Household Semi-public Goods and the Estimation of Consumer Equivalence Scales: Some First Steps

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Abstract

Models of household consumption used to estimate the relative needs of people living in different family types need to take account of economies of household size, price-like substitution effects and the allocation of consumption among the individuals of the household. No existing estimation method tackles all three of these issues in a simultaneous and transparent fashion. Partly because of this, consumption-based estimates of consumer equivalence scales have had little direct application in social and economic policies.

This paper combines a household production model introduced by Lau (1985) with a Samuelson-type (1956) household welfare function to develop a consumption model which is both general and amenable to the incorporation (and testing) of a range of additional identifying information. The latter can include ‘expert judgements’ of the technology of household production (the scale economies) of different goods. A simplified version of this model is used to estimate some preliminary equivalence scales and intra-household allocations for aged couples and singles.

1 Introduction

Equivalence scales are indices which describe the relative incomes required by people in different household types in order to reach the same living standard. Analogous to true cost of living indices, they have applications in both distributional analysis and tax/transfer policy.

However there is little agreement as to how to estimate such scales. Indeed many argue that, because people may derive intrinsic pleasure from their family circumstances, it is meaningless to examine only the expenditure costs of different family types. Several arguments can be advanced against this position. The most straightforward is simply that policy-makers concerned with poverty alleviation are only interested in living standards defined with respect to consumption alone (see Nelson, 1993 and Bradbury, 1995 for justifications of this position). Equivalence scales thus serve as an important input into the policy making process and into wider social welfare calculations. This narrow focus on consumption-based well-being is maintained here.

Even when attention is restricted to living standards defined with respect to purchased commodities, there is still no generally accepted estimation method. Since we are interested in expenditure requirements, household expenditures would seem the natural place to start, and there is a large literature on methods of estimating equivalence scales from household expenditure data. But all these methods are based upon more or less arbitrary identifying restrictions. These restrictions have varying degrees of plausibility and ease of interpretation. Unfortunately those identifying restrictions which have most economic plausibility, have the most severe data requirements, and even then do not take account of all likely influences of family composition on expenditure patterns.

In the light of these difficulties, research has proceeded in two directions. On the one hand, theoretical research into more sophisticated models of household consumption continues unabated. Other researchers with a more practical bent, however, have tended to abandon the economic approach and instead turned back to expert budget studies as a means of establishing equitable income support relativities (Bradshaw, 1991; Bradshaw, Hicks and Parker, 1992). In these studies, 'experts' are asked to judge how much of each commodity families of different composition require in order to reach some given living standard (e.g. 'modest but adequate'). Though the budgets are compared with expenditure data to identify major anomalies, the intention of these studies is to establish a normative standard rather than to be consistent with actual expenditure patterns.

The goal of this paper is to introduce a new model of household welfare and allocation which provides a clear basis for comparing the commodity-based welfare levels of individuals living in different situations. Using a household 'purchase function' framework, this model encompasses many of the previously used methods as special cases, whilst providing a more general framework for considering questions of intra-household resource allocation, scale economies and price-like substitution responses. It does this by addressing directly the question of household public goods (that is, goods potentially shared by the members of the household) – an issue largely omitted from previous research.[2]

Being a general model of inherently unobservable variables, it is necessary to impose *a priori* restrictions in order to ensure identification. Indeed, if it is desired to obtain conclusions with any degree of precision, these restrictions must be accorded considerable weight. If these restrictions are transparent enough, however, they will be easier to justify. More importantly, the generation of these restrictions may permit an integration of the economic and expert panel approaches to equivalence scale estimation.

In this paper, a simplified version of the model is used with Australian expenditure survey data (and with a set of scale economy assumptions provided by the author) to examine the intra-household resource allocation of retired couples, and to estimate equivalence scales for these couples compared to single men and women. The paper draws

2 See Pollack and Wales (1992: 97). The work of Nelson (1986, 1988, 1993) on pure public household goods is an important exception.

the tentative conclusion that the current Australian pension relativities may be insufficiently generous towards single retired people.

2 A General Household Welfare Model

A general way of representing the economies of sharing in households has been proposed by Lau (1985) as an extension of the household welfare model first proposed by Samuelson (1956). Assume that consumption decisions in a household of J individuals are made so as to maximise a current period household welfare function

$$\begin{aligned} & u = U(u_1, u_2, \ldots, u_J) \\ & \text{where } u_j = U_j(q_{1j}, q_{2j}, \ldots, q_{Ij}) \quad j = 1,2,\ldots,J \\ & \text{subject to a budget constraint} \\ & y = \textstyle\sum_i p_i h_i(q_{i1}, q_{i2}, \ldots, q_{iJ}) \end{aligned} \tag{2.1}$$

The welfare function $U_j(.)$ represents the preference ordering of person j over q_{ij}, the service flow received by this person from commodity i, whilst the overall welfare function $U(.)$ is assumed quasi-concave in the individual service flows, q_{ij}. The function $h_i(q_{i1}, q_{i2}, \ldots, q_{iJ})$ represents the household purchase requirements for commodity q_i, and reflects the fact that commodities may have some degree of ‘publicness’, with the total household purchasing requirement for a commodity possibly being less than the total of each individual’s consumption.

The household welfare function $U(.)$ can be interpreted in several ways. Most straightforwardly, it might be considered to represent the preferences of a ‘caring’ but ‘non-paternalistic’ household head who controls household consumption. Becker (1981) shows that this interpretation can hold even when the other individuals have some control over their own consumption.

More generally, we might consider the situation where each person in the household has their own set of preferences defined over the welfare levels of the individuals in the household. A Pareto efficient choice by each individual of their own consumption (for some fixed values of u_j for the other people) will imply a set of first order conditions which must also be met by the above problem. The function *U*(.) in equation (2.1) can then be interpreted as a summary of the relative 'bargaining strengths' of the individuals in the household. In general we might thus expect *U*(.) to also be a function of variables influencing bargaining within the household such as wage rates, private incomes, and social norms of within-household distribution. Incorporation of these would make the present model similar to that of the 'collective consumption' literature.[3] These variables are omitted here because, in contrast to that literature, the focus is on the implications of semi-public goods, and for the empirical example chosen in Section 3 there is little observable variation in bargaining-related variables.

Similarly, a more complete statement of the household decision making process would include the influence of leisure, home production and intrinsic preferences for different household compositions (e.g. the joys of parenthood, or the peace of solitude). As noted in the introduction, the direct contribution of demographic circumstances to welfare is not of interest here and similar justifications can be advanced for the exclusion of time-related inputs to well-being. Policy makers often appear to place a zero value on leisure for people excluded from the labour market when considering pension entitlements. Indeed, some might ascribe a negative value to idleness. As will be seen below, however, there is evidence that retired people do place a positive value on their leisure. A full examination of these issues is left for future research.

If, in reality, these other factors are separable from commodity-based influences on welfare then the analysis here will still be an appropriate analysis of commodity-based welfare. Whilst separability is only likely to be approximately accurate (particularly for home production) this is the most straight-forward way of providing a useful input into the policy-making process.

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3 See for example Chiappori (1988, 1992), Browning et al. (1993), Fortin and Lacroix (1993) and Apps and Rees (forthcoming).

2.1 The Joint Consumption Technology

Lau suggests the following as plausible axioms for the household purchase function … .

1 **Single-consumer equivalence**. If only one person consumes a commodity, then the quantity purchased is equal to that person's consumption (if no-one consumes then the purchase requirement is zero). In other words, there are no economies of scale in single person households.

2 **Monotonicity.** An increase in the quantity consumed by one individual (only) must lead to the household purchase requirements not decreasing.

3 **Symmetry.** The purchase requirement function is independent of the consumer of the service. For example, in a two person household, $h_i(q_{i1}, q_{i2}) = h_i(q_{i2}, q_{i1})$. This simplifying assumption rules out phenomena such as only same-sex siblings being able to share bedrooms.

4 **Quasi-convexity**. The function $h_i(q_{i1}, q_{i2}, \ldots, q_{iJ})$ is quasi-convex. In the context of the previous axioms, this is equivalent to assuming that, for a given quantity of a good purchased by the household, as personal consumption levels are made more equal the total of individual consumption will increase (or stay the same). This rules out certain situations of congestion – for example the noise interference caused by two radios being used simultaneously in the same room.

5 **Homogeneity**. If the quantities of services provided to each individual are increased by the same proportion, then the quantity purchased is increased by the same proportion. This rules out 'economies of scale' of the production function type. That is, goods retain the same degree of 'publicness' as the total quantity consumed increases. In this paper 'economies of scale' henceforth refers to the scale economies flowing from increasing household size rather than from increased volume of purchase.

From these axioms, the following additional properties can be derived:

6 **Minimum bound.** The quantity purchased by the household must be at least as great as the amount consumed by every individual. That is, $h_i(q_{i1}, q_{i2}, \ldots, q_{iJ}) \geq \max(q_{i1}, q_{i2}, \ldots, q_{iJ})$.

This follows from the single-consumer equivalence and monotonicity properties.

7 **Sub-additivity.** The sum of services received by individuals is always greater than or equal to the quantity purchased. That is $\sum_j q_{ij} \geq h_i(q_{i1}, q_{i2}, \ldots, q_{iJ})$. This follows from the single-consumer equivalence, monotonicity and quasi-convexity properties. If sub-additivity did not apply, it would imply that individuals would have a higher consumption level when living in separate households than when living together.

8 **Convexity**. The function $h_i(q_{i1}, q_{i2}, \ldots, q_{iJ})$ is convex in its arguments. This is implied by quasi-convexity and homogeneity (Berge, 1963: 208). This in turn implies that the overall budget constraint is convex and so the household maximisation problem is 'well-defined' in the sense of the Arrow-Enthoven theorem (e.g. Chiang, 1984: 745).

9 **Sub-differentiability**. If the consumption of all members but one is held constant, then household purchase requirements cannot increase at a faster rate than the increase in the consumption of the remaining individual. That is, if *h*(.) is differentiable, $\partial h_i / \partial q_{ij} \leq 1$. [**Proof for differentiable** *h*(.): Let $q = (q_1, q_2, \ldots, q_J)$ be an arbitrary vector of personal consumption levels of a particular commodity in the household. Let q^0 be another consumption vector for that same commodity with $q_1^0 = q_1$ and $q_j^0 = 0 \; \forall j \neq 1$. Convexity implies that $\mathrm{h}(\mathrm{q}^0) \geq \mathrm{h}(\mathrm{q}) + \sum_\mathrm{j} \frac{\partial \mathrm{h}(\mathrm{q})}{\partial \mathrm{q}_\mathrm{j}} \left(\mathrm{q}_\mathrm{j}^0 - \mathrm{q}_\mathrm{j}\right)$. The right-hand side of this expression simplifies to $h(q) + \frac{\partial h(q)}{\partial q_1} q_1 - \sum_j \frac{\partial h(q)}{\partial q_j} q_j$. But homogeneity implies that $\sum_j \frac{\partial h(q)}{\partial q_j} q_j = h(q)$ and single consumer equivalence requires that $h(q^0) = q_1$. Substituting gives the result $\frac{\partial h(q)}{\partial q_1} \leq 1$]

The sub-differentiability property of the household purchase function permits us to see how the household consumption choice differs from that made when the budget constraint is linear. In the first order conditions for the household maximisation problem in equation (2.1), the expression $p_i \frac{\partial h_i}{\partial q_{ij}}$ will replace p_i and so this can be thought of as the shadow price of commodity i for person j. The sub-differentiability property implies that this will be always less than, or equal to, the market price, and will be lower the more 'public' is the good. When the good is fully public the purchase function will not be differentiable, but similar qualitative conclusions will apply (Nelson, 1986).

Using the homogeneity property (and assuming differentiability), we can re-write the budget constraint in terms of these shadow prices. Applying Euler's theorem to the budget constraint in (2.1) yields

$$y = \sum_j \left(\sum_i q_{ij} \, p_i \frac{\partial h_i}{\partial q_{ij}} \right) = \sum_j y_j \qquad (2.2)$$

This description provides a natural way to describe the allocation of household income amongst household members. It should be noted, however, that the shadow prices in this expression will usually not be fixed but will vary with the consumption levels of all household members.

Examples

Some examples of single parameter functions $h(.)$ include

$$
\begin{array}{lll}
I & q = \sum_j q_{ij} & \text{(private good)} \\
II & q_i = \max_j (q_{ij}) & \text{(public good)} \\
III & q_i = \begin{cases} \left(\sum_j q_{ij}^{1/r_i} \right)^{r_i} & 0 < r_i \leq 1 \\ \max_j (q_{ij}) & r_i = 0 \end{cases} & \\
IV & q_i = s_i \sum_j q_{ij} + (1 - s_i) \max_j (q_{ij}) & \\
V & q_i = \max \left\{ \frac{1}{1 + (J-1)(1-t_i)} \sum_j q_{ij}, \max_j (q_{ij}) \right\} &
\end{array}
\tag{2.3}
$$

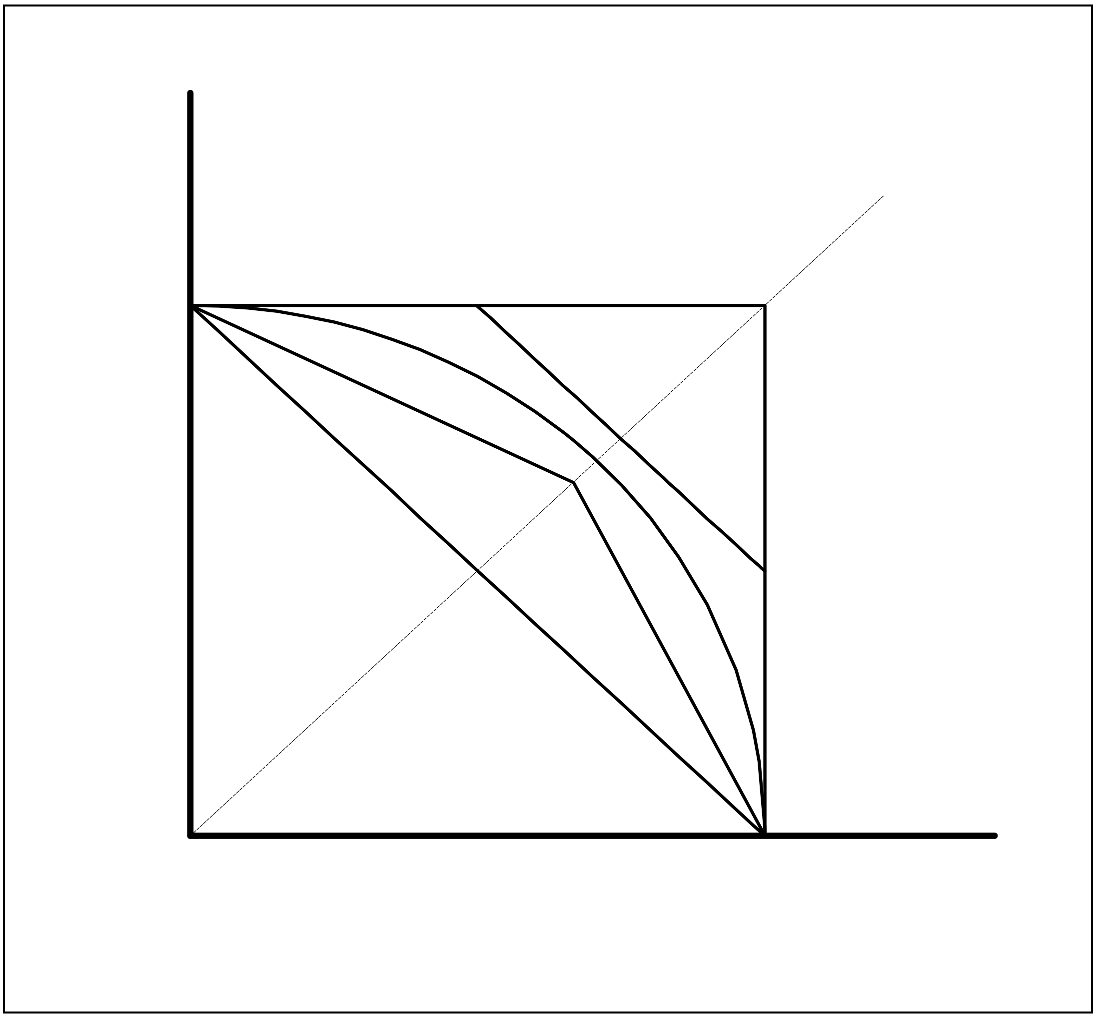
The first formulation is simply the conventional private good assumption. The second describes the situation where the good is 'public' within the household in the sense that the consumption of one member does not detract from the consumption of another. If we assume non-satiation of preferences, then this can be re-written as $q_i = q_{ij} \forall j$. That is, each member of the household consumes the full amount of the public good purchased. However, non-satiation may not always be a reasonable assumption, as there may be some goods which some family members do not consume (e.g. adult-only goods). Nelson (1986) describes the implications of pure private and pure public goods on the household's consumption behaviour.

The last three options are alternative formulations for goods which are neither fully private nor fully public within the household. When the scale parameters *s*, *r* or *t* equal 1, the good is private, and the household demand is simply the sum of the individual demands, whilst when *s* or *r* equal 0, the good is pure public. The different household purchase functions for a single commodity are illustrated in Figure 1 for the two person household case (with $r = s = t = 0.5$).

Equation *IV* is specified as a weighted average of the pure public and pure private good models of *I* and *II*. If person *m* consumes the greatest amount of commodity i *IV* can be written as $q_i = q_{im} + s_i \sum_{j \neq m} q_{ij}$ (if there are equal-maximum consumers, *m* denotes one of these people). The household purchase requirement thus consists of the consumption of person *m*, plus a fraction s_i of the consumption of other family members.

One simple example might be the consumption of housing services for two people living together in a house with a (fully public) common living area and two private bedrooms. Assume that Person 1 is able to make use of all the common living area facilities, but that Person 2 only uses some of these (e.g. Person 1 likes to hold large parties). If s is used to denote the consumption value of the private bedroom of Person 2 relative to the total consumption of that person, then the total household purchase requirement for housing will be $q = q_1 + s\,q_2$ where q_1 and q_2 are the total housing consumption of Persons 1 and 2 respectively.

Figure 1: Household Consumption Possibilities Frontiers for a Single Commodity



An important feature of this purchase function is that the shadow price of the commodity for the primary consumer is equal to the market price (i.e. $\frac{\partial q_i}{\partial q_{im}} = 1$) – so that it is only the second and subsequent household members who benefit from economies of scale. This is clear in the housing example. If the first person needs more space, the household must purchase a larger common area and/or a larger bedroom to satisfy this. On the other hand, if the second person needs more space the household only need purchase the additional requirement for bedroom space since they are not fully utilising the common area.

Note that this example assumes that the second person does not consume as much of the purely public component of the good as Person 1, even though this component of the good is effectively free to them. This can arise in two situations. The first is where Person 2 is constrained to always consume the private and public components of the good in the same proportion. In this case there will be a positive price for Person 2 (though less than the market price) which may mean that they consume less of the pure public component. It is difficult, however, to think of concrete examples where this might hold.

Alternately, Person 2 might consume less of the public good component because, at the quantity demanded by Person 1, their marginal utility from this good is zero. The most plausible case where this might be relevant is for parents and children where the observed commodity (e.g. housing services) actually consists of several different commodities (size, location, view etc.) – some of which are not consumed by children.

An alternative quasi-linear consumption possibility frontier with a more straight-forward interpretation is given by expression *V*. This implies that a fixed fraction (1-t_i) of the household purchase of a commodity is allocated to public consumption within the household. The remainder is allocated to individuals for private consumption. A simple example of this type of allocation might be a television set which is used for two hours a day. In the first hour, everyone in the household agrees that they want to watch the same program, and so consumption by one person is non-rival to that of other people (assuming there are enough seats in the lounge room). In the second hour, however, everyone has a different opinion of what they want to watch. Hence the second hour of TV viewing by one person must be at the expense of the consumption of someone else. Similarly, one could think of the role of common living areas when considering housing costs.

In general, one might expect a less discontinuous pattern of congestion externalities with the probability of being able to watch what you want diminishing in a smooth fashion as the number of people in the household and the amount of viewing by the other people increases. In this case, the model of *III* is probably the most appropriate single parameter model. However, in certain circumstances the quasi-linear character of equations *IV* and *V* lead to a much simpler household decision problem – particularly when we can make assumptions about which sector of the constraint the household lies on.

2.2 Equivalence Scale Identification

Consider a household with J people behaving as implied by equation (2.1) and facing prices p. For each of these individuals we can think of a 'quasi-cost' function describing the level of household expenditure required in order for the jth individual to attain a welfare level of u_j. Denote this as $c_j(u_j, p, J)$. Now consider another household with J^* people, the first J of whom are identical to those in the first household. For these J people the individual preference orderings $U_j(.)$ are assumed to not vary with the demographic composition of their household. Some such assumption of 'demographic separability' is a necessary part of all attempts to identify consumer equivalence scales (Gronau, 1988).

An equivalence scale for the j th person can then be defined as

$$m_j(u_j, p, J^*, J) = \frac{c_j(u_j, p, J^*)}{c_j(u_j, p, J)} \quad j \leq J < J^* \tag{2.4}$$

That is, the equivalence scale is the relative household income required for person j to reach the same welfare level when living in households of different size. Note that, in principle, there may be a different equivalence scale for every person who is in both household types. For example, the costs of a child may be born unequally by the husband and wife.

For the remainder of this paper a simpler model is considered where $J = 1$ and $J^* = 2$. If both people can live individually there are then two scales – comparing the situation of each person when living alone and when together. Alternatively, one might make the assumption that the subscript j indexes groups of similar people (who share a common welfare level u_j). In this case, $m(.)$ can be used to describe the cost of having a given number of children.

Whether we use this simplified form or not, however, identification of equation (2.4) is not guaranteed without additional assumptions. The lack of identification is closely related to the unobservability of the personal consumption levels q_{ij}. This relationship is formalised in the following statement.

Theorem 1: If the household welfare function is of the form (2.1) and the following two pieces of information are both known

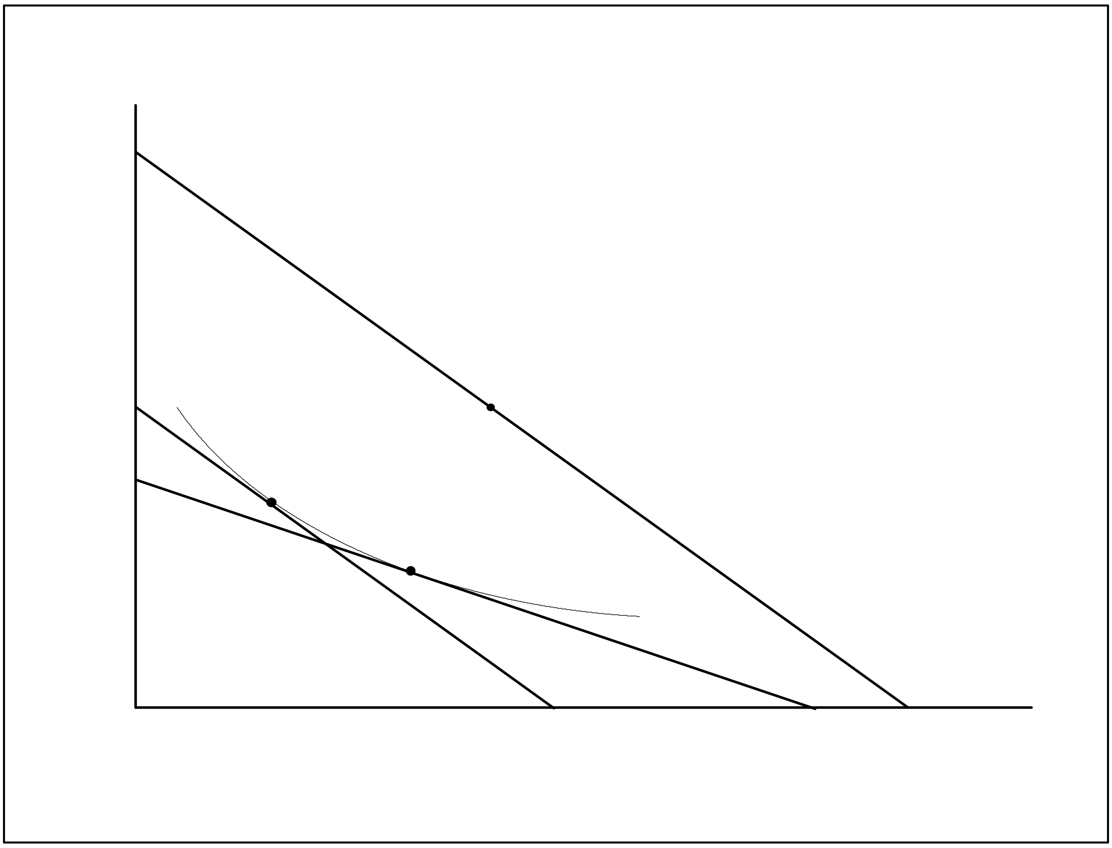
- Full information about consumption behaviour for the single person household (i.e. consumption under a range of price and income conditions), and
- in the larger household, for some price and household income combination, the personal consumption vector of this reference person,

then the equivalence scale can be identified (at that household income and price level).

Proof*:* The first assumption implies that the individual welfare function for Person 1, $u_1 = U_1(q_{11}, q_{21}, \ldots, q_{I1})$, can be recovered up to a monotonic transformation. It also implies that the cost function, $c_1(u_1, p, 1)$, is known for the same normalisation of u_1. Since the vector of individual consumption, $q_{i1} = q_{i1}(y, p)$, is known, this can be substituted into $U_1(.)$ to obtain the welfare level of the person when living in the larger household. This in turn can be substituted into the cost function to obtain the cost of reaching this welfare level when living alone. The ratio of the income of the larger household to this cost gives the equivalence scale. QED

This procedure is illustrated in Figure 2 for two commodities, with the market price of commodity 2 normalised to unity. For a given income, *y*, and market prices, the larger household purchases an amount *H* of commodities q_1 and q_2. When living in this household, the reference person has a personal consumption of *A*, reaching a welfare level labelled *u*. If the person were to live alone, they would need an income of *y** to reach the same welfare level. Note that in this figure, commodity 1 is more 'public' than commodity 2, and so when living in the larger household the effective price of this commodity will be lower, and so there will a tendency for substitution towards consumption of this commodity. Given knowledge of point *A*, however, point *B* can be found from information about market prices and the curvature of the indifference curve through point *A*. This in turn defines the equivalence scale, which is simply *y*/*y**.

Figure 2: Equivalence Scale Identification



Note that the reverse of Theorem 1 does not necessarily apply. That is, knowledge of the equivalence scale, single person household consumption at income *y**, and the shape of the indifference curve *u*, does not permit the identification of personal consumption within the household. To identify point *A*, other information is required. In the two good case, knowledge of the personal consumption of one good is sufficient to identify the other. More generally, the implicit prices faced by the reference person in the household will be sufficient. But the non-linearity of *h*(.) implies that these will not be easy to recover.

2.3 Special Case 1: The Barten Model

One household welfare model that has been widely used in the estimation of equivalence scales is that of Barten (1964). As Nelson (1988) notes, this model can be derived as a special case of (2.1) if it is assumed that individuals within the household are identical, and that the household welfare function is symmetric. In this case, the household maximisation problem simplifies to

$$\begin{aligned} &\max u^* = U^*(q_{1*}, q_{2*}, \ldots, q_{I*}) \\ &\text{subject to y} = \textstyle\sum_{\text{i}} p_i q_i \text{ with } q_i = h_i(q_{i*}, q_{i*}, \ldots, q_{i*}) \end{aligned} \qquad (2.5)$$

where q_{i*} is the (identical) consumption of each person of commodity i. The purchase function now has only one argument and so can be written as simply $h_i(q_{i*})$. For example, if the purchase function takes the form *III* then $q_{i*} = q_i \big/ J^{r_i}$, and if it takes the form of *IV* then $q_{i*} = q_i / (1 + s_i(J-1))$. For the two person household case we can simply represent the denominator of these fractions by m_i (with m_i =1 for the single person). This produces a maximisation problem identical to the Barten maximisation problem

$$\max U\left(\frac{q_1}{m_1}, \frac{q_2}{m_2}, \ldots, \frac{q_I}{m_I}\right) \qquad (2.6)$$

subject to a budget constraint $y = \sum p_i q_i$. The correspondence of the Barten model with this very special case of the welfare model in (2.1) clearly points to its limitations, and explains why the model can imply implausibly high degrees of quasi-price substitution in response to demographic changes such as the addition of a child to the household (see Brown, 1964; Muellbauer, 1977; and Nelson, 1992, 1993, for discussion of the limitations of the Barten model).

2.4 Special Case 2: The Rothbarth Model

A quite different approach is illustrated in the Rothbarth model. This is also known as the adult good, or (generalised) translation model.[4] Two additional assumptions are used to identify equivalence scales. First, it is assumed that all goods are private, so that there are no price-like substitution effects as people move between household types. Second, it is assumed that the personal consumption of (at least) one good is known as some function of observable variables. Most commonly, this takes the form of assuming that there is at least one good which is consumed by adults alone. The ‘reference person’ in this case is the adult(s) and the additional person is a child (or children).

Since private goods imply a linear budget constraint, the solution to equation (2.1) can be considered in two stages, a division of income between the different members, and the allocation of consumption by each member as a function of their own income, market prices and preferences. To identify the division of income all that is needed is information about the Engel curve for the adult commodity in the two household types. The intersection of the Engel curve for the family without children with the observed consumption of the adult good in the household with children can then be used to read off y_A, the income required by the adult-only household to reach the same welfare level as the family with children.

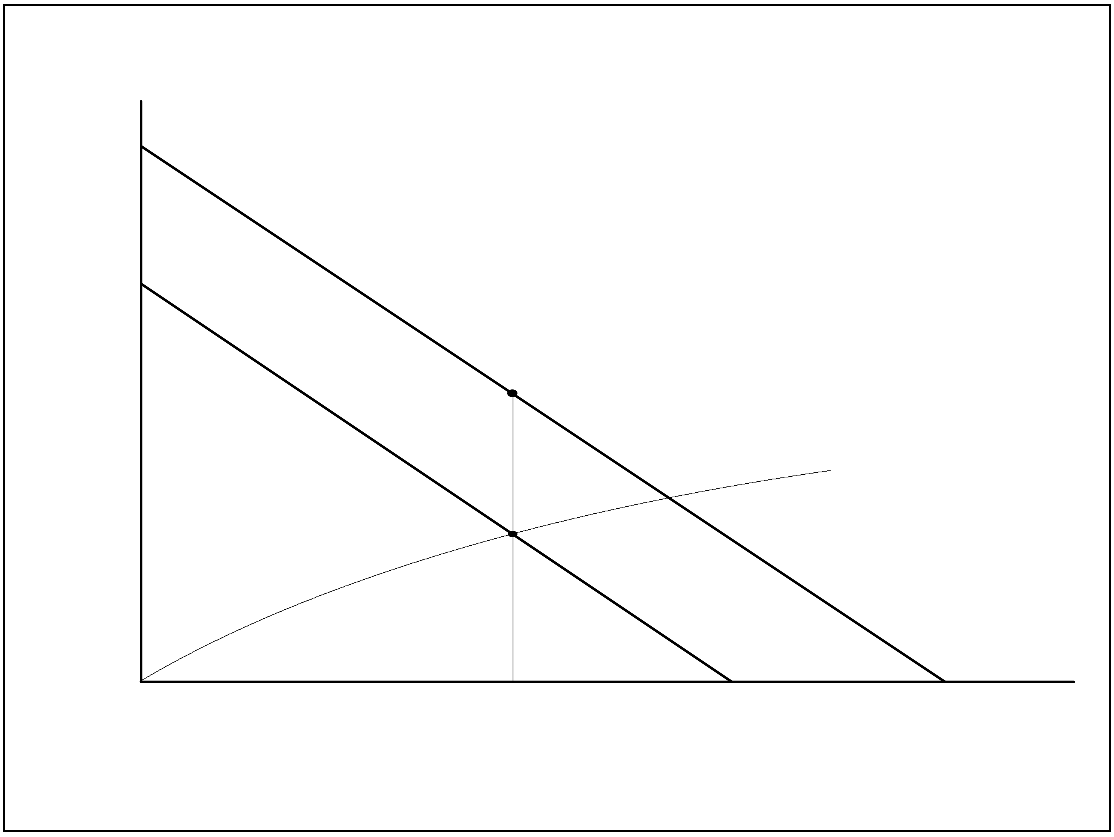
This procedure is illustrated in Figure 3 Since q_1 is consumed only by adults, the household consumption is also the adult consumption. If the income expansion path of the adult good consumption in the adult-only household is known, then the intersection of this with the observed consumption of the adult good in the larger household can then be used to read of y_A.

4 See Rothbarth (1943), Deaton and Muellbauer (1986), Gronau (1988) and Bradbury (1992). The translation model of Pollack and Wales (1981) is closely related, but does not permit child costs to vary with income.

Rothbarth Bias

Whilst the main econometric limitation of the Rothbarth model stems from the need to observe goods consumed only by adults, the assumption that all goods are private is theoretically of more concern as it seems to deny the very essence of household economies of scale. Nelson (1992) shows that, when some goods in the household are pure public, price-like substitutions will mean that the Rothbarth model will produce biased estimates of the cost of children. The same result applies when there are semi-public goods. Unless there are significant complementarity relationships between different goods the Rothbarth method will tend to overestimate the cost of children.

Figure 3: Identification in the Rothbarth Model



To see this, assume there are two (classes of) people in each household: adult(s) and child(ren), and two classes of goods, the pure adult good, *A*, and other goods, *O*. Only adults consume the former, and both adults and children consume the latter. The single-consumer equivalence property of the household purchase function implies that for the adults in the household with children, the implicit (or shadow) price of the adult good will equal its market price. The sub-differentiability property, on the other hand, implies that, for the adults the implicit price of all other goods which are not private will be less than or equal to their market price (usually the implicit price will be strictly lower, an exception is considered below). Hence other goods are relatively cheaper and unless the pure adult good is a (Hicks-Allen) complement to one or more of the other goods, the parents will tend to substitute away from the more expensive pure adult good (adult welfare held constant). Since all other goods may become relatively cheaper, and since Hicks-Allen substitutes must be 'more prevalent' than substitutes, it would be unusual if complementarity were such as to actually lead to the parents substituting towards the adult good.

This substitution relationship is illustrated in Figure 4 (for a single 'other good' – in which case there can be no ambiguity about the direction of substitution). The consumption point of the adults will occur at a point on their indifference curve where their marginal rate of substitution between the two goods is greater than market prices – denoted *A* in the figure. When the adults have no children (and hence face market prices) they will reach this same welfare level when they consume at point *B* – where they have an income of y_A and consume more of the adult good.

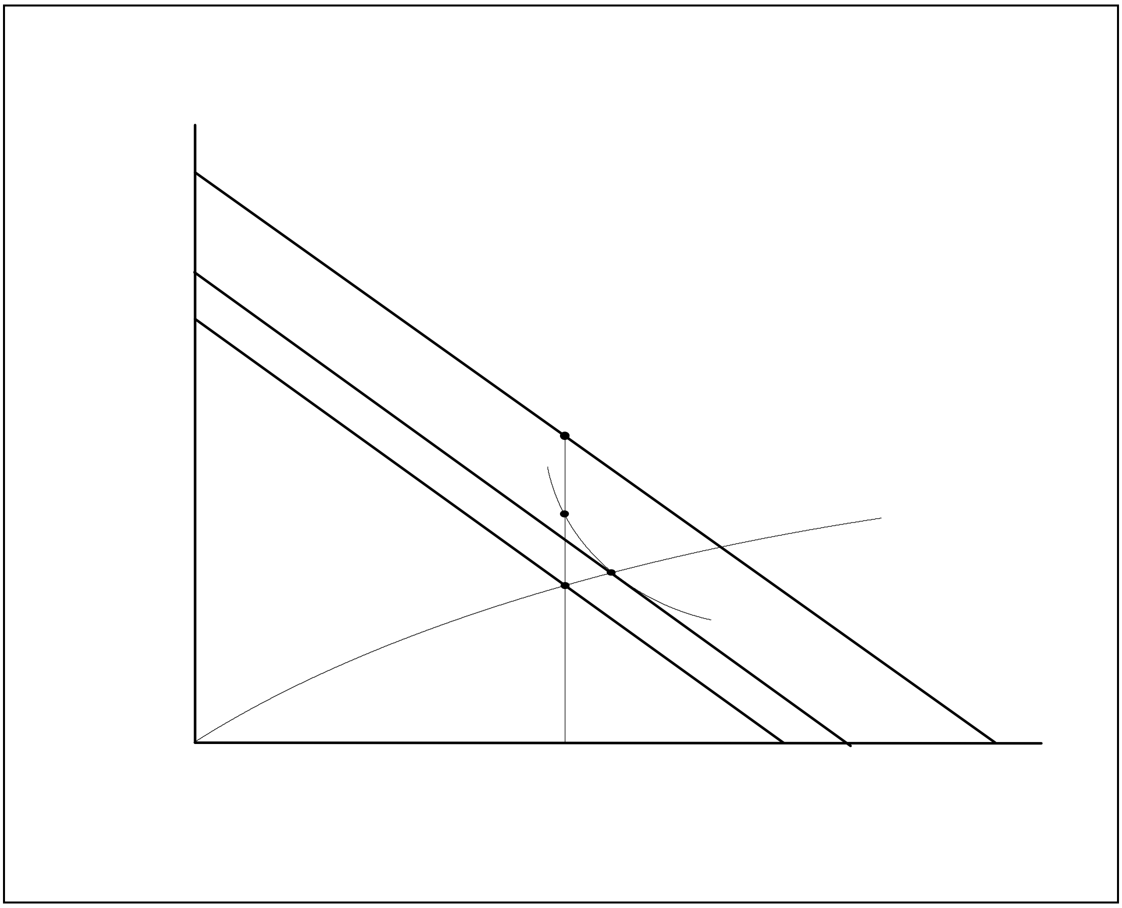
However the Rothbarth method looks for the income level, y^*, that will permit the childless adults to have the same adult good consumption as they had at point *A*. Since adult good consumption at point *A* is less than at point *B* (and if the adult good is normal, which is usually the case), this implies that y^* must be less than y_A. Hence the equivalence scale estimated by the Rothbarth method (y/ y^*) must be larger than the true equivalence scale (y/ y_A).

It is interesting to compare this analysis with the shadow prices implied by the Barten model. In that case, the effective price of other goods is assumed to **rise** when children enter the household – 'a penny bun costs threepence when you have a wife and child'. Even when the goods are more public than penny buns, like home heating, the Barten model still treats the additional expenditure as a price effect. The different interpretation arises because, by effectively assuming that individuals in the household are identical, the Barten model conflates the within-household distribution effect and price effects of children into a single price effect. The more general welfare model proposed here, however, confines price effects to the public good nature of consumption, and permits income effects to enter more directly via the allocations between individuals flowing from the household welfare function *U*(.).

Quasi-linear Purchase Functions and Rothbarth Bias

One of the assumptions required for the conclusion that the Rothbarth method is biased is that the implicit price of semi-public goods will be strictly less than their market price. This will be the case if the purchase function is strictly quasi-convex, but in some cases linear segments in the

Figure 4: Rothbarth Model Bias



purchase function can imply an implicit price equal to the market price (and hence no bias in the Rothbarth method). In particular, the Rothbarth method will be unbiased if it is assumed that the purchase function takes the form *IV*, and it can be assumed that the adult(s) will be the main consumers of all commodities. For a sufficiently broad grouping of goods, the latter condition may be a quite reasonable assumption, though the implications of the purchase function of form *IV* may perhaps be less plausible. This result follows from the fact noted above that the shadow price for the main consumer will be equal to the market price.

2.5 Identification with a Quasi-linear Budget Constraint

An alternative simplifying assumption is that the household purchase function takes the form *V*. That is, purchase requirements for each commodity are defined as if part of the commodity were a pure public good, and part a pure private good. This implies that, unless a good provides zero marginal utility to an individual, individual consumption of each commodity must be 'reasonably' equal.

More specifically, for a two person household, the purchase function will take the form $q_i = \max\{(q_{i1} + q_{i2}) / n_i, \max(q_{i1}, q_{i2})\}$ where $n_i = 2 - t_i$ and hence $1 \le n_i \le 2$. The lower bound for n_i applies to a pure private good, and the upper bound to a pure public good. Inspection of Figure 1 shows that unless they have zero marginal utility for a good, each individual will consume at least $(n_i - 1)q_i$. For pure private goods this imposes no restriction, but the closer the good is to a public good, the more equal will individual consumption become.

Whilst this is a reasonably plausible assumption, it is clear from Figure 1 that it imposes more restrictions on consumption patterns than do alternative purchase functions such as the iso-elastic function (*III*). It is also clear from that figure that this quasi-linear purchase function will be closest to the iso-elastic form when individual consumption levels are equal. This suggests that if we consider the iso-elastic form to be the most plausible simple purchase function, then this quasi-linear function will best serve as a simplifying assumption when individuals are likely to have similar consumption levels. This will be most likely in multi-adult household, rather than in households comprising both adults and children.[5]

The great advantage of a quasi-linear purchase function is that it ensures constant shadow prices (for non-boundary cases) and this simplifies the analysis considerably. If we assume that this functional form is correct, and that everyone gets positive marginal utility from

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5 This functional form is less restrictive than it might appear. For example, individuals in a household might have very different consumption patterns for men's and women's clothing. But since clothing is a pure private good, the purchase function places no discontinuous lower bound on consumption levels. It is only when we might want individuals to have to very different consumption levels for near-public goods that the quasi-linear model becomes inappropriate.

each commodity then the above discussion implies that the effective part of the household purchase function is simply $q_i = (q_{i1} + q_{i2}) / n_i$ together with the constraints

$$q_{i1} \le q_i \text{ and } q_{i2} \le q_i \tag{2.7}$$

Or equivalently,

$$n_i - 1 \le \frac{q_{i2}}{q_{i1}} \le \frac{1}{n_i - 1} \tag{2.8}$$

If we assume that preferences for semi-public goods are similar enough so that none of these constraints is binding then we can consider a single budget constraint $y \ge \sum_i \frac{p_i}{n_i}(q_{i1} + q_{i2})$. The resulting consumer demand model can be thought of as a hybrid of the Barten and Rothbarth models. On the one hand, the publicness of the good leads to a fixed decrease in the implicit price of the commodity via the n_i term – as in the Barten model. But on the other hand, there is no assumption of equal consumption by household members and the equivalence scales estimated for each household member may be different from one another. This suggests that identification of this model will require information on both the price variation in consumption and Rothbarth-type assumptions about within-household allocations.[6]

It is useful to consider some information-rich cases where the equivalence scale can be clearly identified. Assume to begin with that all the n_i terms, and hence all the shadow prices facing each individual, are known. If we can observe the demand function of the person when they are living alone, we can then map out an income expansion path describing the individual's consumption of each commodity when facing the shadow rather than market prices. If we also know the

6 This model is also related to the Gorman-Barten model (Gorman, 1976). In this model a fixed cost element is added to the Barten cost function. The present case can be interpreted in terms of a variable additional cost being added to each adult's cost function, where the additional cost is a function of the within-household allocation.

personal consumption of one good, then we can pin down the welfare level. By calculating the cost of reaching that welfare level when living alone and facing market prices, the equivalence scale can be derived.

The two key assumptions associated with this procedure are that, first we know the degree of publicness of each good, and second, that the individual consumption of one good is known. In the multiple-adult household case, two possible cases where it might be reasonable to make assumptions regarding the latter are that 1) some good is only consumed by one individual (e.g. men's clothing in a couple) or 2) some good is pure public.

The case where there is one pure public good and one semi-public good is illustrated in Figure 5. As before, the household has an income of y and consumes at point H. The reference individual's consumption of the public good will be identical to the household's purchase, q^*. The intersection of this with the estimated expansion path for an individual facing the shadow prices determines the individual's welfare level in the two adult household. The cost of reaching this welfare level when living alone and facing market prices, y_A can then be compared with the household's expenditure to calculate an equivalence scale.

In fact, in a multi-person household, it is not necessary to make the Rothbarth-type assumption described above. This is because there is the additional identifying information of the second person's consumption patterns which must be consistent with those of the first person and the household purchases. The intra-household allocation decision when the scaling factors (n_i) are known can be represented in an 'Edgeworth box' as in Figure 6.

If the household purchases quantities q_1 and q_2 , and the scaling parameters are n_1 and n_2 then the total consumption available to the individuals in the household is given by $n_1 q_1$ and $n_2 q_2$. This is thus the size of the Edgeworth box in Figure 6. The scaling parameters, together with market prices (to simplify presentation these are assumed to be unity) determine the effective price facing each individual (drawn as dotted lines in Figure 6). The income expansion paths (drawn as curved lines) can then be predicted on the basis of the individual demand functions estimated for people living alone. The intersection of these two curves thus determine the intra-household allocation of consumption.

Figure 5: Single Person Identification in the Quasi-linear Model

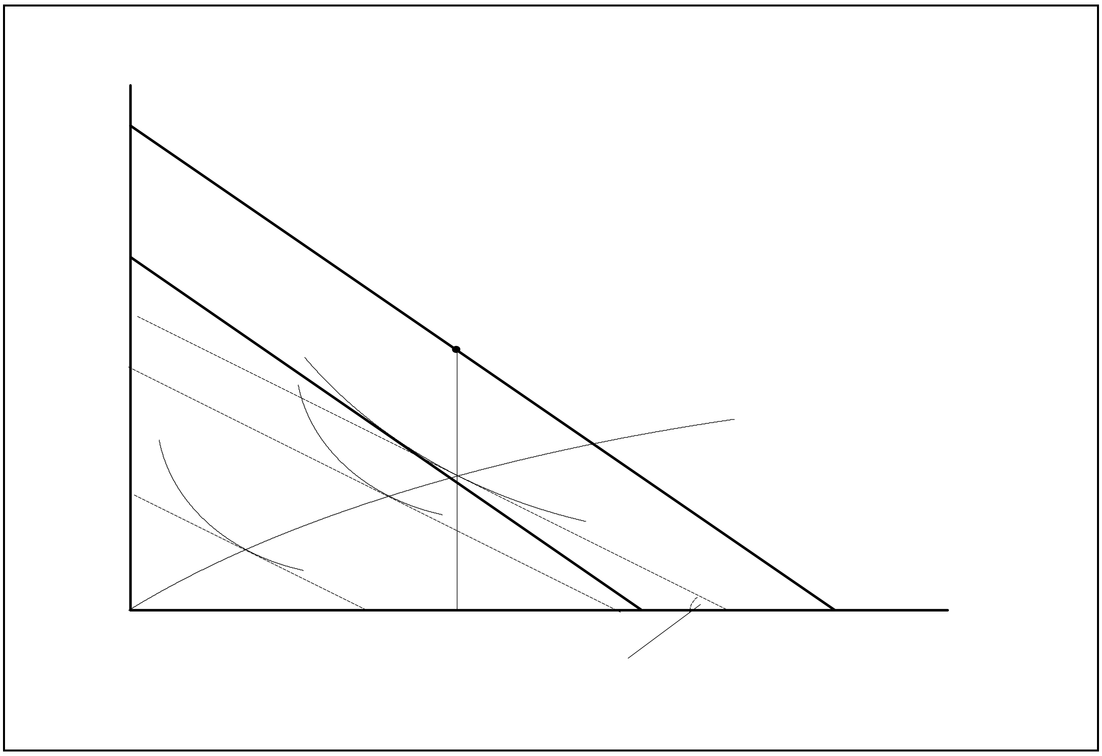
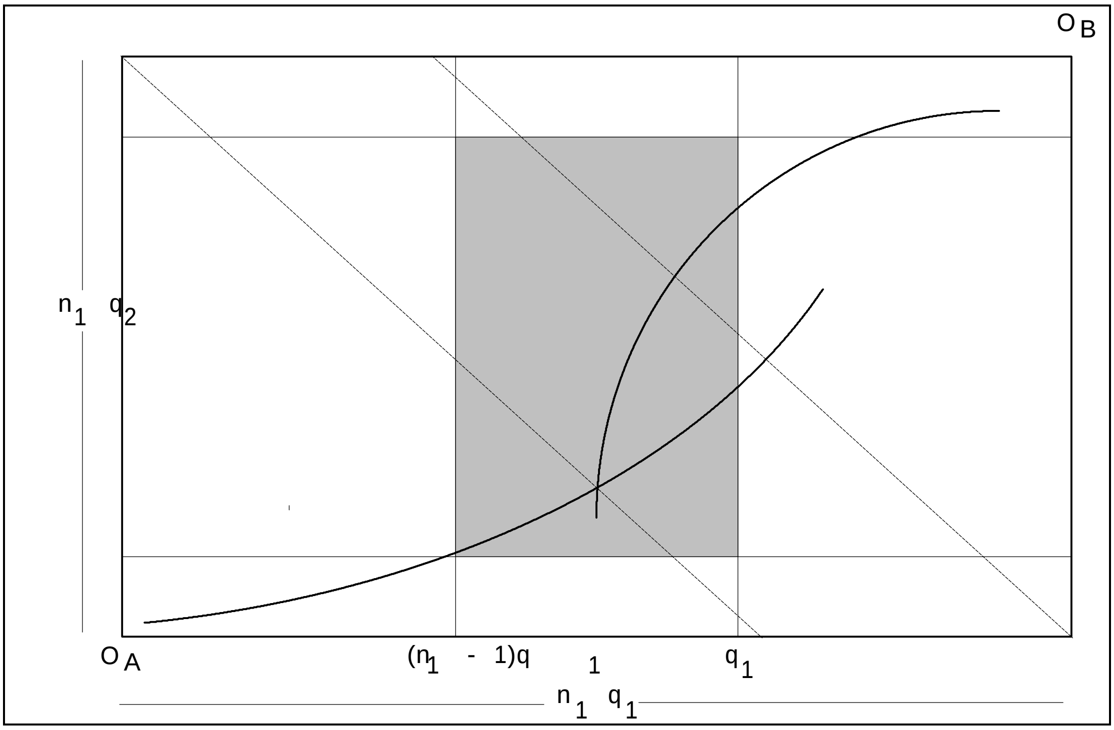


Figure 6: Allocation Between Two Household Members When Scaling Parameters are Fixed



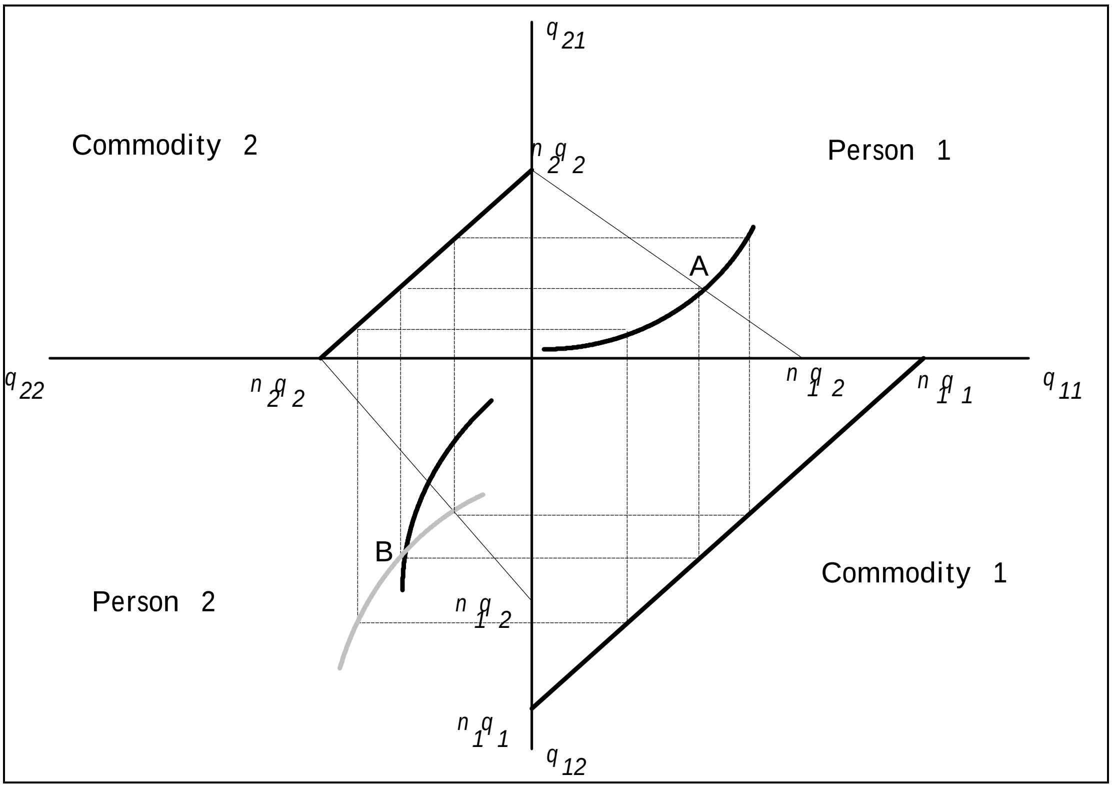
From Theorem 1, the equivalence scale for each individual can then be derived. Note that the quasi-linear purchase function requires that the solution to this allocation lie within the shaded box (or else at the boundary with one person not consuming a commodity).

Thus, in principle, the interaction of two individual's preferences can be used to recover the intra-household allocation, provided the household consumption technology is known. Note that there is no apparent requirement in Figure 6 that the two expansion curves must intersect, nor that they should intersect only once (though the latter will occur only if the expansion paths are very curved). However if the curves did not intersect (e.g. because neither person wanted to consume commodity 2) this simply means that the household will not purchase the amount of commodity 2 assumed by the diagram. Alternately, if we actually do observe the household purchasing an amount that is not consistent with observed individual preferences then we can infer that the assumed technology (or the general model) is incorrect.

An alternative, and somewhat more flexible, way of illustrating the same intra-household allocation decision is shown in Figure 7. This figure shows both allocations between people (in the NE-SW direction) and between commodities (in the NW-SE direction). The bottom-right quadrant shows the allocation of commodity 1 between the two people. If household consumption is observed, and n_1 known, then the potential allocations are restricted to those on the bold line. Strictly speaking this line should be drawn as line *V* in Figure 1, here it is continued out to the axes to show the total amount available for consumption as $n_1 q_1$. Similarly, the bold line in the top-left quadrant is the ‘known’ set of potential allocations of commodity 2. As drawn, the household purchases more of commodity 1 (which is also more public than commodity 2).

Now the n_i parameters also reflect the shadow prices of the two commodities facing each member of the household, and because of the quasi-linear budget constraint, these shadow prices will be the same, and constant, for each individual. The slope of the straight lines in the top-right and bottom-left quadrants thus shows the prices facing each individual as they choose their personal consumption levels. If we know

Figure 7:Alternative Representation of Two-Person Allocation



the demand function for each individual when they are living alone we can use this price ratio to predict the income expansion path along which their consumption in the joint household will be located. This is indicated by the bold curved lines in the top-right and bottom-left quadrants.

However, for each level of personal consumption for Person 1, there will only be one corresponding level of personal consumption for Person 2 that will lead to the observed level of household consumption. The income expansion path for Person 1 thus has a corresponding required consumption level for Person 2 – as indicated by the grey curve in the bottom-left quadrant. But the consumption expansion path for Person 2 will not necessarily be identical to this curve – as this is assumed to reflect the preferences of Person 2 which would apply whether or not they were living with Person 1. The household budget constraint will only be met if individual consumption levels are located at the intersection of the two curves. That is, with Person 1 consuming at A and Person 2 at B.

Whilst this figure conveys the same information as Figure 6, it is more easily generalised to the case where the purchase function is not linear. In general, concavity of the budget constraint implies that the price lines in each quadrant of the diagram will be concave to the origin. With this modification, the discussion of the previous paragraph can be mirrored for more general technologies (though the analytic solution will be very non-linear).

Finally, it is of interest to compare the identification of the intra-household allocation process described here with that in the literature on household collective labour supply and consumption.[7] There, the household sharing rule is recovered only up to an additive constant. Here, sharing of consumption, and hence personal income and the sharing rule, is recovered in absolute terms. The difference arises because of the assumption made here (as in all equivalence scale calculations) that preferences are stable between different family compositions. In particular, consumption patterns of individuals living alone are necessary to recover the sharing rule. The collective consumption literature does not make this assumption nor draw upon individual consumption data, and hence is unable to recover the full sharing rule.

3 An Application: Single and Dual Adult Households

In this section an application of the approach outlined above is applied to single and two adult retired households. Equivalence scales for these two family types are particularly relevant for policy decisions such as the setting of pension rates. It is assumed that the household purchase function is of the quasi-linear form V, which appears plausible for such households. As long as consumption is not at a vertex of this budget constraint, this will imply a separable household maximisation problem which can be considered in two stages. At the first stage, the household income y is allocated between the consumption of each person, so that $y_1+y_2=y$. Given this upper-level allocation, each person

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7 For references see footnote 3.

then choses their individual consumption as a function of their income allocation and the implicit prices faced by them.

The upper level of this allocation will, in general, be a function of the household-level welfare function, individual preferences, market prices and the household consumption technology. One special case where this is simplified is when individual preferences take the form of the Gorman generalised polar form and the household welfare function is directly additive.[8] Here it is assumed that individual preferences can be represented in a special case of this, the PIGLog/AID system proposed by Deaton and Muellbauer (1980b). This has the following cost and indirect utility functions

$$\log c_j(u_j, p) = (1 - u_j)\log a_j(p) + u_j \log b_j(p) \tag{3.9}$$

$$v_j(y_j, p) = \log\left(y_j / a_j(p) \right) \Big/ \left(\log b_j(p) - \log a_j(p) \right) \tag{3.10}$$

where p is the vector of prices facing the individual and

$$\begin{aligned} \log a_j(p) &= \alpha_{0j} + \sum_k \alpha_{kj} \log p_k + \tfrac{1}{2} \sum_k \sum_l \gamma^*_{klj} \log p_k \log p_l \\ \log b_j(p) &= \log a_j(p) + \beta_{0j} \prod_k p_k^{\beta_{kj}} \end{aligned} \tag{3.11}$$

When person j is living alone, their budget share demand functions are then

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8 Deaton and Muellbauer (1980a: 131) show this in the context of commodity groupings.

$$w_{ij}(y,p) = \alpha_{ij} + \sum_k \gamma_{ikj} \log p_k + \beta_{ij} \log(y / P_j)$$

where

$$\log P_j = \alpha_{0j} + \sum_k \alpha_{kj} \log p_k + \tfrac{1}{2} \sum_l \sum_k \gamma_{klj} \log p_k \log p_l, \quad (3.12)$$

$$\gamma_{ikj} = \gamma_{kij} = \tfrac{1}{2}(\gamma^*_{ikj} + \gamma^*_{kij}) \quad \text{and}$$

$$\sum_i \alpha_{ij} = 1, \quad \sum_i \gamma_{ikj} = \sum_k \gamma_{ikj} = \sum_i \beta_{ij} = 0$$

For the combined household, the household welfare function is assumed to be a simple weighted average[9] of the individual welfare levels

$$u = \delta_1 u_1 + \delta_2 u_2 \quad (3.13)$$

This is maximised subject to a global budget constraint $y = \sum_i p_i q_i$. With a household purchase function of the form *V*, $q_i = (q_{i1} + q_{i2}) / n_i$ and so the budget constraint can be written (assuming a non-boundary solution) as

$$y = y_1 + y_2 = \sum_i p_i^* q_{i1} + \sum_i p_i^* q_{i2} \quad (3.14)$$

where $p_i^* = p_i / n_i$

Household and personal welfare maximisation now takes place in the context of shadow rather than market prices (recall n_i=1 for pure private goods and =2 for pure public goods).

If we define $z_j = y_j/a_j(p^*)$ then the upper level of the household maximisation problem can be written as

9 This model could be extended by making δ_j a function of external influences on household decision making (such as private incomes and wage rates) as in the bargaining literature.

$$\max u = \left(\frac{\delta_1}{\beta_{01}}\prod_k p_k^{-\beta_{k1}} n_k^{\beta_{k1}}\right)\log z_1 + \left(\frac{\delta_2}{\beta_{02}}\prod_k p_k^{-\beta_{k2}} n_k^{\beta_{k2}}\right)\log z_2$$

subject to (3.15)

$$y = a_1 z_1 + a_2 z_2$$

This yields a solution which can be expressed as, $= \Theta$ where

$$\Theta \;=\; / \left(+ \frac{\delta}{\delta} \frac{\beta}{\beta} \prod \quad \beta \;\; -\beta \quad\; \beta \;\; -\beta \quad \right) \tag{3.16}$$

$$\Theta \;=\; -\Theta$$

Note that whilst the household welfare function is directly additive in welfare, it will be concave in real income (z). One generalisation of this approach would be to use an individual welfare functions of the PIGL form rather than the PIGLog (see Deaton and Muellbauer, 1980a). This introduces an inequality aversion parameter which could, in principle, be estimated from the curvature of the Engel curves.

Within the household, individual consumption will be given as in (3.12), but with y_j replacing y and p^* replacing p. That is,

$$w_{ij}^* = p_i^* q_{ij} / y_j = w_{ij}\left(\Theta_j y, p^*\right) \tag{3.17}$$

Since $q_i = (q_{i1} + q_{i2}) / n_i$, the household budget shares are derived as

$$w_i = p_i q_i / y = \Theta_1 w_{i1}\left(\Theta_1 y, p^*\right) + \Theta_2 w_{i2}\left(\Theta_2 y, p^*\right) \tag{3.18}$$

It is assumed that commodity 1, men's clothing, is only consumed by the husband (person 1), and commodity 2, women's clothing, is only consumed by person 2 (i.e. $w_{12}(.) = w_{21}(.) = 0$). This requires that $\alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = 0$ and $\gamma_{1k2} = \gamma_{i12} = \gamma_{2k1} = \gamma_{i21} = 0 \forall i, k$. In addition, since little is known about price response patterns, it is assumed that the (approximate) compensated price effects are the same for men and women (apart from these restrictions on clothing). The γ matrices for the husband and wife are thus identical except that the husband has zeros in the second row and column, whilst for the wife the first row and column are zeros.

With estimates of the parameters of this system we can recover equivalence scales. For person *j* this proceeds as follows. When the household has an income of *y*, and the person faces implicit prices p^* they will reach a welfare level $u_j=v_j(\Theta_j y, p^*)$ where $v_j(.)$ has the form given in (3.10). The income required by the person to attain this same welfare level when they face market prices will be given by the cost function defined over market prices $c_j(u_j,p)$. Substituting $v_j(.)$ into this cost expression, and dividing the result by the income of the two person household, *y*, gives the equivalence scale for living alone relative to living in the household as

$$m_j = y^{\left(n_j^* - 1\right)} \Theta_j^{n_j^*} a_j(p^*)^{-n_j^*} a_j(p) \qquad (3.19)$$

where $n_j^* = \prod_k n_k^{\beta_{kj}}$. At reference prices (p=1) this expands as

$$m_j = \Theta_j^{n_j^*} \exp\left(\begin{array}{l} (n_j^* - 1)(\log y - \alpha_{0j}) + \\ n_j^* \left(\sum_k \alpha_{kj} \log n_k - \frac{1}{2} \sum_k \sum_l \gamma_{klj} \log n_k \log n_l \right) \end{array} \right) \qquad (3.20)$$

Since the β_{kj} parameters sum to zero, n_j^* will be approximately unity. Using this approximation and ignoring second order price effects, gives an approximate value for the equivalence scale of

$$m_j \approx \Theta_j \bar{n}_j \qquad (3.21)$$

where $\bar{n}_j = \prod_k n_k^{\alpha_{kj}}$ – the geometric mean of the scale parameters weighted by the autarchic budget shares at minimum income.

3.1 Empirical Results

Some results based upon single and married people of age pension age are presented in this section. This analysis only requires data from a single household expenditure survey. Men are included if aged 65 or more, women if aged 60 or more, and couples if both husband and wife have reached these ages.[10] So as to control for wealth effects on consumption patterns, only home-owner households are included (this includes the majority of households and also includes some paying off mortgages). Since there is no price variation, all market prices are fixed at unity, and assumed values are used for the price response parameters, γ, as well as the scale economy parameters, n. Because there are significant variations in consumption patterns with age, we also include age of husband and wife as controlling factors.

Let h_0, h_1, h_2 be indicator variables which take the value 1 when the household comprises a retired couple, single male or single female respectively. Households including other people are not examined. Then equations (3.12) and (3.18) can be combined into a single demand system as follows (at unit prices):

$$w_i = h_1 w_{i1}(y) + h_2 w_{i2}(y) + h_0 \left(\Theta_1 w_{i1}\left(\Theta_1 y, p^*\right) + \Theta_2 w_{i2}\left(\Theta_2 y, p^*\right)\right)$$

where

$$w_{ij}(y) = \alpha_{ij} + \varepsilon_{ij} a_j + \beta_{ij}(\log y - \alpha_{0j}) \tag{3.22}$$

$$w_{ij}\left(\Theta_j y, p^*\right) = \alpha_{ij} + \varepsilon_{ij} a_j - \sum_k \gamma_{ikj} \log n_k + \beta_{ij}(\log(\Theta_j y) - \log P_j^*)$$

$$\log P_j^* = \alpha_{0j} - \sum_i (\alpha_{ij} + \varepsilon_{ij} a_j) \log n_i + \tfrac{1}{2} \sum_i \sum_k \gamma_{ikj} \log n_i \log n_k$$

and where a_1 and a_2 are male and female ages respectively (minus 70). The system of equations must also satisfy the constraints

$$\sum_i \alpha_{ij} = 1,$$
$$\sum_i \gamma_{ik} = \sum_k \gamma_{ik} = \sum_i \beta_{ij} = 0,$$
$$\gamma_{ik} = \gamma_{ki}$$
$$\Theta_2 = (1 - \Theta_1)$$

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10 These are the eligibility ages for the Australian Age Pension. Employed people are not explicitly excluded.

together with the constraints for male and female clothing described above. Results are presented for the case with both husband and wife aged 70 (near the mean of the sample).

The key to incorporating sensible assumptions about the household purchase function is the separate identification of commodities with different scale economies. In this example, a 17 category disaggregation is used. These categories are generated from aggregations of the detailed three digit commodity codes of the 1988-89 Household Expenditure Survey conducted by the ABS. A summary of the commodities in each category is provided in Table 1 and budget shares are shown in Table 2, along with a set of assumed commodity-specific scales.

Table 1: Commodity Categories

	Title	Notes
1	Men's clothing	
2	Women's clothing	
3	Housing	Includes repairs and maintenance, mortgage interest but not mortgage principal repayments.
4	Fuel	Household fuel and energy consumption (not including transport fuels).
5	Food - high economies of scale	Includes foods that require substantial preparation and/or are perishable such as flour, rice, pasta, vegetables, bread, unprocessed meat and milk.
6	Food - low economies of scale	Includes cakes, biscuits, confectionary, processed meat, dairy products (other than milk), fruit, spreads, tea, coffee, non-alcoholic drinks and food consumed outside the household.
7	Alcohol	
8	Tobacco	
9	Furnishings and equipment	
10	Household services and operation	
11	Medical care and health	
12	Public transport	Not including holiday fares.
13	Private transport	Mainly motor vehicle operation.
14	Recreation - high economies of scale	Capital goods such as televisions, stereos and associated supplies; pets, holiday expenditures.
15	Recreation - low economies of scale	Books, magazines, gambling, sports, admission charges, and holiday travel.
16	Personal care	
17	Miscellaneous	Includes consumer interest charges. The small amount of observed clothing expenditures for members of the opposite sex in single person households is included here.

Table 2: Expenditure Shares: Singles and Couples Over Pension Age

	Expenditure Shares				Assumed Scale Economies (n)
	Couples	Single Men	Single Women	All House-holds	(1=private good, 2=public good)
Number of Cases	445	94	387	926	
Men's clothing	0.013	0.024	0.000	0.009	1.00
Women's clothing	0.023	0.000	0.038	0.027	1.00
Housing	0.091	0.143	0.137	0.115	1.80
Fuel	0.045	0.053	0.056	0.051	1.60
Food-high	0.133	0.103	0.111	0.121	1.30
Food-low	0.153	0.131	0.136	0.144	1.10
Alcohol	0.033	0.064	0.008	0.026	1.10
Tobacco	0.012	0.016	0.009	0.011	1.00
Furnishings	0.062	0.062	0.062	0.062	1.80
HH operation	0.068	0.069	0.094	0.079	1.80
Health	0.064	0.058	0.062	0.062	1.10
Public transport	0.004	0.012	0.006	0.006	1.00
Private transport	0.104	0.111	0.066	0.089	1.90
Recreation-high	0.051	0.045	0.041	0.046	1.80
Recreation-low	0.064	0.048	0.067	0.064	1.20
Personal care	0.025	0.010	0.033	0.027	1.10
Miscellaneous	0.056	0.051	0.072	0.062	1.50
Total Expenditure ($/week)	288	179	184	233	
Simple Average Equivalence Scale[a]					
Men					0.59
Women					0.58

Note: a) Simple Average scale defined as half the geometric mean of the commodity scales, weighted by budget shares.

It is apparent that expenditure patterns do differ between these three family types. To begin with, there are clear taste differences between single men and women. Men spend more on alcohol, tobacco and private transport, and women spend more on clothing. Comparing singles with couples, the variations seem to be in accord with the theory. Equation (3.22) implies that couples will tend to spend a lower proportion of their total expenditure on commodities with strong economies of scale such as housing. This is because the household purchase requirement of commodities with low scale economies (such as food) will be much higher for a couple than for a single person, whilst the purchase requirement for housing will be only slightly greater. Only when the compensated own-price elasticity is large (not likely for housing), will substitution towards the good which is cheaper because of scale economies outweigh this effect.[11] The lower budget share for housing, and the higher budget shares for food for couples is thus consistent with this reasoning.

Initial estimates of equation (3.22) were generated by assuming the values of the scale economy parameters, n_i, as shown in Table 2, and using assumed values for the cross price parameters, γ_{ik}. These were assumed to have the 'substitution-independent' structure introduced by Keller (1984). When this is the case, the cross-price responses can be implied from estimates of the own-price elasticities. The latter were calculated from Rimmer and Powell (1992) who estimate an Australian time-series demand system for six commodities. The calculation method and resulting γ matrix is described in more detail in Appendix One.

One commonly encountered problem in estimating the AID system is that the parameters α_{01} and α_{02} are hard to separately identify from the other α parameters. This was found to be the case here, and these two parameters have been arbitrarily set to a value of four – implying that $55 per week is required in order to produce a minimal utility level.

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11 The parameter γ_{ii} will be (approximately) positive when the absolute value of the compensated own-price elasticity is less than 1.

Estimation of equation (3.22) with these assumptions requires the estimation of 91 parameters, compared with the 154 parameters that would be required if there were no constraints between the demand functions for the different household types.[12] It is therefore of interest to see if the restriction imposed by the theory (together with the assumptions about the *n* and γ parameters) is rejected by the data. Though the constrained model cannot be nested as an explicit function of parameters within the unconstrained model, a likelihood ratio test strongly suggests rejection of the hypothesis with the difference in twice log-likelihoods 166 for 63 degrees of freedom.

A simple means of evaluating the fit of the equations is presented in Table 3.[13] For each commodity this shows the R^2 for an unrestricted estimation (i.e. with couple demands unrelated to single person demand patterns) together with the equation-specific R^2 obtained by fitting equation (3.22). The difference between these two is an approximate indicator of the contribution of the commodity to the lack of fit of the overall restriction (this is only approximate because of the cross-commodity restrictions imposed through the common price index). In addition, the table shows the mean residuals for couples, single men and single women.

The greatest drop in R^2 is for the commodities ‘food-high’, housing and public transport. The first two of these also have the largest positive and negative mean residuals for couples’ income shares respectively.

Considering housing first, the residuals show that the model overestimates the expenditure share for aged couples by 1.5 percentage points (the mean residual for couples). One potential explanation for this is that the assumed values for the *n* or γ parameters are incorrect. If the γ matrix takes the ‘substitution independent’ form, then the off-

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12 The difference represents four parameters for each commodity describing the independent consumption of couples (a constant term, and parameters for income and the ages of husband and wife) less theΘ parameter.

13 Parameter estimates were obtained using the nonlinear SUR method of the SAS model procedure. Log-likelihood statistics were calculated using the (more expensive) ML method.

diagonal elements will be small relative to the diagonal (see Appendix One). This

Table 3: Estimation Results

Commodity		Parameters				Mean Residuals (actual predicted)			R-squared			Personal consumption relative to household purchases (at y = $300)	
		Men		Women									
Commodity		α	β	α	β	Couples	Men	Women	Uncon-strained	Con-strained	Differ-ence	Men	Women
Men's clothing	1	0.011	0.010	0.000	0.000	0.000	0.000	0.000	0.077	0.072	0.005	1.00	0.00
Women's clothing	2	0.000	0.000	0.021	0.018	0.001	0.000	-0.001	0.049	0.049	0.001	0.00	1.00
Housing	3	0.130	-0.031	0.128	-0.002	-0.015	0.037	0.009	0.067	0.034	0.033	0.73	1.07
Fuel	4	0.076	-0.024	0.093	-0.036	0.000	-0.001	0.000	0.239	0.231	0.008	0.77	0.83
Food-high	5	0.202	-0.065	0.181	-0.058	0.013	-0.031	-0.008	0.240	0.194	0.046	0.67	0.63
Food-low	6	0.192	-0.048	0.199	-0.057	0.005	-0.012	-0.003	0.173	0.163	0.009	0.55	0.55
Alcohol	7	0.052	0.009	0.002	0.005	-0.003	0.006	0.001	0.126	0.122	0.004	0.94	0.16
Tobacco	8	0.022	-0.005	0.014	-0.005	-0.001	0.001	0.000	0.021	0.018	0.003	0.64	0.36
Furnishings	9	0.002	0.051	-0.027	0.082	-0.003	0.008	0.002	0.103	0.098	0.006	0.81	0.99
HH operation	10	0.075	-0.006	0.124	-0.029	0.001	-0.002	0.000	0.123	0.118	0.004	0.75	1.05
Health	11	0.001	0.037	0.068	-0.006	0.000	-0.001	0.000	0.051	0.042	0.010	0.50	0.60
Public transport	12	-0.002	0.008	0.009	-0.004	-0.003	0.006	0.002	0.028	0.007	0.021	0.66	0.34
Private transport	13	0.125	0.008	0.064	0.007	0.004	-0.007	-0.002	0.077	0.069	0.008	1.22	0.68
Recreation-high	14	0.051	0.012	0.025	0.017	0.004	-0.010	-0.002	0.025	0.019	0.006	1.05	0.75
Recreation-low	15	0.014	0.037	0.018	0.045	-0.003	0.004	0.001	0.070	0.062	0.008	0.53	0.67
Personal care	16	0.018	-0.005	0.031	0.002	0.000	-0.001	0.000	0.038	0.036	0.002	0.29	0.81
Miscellaneous	17	0.031	0.013	0.051	0.019							0.56	0.94
Number of parameters							154	91	63				
Log-likelihood							-22490	-22407		83			
Θ												0.49	0.51
Equivalence scale (at $300)												0.70	0.73

Table 4: Diagnostic Results (alternative model)

Commodity		Mean Residuals (actual-predicted) Couples	Men	Women	R squared	Personal consumption relative to household purchases (at y=$300) Men	Women	n
Men's clothing	1	0.001	-0.002	0.000	0.071	1.00	0.00	1
Women's clothing	2	-0.001	0.000	0.000	0.049	0.00	1.00	1
Housing	3	0.001	0.001	0.000	0.058	0.88	1.12	2
Fuel	4	-0.001	0.002	0.000	0.230	0.72	0.88	1.6
Food-high	5	0.006	-0.014	-0.004	0.224	0.51	0.59	1.1
Food-low	6	0.003	-0.006	-0.002	0.167	0.51	0.59	1.1
Alcohol	7	-0.002	0.004	0.001	0.124	0.90	0.20	1.1
Tobacco	8	-0.001	0.001	0.000	0.017	0.62	0.38	1
Furnishings	9	-0.004	0.009	0.002	0.098	0.68	1.12	1.8
HH operation	10	-0.001	0.002	0.000	0.119	0.67	1.13	1.8
Health	11	0.000	0.000	0.000	0.044	0.43	0.67	1.1
Public transport	12	-0.003	0.005	0.002	0.009	0.65	0.35	1
Private transport	13	0.005	-0.009	-0.003	0.068	1.13	0.77	1.9
Recreation-high	14	0.004	-0.008	-0.003	0.021	0.92	0.88	1.8
Recreation-low	15	-0.004	0.008	0.002	0.061	0.44	0.76	1.2
Personal care	16	-0.001	0.002	0.001	0.035	0.24	0.86	1.1
Miscellaneous	17					0.47	1.03	1.5
Number of parameters					91			
Log-likelihood					-22434			
Θ						0.45	0.55	
Equivalence scale (at $300)						0.64	0.78	

means that the main influence of these parameters on the budget share of a commodity i is via the term $\gamma_{ii} \log n_i$ in equation (3.22). Using the expression for the compensated price elasticities in the AID system shown in Appendix One, it can be seen that γ_{ii} will approach the budget share of the commodity when the price elasticity is zero, and will approach zero with a unit price elasticity (and be negative for higher elasticities). Hence the overestimation of the housing share could be rectified by either increasing the n parameter or by decreasing the assumed own-price elasticity.

In cross-sectional data, it will generally not be possible to identify which of these will be the appropriate adjustment (and may even be difficult with data containing some price variation). However both these parameters have natural bounds – our knowledge of housing consumption of elderly people might lead us to assume that n lies in the range, [1.7,2.0], and γ can be no larger than (approximately) the budget share (i.e. the compensated own price elasticity cannot be positive). Such knowledge can sometimes aid identification.[14] In fact, in the present case, the prediction error is so large that both increasing n to 2.0 and setting the own price elasticity for housing to (approximately) zero only slightly overcompensates for this bias in the housing residual. This is done for the alternative estimation presented in Table 4. A zero price elasticity could be justified on the assumption that housing costs (mainly land taxes and maintenance costs for this sample) largely reflect decisions made in other periods and are not flexible. Similarly, for many elderly people, the social norm is to stay on in the family home after their partner dies – thus implying full economies of scale in housing costs.

The other change made to estimate the alternative model in Table 4 is to reduce n for ‘food-high’ down to the same level as for ‘food-low’ (i.e. to 1.1). This is essentially an ad-hoc adjustment made to reduce the large residual for this commodity. Whilst this does reduce the residual (and increase the R^2), this equation still fits only relatively

14 Whilst the structure of the problem as presented here would suggest a role for a Bayesian approach the computational burdens of dealing with disaggregated consumption patterns are already very large.

poorly (and the log-likelihood still rejects the overall restricted model). The reason for this poor fit probably lies in the exclusion of any consideration of home production in the model. Thus the commodities in the ‘food-high’ group generally require the input of considerable time (and skill) inputs to become useable. Hence we might expect that in couple households with greater time resources there will be a greater consumption of these commodities.

The lack of direct accounting for these effects is one of the main reasons for the poor fit of the present model and is an obvious candidate for future extensions. However a model which does include such effects will have to directly address the question of whether policy makers wish to place the same valuation on time as is apparently made by retired people themselves. This does not automatically follow.

Finally, the other main limitation of this example is that there is no explicit imposition of the minimum bound restriction. The violation of this property for goods with high economies of scale is show in the last columns of Table 3 and 4. For the minimum bound property to hold, personal consumption must always be no greater than household consumption. The sum of the male and female ratio of personal to household consumption will equal n and so this restriction will be more likely to be violated as n approaches 2 and as tastes between men and women are more different. In Table 3 the main violation is for private transport with weak violations for housing, household operation and ‘recreation-high’, whilst in Table 4 the higher scale economies imply a strong violation for housing.

Explicit implementation of the minimum bound restriction would thus seem to be warranted. This could be done by either employing appropriate inequality restrictions on commodity demands, or by using a non-linear budget constraint such as form *III*. Either approach will complicate the analysis considerably, as the budget constraint will no longer be separable.

Whilst the models estimated here are thus still far from ideal, it is of interest nonetheless to consider the final estimates of the sharing parameter Θ and the overall equivalence scales. These are shown at the foot of Tables 3 and 4. The original model in Table 3 has a sharing parameter of almost exactly half (the standard error is about 0.027 in both models) – and an overall equivalence scale of 0.70 for men and 0.73 for women.[15] That is, single men need 70 per cent of the income of couples to have the same living standard. This scale, whilst obviously largely determined by the assumptions of the *n* parameters, is quite different from the simple weighted average of these parameters (see Table 2).

For the alternative (and better fitting) model shown in Table 4, the sharing parameters show women receiving slightly more than half the household income. The average of the two equivalence scales is much as before, but now women need 78 per cent of the income of a couple, and men need only 64 per cent. The average ratio between singles and couples, at around 71 per cent in both models, is somewhat more than the actual ratio employed in the Australian pension system of only 60 per cent.

There are three points that need to be clearly understood in interpreting these results. First, as the differences between Tables 3 and 4 should make clear, the sharing parameter is not very robustly estimated, and further exploration using models that fit the data better are required before drawing firm conclusions on this matter. Second, nothing is said here about non-monetary inputs to well-being (such as who does the housework).

Finally, it is important to understand that statements such as ‘women need 78 per cent of the income of a couple’ describe the relative incomes required for the average woman to have the same (expenditure-based) living standard in the two household types. It says nothing about the living standard of **the couple** as such – since a couple consists of two separate individuals with (potentially) different living standards. This means that, without further assumptions, one cannot use the results above to conclude that single women need more than single men to reach the same living standard. It could be the case

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15 This is for a couple household income (total expenditure) of $300 per week. Between the income levels of $55 and $500, the scales range respectively from 0.700 to .705 (men) and 0.697 to 0.734 (women).

that single women actually need less than single men, but that when living in a couple women get much more than their share of the household's consumption. Since they are so well-off when married, their equivalence scale is high.

The approach used here is all about comparing the living standard of the same individual (or type of individual) living in different circumstances. To compare different people additional assumptions are required.[16] One possible assumption might be that husbands and wives have the same welfare level and that if they share income unequally, this is because one has greater needs. **If** this assumption is held, then the result that women have a greater share of household income **does** imply that single women have greater needs than single men.

On the other hand, one might reject this assumption but wish to make explicit assumptions about how different types of people have different needs. For example, it might be assumed that men require a greater amount of food to reach the same living standard as women. This could be incorporated by employing a Barten-like scaling of the cost function for men, inflating the food price variable by some positive factor. This might be a useful approach when examining households containing both adults and children.

4 Conclusions

There is never likely to be an easy answer to the question of how to estimate consumer equivalence scales. The goal of this paper has been to introduce a new model of household consumption decision making that can take account of the semi-public nature of most consumption goods, taste differences between household members, and the intra-household allocation of economic resources. Since there is unlikely to ever exist a single data set which could be used to estimate all these factors simultaneously, the model presented here attempts to portray these different factors in as transparent manner as possible.

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16 This is due the fact that in equation (3.16) δ_1/δ_2 cannot be separately identified from β_{01}/β_{02}.

Transparency is important if we wish to derive policy-relevant estimates, because such estimates must inevitably be derived from a combination of external assumptions combined with observations of consumer behaviour. Section 3 of this paper provided an illustrative example of how assumptions and data can be combined in this regard – but this is clearly only a first step. The results of this section suggest that the rate of pension paid to single age pensioners might not be sufficient to enable them to reach the same living standards as couples. The degree of income sharing seems to be less robustly estimated.

These results should be treated very cautiously, as there are clearly deficiencies in the model as estimated. Two additional aspects of consumption patterns that need to be incorporated are the direct imposition of the minimum bound property, and the incorporation of the influence of home production on purchase patterns.

Appendix One: Generation of Cross-price Parameters

Since there exist no estimates of the full set of price substitution relationships for the 17 commodities considered here, the following procedure is used to generate a synthetic matrix of γ_{ij} parameters. It is assumed that the demand system takes the form of the Substitution Independent AID system (AIDS/SI) of Keller (1984).[17] This implies that the $I(I-1)/2$ independent elements of the γ matrix can be represented by I independent parameters as

$$\gamma_{ij} = \varepsilon\gamma_i(\delta_{ij} - \gamma_j) \qquad \text{(A.23)}$$

where and δ_{ij}=1 when $i=j$ and 0 otherwise. This structure permits own-price elasticities to vary, but ensures relatively small cross-price effects without imposing the assumption of additivity. The γ_i and ε parameters are chosen so as to replicate a set of externally provided own-price elasticities whilst satisfying the adding-up requirement that $\sum\gamma_i = 1$. Matching to external elasticities is made using the relationship

$$\gamma_{ij} \approx w_i\left(e_{ij} - w_j + \delta_{ij}\right) \qquad \text{(A.24)}$$

where w_i is the budget share of the i th commodity and e_{ij} is the compensated price elasticity of commodity i with respect to the price of commodity j. The approximation is exact at when estimated at minimum income or when the β_i parameter for either commodity is zero.

The price elasticities were obtained from Rimmer and Powell (1992) who estimated an Australian time-series demand system for six commodities (food, alcohol and tobacco, clothing, durables, rent, other). Elasticities for the 16 commodity groups used here were

17 Keller considers a differential version of the model described here.

derived from the closest matching commodity used by Rimmer and Powell. The ε and 15 independent γ_i parameters were identified from these 16 elasticities using a simple iterative procedure.

Table A shows the resulting 16x16 γ matrix created treating clothing as a single good. The 17x17 γ matrices including men's and women's clothing were created from this by adding rows and columns of zeros in the second and first positions respectively. The imposed structure implies that all goods are Hicks-Allen substitutes. All the eigenvalues of the implied Slutsky matrix at minimum incomes are non-positive. The γ matrix for the alternative model was generated in a similar fashion, but with the own price elasticity for housing set to zero.

Table A: Assumed Gamma Matrix

		Gamma (x100)															
		1,2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Clothing	1,2	0.54															
Housing	3	-0.02	1.44														
Fuel	4	-0.04	-0.10	2.44													
Food-high	5	-0.09	-0.25	-0.45	5.31												
Food-low	6	-0.11	-0.30	-0.52	-1.30	5.99											
Alcohol	7	-0.01	-0.04	-0.06	-0.16	-0.18	0.89										
Tobacco	8	-0.01	-0.02	-0.03	-0.07	-0.08	-0.01	0.39									
Furnishings	9	0.00	-0.01	-0.01	-0.03	-0.03	0.00	0.00	0.17								
HH operation	10	-0.06	-0.16	-0.28	-0.68	-0.79	-0.10	-0.04	-0.02	3.56							
Health	11	-0.05	-0.13	-0.22	-0.54	-0.63	-0.08	-0.03	-0.01	-0.33	2.90						
Public transport	12	0.00	-0.01	-0.02	-0.05	-0.06	-0.01	0.00	0.00	-0.03	-0.03	0.31					
Private transport	13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01				
Recreation-high	14	-0.03	-0.09	-0.17	-0.40	-0.47	-0.06	-0.03	-0.01	-0.25	-0.20	-0.02	0.00	2.23			
Recreation-low	15	-0.05	-0.13	-0.23	-0.55	-0.65	-0.08	-0.03	-0.02	-0.34	-0.27	-0.03	0.00	-0.21	2.98		
Personal care	16	-0.02	-0.06	-0.10	-0.24	-0.28	-0.03	-0.01	-0.01	-0.15	-0.12	-0.01	0.00	-0.09	-0.12	1.36	
Miscellaneous	17	-0.05	-0.13	-0.22	-0.54	-0.63	-0.08	-0.03	-0.01	-0.33	-0.27	-0.03	0.00	-0.20	-0.27	-0.12	2.90
Compensated own-price elasticities		-0.81	-0.76	-0.47	-0.44	-0.44	-0.63	-0.63	-0.91	-0.47	-0.47	-0.47	-0.91	-0.47	-0.47	-0.47	-0.47

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