving invest. Furthermore, it immediately follows that for any stationary distribution, $\mu \in \mathcal{M}$, there exists an initial distribution, μ_1, such that $\mu_t \xrightarrow{v} \mu$. More importantly, these stationary distributions are stable in the sense that if a mutation occurs, the population distribution will converge to another stationary distribution.[6] These stationary distributions have further implications regarding Knight's theory that aggregate profits from entrepreneurial activities are negative since in the long run, the expected return from investing is identical to the sure or 'normal' return from not investing—hence in the long run, expected economic profits are zero. Evolutionary selection over risk preferences in a Knightian investment framework, implies then that agents that are the least risk-averse become entrepreneurs, with the population of risk-lovers adjusting until only and all risk-lovers invest and the expected return from investing is identical to the sure return from not investing.

Although risk-neutral agents maximize expected fitness, they do not, as one might expect, eventually take over and dominate the population. The reasoning behind this is as follows. Given some initial, non-stationary distribution, we know from the proof of the

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[6] Mutations are not formally modeled, however, I conjecture that for vanishing mutation rates, the convergence result is identical except for the additional result that the set of stationary distributions is a singleton. The without discussing the convergence properties of such a model, it can be seen that the shape of a stationary distribution will determined by how mutations are distributed. For example, suppose that after being born, each agent has an ϵ probability of independently and uniformly mutating into another type. Assuming and invoking the law of large numbers, a proportion ϵ of each type mutates. This implies that for a given ϵ, an ϵ-stationary distribution, μ_ϵ, must be such that there exists a p where μ_ϵ is uniform on $[\underline{\alpha}, \alpha^*(p))$ and on $(\alpha^*(p), \bar{\alpha}]$ with

$$\mu_\epsilon[\alpha^*(p), \bar{\alpha}] = \frac{\sigma}{p\mu_\epsilon[\underline{\alpha}, \alpha^*(p)] + \sigma\mu_\epsilon[\alpha^*(p), \bar{\alpha}]}\mu_\epsilon[\alpha^*(p), \bar{\alpha}](1-\epsilon) + \frac{\bar{\alpha} - \alpha^*(p)}{\bar{\alpha} - \underline{\alpha}}\epsilon.$$

That is, in a stationary population, the net losses (gains) from mutation must be just offset by the gains (losses) awarded by the replicator dynamic. For sufficiently small ϵ, such p exists and as $\epsilon \longrightarrow 0$, $\mu_\epsilon[1, \bar{\alpha}] \to x_2^*(\sigma)$.

Theorem that either $p_t < \sigma$ for all t or else $p_t > \sigma$ for all t. In the case of the former, the risk-neutral individuals will always invest everything, however, so will the risk-loving individuals. This implies that the risk-lovers always do as well as the risk-neutrals and therefore the risk-neutrals never take over. Similarly if $p_t > \sigma$.

A standard result with finite type replicator dynamics is that stable points constitute a Nash equilibrium of the game (see van Damme (1991)). Here also, with the continuum replicator dynamic, a stable and stationary distribution constitutes a Nash equilibrium of a game in which agents choose their actions to maximize their expected utility but in an earlier stage, choose their attitude towards risk to maximize expected fitness. Hence the model can also be interpreted as a game in which agents have the ability to first choose their preferences prior to actually playing the game. Examples of such games include Brander and Spencer (1985), and Fudenberg and Tirole (1983, 1984) where governments or firms have the ability to modify the profit structure using taxes or investments as a commitment device. One possible interpretation here might be that each agent represents the manager of a firm. Each owner of a firm then selects a manager with the appropriate attitude towards risk or else each owner chooses an incentive scheme such that the manager's behavior reflects the 'correct' risk attitude.

It is interesting to note that the aggregate investment at these stationary points is identical to the aggregate equilibrium investment which would occur in a population consisting entirely of risk-neutral agents. What happens then if this population is invaded by mutants? The aggregate investment under the new population distribution will still be the same as that under stationary population distributions.

Corollary 1 *In population consisting entirely of risk-neutral agents, aggregate equilibrium investment is identical to that under any stationary population, $\mu \in \mathcal{M}$. In addition, if this population of risk-neutrals is invaded by a sufficiently small contingent of non-risk-neutrals, aggregate investment under the new population distribution remains the same.*

Proof: In a population of risk-neutrals, equilibrium is such that mass $1 - x_2^*(\sigma)$ agents do not invest and $x_2^*(\sigma)$ agents invest with market clearing implying that $p = \sigma$. Any other distribution of investments does not constitute an equilibrium. For example, if less than $x_2^*(\sigma)$ agents invest, there is an abundance of good 1 so market clearing implies that $p < \sigma$. But in this case, risk-neutral individuals would strictly prefer to invest and hence all agents would invest—a contradiction. Since $p = \sigma$, this population distribution is stationary.

Now, suppose that this population is perturbed by the entry of ϵ^{RA} risk-averse agents and ϵ^{RL} risk-lovers so that μ is such that $\mu[\underline{\alpha}, 1) = \epsilon^{RA}$, $\mu(1, \bar{\alpha}] = \epsilon^{RL}$, and $\mu\{1\} = 1 - \epsilon^{RA} - \epsilon^{RL}$. If these invading populations are small in the sense that $\epsilon^{RA} < 1 - x_2^*(\sigma)$ and $\epsilon^{RL} < x_2^*(\sigma)$ then μ is a stationary population distribution—$p = \sigma$ with all risk-lovers investing, $x_2^*(\sigma) - \epsilon^{RL}$ risk-neutrals investing and the remainder of the population not investing. ■

It is also useful to consider population distributions with support over some subset of $\bar{A}$. In particular, consider subsets of the following type, $A_1 \subset [\underline{\alpha}, 1)$ and $A_2 \subset (1, \bar{\alpha}]$. The following result follows immediately from Theorem 2 and demonstrates that with the possibility of mutation, no distribution with support A_1 or A_2 can be a stable distribution.

Corollary 2 *Let* μ^* *be an atomless distribution with support* $A_1 \subset [\underline{\alpha}, 1)$ *or support* $A_2 \subset (1, \bar{\alpha}]$*. Let* μ_1 *be an atomless perturbation of* μ^* *with support* $\bar{A}$ *and where* $\mu_1(A_i) = \mu^*(A_i) - \epsilon$ *and* $\mu_1(\bar{A} - A_i) = \epsilon$*. If* μ_1 *follows (9) then* $\mu_t \rightarrow \mu \in \mathcal{M}$*.*

The conclusion to be drawn from Theorem 2 and Corollaries 1 and 2 is that the evolutionary mechanism, in conjunction with the exchange market, protects preference types that might be considered inefficient or 'irrational' and encourages diversity by allowing mutant preference types to successfully invade the population.

The current setting corresponds to Robson's (1994a) scenario of 'idiosyncratic risk.' He refers to idiosyncratic risk as a setting in which gambles are the same in every period but independent between individuals. Of particular interest is the fact that, with the model of the current paper, more than one type is represented in steady state distributions. In a general equilibrium framework the evolutionary mechanism does not select out agents who do not maximize fitness (i.e., non-risk-neutrals). This is in contrast to Robson's result where individuals that maximize expected fitness are selected for and hence their attitudes towards risk are embodied in the fitness function. The critical feature leading to these contrasting results is that Robson assumes an autarky environment in which the fitness payoff each type receives is independent of the relative representation of the other types. In a market setting, if there is a relatively large number of risk-averse types then the market fitness payoff to the risk-loving types will be relatively high and their representation in the population will increase. In the next generation, there will be fewer risk-averse types with the result that their fitness payoff increases. As the population distribution converges to a steady state, the expected fitness payoffs for all types equalize and although almost all agents do not maximize expected fitness, neither are they selected out.

Finally, given the model of investment and exchange, the distribution over preferences can be examined for those which are optimal among all feasible distributions. As I now demonstrate, all stationary, atomless distributions are, in some sense, 'optimal.'

Theorem 3 *An atomless population distribution, μ, maximizes average population fitness if and only if $\mu \in \mathcal{M}$.*

Proof: Let $\mu \in \mathcal{M}$. In a market economy, with identical, homogeneous fitness functions, all agents consume goods 1 and 2 in the same proportion. Hence if aggregate consumption maximizes aggregate fitness then average fitness is also maximized. With the law of large numbers, the expected production of good 2 is equal to realized production so let σE_2 be the production of good 2 when E_2 is aggregate investment. Optimal E_2 satisfies, $\sigma = f_1/f_2$. This holds at any stationary point since $p = f_1/f_2$ when consumers maximize utility and $p = \sigma$ at all stationary points. Hence aggregate consumption at stationary distributions maximizes aggregate fitness and therefore average fitness.

Suppose μ is atomless and that average fitness is maximized with population distribution μ. Since μ maximizes average population fitness, $f_1/f_2 = \sigma$. Consumer utility maximization implies that $p = f_1/f_2$. Therefore $p = \sigma$ and $\mu \in \mathcal{M}$. ∎

Hence, not only does this evolutionary system converge to a stationary distribution but these stationary population distributions are optimal in the sense that average population fitness is maximized. In Kihlstrom and Laffont (1979) they showed, that for a given distribution of preferences, the equilibrium is efficient only if all entrepreneurs are risk-neutral and hence, for most distributions, their equilibrium is inefficient. In general, for an arbitrary distribution of preferences, the static equilibrium of the current paper is also inefficient. However, the introduction of evolutionary dynamics to the general equilibrium model allows risk preferences to adjust over time, towards an efficient distribution.

6 An Example with Two Types

I now consider an example with a finite type space (one type of risk-averse agent and one type of risk-loving agent). With two types, comparative statics results are possible and some additional welfare results can be easily attained. It is also useful for constructing a more precise illustration of the behavior of the evolutionary dynamic.

Assume that there is a single risk-averse type with $\alpha^{RA} < 1$ and a single risk-loving type with $\alpha^{RL} > 1$. Analogous to the definitions in Section 4, let n^{RA} be the proportion of risk-averse types and let n^{RL} be the proportion of risk-loving types. Later, once I consider evolutionary dynamics, I add a t subscript. With a distribution consisting of two atoms, it is no longer true that each type chooses the same action. Let $\tilde{n}^{RA}$ and $\tilde{n}^{RL}$ be the equilibrium masses of risk-averse types and risk-loving types that invest. All other assumptions remain the same.

With two types, aggregate wealth is equal to the sum of the wealth of the risk-averse agents who do not invest $(n^{RA} - \tilde{n}^{RA})p^e$, the wealth of the risk-averse who invest, $\tilde{n}^{RA}\sigma$, the wealth of the risk-loving who do not invest, $(n^{RL} - \tilde{n}^{RL})p^e$, and the wealth of the risk-loving who invest, $\tilde{n}^{RL}\sigma$. Similarly, aggregate supply of goods 1 and 2 are $(n^{RA} - \tilde{n}^{RA}) + (n^{RL} - \tilde{n}^{RL})$ and $\sigma(\tilde{n}^{RA} + \tilde{n}^{RL})$. Hence the market clearing conditions for goods 1 and 2 are:

$$x_1^*(p^e)[(n^{RA} - \tilde{n}^{RA})p^e + \tilde{n}^{RA}\sigma + (n^{RL} - \tilde{n}^{RL})p^e + \tilde{n}^{RL}\sigma] = (n^{RA} - \tilde{n}^{RA}) + (n^{RL} - \tilde{n}^{RL}) \quad (18)$$

$$x_2^*(p^e)[(n^{RA}-\tilde{n}^{RA})p^e+\tilde{n}^{RA}\sigma+(n^{RL}-\tilde{n}^{RL})p^e+\tilde{n}^{RL}\sigma]=\sigma(\tilde{n}^{RA}+\tilde{n}^{RL}). \tag{19}$$

Just as when there are a continuum of types, gross substitutability implies existence and uniqueness of the competitive exchange equilibrium.

A Nash equilibrium in the investment stage is a triple $(\tilde{n}^{RA},\tilde{n}^{RL},p)$ such that i) $(\tilde{n}^{RA},\tilde{n}^{RL})$ are such that all agents are maximizing (4) subject to (5) and (6), given p, ii) $p=p^e$ where p^e satisfies (18) and (19), given $(\tilde{n}^{RA},\tilde{n}^{RL})$.

The two-type analog to Theorem 1 is:

Proposition 1 *There exists a unique equilibrium, $(\tilde{n}^{RA},\tilde{n}^{RL},p)$, of the investment stage where:*

i) $p\in[\underline{p},\bar{p}]$ *where* $\underline{p}=\sigma^{1/\alpha^{RA}}$ *and* $\bar{p}=\sigma^{1/\alpha^{RL}}$,

ii) $$p\left\{\begin{matrix} < \\ = \\ > \end{matrix}\right\}\sigma \text{ as } n^{RL}\left\{\begin{matrix} < \\ = \\ > \end{matrix}\right\}x_2^*(\sigma),$$

iii) $$\text{if}\left\{\begin{matrix} p=\underline{p} \\ p\in(\underline{p},\bar{p}) \\ p=\bar{p} \end{matrix}\right\}\text{then}\left\{\begin{array}{l} \tilde{n}^{RL}=n^{RL} \text{ and } \tilde{n}^{RA} \text{ is such that } \underline{p} \text{ clears the market.} \\ \tilde{n}^{RA}=0 \text{ and } \tilde{n}^{RL}=n^{RL}. \\ \tilde{n}^{RA}=0 \text{ and } \tilde{n}^{RL} \text{ is such that } \bar{p} \text{ clears the market.} \end{array}\right.$$

The proof is similar to the continuum version and the main difference is in part iii) where, not all agents of the same type choose the same action. Note that when $p=\underline{p}$ or $p=\bar{p}$, investment is a constant and the respective investments of the risk-averse and risk-loving are just enough to ensure that those investments hold.

Using the properties of the investment and exchange equilibrium, the two types version of the population dynamics equation is:

$$n_{t+1}^{RL}=\frac{(n_t^{RL}-\tilde{n}_t^{RL})p_t+\tilde{n}_t^{RL}\sigma}{(n_t^{RA}-\tilde{n}_t^{RA})p_t+\tilde{n}_t^{RA}\sigma+(n_t^{RL}-\tilde{n}_t^{RL})p_t+\tilde{n}_t^{RL}\sigma}. \tag{20}$$

Correspondingly, I have the following stability result.

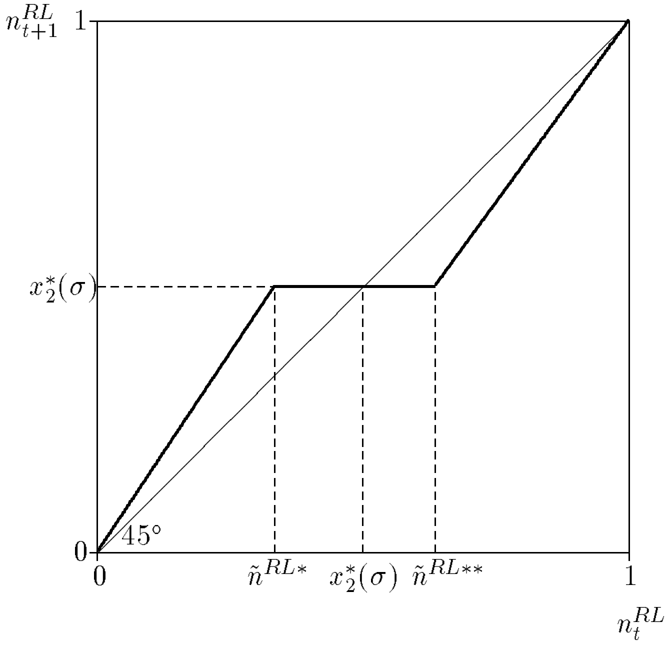

Figure 3: Population Dynamics

Proposition 2 *The population dynamics equation (20) has three fixed points: 0, 1 and* $x_2^*(\sigma)$*. The interior fixed point* $x_2^*(\sigma)$ *occurs when* $p = \sigma$ *and is the unique globally stable fixed point.*

The proof is similar to the proof of Theorem 2 and that the stable distribution is essentially the same as the stable distributions from Theorem 2 is unsurprising. Although with only two types there are two additional fixed points, neither is stable and thus they are susceptible to invasion by mutants.

Using a Cobb-Douglas fitness function (again $f(x_1, x_2) = (x_1 x_2)^{1/2}$), a graphic example of the population dynamic is given in Figure 3. Notice that the dynamic has two kinks occurring at the critical points where $n_t^{RL} = \tilde{n}^{RL*}$ and $n_t^{RL} = \tilde{n}^{RL**}$.[7] Between these critical points, with Cobb-Douglas fitness, the population moves immediately to the

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[7] For distributions where n_t^{RL} is between the critical points, $\underline{p} < p_t < \bar{p}$ so all risk-averse individuals do not invest and all risk-lovers invest. Outside the critical points, either risk-averse agents or risk-lovers, respectively, are indifferent between investing and not investing.

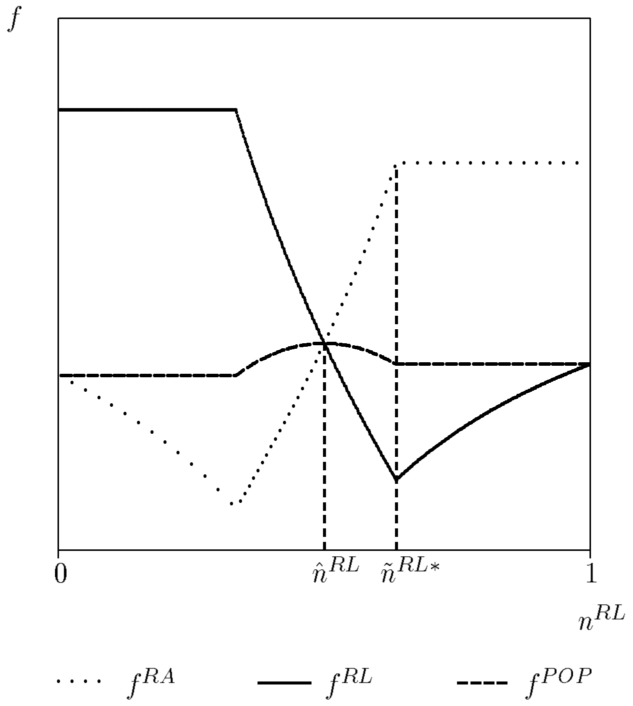

Figure 4: Average Fitness vs. Population Proportion

stationary distribution and hence convergence is attained in finite time.

Like the case when there is a continuum of types, the same proof demonstrates that the stable stationary point maximizes average fitness.

Proposition 3 *Average population fitness is maximized if and only if* $n^{RL} = x_2^*(\sigma)$.

Again using Cobb-Douglas fitness, Figure 4 provides an illustration of this—the average risk-averse, average risk-loving, and average population fitnesses are plotted against the population proportion of the risk-loving agents, n^{RL}.

Proposition 4 *When* $n^{RL} = 0(1)$, *the average fitness of risk-averse (risk-loving) agents decreases if a sufficiently small contingent of risk-loving (risk-averse) individuals enters the population.*

Proof: When $n^{RL} = 0$ we know that $p < \sigma$. The average fitness of risk-averse individuals when $p < \sigma$ is $g(p)((n^{RA} - \tilde{n}^{RA})p + \sigma\tilde{n}^{RA})$. When $n^{RL} < \tilde{n}^{RL*}$, $\tilde{n}^{RA} = \tilde{n}^{RL*} - n^{RL}$ and expected fitness becomes $g(p)((1-\tilde{n}^{RL*})p+(\tilde{n}^{RL*}-n^{RL})$. Differentiating this with respect to n^{RL} yields:

$$[g'(p)((1-\tilde{n}^{RL*})p+(\tilde{n}^{RL*}-n^{RL})\sigma+g(p)(1-\tilde{n}^{RL*}))]\frac{dp}{dn^{RL}}-g(p)\sigma.$$

The first term is zero since when $n^{RL} < x_2^*(\sigma)$, price is constant at $\underline{p}$. The second term is always negative and hence for small n^{RL}, the risk-averse types are made worse off by the entry of a sufficiently small contingent of risk-loving types. The proof for $n^{RL} = 1$ is identical. ■

Hence in an economy with only risk-averse (risk-loving) agents, the introduction of a small contingent of risk-loving (risk-averse) mutants makes the risk-averse agents worse off in terms of expected fitness. On the other hand, since price is constant at $\underline{p}$, risk-averse (risk-loving) agents are indifferent between investing and not investing and hence they are no worse off, subjectively, by the entry of small contingents of risk-loving (risk-averse) agents. By investing, risk-loving entrants provide insurance to the risk-averse agents, resulting in no reduction of subjective utility, however, risk-averse agents are worse off in terms of objective fitness. When $n^{RL} > x_2^*(\sigma)$ there is an under-provision of insurance, when $n^{RL} < x_2^*(\sigma)$ there is an over-provision of insurance and only when $n^{RL} = x_2^*(\sigma)$ is this insurance perfect.

Finally, a population of risk-averse (risk-loving) types, invaded by risk-lovers (risk-averse), although initially they are made worse off, over time, the population adjusts and:

Proposition 5 *At the stable population distribution, the risk-averse (risk-loving) agents are better off (in both an expected utility and an expected fitness sense) than if there were no risk-loving (risk-averse) individuals.*

Proof: When $n^{RL} < x_2^*(\sigma)$, $p < \sigma$. It is easy to see that $g(p)$ must be strictly decreasing in p. This implies that $g(p)\sigma > g(\sigma)\sigma$ or a risk-lover's fitness is greater than her steady state fitness. Proposition 3 implies that average population fitness is maximized when $p = \sigma$. Since at $p < \sigma$, risk-lovers earn fitness greater than their steady state fitness, the risk-averse must be earning below population average fitness which is less than the steady state average fitness. Furthermore, since at the steady state, a risk-averse agent achieves, with certainty, a fitness greater than her expected fitness at $n^{RL} = 0$, her expected utility at the steady state must be greater as well. ■

Thus although risk-averse agents are initially made objectively worse off by the entry of risk-loving mutants, they are strictly better off as the population converges to the steady state (see Figure 4 for an illustration).

7 Conclusion

A Knightian model of investment and trade was developed to investigate entrepreneurial self-selection in conjunction with the evolutionary selection of risk attitudes. Using this model, I characterized the equilibrium in the exchange economy, solved for the optimal investment strategies and then examined the properties of an evolutionary system in which preferences change over time. The main result is that for a given initial distribution of preference types, the evolutionary system converges to a stationary point at which both risk-averse and risk-loving types are represented and where risk-lovers invest and the risk-averse don't.

The evolutionary and market mechanisms in this model are efficient in the sense that average population fitness is maximized at any stable and stationary point. As seen in Kihlstrom and Laffont (1979), the market mechanism by itself does not achieve this. In a particular example with two types, if the population initially consists entirely of risk-averse agents, the risk-averse agents are made objectively worse off with the introduction of risk-loving mutants. However, as the system converges to the steady state, they are eventually made better off both objectively and subjectively.

In summary, although non-risk-neutral preferences are 'irrational' in the sense that agents without risk-neutral preferences do not maximize expected fitness, the evolutionary and market mechanisms allow and encourage the existence of a diverse population where different types specialize in different activities and population wealth is maximized. Risk-loving individuals provide insurance to the risk-averse individuals and at the steady state, this insurance is perfect. Hence although agents choose their investments to maximize expected utility, not expected fitness, in the steady state these choices do in fact maximize expected fitness.

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