PRODUCTIVITY AND UNDESIRABLE OUTPUTS: A DIRECTIONAL DISTANCE FUNCTION APPROACH

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Y. Chung
and
R. Färe

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Mailcode 4515
Department of Economics
Southern Illinois University
Carbondale, IL 62901

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Abstract

Undesirable outputs are often produced together with desirable. This joint production of good and bad outputs bring about a difficulty for productivity measurement. Here we introduce a directional distance function and use it as a component in a new productivity index. This index, as an empirical example shows, seems to solve the problem caused by the joint production of good and bad outputs.

PRODUCTIVITY AND UNDESIRABLE OUTPUTS:

A DIRECTIONAL DISTANCE FUNCTION APPROACH

by

Yangho Chung and Rolf Färe*

Undesirable outputs are often produced jointly with desirable outputs, i.e., good output cannot be produced without producing some bad output. This joint production of good and bad outputs has gained attention in the literature on efficiency and productivity.[1] One can distinguish between two lines of inquiry: (i) how to model joint production of good and bad outputs and (ii) how to account for reductions of bad outputs. The first issue is fairly well understood, but the second has not yet been resolved satisfactorily.

To model joint production it is usually assumed that outputs are weakly disposable as defined by Shephard (1970). This means that if a given output vector is feasible, i.e., it can be produced using some input vector, then any proportional reduction of that output vector is also feasible. If in addition to weak disposability one assumes that the good outputs are nulljoint with the bad,[2] i.e., zero production of bad outputs is feasible only if good outputs are zero, then the joint production of good and bad outputs are quite well modelled.

A short list of papers includes: Pittman (1983), Färe, Grosskopf and Pasurka (1986, 1989), Färe and Grosskopf (forthcoming), Ball, ell, Nehring and Somwaru (1994), Haynes, Patick, Bowen and Cummings-Saxton (1993), Tyteca (1995).

See Shephard and Färe (1974).

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In general there is no market prices on undesirable outputs which make the Törnqvist and Fisher productivity indexes difficult to apply when such outputs are present. Pittman (1983) showed by using constructing shadow prices how this shortcoming could be overcome.[3] Färe, Grosskopf, Lovell and Pasurka (1989) took another approach and introduced a hyperbolic Farrell-type efficiency measure. Their measure expands good outputs and contracts bad outputs using a single scalar. This makes their measure nonlinear, a property which is undesirable for an efficiency or a productivity measure.[4]

In this paper we introduce a directional output distance function that has the properties of being radial and accounts for reduction in undesirable outputs, thus it has the two desirable properties. We show how the directional output distance function is related to Shephard's output distance function and we apply it as a component in a Malmquist type productivity measure. Data from the Swedish paper and pulp industry are used in computing this new and the usual Malmquist productivity indexes.

The directional output distance function is the output-oriented version of the benefit function introduced by Luenberger (1992a, b, 1994a, b, 1995a, b). Luenberger has

See Färe, Grosskopf, Lovell and Yaisawarng (1993) for how the output distance function can be applied in deriving shadow prices for esirable outputs.

For other measures see Färe, Grosskopf and Pasurka (1986, 1989), Färe and Grosskopf (forthcoming) and Haynes et. al. (1993).

applied his benefit function to various areas in economics, including general equilibrium models and externalities.[5]

1. The Directional Output Distance Function

The directional output distance function, as any distance function, is a functional representation of the technology. To show this, we model the technology in terms of the output sets P(x), where $x \in \mathbb{R}_+^N$ denotes inputs and

(1.1) $P(x) = \{(y,b) : x \text{ can produce } (y,b)\}$.

We denote good outputs by $y \in \mathbb{R}_+^M$ and bad by $b \in \mathbb{R}_+^I$, and we assume that outputs are weakly disposable, i.e.,

(1.2) $(y,b) \in P(x)$ and $0 \le \theta \le 1$ imply $(\theta y, \theta b) \in P(x)$.

In addition, we assume that the good or desirable outputs are freely disposable, i.e.,

(1.3) $(y,b) \in P(x)$ and $y' \le y$ imply $(y',b) \in P(x)$.

Our final condition on outputs is that the good outputs are nulljoint with the bads. This condition is modelled by

(1.4) if $(y,b) \in P(x)$ and $b = 0$ then $y = 0$.

In words, (1.4) says that the good outputs are nulljoint with the bad outputs whenever zero production of the latter occurs only when the former is also zero. Alternatively, this means that if a good output is produced in a positive amount some bad output must also be produced. Conditions (1.2) - (1.4) will be incorporated into our

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See Chambers, Chung and Färe (1995) regarding the relation between the input distance function and the benefit function.

computational model discussed in Section 2. To continue, Shephard's (1970) output distance function is defined on the output set P(x) as

(1.5) $D_o(x,y,b) = \inf\{\theta: ((y,b)/\theta) \in P(x)\}.$

This function expands the good and bad outputs (y,b) proportionally as much as is feasible. It does not credit reduction of bads, since both outputs are expanded. It is of course a complete characterization of the technology, and it was shown by Färe and Primont (1995) that under weak disposability of outputs,

(1.6) $(y,b) \in P(x) \Leftrightarrow D_o(x,y,b) \leq 1.$

Besides being a complete characterization of the technology, the distance function is homogeneous of degree +1 and concave in outputs, see Färe and Primont. Its reciprocal is known as the Farrell (1957) output measure of technical efficiency. This measure has been thoroughly investigated, see Färe, Grosskopf and Lovell (1994) for a summary.

To illustrate the directional output distance function and to compare it to Shephard's output distance function, let us again represent the technology by an output set. If we impose conditions (1.2) - (1.4) on this set, it may take the form in Figure 1.

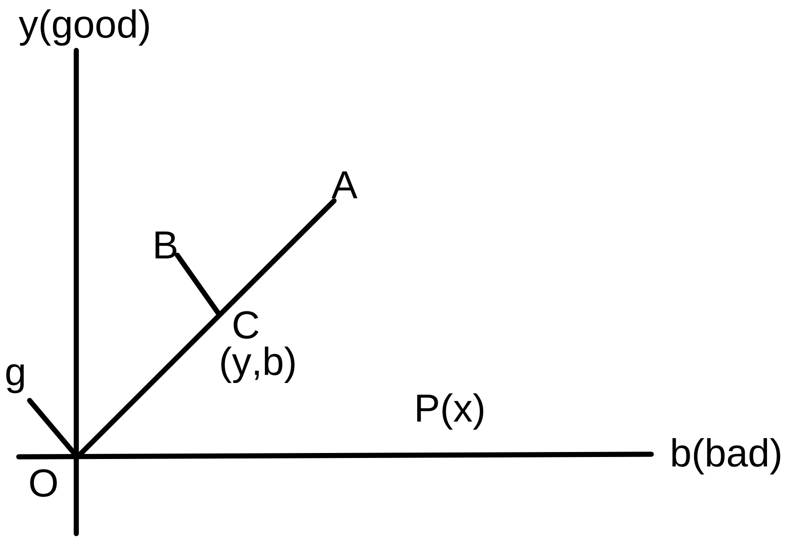

Figure 1 Distance functions

The output set is denoted by P(x), good output by y and the bad by b. The outputs (y,b) are weakly disposable and y by itself is strongly disposable. Moreover, the good output y is nulljoint with b, since if b = 0, then the only y with (y,b) ∈ P(x) is y = 0.

Shephard's distance function applied to the output vector (y,b) places it on the boundary of P(x) at A. The directional output distance function on the other hand takes (y,b) in the "g" direction and places it on the boundary at B. Here the directional distance function increases the good output and decreases the bad. Formally, it is defined as,

(1.7) $\vec{D}_o(x,y,b;g) = \{\beta: (y,b) + \beta g \in P(x)\},$

In Figure 1 this amounts to the ratio of the distances (BC/0g).

One can prove[6] that under g-disposability,

See Chung (forthcoming).

(1.8) $\vec{D}_o(x,y,b;g) \geq 0$ _ $(y,b) \in P(x)$,

where by g- disposability we mean that

(1.9) $(y,b) \in P(x) \Rightarrow (y,b) - \alpha g \in P(x), \alpha \geq 0,$

and $((y,b) - \alpha g) \in$ _$_+^{M+I}$. ***Expression (1.8) gives our functional representation of the technology by*** D_o ***in the same fashion as (1.6) does for*** D_o. ***Thus both the directional and Shephard's output distance functions can be employed as models of technology. In the Shephard case the "cost" is weak disposability whereas g-disposability suffices for the directional distance function.***

In order to relate the two distance functions to each other, let g = (y,b), then through (1.6), we get

$$(1.10) \quad \vec{D}_o(x,y,b;y,b) = \{\beta: D_o(x, (y,b) + \beta(y,b)) \leq 1\}$$

$$= \{\beta : (1+\beta) D_o(x,y,b) \leq 1\}$$

$$= \{\beta : \beta \leq (1 / D_o(x,y,b)) - 1\}$$

$$= (1 / D_o(x,y,b)) - 1$$

This expression shows that Shephard's output distance function is a special case of the directional distance function. The relation between the two can be written as

$$(1.11) \quad \vec{D}_o(x,y,b;y,b) = (1 / D_o(x,y,b)) - 1$$

or equivalently

$$(1.12) \quad D_o(x,y,b) = 1/(1+ \vec{D}_o(x,y,b;y,b)).$$

2. Productivity Measurement

Färe, Grosskopf, Lindgren and Roos (1989) defined a productivity index based on Shephard's output distance function. Their index is the geometric mean of two Malmquist productivity indexes, which were introduced by Caves, Christensen and Diewert (1982). Caves, Christensen and Diewert named their index after the Swedish statistician Sten Malmquist, who used the distance function in 1953 in defining input quantity indexes.

In this section we give a short presentation of the output-oriented Malmquist productivity index and we introduce our new productivity index based on the directional distance function. We call this the Malmquist-Luenberger productivity index.

Suppose there are t = 1,...,T time periods, then the Färe, Grosskopf, Lindgren and Roos (FGLR) output-oriented Malmquist productivity index is defined by

$$(2.1) \qquad M_t^{t+1} = \left[\frac{D_o^t(x^{t+1}, y^{t+1}, b^{t+1})}{D_o^t(x^t, y^t, b^t)} \; \frac{D_o^{t+1}(x^{t+1}, y^{t+1}, b^{t+1})}{D_o^{t+1}(x^t, y^t, b^t)} \right]^{1/2},$$

and it is the geometric mean of two indexes as originally defined by Caves, Christensen and Diewert (1982). The Malmquist index (2.1) can be decomposed into two component measures, one accounting for efficiency change (MEFFCH), and one measuring technical change (MTECH). These are

$$(2.2) \qquad MEFFCH_t^{t+1} = \frac{D_o^{t+1}(x^{t+1}, y^{t+1}, b^{t+1})}{D_o^t(x^t, y^t, b^t)}$$

and

$$(2.3) \qquad MTECH_t^{t+1} = \left[\frac{D_o^t(x^{t+1}, y^{t+1}, b^{t+1})}{D_o^{t+1}(x^{t+1}, y^{t+1}, b^{t+1})} \; \frac{D_o^t(x^t, y^t, b^t)}{D_o^{t+1}(x^t, y^t, b^t)} \right]^{1/2}.$$

The product of the component measures exhausts the productivity measure so that

(2.4) $$M_t^{t+1} = MEFFCH_t^{t+1} \bullet MTECH_t^{t+1}.$$

In the empirical part of the paper we compute the Malmquist index as well as its component measures.

To define an output-oriented Malmquist-Luenberger (ML) productivity index we need to specify in what direction the index should be oriented. Clearly, we want to account for increases in good outputs and decreases in bads. A natural choice is to take g = (y,-b). We also want the Malmquist-Luenberger index to be comparable to the Malmquist index. These considerations lead us to following definition.

<u>Definition (2.5)</u>: The output-oriented Malmquist-Luenberger productivity index, with undesirable output is

$$ML_t^{t+1} = \left[\frac{(1+\vec{D}_o^t(x^t,y^t,b^t;y^t,-b^t))}{(1+\vec{D}_o^t(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1}))} \frac{(1+\vec{D}_o^{t+1}(x^t,y^t,b^t;y^t,-b^t))}{(1+\vec{D}_o^{t+1}(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1}))}\right]^{1/2}.$$

Our definition is such that when the direction g is (y,b) rather than (y,-b), the Malmquist-Luenberger index coincides with the Malmquist index. As in the case of the Malmquist index, the new index can also be decomposed into two components, namely

(2.6) $$MLEFFCH_t^{t+1} = \frac{1+\vec{D}_o^t(x^t,y^t,b^t;y^t,-b^t)}{1+\vec{D}_o^{t+1}(x^{t+1},y^{t+1},b^{t+1};y^{t+1},-b^{t+1})}$$

(2.7) $$MLTECH_t^{t+1} = \left[\frac{\{1+\vec{D}_o^{t+1}(x^t,y^t,b^t,y^t,-b^t)\}}{\{1+\vec{D}_o^{t+1}(x^t,y^t,b^t,-b^t)\}} \frac{\{1+\vec{D}_o^t(x^{t+1},y^{t+1},b^{t+1},y^{t+1},-b^{t+1})\}}{\{1+\vec{D}_o^{t+1}(x^{t+1},y^{t+1},b^{t+1},y^{t+1},-b^{t+1})\}}\right]^{1/2}$$

and their product equals ML^{t+1}_{t} The Malmquist-Luenberger measure and the Malmquist measure indicate productivity improvements if their values are greater than one and declines in productivity if the values are less than one.

Our next task is to develop a procedure for calculating the two indexes and their decompositions. This requires the computation of four distance functions for each index. We assume that at each time t = 1,...,T, there are k =1,...,K observations of inputs and outputs,

(2.8) $$(x^{t,k}, y^{t,k}, b^{t,k}),\ k = 1,\dots,K,\ t = 1, \dots,T.$$

In our example k is a paper and pulp mill.

Following Färe, Grosskopf and Lovell (1994) the output set that meets conditions (1.2) - (1.4) and is derived from the data (2.8) is

(2.9) $$P(x) = \{(y,b): \sum_{k=1}^{K} z_k y^t_{km} \geq y^t_m,\ m = 1,\dots,M,$$
$$\sum_{k=1}^{K} z_k b^t_{ki} = b^t_i,\ i = 1,\dots,I,$$
$$\sum_{k=1}^{K} z_k x^t_{kn} \leq x^t_n,\ n = 1,\dots,N,$$
$$z_k \geq 0,\ k = 1,\dots,K\}.$$

This activity analysis model (2.9) also satisfies constant returns to scale, i.e.,

(2.10) $$P(\lambda x) = \lambda P(x),\ \lambda > 0$$

and strong disposability of inputs,

(2.11) $x' \geq x \Rightarrow P(x') \supseteq P(x).$

The inequalities for inputs in (2.9) makes them freely disposable, and the same holds for the good outputs. The bad outputs are modelled with equalities; this makes them not freely disposable. Finally, the nonnegativity constraints on the intensity variables z_k allow the model to exhibit constant returns to scale.

For each observation the distance functions in the Malmquist index are computed as the solutions to a linear programming problem. For example, for k′,

(2.12)
$$\left(D_o^t(x^{t,k'}, y^{t,k'}, b^{t,k'})\right)^{-1} = \max \theta$$
$$\text{s.t.} \quad \sum_{k=1}^{K} z_k y_{km}^t \geq \theta \; y_{k'm}^t, \; m = 1,\dots,M,$$
$$\sum_{k=1}^{K} z_k b_{ki}^t = \theta \; b_{k'i}^t, \; i = 1,\dots,I,$$
$$\sum_{k=1}^{K} z_k x_{kn}^t \leq x_{k'n}^t, \; n = 1,\dots,N,$$
$$z_k \geq 0, \; k = 1,\dots,K.$$

To use the Malmquist productivity index when undesirable outputs are present may create a problem, which we illustrate in Figure 2.

The technology is based on t period data as indicated by P^t. The observation is from period t + 1, $(x^{t+1}, y^{t+1}, b^{t+1})$. If we try to compute the mixed period distance function

(2.13) $D_o^t(x^{t+1}, y^{t+1}, b^{t+1}),$

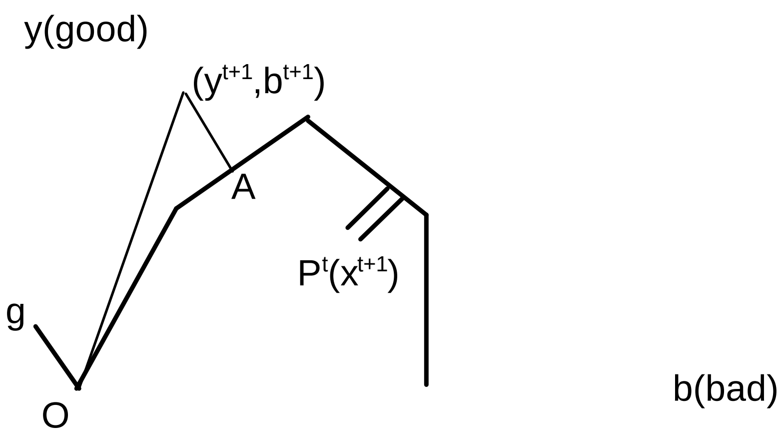

Figure 2 A mixed period distance function

its value will be +∞, in which case the Malmquist index does not yield a relevant indication. In our empirical section we encounter this problem for a third of our observations.

The directional distance functions may also be calculated as solutions to linear programming problems. Again as an example,

$$\textit{(2.14)}\qquad \vec{D}_o^t(x^{t,k'}, y^{t,k'}, b^{t,k'}; y^{t,k'}, -b^{t,k'}) = \max \beta$$

$$\begin{aligned}
\text{s.t.}\quad & \sum_{k=1}^{K} z_k y_{km}^t \geq (1+\beta)\, y_{k'm}^t,\ m = 1,\ldots,M,\\
& \sum_{k=1}^{K} z_k b_{ki}^t = (1-\beta)\, b_{k'i}^t,\ i = 1,\ldots,I,\\
& \sum_{k=1}^{K} z_k x_{kn}^t \leq x_{k'n}^t,\ n = 1,\ldots,N,\\
& z_k \geq 0,\ k = 1,\ldots,K.
\end{aligned}$$

This shows that the directional distance function may also be computed using linear programming. We note that in the mixed period problem illustrated in Figure 2, the directional distance function places (y^{t+1}, b^{t+1}) on the output set $P^t(x^{t+1})$ at A. Thus in this case we do not encounter any problems with the Malmquist-Luenberger index. The same is true in our empirical example.

3. Data and Results

We measured productivity changes in the Swedish pulp and paper industry. We use the same panel data as in Brännlund, Färe and Grosskopf (1995) and Brännlund, Chung, Färe and Grosskopf (1995). The data sources are primary data for the pulp and paper industry gathered by Statistics Sweden and the Swedish Environmental Protection Board. The part of the data used here is annual data on quantities of outputs and inputs from 39 paper and pulp mills for the period 1986-1990.[7] To produce desirable output pulp, y, we observe four inputs, namely: labor (x_1), wood fiber (x_2), energy (x_3) and capital (x_4). The desirable output y is jointly produced with some bads, which are Biological Oxygen Demand (BOD), Chemical Oxygen Demand (COD) and Suspended Solids (SS).

Descriptive statistics for the data are presented in Table 1. The data shows that the amount of inputs and outputs increased between 1986 and 1988 and decreased thereafter.

Only the data of the firms which operated during the whole five years are included here.

Table 1. Descriptive Statistics

T	Variable	Mean	SD	Min	Max
1986 (K=39)	y (tons)	233,851.0	174,073.6	2,718.0	824,250.0
	b_{BOD} (tons)	3,013.6	2,640.3	110.7	10,740.0
	b_{COD} (tons)	12,996.2	11,976.3	254.5	47,256.0
	b_{SS} (tons)	1,309.1	1,209.7	51.9	4,400.0
	x_1 (hrs)	828,897.4	129,386.8	107,000.0	2,383,000.0
	x_2 (m^3)	1.01E + 06	7.17E + 05	7,000.0	2.5E + 06
	x_3 (kwh)	3.54E + 08	3.19E + 08	1.61E + 07	1.26E + 09
	x_4 (mil)	454.7	330.2	6.5	1,585.9
1987 (K=39)	y	249,792.0	186,907.4	2,974.0	853,520.0
	b_{BOD}	3,098.8	2,631.8	121.8	11,993.0
	b_{COD}	13,019.8	11,826.1	316.7	47,256.0
	b_{SS}	1,346.4	1,481.8	69.6	7,318.5
	x_1	826,435.9	524,013.3	102,000.0	2,332,000.0
	x_2	1.05E + 06	7.25E + 05	8,000.0	2.52E + 06
	x_3	3.81E + 08	3.63E + 08	1.37E + 07	1.46E + 09
	x_4	435.0	315.4	6.4	1,515.1
1988 (K=39)	y	257,866.3	191,670.6	2,346.0	878,000.0
	b_{BOD}	3,197.5	3,167.3	115.8	14,040.0
	b_{COD}	12,703.3	11,664.8	247.1	47,060.0
	b_{SS}	1,756.1	2,704.0	49.4	14,480.0
	x_1	829,717.9	528,028.4	102,000.0	2,345,000.0
	x_2	1.27E + 06	1.75E + 06	7,000.0	1.11E + 07
	x_3	4.00E + 08	3.83E + 08	1.40E + 07	1.50E + 09
	x_4	419.4	303.9	6.6	1,434.9
1989 (K=39)	y	257,977.9	194,005.7	2,077.0	872,610.0
	b_{BOD}	2,825.8	2,714.6	120.0	11,067.0
	b_{COD}	11,788.2	11,773.4	257.7	49,980.0
	b_{SS}	1,775.4	2,735.7	42.4	14,994.0
	x_1	827,846.2	526,682.6	107,000.0	2,268,000.0
	x_2	1.06E + 06	7.31E + 05	6,000.0	2,543,000.0

	x_3	4.17E + 08	4.23E + 08	1.50E + 07	1.58E + 09
	x_4	408.6	299.7	6.3	1,368.5
1990 (K=39)	y	251,887.3	180,568.9	1,964.0	855,080.0
	b_{BOD}	2,383.8	2,196.4	110.7	9,632.0
	b_{COD}	10,028.6	9,606.3	252.6	38,880.0
	b_{SS}	1,657.9	2,538.1	4.9	12,384.0
	x_1	809,846.2	509,687.0	96,000.0	2,202.000.0
	x_2	1.01E + 06	6.94E + 05	6,000.0	2.25E + 06
	x_3	4.05E + 08	4.15E + 08	1.52E + 07	1.51E + 09
	x_4	403.1	292.3	6.25	1,325.6

The maximization problems in (2.11) and (2.13) are solved by using linear programming. The solutions for each observation are then used to obtain the Malmquist-Luenberger productivity indexes as well as their components. The solutions for the maximization problems for the mixed period distance functions yield the "optimal value" of $+\infty$ for almost one third of the observations. Thus, construction of the Malmquist productivity indexes in (2.1) and technical change (2.3) for those observations is meaningless. This is in contrast to the case of the directional distance functions; no observation reports an infinite "solution."

The average values for the Malmquist-Luenberger productivity indexes and its components are reported in Table 2.

Table 2. The Malmquist-Luenberger Productivity Indexes and Components*

	ML_t^{t+1}	$MLEFFCH_t^{t+1}$	$MLTECH_t^{t+1}$
86/87	1.02710	0.98436	1.04342
87/88	1.08679	1.01130	1.07465
88/89	1.03051	0.99529	1.03539
89/90	1.05889	0.88188	1.20072

**The geometric mean of individual indexes*

This result shows that the productivity in this industry has improved over the whole period of time. The main source of the productivity improvements is technological advance rather than efficiency improvement. In fact, technical efficiency fell every period except during the 87/88 period.

The Malmquist productivity index is not defined for all observations, yet we still want to see whether it is significantly different from the Malmquist-Luenberger index. Thus, we limit the comparison to those observations for which the Malmquist index is well-defined. The null hypothesis is that the Malmquist productivity index is the same as the Malmquist-Luenberger index. The t-test statistics for the difference of the two indexes are presented in Table 3.

Table 3. Test Statistics for H_o: $M_t^{t+1} = ML_t^{t+1}$

K	Mean	S.E.	t-value	p-value
104	0.03099	0.01989	1.558	0.1223

We cannot reject the null hypothesis at the significance level $\alpha = .05$. That is, there is not sufficient evidence that the two indexes are significantly different. This might imply that the Malmquist-Luenberger index is a possible substitute for the Malmquist index when bad outputs are present.

4. Conclusion

This paper introduces the directional output distance function to credit reduction of bads as well as expansion of goods. This function is used in the construction of a Malmquist type productivity measurement. The new index, termed the Malmquist-Luenberger index, which accounts for reduction of bads, can be *decomposed into two parts: efficiency change and technology change. The empirical results indicate that it might be more appropriate to use the Malmquist-Luenberger productivity index in the presence of bad outputs.*

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